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THE INITIAL MASS FUNCTION AND STELLAR BIRTHRATE IN THE SOLAR NEIGHBORHOOD

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ABSTRACT

We examine the initial mass function (IMF) and time history of the stellar birthrate in the solar neighborhood assuming a time-independent IMF. The present-day mass function is constructed from the luminosity function, and a discussion of all the relevant observational quantities (luminosity function, mass-luminosity relation, scale heights, correction for non–main-sequence stars, and correction for main-sequence brightening) and the associated uncertainties is given. The observed total mass is not in serious disagreement with the dynamical mass (Oort limit) when the uncertainties are considered, and so there is no strong evidence for a local "missing mass" problem. The primary constraint on the birthrate history is the requirement that the derived IMF not exhibit an unphysical discontinuity at the mass where the main-sequence lifetime equals the age of the galactic disk. Considering the above uncertainties, the average past birthrate may be at most about 5 times larger or 3 times smaller than the present birthrate according to this constraint, and we favor a variation of less than a factor of 2. A discussion of 12 secondary constraints on the birthrate history is given, and it is concluded that these constraints are either consistent with the continuity constraint or else indeterminate. There is, in particular, no evidence for a birthrate which depends on the square of the mean gas density. If the birthrate depends on some power of the mean gas density, the exponent must be small ($\lesssim 0.5$). Theoretical models for the birthrate history are also examined. Most theories predict birthrates which are expected to be constant or change by relatively small amounts over the history of the disk, in agreement with the observational evidence.

IMFs are derived for a range of birthrate histories consistent with the continuity constraint. The resulting IMFs are smooth and can be well approximated by a half-Gaussian distribution in $\log M$; i.e., mass is distributed lognormally. The slope of the IMF at a given mass is $d \log \xi / d \log M = -(1 + \log M)$. Tables of various useful quantities such as cumulative number and mass distributions, present birthrates, and mass consumption rates are given. The present total rate of mass consumption is found to lie between 3 and $7 \times 10^{-9}\ M_\odot\ \mathrm{pc}^{-2}\ \mathrm{yr}^{-1}$. The corresponding time scales for gas depletion suggest that star formation is balanced by infall and/or that the star formation rate is partly controlled by stochastic factors such as supernova explosions or galactic encounters. The derived field star IMF is compared with previous determinations, the available data on open clusters and associations, and theoretical models for the IMF. Within the observational uncertainties, the cluster IMF agrees with the field star IMF for $1\ M_\odot \lesssim M \lesssim 10\ M_\odot$, but some clusters are deficient in lower-mass stars. The IMF of the Orion OB1 association agrees well with the field star IMF for $M \gtrsim 2.5\ M_\odot$. Various theoretical models for the mass distribution resulting from fragmentation agree roughly with the shape of the IMF for $M \gtrsim 2\ M_\odot$. The main deficiency of these models of fragmentation is their inability to account for the flattening of the observed IMF at low masses or the low-mass turnovers seen in some open cluster IMFs. Models in which the mass distribution is controlled by interactions of fragments with each other and with ambient gas are more successful in accounting for these two features. Finally, we summarize observational and theoretical arguments concerning the basic assumptions of this paper that the IMF is independent of time and a continuous function of mass.

Subject headings: luminosity function — mass-luminosity relation — stars: evolution — stars: formation — stars: stellar statistics

I. INTRODUCTION

There are two functions related to the process of star formation which are of fundamental importance and which have application to many fields of current research: the spatially averaged rate of star formation in a galaxy (the stellar birthrate) and the frequency distribution of stellar masses at birth (the initial mass function, or IMF). Applications which depend on these functions include analyses of the stellar contents of galaxies using photometric properties (e.g., Turnrose 1976; Williams 1976; Huchra 1977), chemical evolution in our own Galaxy and other galaxies (for reviews

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see Trimble 1975; Audouze and Tinsley 1976; Tinsley 1977*a*), models for metal-rich gas in clusters of galaxies (De Young 1978), and constraints on the evolutionary status of red giants with peculiar abundances (Scalo and Miller 1978, 1979). Additionally, determinations of the stellar birthrate and IMF are fundamental constraints on theories of star formation.

Since the pioneering work of Salpeter (1955) and Schmidt (1959), there have been numerous papers (discussed below) on the birthrate and IMF; nevertheless, there exist in the literature conflicting statements about, and misuse concerning, the form of these functions and a disregard for the observational uncertainties. This paper therefore has the following goals: (1) to collect and discuss in one place the data required for the determination of the IMF; (2) to evaluate the uncertainties in the input data and their effect on the results; (3) to examine the available evidence concerning the history of the birthrate, in order to firmly establish the range of the maximum allowable time variation of the birthrate in the solar neighborhood; (4) to derive self-consistent IMFs from the above data.

We have sacrificed some conciseness in order to make clear the sources of, and uncertainties in, the observational input data and to make the presentation reasonably complete and self-contained. An independent review of evidence relating to the stellar birthrate is given by Tinsley (1977*a*), and a general review of the IMF in our Galaxy and other galaxies is presented by Scalo (1979).

The organization of the paper is as follows. Section II deals with the observational determination of the present-day frequency distribution of stellar masses. We discuss in turn each of the observed quantities which enter the construction of this distribution, emphasizing the possible sources of uncertainty. Constraints on the history of the stellar birthrate are discussed in § III. We first introduce the basic relations between the present-day distribution of masses, the history of the birthrate, and the IMF, and use several different forms of the birthrate to illustrate the relation. Several of the basic ideas and methods of § III were introduced by Salpeter (1955) and Schmidt (1959). Following Tinsley (1977*a*), we then examine the constraints on the birthrate history imposed by the requirement that the derived field star IMF not exhibit an unphysical discontinuity. The allowed range of birthrate histories which results is within, but smaller than, the range given by Tinsley (1977*a*), and suggests that the birthrate has been roughly constant to within a factor of from 2 to 4 over the age of the galactic disk. A number of secondary constraints on the birthrate history are examined next, and are found to be either indeterminate or else consistent with the continuity constraint. Theoretical considerations related to the time history of the birthrate are also discussed. We emphasize that there is no evidence for a strongly decreasing birthrate, and that even a birthrate which *increases* somewhat with time cannot be ruled out on either observational or theoretical grounds. In § IV we derive field star IMFs for a range of birthrate histories consistent with the continuity constraint. It is found that the IMF is smooth and can be approximated by a half-Gaussian distribution. We present and discuss some useful derived quantities, such as cumulative number and mass distributions, present birthrates, and mass consumption rates. The derived field star IMFs are compared with previous determinations and the available data on open cluster and association IMFs and with theoretical models for the IMF. A summary and discussion of the results and basic assumptions are given in § V.

