Project Assignment 1A

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Problem description

The telecommunication company Nett provides capacity between cites in Europe. The cities are Stockholm, Berlin, London, Warsaw, Paris, Madrid and Rome. Some of the cities are connected in which traffic can be sent in both directions. The connections and the maximum traffic which can be sent between the cites respectively can be seen in Figure 1.

Nett wishes to provide 50 Gbit/s between Stockholm and Rome at the same time they provide 40 Gbit/s between London and Warsaw. They need help in routing the traffic because of the capacity limitations. Also, they would like to investigate if there is any slack in the network, potential to adding another traffic route and how to handle fluctuations in capacity.
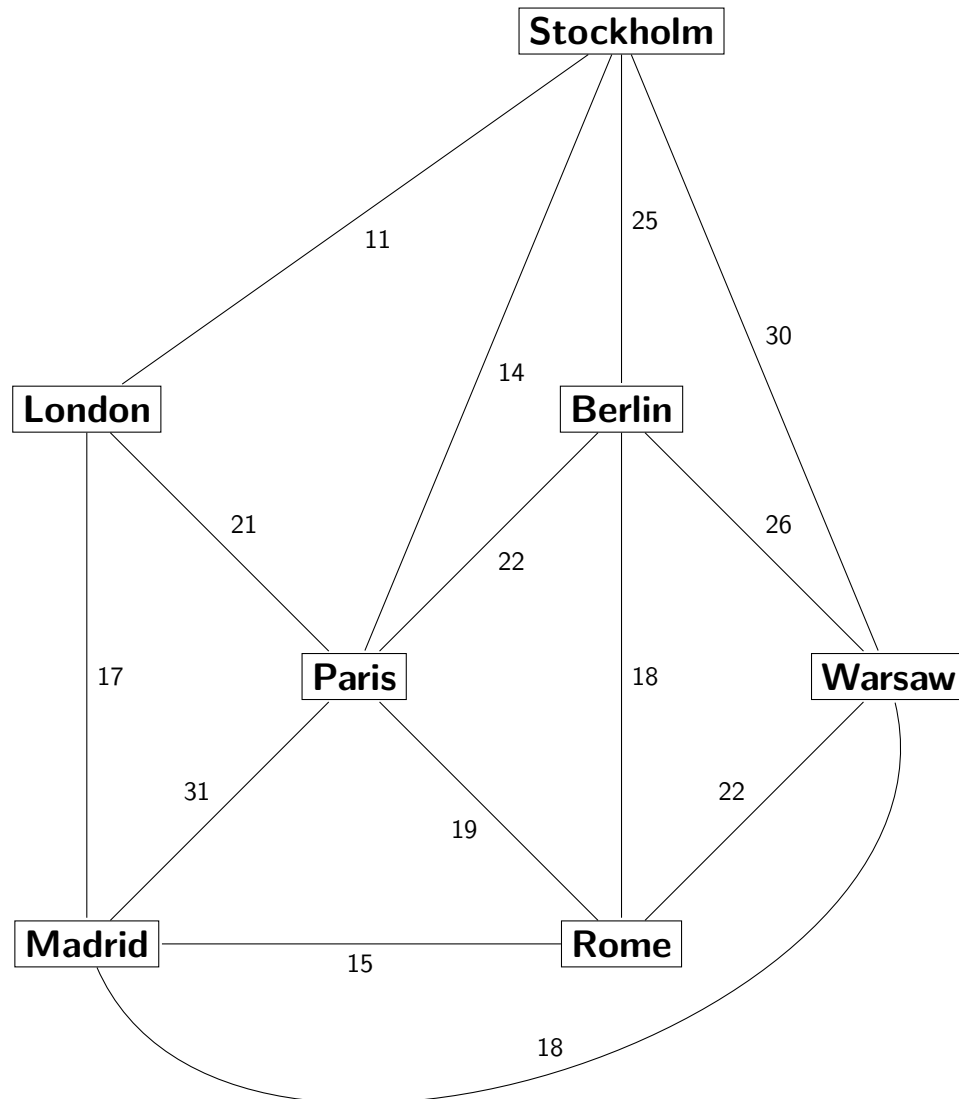

Figure 1. System modelled as a network where maximum capacity is marked out on the arcs.

1   Routing the traffic

In this section we investigate the best way to sent the traffic in the network. 50Gbit/s must be sent from Stockholm to Rome at the same time 40Gbit/s is sent from London to Warsaw without violating the capacity limitations. We would like to spread the traffic evenly. Therefore, we minimize the strain on any link in the network, i.e. traffic divided by the capacity.

1.1   Mathematical formulation

To solve this we treat the cities and links as a network and the traffic as a flow throughout the network. The cities are nodes connected by links which are arcs. Stockholm is a source node where 50Gbit/s comes in which will be distributed to the sink node Rome. The same goes for London and Warsaw respectively.

