It's the title bitchess! !!

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1 Stability of Numerical Schemes

We have the general scheme

$$\begin{aligned} U^{n+1} &= Q(t_n)U^n + \Delta t F^n \\ U^0 &= g \end{aligned}$$

where $U^n \in \mathbb{R}^d$.

1.1 Duhamel's Principle

We are given the following discrete Duhamel's Principle:

$$U^n = S_h(t_n, 0)g + \Delta t \sum_{\nu=0}^{n-1} S_h(t_n, t_{\nu+1})F^\nu, \tag{1}$$

where $t_n = n\Delta t$, and

$$\begin{aligned} S_h(t,t) &= I, \quad t \in \mathbb{R} \\ S_h(t_{n+1,t_\mu}) &= Q(t_n)S_h(t_n, t_\mu). \end{aligned}$$

We begin by showing that (1) holds by induction.

Base Case: $n = 0$

$$\begin{aligned} U^0 &= S_h(0,0)g + \Delta t \sum_{\nu=0}^{-1} S_h(0, t_{\nu+1})F^\nu \\ &= 0 \end{aligned}$$

which fits the general scheme. Now we assume that the (1) fits the general scheme at step n, and we want to show that this implies that it fits for step $n + 1$.

$$\begin{aligned} U^{n+1} &= S_h(t_{n+1}, 0)g + \Delta t \sum_{\nu=0}^{n} S_h(t_{+1}n, t_{\nu+1})F^\nu \\ &= Q(t_n)S_h(t_n, 0)g + \Delta t Q(t_n) \sum_{\nu=0}^{n-1} S_h(t_n, t_{\nu+1})F^\nu + S_h(t_{n+1}, t_{n+1})F^n \\ &= Q(t_n)(S_h(t_n, 0)g + \Delta t \sum_{\nu=0}^{n-1} S_h(t_n, t_{\nu+1})F^\nu) + F^n \\ &= Q(t_n)U_n + F^n \quad \text{by induction assumption} \end{aligned}$$

which fits our general scheme.

1.2 Bound in the h-norm