Throughout this paper it is assumed that the IMF in the solar neighborhood has been independent of time over the lifetime of the galactic disk. It should be noted that there is no compelling observational or theoretical evidence for or against this assumption. We make this assumption for the purpose of using the simplest theory to describe the observations. The question of how (or whether) the IMF may have changed over the age of the Galaxy has not been resolved (see Trimble 1975; Audouze and Tinsley 1976; Ostriker and Thuan 1975), and arguments relating to this problem are discussed in § V.

This paper is concerned with the disk population in the solar neighborhood, which has been taken to be a cylindrical volume with its axis passing through the Sun and perpendicular to the plane of the Galaxy. Following, for example, Tinsley (1974*a*), in order to attempt to satisfy the conflicting requirements of using a volume large enough that any statistical fluctuations are averaged out and a volume small enough that observations are reliable and a mixture of other population types is avoided, we take the radius of this cylinder to be about 1 kpc and to extend several hundred parsecs on either side of the plane. All quantities are integrated over the height of this cylinder and given per square parsec.

II. PRESENT-DAY MASS FUNCTION

a) *Construction of the Present-Day Mass Function*

The present-day mass function of main-sequence field stars in the solar neighborhood (PDMF) is the observational foundation of the present investigation. In this section the observational data leading to the PDMF are examined in detail. The uncertainties involved are estimated in order to find the resulting uncertainties in the birthrate and IMF and also to indicate areas where improved observations are needed. The PDMF $\phi_{ms}(\log M)$ is defined as the number of main-sequence stars per unit logarithmic mass interval per square parsec in the solar neighborhood. Note that $\phi_{ms}(\log M)$ is given per square parsec, that is, integrated perpendicular to the disk, in order to account for the fact that high-mass stars are strongly concentrated toward the disk while low-mass stars are found several hundred parsecs outside the disk. The relative numbers of the low-mass stars in the solar neighborhood would be underestimated if the PDMF were defined in terms of space densities in the galactic plane.

The PDMF of main-sequence field stars is related

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to the luminosity function of field stars $\phi(M_v)$, which gives the number of stars of all types (not just main sequence) per unit absolute magnitude and per cubic parsec in the plane of the Galaxy. The PDMF is derived from the luminosity function by

$$\phi_{\mathrm{ms}}(\log M) = \phi(M_v)\left|\frac{dM_v}{d\log M}\right|2H(M_v)f_{\mathrm{ms}}(M_v)\,. \quad (1)$$

In equation (1), $dM_v/d\log M$ is the slope of the (absolute magnitude, mass)-relation and converts the luminosity function to a mass function. The term $2H(M_v)$ is the result of integrating the luminosity function perpendicular to the plane of the Galaxy, assuming an exponential distribution with scale height $H(M_v)$. The factor $f_{\mathrm{ms}}(M_v)$ gives the fraction of stars at a given magnitude which are on the main sequence. The observations relevant to each of these factors are discussed below.

b) *Luminosity Function*

All methods for determining the luminosity function of field stars involve counting the number of stars as a function of apparent magnitude and determining distances to the stars by various means. From this, the absolute magnitudes and volume density can be calculated. An excellent review of the methods and results of early investigations is given by McCuskey (1966). A brief summary and update are given here.

The basic data used in obtaining star densities from the method of trigonometric parallax are a catalog of the number of stars as a function of apparent magnitude, proper motion, and parallax. Corrections are applied for incompleteness and random errors in the parallaxes and for incompleteness in proper motions (see McCuskey 1966 for details). This method was used by van Rhijn (1925) and more recently by Muzzio (1973). Wielen (1974) and Mazzitelli (1972) have derived the luminosity function by this method, but have circumvented the need for the incompleteness factors by using the most complete catalog of stars available (Gliese 1957, 1969) and using small enough distances (5–20 pc) that the catalog is not seriously incomplete. On the other hand, the determinations of Wielen (1974) and Mazzitelli (1972) do not include intrinsically bright stars. Another method for the determination of the luminosity function, which was developed by J. C. Kapteyn and P. J. van Rhijn in the first third of this century, is the use of mean (statistical and secular) parallaxes to obtain distances. At that time, there were few stars with measured trigonometric parallaxes and distances were inferred from proper motions and radial velocities. The method of determining the luminosity function from "mean absolute magnitudes" was also developed because of the small number of trigonometric parallaxes available to early investigators. It is based observationally on blink surveys of proper motion and has been applied by Luyten (1938, 1968) and Wanner (1972). Wanner presented a lucid discussion of this method. He also considered the uncertainties produced by the details of the computational methods and found that such consideration results in an uncertainty of about $\pm 30\%$ in the derived luminosity function. The methods of spectroscopic and photometric parallax use spectral class and color, respectively, as a measure of absolute magnitude. McCuskey (1956, 1966) has used spectroscopic parallax to investigate variations of the luminosity function in the galactic plane, while Upgren (1963) has examined the luminosity function at different heights above the plane. Weistrop's (1972) luminosity function uses the method of photometric parallaxes.