We start by introduction the set I :

$$I = \{Sto, Lon, Ber, War, Par, Rom, Mad\}$$

The set I make up for the network's node and an arc between node i and j is denoted as (i,j). The flows from Stockholm and London can be sent in the same link in either direction. However, the two flows must be treated independently. That being said, the 50Gbit/s that comes in from Stockholm is the same 50Gbit/s that arrives at Rome. We must keep track of the flows which is why we introduce another index k. The flow from Stockholm to Rome is $k = 1$ and the flow from London to Warsaw is $k = 2$.

For each arc in the network, there will be a corresponding variable $x_{i,j,k}$ which denotes the flow from node i to j of flow k. Solving for the variable x will provide the strategy for how to send the traffic between the cites. Though, there are two constraints which must be taken into account.

The first constraint deals with conservation of flow, i.e. the flow into a node is equal the flow out.

$$\sum_{j \in I} x_{i,j,k} - \sum_{j \in I} x_{j,i,k} = b_i$$

$$\text{where } b_i = \begin{bmatrix} b_{Sto} \\ b_{Ber} \\ b_{War} \\ b_{Lon} \\ b_{Par} \\ b_{Mad} \\ b_{Rom} \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ -40 \\ 40 \\ 0 \\ 0 \\ -50 \end{bmatrix}$$

The other constraint is that both types of traffic, in either direction, must not exceed the capacity in each link. If there is no direct connections between two cities, the capacity is zero and no traffic can be sent in this arc.

$$\sum_{k=1}^{2} x_{i,j,k} + \sum_{k=1}^{2} x_{j,i,k} \leq c_{i,j}$$

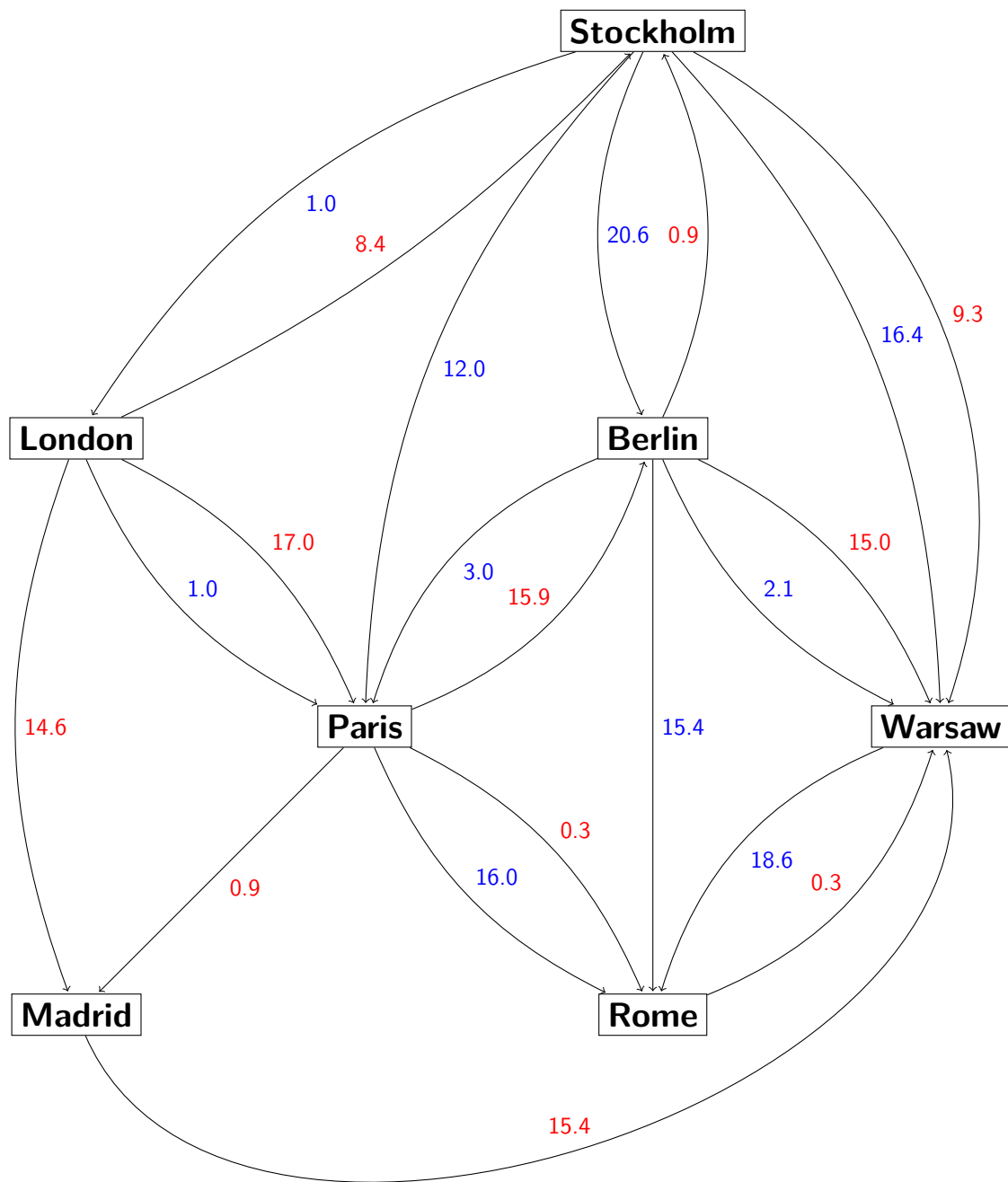where $c_{i,j}$ is the maximum capacity in arc (i,j)

