Numerical Solutions of Differential Equations : HW1

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# Introduction

This is the introduction bitches!

# 1 Conservation laws

blabla

# 2 Heat Equation

## 2.1 Flux vector

## 2.2 Q(t)

We are now interested in computing :

$$Q(t) = \int_0^1 \int_0^1 q(x,y,t)dxdy$$

Using the fundamental theorem of calculus, we have :

$$Q(t) = Q(0) + \int_0^t Q'(\tau)d\tau$$

The initial condition gives :

$$Q(0) = \int_0^1 \int_0^1 q(x,y,0)dxdy = \int_0^1 \int_0^1 0\,dxdy = 0$$

We also know that (using the definition of $Q$) :

$$Q'(t) = \frac{d}{dt}(\int_0^1 \int_0^1 q(x,y,t)dxdy) = \int_0^1 \int_0^1 q_t(x,y,t)dxdy$$

We are now going to use the PDE to substitute $q_t$. This yields :

$$Q'(t) = \int_0^1 \int_0^1 q_t(x,y,t)dxdy = \int_0^1 \int_0^1 \nabla \cdot (\nabla q(x,y,t))dxdy + \int_0^1 \int_0^1 S(x,y,t)dxdy$$

With divergence theorem, and if $S$ means the boundary of the domain and $\mathbf{n}$ the outward unit normal to the boundary, we have :

$$Q'(t) = \oint_S \nabla q(x,y,t) \cdot \mathbf{n}\,ds + \int_0^1 \int_0^1 S(x,y,t)dxdy$$

Because the given boundary condition given is :

$$\nabla q(x,y,t) \cdot \mathbf{n} = 0$$

We finally have :

$$Q'(t) = \int_0^1 \int_0^1 S(x,y,t)dxdy$$

This yields an expression for $Q$ as a function of $t$.

$$Q(t) = \int_0^t \int_0^1 \int_0^1 S(x,y,\tau)dxdyd\tau$$

Because $S$ is a given function, this expression can always be computed as a function of $t$.

# 3 Discretization and implementation

blabla yo!

# 4 Numerical results

blabla bitches

# 5 Refinements

blabla