The results of these myriad investigations are in surprisingly good agreement. In Figure 1 the luminosity functions of McCuskey (1966), Wielen (1974), and Luyten (1968) are shown, along with the luminosity function adopted in the present paper. The luminosity functions of Wielen and McCuskey show a small dip around $M_v = 7$. We attach no physical significance to this dip because its size is within the errors of the determination of the luminosity function and this dip is not found by all investigators. The turnover or flattening at $M_v \approx 15$ appears to be a real effect, although its precise location is not known. It should be noted that the luminosity function is uncertain for stars with $M_v \lesssim -3$ because of the scarcity of these stars. Further uncertainties for these bright stars are discussed in § II*f* below. The luminosity functions in Figure 1 agree well with the luminosity functions of others, except for those of Wanner (1972) and Weistrop (1972). Wanner finds a slightly larger value for the luminosity function for the stars with $M_v < 11$ and a slightly smaller value for fainter stars. He states that his computational methods tend to overestimate the number of intrinsically bright stars and underestimate the number of faint stars. Weistrop's (1972) luminosity function is in disagreement with those given in Figure 1 only for stars fainter than $M_v = 10$, and the resolution of this discrepancy is discussed below. Because of the agreement between luminosity functions derived by different methods and for different volumes of space, it can be concluded that there exists a meaningful average luminosity function which represents the disk population in the solar neighborhood.

In her determination of the luminosity function, Weistrop (1972) found that the reddest M dwarfs were 5–10 times more numerous than indicated in Figure 1 and that they were concentrated in a narrow layer in the galactic plane. Several other concurrent papers also gave evidence for an enhanced low-velocity M dwarf population: Murray and Sanduleak (1972), Pesch (1972), Gliese (1972), and Jones (1972). Subsequent investigations have not supported these results. Faber *et al.* (1976) and Weistrop (1976*a*) found errors in Weistrop's (1972) photographic photometry. Colors were systematically measured to be too red, so the absolute magnitudes assigned were too faint. The stars were thought to be closer than they actually are, with a resulting increase in the derived density. Additionally, investigations by Weistrop (1976*b*, 1977*a*, *b*), Koo and Kron (1975), Jones and Klemola (1977), and

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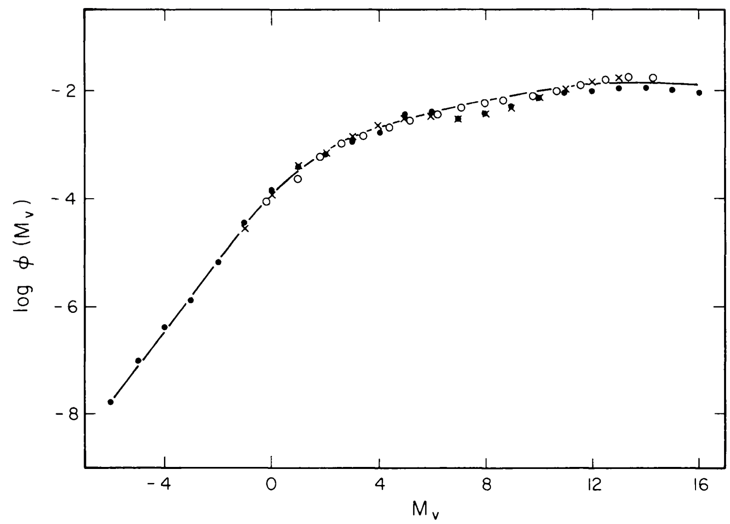

FIG. 1.—The luminosity function $\phi(M_v)$ given as the number of stars per unit absolute visual magnitude M_v and per pc^3 in the plane of the Galaxy. ●, McCuskey (1966); ○, Luyten (1968); ×, Wielen (1974); *solid line*, adopted relation.

Warren (1976) show that there is no evidence for a large number of low-velocity M dwarfs. It is concluded that Figure 1 is a good representation of the faint end of the luminosity function.

The luminosity function has been given for stars as faint as $M_v = 16$ (corresponding to 0.1 $M_\odot$). Although determinations of the luminosity function for fainter stars do exist (e.g., Luyten 1968 goes to $M_{pg} = 24$!), these data cannot be used to infer a mass spectrum for several reasons. First, the observational mass-luminosity relation is unknown for such faint stars and any theoretical relation would be extremely uncertain because of difficulties in model construction and uncertain bolometric corrections. Second, the scale height of faint dwarfs is unknown. Third, the fraction of these very faint stars which are white dwarfs and objects too small to ever fuse hydrogen (rather than main-sequence stars) is unknown. Finally, as emphasized by Hartmann (1970), there are incompleteness problems caused in part by the fact that these stars were discovered in proper motion surveys, so stars of small proper motion may not have been adequately accounted for. It is clear that the PDMF cannot be established by the present methods below about 0.1 $M_\odot$. It should be noted that $M_v = 16$, or 0.1 $M_\odot$, is in rough agreement with theoretical predictions of the lowest possible mass for a main-sequence star ($\sim$0.08 $M_\odot$ [Grossman and Graboske 1971]) and the lowest observed masses ($\sim$0.06 $M_\odot$ [Heintz 1972; Lippincott and Hershey 1972]).

Hartmann (1970) pointed out that the existence of unresolved multiple stars affects the definition of the luminosity function (and derived IMF), which must refer to all discrete mass entities whether or not they are found in multiple systems. Since unresolved multiple systems conceal stars of low luminosity, the apparent luminosity function needs to be weighted toward the fainter stars if one is to obtain the "true" luminosity function (F. L. Whipple 1978, personal communication). In principle such a correction could be attempted, but the uncertainties would be large because of our lack of knowledge concerning the distributions of mass ratios and separations for binary stars. Fortunately, Hartmann's (1970) comparison of the luminosity function of nearby and distant stars suggests that the effect is small.

c) *Mass-Luminosity Relation*

Having justified the validity of the adopted luminosity function, we now proceed to convert this luminosity function to a mass distribution. In order to accomplish this, the relationship between mass and mean absolute visual magnitude on the main sequence is needed. Figure 2 shows masses and magnitudes for visual and eclipsing main-sequence binaries as compiled by Lacy (1977) and for main-sequence spectroscopic binaries (Cester 1965). The dashed line is the magnitude-mass relation derived from Stothers's (1974) theoretical mass-luminosity relation and Code's (1975) bolometric corrections. The relation adopted in the present work is shown by the solid line. For low masses ($\lesssim$0.6 $M_\odot$) the adopted relation is the same as the theoretical relation of Copeland, Hensen, and Jorgensen (1970) as given by Lacy (1977).