Finally, we define the objective function as the utility in a link.

$$\frac{\sum\limits_{k=1}^{2} x_{i,j,k} + \sum\limits_{k=1}^{2} x_{j,i,k}}{c_{i,j}}$$

Taking all the above into consideration, the mathematical formulation of the network flow optimization problem becomes:

$$
\begin{aligned}
\min \quad & \frac{\sum\limits_{k=1}^{2} x_{i,j,k} + \sum\limits_{k=1}^{2} x_{j,i,k}}{c_{i,j}} \\
\text{s.t.} \quad & \sum_{j \in I} x_{i,j,k} - \sum_{j \in I} x_{j,i,k} = b_i \\
& \sum_{k=1}^{2} x_{i,j,k} + \sum_{k=1}^{2} x_{j,i,k} \le c_{i,j} \\
& x_{i,j,k} \ge 0 \quad \forall\, i, j \in I
\end{aligned}
$$

2 Model 2

2.1 Problem description

The problem in the previous section is now to be extended such as to calculate the maximum avialable traffic between Berlin and Madrid under the same conditions as in the previous section, ie the capacity limits given in figure 1 as well as 50 Gbit/s of traffic Stockholm-Rome and 40 Gbit/s London-Warsaw.

2.2 Mathematical model

Once again we introduce the set I :

$$I = \{Sto, Lon, Ber, War, Par, Rom, Mad\}$$

Representing the nodes of the system.

The flow from Stockholm to Rome is denoted $k = 1$ and the flow from London to Warsaw is $k = 2$. Thus, the added flow from Berlin to Rome is $k = 3$. The variable $x_{i,j,k}$ represents the flow of traffic type k from node i to node j.

As before, the first constraint treats conservation of flow:

$$\sum_{j \in I} x_{i,j,k} - \sum_{j \in I} x_{j,i,k} = b_i$$

The transmitted data is not to exceed the capacity between each node, which may be modelled as a second constraint:

$$\sum_{k=1}^{2} x_{i,j,k} + \sum_{k=1}^{2} x_{j,i,k} \leq c_{i,j} + c_{j,i}$$

3 Model 3

3.1 Presentation of the problem

In this section, we are going to generalize the model a little bit. Up to this point, we have considered that each link has a known capacity and that this quantity is exact. This is, however, not true in general. We can expect the actual capacities to be close to the numbers given but we should take into account that they can fluctuate around those values. Our new model will thus have to deal with this uncertainty. Because the value of the capacities can be considered as an unknown parameter, we will use stochastic programming.

We also know that the 50 Gbit/s-demand can be rerouted after the actual values are known but the 40 Gbit/s-demand must be determined on beforehand (and thus not cannot be rerouted after knowing the actual capacities).

3.2 Mathematical model

Just as in the previous models, we introduce the set I :

$$I = \{Sto, Lon, Ber, War, Par, Rom, Mad\}$$

The main difference with previous models is the presence of uncertainty. As usual, we will handle this by discretizing this uncertainty into different scenarios s with an associated

probability p_s. Each scenario corresponds to a complete set of known capacities. We can for example say that the second scenario is the one where the capacities are exactly those given. Let us assume that we have S scenarios and define the parameters :

$$c_{i,j,s} \qquad \text{capacity in scenario } s \text{ from city } i \text{ to } j$$
$$i,j \in I \text{ and } s = 1,...S$$

Let us also introduce the variables x, depicting the 40 Gbit/s-flow. Because this flow must be determined on beforehand, it cannot depend on the scenarios.

$$x_{i,j} \qquad \text{flow in Gbit/s going from } i \text{ to } j \text{ for the 40 Gbit/s-flow}$$
$$i,j \in I$$

Similarly, let us define the variables y for the 50 Gbit/s-flow. This can be rerouted and thus we could potentially have different flows for different scenarios.

$$y_{i,j,s} \qquad \text{flow in Gbit/s going from } i \text{ to } j \text{ for the 50 Gbit/s-flow and scenario } s$$
$$i,j \in I \text{ and } s = 1,...,S$$