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Applied Numerical Methods, SF2520 and SF1536
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Computer Lab 5
Numerical solution of elliptic PDE-problems with finite differences and Comsol Multiphysics

This lab is an exercise in solving an elliptic PDE, the applicational background of which is stationary heat conduction in 2D.

The physical background is as follows: Heat is conducted through a rectangular metal block, being the region $\Omega = [0 \leq x \leq 4, 0 \leq y \leq 2]$ when placed in a xy-coordinate system. Depending on the boundary conditions, the temperature distribution $T(x, y)$ in the block will be different.

The following elliptic problem is formulated:

$$\triangle T = 0, \quad (x, y) \in \Omega$$

$$T(0, y) = 300, \quad T(4, y) = 600, \quad 0 \leq y \leq 2$$
$$\frac{\partial T}{\partial y}(x, 0) = 0, \frac{\partial T}{\partial y}(x, 2) = 0, \quad 0 < x < 4$$

1) Solve the elliptic partial differential equation problem Use the finite difference method as described in the lecture notes. Discretize the rectangular domain into a quadratic mesh with the following two stepsizes: $h = 0.2$, i.e. $N = 19$, $M = 11$ and $h = 0.1$, i.e. $N = 39$, $M = 21$, see figure in the next page.

Solve the problem for these two stepsizes and visualize the solution $T(x, y)$ with colors, using the Matlab function `imagesc`. What is the T-value at $(2, 1)$ for the two stepsizes? Derive the analytic solution. Why is the numerical solution almost exact?

2) Solve the same problem with Comsol Multiphysics. Follow the session example given out at the lecture and the demonstration. (General recommendation: if the session turns out to be too messy, start the session again). Check your numerical result from 1): what is the T-value at the point $(2, 1)$? How many nodes and triangels have been generated in the mesh? Make a refinement of the grid and find again $T(2, 1)$. How many nodes and triangles are there now?
Start a new session. Make the whole session again: draw geometry, impose boundary values, set the PDE-values, initialize the mesh, solve the problem, plot a 2D-graph of the solution.

3) Go back to Comsol and generate the following L-shaped area.

Impose the boundary conditions. In the figure above, the walls are heat insulated, i.e. the normal derivative is equal to zero along these boundaries. Set the PDE-parameters and initialize the mesh. How many nodes and triangles are generated? Solve the problem. What is the value of T in the points $(1, 1)$ and $(2, 2)$? Refine the mesh and solve again. How many nodes and triangles? What is $T(1, 1)$ and $T(2, 2)$?
Initialize the mesh again, and then refine the mesh only in the area around the point $(2, 2)$. What is $T(1, 1)$ and $T(2, 2)$?

Submit your results in a report. Attach programs, solution graphs, required numerical values and questions answered.