The adopted (mass, absolute magnitude)-relation requires further explanation. The magnitude of a main-sequence star decreases as the age of the star increases, so it is necessary to specify the age of the stars to which the (mass, absolute magnitude)-relation applies, e.g., the (mass, zero-age main-sequence magnitude)-relation. The relation we have chosen (denoted the [mass, mean absolute magnitude]-relation) is for a star of average age, that is, the relation between mass and the average absolute magnitude of main-sequence stars of all ages at a given mass. For stars with main-sequence lifetimes less than the age of the Galaxy, the mean relation is roughly 0.5 mag below that of the zero-age relation since stars brighten by roughly 1 mag during their lifetime. For stars with main-sequence

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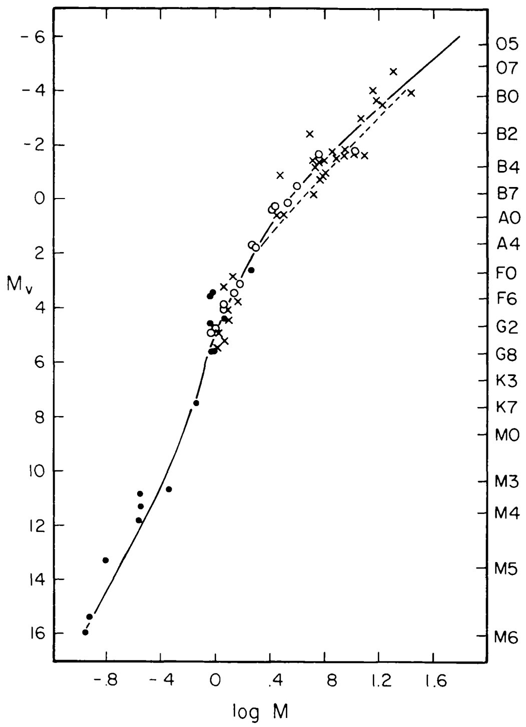

FIG. 2.—Relationship between absolute visual magnitude M_v and mass M ($M_\odot$) for main-sequence stars. ●, visual binaries (Lacy 1977); ○, eclipsing binaries (Lacy 1977); ×, spectroscopic binaries (Cester 1965); *dashed line*, theoretical mass-luminosity relation (Stothers 1974); *solid line*, adopted relation.

lifetimes greater than the age of the Galaxy, the mean and zero-age relations are nearly equal. The motivation for this choice is as follows. The brightening of a star during its main-sequence lifetime has an effect on the PDMF and IMF. Above 1 $M_\odot$, more stars are seen at a given luminosity than would be seen if no brightening occurred. The effects of brightening are considered in detail in the Appendix, where it is shown that the use of the (mass, mean absolute magnitude)-relation compensates for the effects of main-sequence brightening on the PDMF and IMF.

The relationship between spectral class and mean absolute visual magnitude on the main sequence, which enters in a variety of applications, is also shown in Figure 2. This was found by plotting published determinations of the (spectral class, magnitude)-relation (listed below) as well as the individual stars of Lacy (1977) and Cester (1965), and drawing a mean relation by hand. The absolute magnitudes of O stars which are members of clusters with reliably determined distances were determined by Upton and Bohannan (1971), Conti and Alschuler (1971), and Walborn (1973). Blaauw (1963) gives absolute magnitudes covering most of the main sequence. Distances were found by zero-age main-sequence fitting for stars earlier than A2 and from trigonometric and mean parallaxes for stars later than B7. Blaauw (1973) gives improved values for B8–G8 stars based on secular parallaxes (Jung 1970). Trigonometric parallaxes were used by Joy and Abt (1974) for M stars and Woolley (1976) for F7–K7 stars.

d) *Scale Heights*

It is well known that the spatial distribution of stars perpendicular to the plane of the Galaxy varies with spectral type. O stars are only found relatively close to the plane, while M stars are found in substantial numbers at much greater distances. If no attempt were made to account for this effect in deriving the PDMF, the numbers of massive stars would be overestimated relative to the number of low-mass stars. It is usually assumed that the luminosity function of stars perpendicular to the plane is given by an exponential function

$$\phi(z) = \phi(z = 0)e^{-|z|/H}\,, \qquad (2)$$

where z is the distance from the plane, $\phi(z)$ is the number of stars per cubic parsec at height z (and per magnitude or mass interval), and H is the scale height. Examination of the existing literature on the distribution of stars perpendicular to the plane (see below) shows that an exponential distribution is at best a crude representation of the true one. The existing observations do not, however, allow a better representation either numerically or analytically, so an exponential distribution is used.

Figure 3 shows scale heights from Schmidt (1959, 1963), Allen (1973), Weistrop (1972), McCuskey (1966), Upgren (1963), and the work of Elvius (1951, 1962) and Bok and MacRae (1941) (as given in Elvius 1965). Only Schmidt and Allen published values for the scale height. For the other papers, scale heights were found from graphs of log $\phi(z)$ versus z by taking a mean slope. The adopted relation is shown by the solid line in Figure 3. The conversion from spectral type to mass was performed using the relation in Figure 2. Figure 3 shows that the scale heights of all but the brightest stars are not well determined. Additionally, the values of the scale heights for the faintest stars are extrapolations. The distribution of faint main-sequence stars perpendicular to the galactic plane is not known. (The determination by Weistrop 1972 cannot be used because of the errors in her photometry for faint stars.) In the absence of observations for these stars, we adopt an extrapolation similar to that of Schmidt (1959, 1963) and Allen (1973). It should be noted that the uncertainty introduced by integrating over the (poorly determined) z distribution of stars is not as large as the error that would be introduced if only stellar densities in the galactic plane were considered.

Table 1 summarizes the values of the luminosity function, masses, $dM_v/d\log M$, and scale heights adopted in the present investigations. Also given in Table 1 are main-sequence lifetimes extracted from the theoretical calculations of Iben (1965*a*, *b*, 1966, 1967*a*, *b*, *c*), Thomas (1967), Hofmeister, Kippenhahn, and Weigert (1964*a*, *b*, *c*), Hofmeister (1967), Kippenhahn, Thomas, and Weigert (1965), and Faulkner and Cannon (1973). These lifetimes will be needed to

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TABLE 1

ADOPTED RELATIONSHIPS

M_v	$\phi(M_v)$ (stars pc^{-3} mag^{-1})	log $M/M_\odot$	$\frac{-dM_v}{d \log M}$	$2H$ (pc)	log T_{ms} (yr)
−6..........	1.78 (−8)	+1.79	4.0	180	6.50
−5..........	7.94 (−8)	+1.55	4.2	180	6.57
−4..........	3.55 (−7)	+1.33	4.6	180	6.76
−3..........	1.58 (−6)	+1.11	4.9	180	7.02
−2..........	7.08 (−6)	+0.91	5.3	180	7.32
−1..........	3.16 (−5)	+0.72	5.8	180	7.68
0..........	1.20 (−4)	+0.54	6.4	180	8.11
+1..........	3.31 (−4)	+0.40	7.1	185	8.50
+2..........	7.24 (−4)	+0.27	8.1	210	8.90
+3..........	1.41 (−3)	+0.16	9.2	300	9.28
+4..........	2.19 (−3)	+0.07	10.6	465	9.63
+5..........	2.95 (−3)	−0.01	12.8	590	9.93
+6..........	3.98 (−3)	−0.08	15.4	635	10.18
+7..........	5.25 (−3)	−0.14	15.6	650	...
+8..........	6.61 (−3)	−0.21	15.0	650	...
+9..........	8.13 (−3)	−0.27	13.5	650	...
+10..........	9.55 (−3)	−0.35	11.6	650	...
+11..........	1.12 (−2)	−0.44	10.4	650	...
+12..........	1.26 (−2)	−0.54	10.0	650	...
+13..........	1.41 (−2)	−0.65	10.0	650	...
+14..........	1.41 (−2)	−0.75	10.0	650	...
+15..........	1.38 (−2)	−0.85	10.0	650	...
+16..........	1.26 (−2)	−0.96	10.0	650	...

convert the PDMF to the IMF. It is important to realize that a great deal of faith must be placed on these theoretical lifetimes because of the lack of observational constraints. Roxburgh (1978) has suggested that the lifetimes of stars with substantial convective cores ($M \gtrsim$ 2–3 $M_\odot$) are uncertain by about 70% because of uncertainties in convection theory.

e) *Correction for Non–Main-Sequence Stars*

The last term in equation (1) is f_{ms}, the fraction of stars at a given magnitude that are on the main sequence. This quantity is needed to correct the luminosity function for the presence of evolved stars.

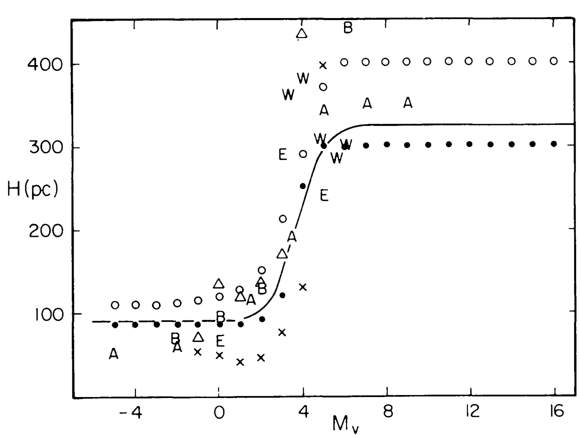

FIG. 3.—Relationship between scale height H (pc) and absolute visual magnitude M_v. ●, Schmidt (1963); ○, Schmidt (1959); ×, Upgren (1963); ▲, McCuskey (1966); *W*, Weistrop (1972); *B*, Bok and MacRae (1941); *E*, Elvius (1951, 1962); *A*, Allen (1973); *solid line*, adopted relation.

For example, both an A0 V star and a K2 III star contribute to the luminosity function at $M_v = 0$. Table 2 lists several determinations of f_{ms}, and it can be seen that there is some disagreement for brighter stars. Salpeter (1955) and Sandage (1957) determined f_{ms} from data in Trumpler and Weaver (1953); Schmidt (1959) used Trumpler and Weaver (1953) and McCuskey (1956); Upgren (1963) determined f_{ms} from his own observations of spectral types; and McCuskey (1966) used the observations of Starikova (1960).

The f_{ms} relation of Sandage will be used for $M_v \leq 3$, and a check on the accuracy of this relation is given below. For fainter stars, $f_{ms} = 1.0$ will be adopted. At magnitudes fainter than $M_v \approx 10$, f_{ms} must be less than 1.0 because of the presence of white dwarfs. As the relative fraction of white dwarfs is small in

TABLE 2

FRACTION (f_{ms}) OF STARS OF A GIVEN ABSOLUTE MAGNITUDE ON THE MAIN SEQUENCE

M_v	Salpeter 1955	Sandage 1957	Schmidt 1959	Upgren 1963	McCuskey 1966
−6......	...	0.46	...	...	0.40
−5......	...	0.48	0.41	...	0.42
−4......	0.18	0.48	0.41	...	0.43
−3......	0.36	0.50	0.46	...	0.44
−2......	0.50	0.51	0.48	...	0.45
−1......	0.47	0.53	0.52	0.12	0.47
0......	0.41	0.56	0.46	0.29	0.51
+1......	0.47	0.62	0.33	0.62	0.56
+2......	0.65	0.71	0.69	0.91	0.66
+3......	0.76	0.86	0.87	1.00	0.82
+4......	0.95	1.00	1.00	1.00	0.98
+5......	1.00	1.00	1.00	1.00	1.00

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TABLE 3

PDMF of Main-Sequence Field Stars

$\log M/M_\odot$	$\phi_{ms}(\log M)$*
+1.79	5.85 (−6)
+1.55	2.91 (−5)
+1.33	1.39 (−4)
+1.11	6.99 (−4)
+0.91	3.46 (−3)
+0.72	1.75 (−2)
+0.54	7.77 (−2)
+0.40	2.71 (−1)
+0.27	8.62 (−1)
+0.16	3.34 (0)
+0.07	1.08 (+1)
−0.01	2.24 (+1)
−0.08	3.89 (+1)
−0.14	5.33 (+1)
−0.21	6.44 (+1)
−0.27	7.14 (+1)
−0.35	7.22 (+1)
−0.44	7.60 (+1)
−0.54	8.18 (+1)
−0.65	9.19 (+1)
−0.75	9.19 (+1)
−0.85	8.97 (+1)
−0.96	8.18 (+1)

* In units of main-sequence stars pc^{-2} log M^{-1}.

comparison to the uncertainties ($f_{ms} \approx 0.9$ [Salpeter 1955]) and is very poorly determined, the contribution of white dwarfs will be neglected.

f) Resulting PDMF and Uncertainties

Having examined the observations relevant to each term in equation (1), we give the PDMF in Table 3 and Figure 4.

It is important to estimate the uncertainties in the PDMF due to the uncertainties in the observational data discussed above. This is a difficult task since most authors either do not estimate errors or else consider only the internal accuracy of their results. What is needed here is an estimate of the total uncertainty in the adopted PDMF. The uncertainties for large masses are considered first.

The above calculation of the PDMF is based on counts of the number of stars at a given absolute magnitude. This procedure may be thought of as taking horizontal "slices" of the H-R diagram and counting stars in each slice. When this is done, a sizable correction must be applied at bright magnitudes for the presence of evolved stars because these slices of constant magnitude intersect the giant and supergiant regions as well as the main sequence. As indicated above, this correction (f_{ms}) is not well determined. The need for this correction can be avoided by counting stars as a function of spectral type, that is, by taking vertical slices of the H-R diagram, since most O and early B stars are on the main sequence. Three recent investigations have examined the numbers of O and early B stars, and therefore offer a check on the PDMF given in Table 3.

In an investigation of the ionization of the interstellar medium by O and early B stars, Torres-

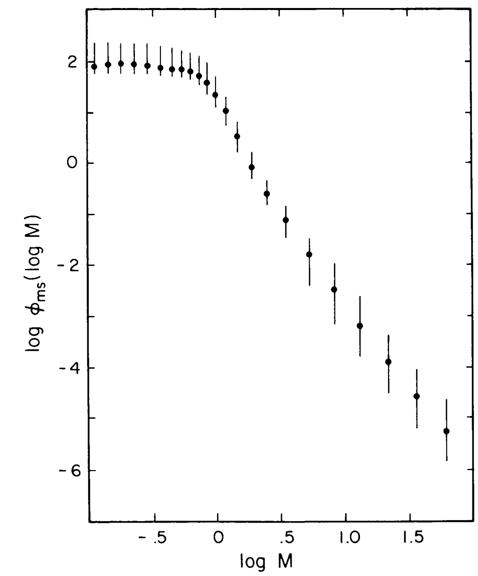

FIG. 4.—The PDMF of main-sequence field stars $\phi_{ms}(\log M)$ given as the number of stars pc^{-2} log M^{-1} in the solar neighborhood. Uncertainty bars are from Table 5.

Peimbert, Lazcano-Araujo, and Peimbert (1974) examined 2883 stars in the catalog of Blanco *et al.* (1968). They derived the space densities of these stars and also compared their results with those of Prentice and ter Haar (1969). Prentice and ter Haar's space densities of early-type stars were based on the catalog of Bečvář (1964). In a paper on the birthrate of massive stars, Ostriker, Richstone, and Thuan (1974, hereafter ORT) give densities of O and early B stars based on the work of Richstone and Davidson (1972), who counted 350 stars from various catalogs. By use of the relations given in this section, the numbers given in these papers have been converted to the PDMF and are shown in Figure 5 along with our PDMF. The agreement between the results of Prentice and ter Haar,

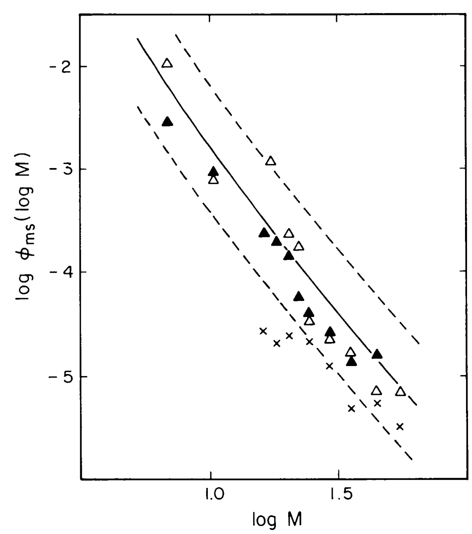

FIG. 5.—Comparison of the adopted PDMF and counts of stars versus spectral class. ▲, Torres-Peimbert *et al.*; △, Prentice and ter Haar (1969); ×, ORT; *solid line*, adopted PDMF; *dashed line*, uncertainty limits.

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Torres-Peimbert *et al.* and the PDMF adopted here confirms the validity of the adopted relation for the earliest-type stars. The PDMF of ORT for these massive stars is consistently smaller than that of the others but is marginally within the lower uncertainty limit. More important, the PDMF of ORT has a substantially different shape. This discrepancy may be due to the small number of stars in their sample or to problems in correction for extinction. ORT's estimates of the birthrate of massive stars based on these numbers are also affected. Note also that the probably incorrect turnover apparent in the data of ORT explains the plateau in the IMF given by Audouze and Tinsley (1976).

ORT also calculate the PDMF and birthrate for stars with $M \lesssim 12\ M_\odot$, and a comparison with the PDMF of this paper is warranted. For these stars, ORT derive the PDMF from the luminosity function as discussed in § II*a*. Their input data are similar to ours except for the values of f_{ms}. ORT used the f_{ms} relation of Upgren (1963), which is about a factor of 4 smaller than that adopted here, resulting in a correspondingly smaller PDMF. In estimating the uncertainties in our PDMF below, we have taken into account this uncertainty in f_{ms}, with the result that for $3\ M_\odot \lesssim M \lesssim 12\ M_\odot$, the PDMF of ORT is within the lower uncertainty limit of the PDMF of this paper; in particular, they have similar shapes.

There are two additional sources of uncertainty concerning the luminosity function of high-mass stars. Massive stars are observed to be losing mass (e.g., Snow and Morton 1976). If the mass-loss rates are very large, as adopted in some of the evolutionary calculations of Dearborn and Eggleton (1977), many of these stars will be observed at luminosities smaller than the luminosity corresponding to the initial masses of the stars. Such a "pileup" of high-mass stars at lower luminosities seems unlikely because then the smooth behavior of the luminosity function at high masses will require a very peculiar form of the "real" IMF. If mass loss is not so rapid, then high-mass stars may evolve at roughly constant luminosity as in the calculations of Chiosi, Nasi, and Sreenivasan (1978). In this case the only error in the IMF will be the correction of the luminosity function for the fraction of stars which are not main-sequence stars, f_{ms}. Since this fraction is usually estimated as about $\frac{1}{2}$ for high-mass stars, this effect can only increase the IMF by a factor of 2. We therefore consider it unlikely that mass loss has disguised many initially high-mass stars.

Another possibility is that high-mass stars remain hidden within molecular clouds or circumstellar dust shells for most of their lives since luminous infrared sources are commonly interpreted as newly born O stars embedded in placental dust shrouds. The problem may not be too severe because an O star should rapidly ionize an H II region, which can be detected at radio wavelengths. Mezger and Smith (1976) have used the observed numbers and other properties of H II regions to estimate that only 20% ± 5% of O and early B stars are unobserved owing to surrounding dust.

Table 4 gives our estimates of the uncertainties in the luminosity function, (mass, absolute magnitude)-relation, scale heights, and f_{ms} correction. These uncertainties are chiefly a measure of the agreement between different authors or of the scatter in the observations. The combined uncertainty in $\log \phi$ and $\log f_{ms}$ for $\log M > 0.8$ was estimated from Figure 5, while the uncertainties in $\log \phi$ and $\log f_{ms}$ for lower masses were found from Figure 1 and Table 2, respectively. The uncertainties in the (M_v, M)-relation and in $dM_v/d \log M$ were found from Figure 2. The uncertainties in the scale heights were estimated from Figure 3. Note that this uncertainty takes into account the possibility that the scale heights of low-mass stars could be substantially larger than adopted. The total uncertainty in the logarithm of the PDMF is taken as the square root of the sum of the squares of the individual uncertainties. These uncertainty limits are also shown in Figures 4 and 5. For low-mass stars the largest uncertainty is introduced by the scale heights, while for the high-mass stars most of the quoted uncertainty is due to the (mass, absolute magnitude)-relation and luminosity function. It is to be emphasized that these are generous uncertainty estimates and that in the absence of any gross errors in the constituent observations the uncertainties derived for the PDMF represent the extremes allowed by the observations.

A referee has pointed out that scatter and systematic variations in chemical composition should be considered as a source of uncertainty in our calculations. The most important effects are on the mass-luminosity relation and the (mass, main-sequence lifetime)-relation. Since we are concerned only with the mean PDMF and IMF, scatter in composition will not affect our results as long as we have used luminosities and lifetimes corresponding to the appropriate mean composition. A purely empirical mass-luminosity relation defined by a large number of stars would automatically reflect the correct mean composition. However, the relation adopted here (Fig. 2) employs a relatively small number of stars and places some weight on a theoretical relation, so it is of interest to estimate the magnitude of the uncertainty. A dependence of mean composition, and hence luminosity, on time of formation in the history of the disk is another source of uncertainty since this affects the slope of the mass-luminosity relation. Again, empirical relations automatically include this effect. On the other hand, theoretical models for the spectrum of stellar masses refer to a single composition, and so a comparison of theory with the empirical IMF must allow for this difference if it is significant. In what follows we assume a standard composition $X = 0.75$, $Z = 0.02$, and consider effects of changes in X by ± 0.05 and in Z by ± 0.01.

It is convenient to use the standard technique of differential corrections (e.g., Stein 1966; Sears and Brownlee 1965) to estimate the magnitude of the above effects. We assume that the rate of nuclear energy generation varies as XZ for $M \geq 2\ M_\odot$ and as X^2 for $M < 2\ M_\odot$, and that the opacity varies as $(1 + X)$ for $M \geq 10\ M_\odot$ and $[Z + (X + Y)/60](1 + X)$ for

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TABLE 4

UNCERTAINTIES IN THE PDMF

		UNCERTAINTY					
$\log M$	M_v	$\log \phi(M_v)$	$\log f_{ms}$	$\log \frac{-dM_v}{d \log M}$	M_v–M Relation	$\log H$	TOTAL UNCERTAINTY IN PDMF
+1.79	−6						
		+0.15, −0.38		±0.07	±0.50	±0.10	+0.54, −0.64
+0.80	−1.5						
		±0.10	±0.15	±0.04	±0.04	±0.10	±0.21
+0.27	+2						
		±0.10	...	±0.04	±0.04	+0.30, −0.12	+0.32, −0.17
−0.08	+6						
		±0.10	...	±0.07	±0.08	+0.38, −0.03	+0.40, −0.15
−0.35	+10						
		±0.15	...	±0.07	±0.08	+0.38, −0.03	+0.42, −0.19
−0.96	+16						

$M < 10\ M_\odot$. These assumptions allow us to calculate $\partial \log L/\partial \log X$ and $\partial \log L/\partial \log Z$. The resulting uncertainties in M_v at a given mass are less than 0.5 mag due to Z and less than 0.4 mag due to X, both uncertainties decreasing with increasing mass. The effects on $dM_v/d \log M$, which enters the PDMF directly, and on the proper identification of each mass with a luminosity in constructing the PDMF are in all cases at least a factor of 2 smaller than the uncertainties we already attached to the $(M_v, \log M)$-relation (Fig. 2) due to observational scatter. We therefore feel that uncertainties in composition will not significantly affect our PDMF within the estimated uncertainties.

The corresponding uncertainties in main-sequence lifetime T_{ms} at a given mass, which affect only the IMF and not the PDMF, can be estimated by assuming $T_{ms} \propto L^{-1}$. The uncertainties in $\log T_{ms}$ are less than ± 0.18 for $M < 10\ M_\odot$ and less than ± 0.08 for $M > 10\ M_\odot$. The effect of these uncertainties on the IMF will be accounted for in § III*b*.

g) *Comparison with Oort Limit*

The PDMF can be used to calculate the mass of main-sequence stars in the solar neighborhood, which can be compared with the local mass density derived from dynamical considerations (the "Oort limit"). The Oort limit is the total amount of matter in the solar neighborhood, as deduced from the observed motions of stars perpendicular to the galactic plane. At various times in the past, determinations of the Oort limit have resulted in densities considerably in excess of the amount of matter that resides in stars, gas and dust. This leaves the "missing mass" and allows one's imagination to invent repositories for this mass (low-velocity M dwarfs, planets, rocks, etc.). Krisciunas (1977) has shown that the Oort limit is not well known (to about $\pm 20\%$) and concludes that, considering uncertainties in the observed densities and in the Oort limit, the discrepancy may not be serious. Jõeveer and Einasto (1977) have emphasized that the Oort limit is subject to large systematic and accidental errors, and they conclude that there is no local missing mass. Krisciunas (1977) finds that the Oort limit lies between 0.11 and 0.17 $M_\odot$ pc^{-3}, and he favors 0.14 $M_\odot$ pc^{-3} as the most likely value. We find that the total amount of mass contained in the PDMF is 27 $M_\odot$ pc^{-2}, and the uncertainties give a range from 20 to 68 $M_\odot$ pc^{-2}. Since the largest contribution to this comes from stars with masses less than about 1 $M_\odot$, dividing by twice the scale height ($2H = 650$ pc for these stars, see Table 1) gives 0.04 $M_\odot$ pc^{-3} (ranging from 0.03 to 0.10 $M_\odot$ pc^{-3}). According to Wielen (1974) and Tinsley (1977*a*) interstellar gas accounts for 0.02 $M_\odot$ pc^{-3}, and Krisciunas has estimated that 0.02 $M_\odot$ pc^{-3} can be accounted for in the form of undetected M3–M8 dwarfs. (Note that this is *not* the large, low-velocity population once advocated by Weistrop 1972 and others [see § II*b*].) Therefore, about 0.07–0.14 $M_\odot$ pc^{-3} can be accounted for, in good agreement with the estimate of Krisciunas. Our results are seen to support the conclusion of Krisciunas and of Jõeveer and Einasto that any discrepancy between the observed mass and the Oort limit is not serious.

III. HISTORY OF THE BIRTHRATE IN THE SOLAR NEIGHBORHOOD

a) *Relation between PDMF, Birthrate, and IMF*

The *stellar creation function* $C(\log M, t)d \log M dt$ is defined as the number of field stars born per unit area in the galactic disk in the mass range $\log M$ to $\log M + d \log M$ during the time interval t to $t + dt$. The term *stellar creation function*, suggested to us by Dr. T. Barnes, is used to emphasize the dependence on both mass and time and also to distinguish it from the stellar birthrate defined below.

The creation function can be related to an observable quantity, the PDMF of main-sequence field stars $\phi_{ms}(\log M)$, which gives the number of main-sequence stars in the solar neighborhood per square parsec and per unit logarithmic mass interval (§ II). For stars with main-sequence lifetimes T_{ms} less than the age of the Galaxy T_0, we observe only those stars born within the last T_{ms} years because all stars born between $t = 0$

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and $t = T_0 - T_{ms}$ have evolved off the main sequence. Thus the observed number of main-sequence field stars $\phi_{ms}(\log M)$ is related to the creation function by

$$\phi_{ms}(\log M) = \int_{T_0 - T_{ms}}^{T_0} C(\log M, t)dt\,, \quad T_{ms} < T_0\,. \tag{3}$$

All stars with main-sequence lifetimes greater than the age of the Galaxy are still on the main sequence, so the observed number of stars is obtained by integrating the creation function over all times:

$$\phi_{ms}(\log M) = \int_0^{T_0} C(\log M, t)dt\,, \quad T_{ms} \geq T_0\,. \tag{4}$$

Equations (3) and (4) are not useful as they stand because of the unknown nature of the creation function. The equations can be simplified by assuming that the creation function can be separated into the product of a function of mass and a function of time, or equivalently that the mass spectrum of newly formed stars is independent of time. We emphasize that the assumption of a separable creation function is made purely for the sake of convenience and simplicity at this point; the validity of this assumption is discussed in § V. Proceeding to separate the creation function, we define the field star IMF, $\xi(\log M)$, as the total number of field stars that have *ever formed* per square parsec and per unit logarithmic mass. If the age of the Galaxy is T_0, then the time-averaged birthrate per unit logarithmic mass is

$$\langle C \rangle = \int_0^{T_0} C(\log M, t)dt/T_0 = \xi(\log M)/T_0\,. \tag{5}$$

The absolute total birthrate (total number of stars born per unit area per unit time) is

$$B(t) = \int_{-\infty}^{\infty} C(\log M, t)d\log M\,, \tag{6}$$

and the absolute time-averaged total birthrate is

$$\langle B \rangle = \int_0^{T_0} B(t)dt/T_0\,. \tag{7}$$

Then for a separable creation function we have

$$\begin{aligned} C(\log M, t) &= \langle C \rangle C/\langle C \rangle \\ &= \langle C \rangle \int_{-\infty}^{\infty} C d\log M \Big/ \int_{-\infty}^{\infty} \langle C \rangle d\log M \\ &= \frac{\xi(\log M)}{T_0}\frac{B(t)}{\langle B \rangle}\,, \end{aligned} \tag{8}$$

or

$$C(\log M, t) = \xi(\log M)b(t)/T_0\,, \tag{9}$$

where the *relative birthrate* $b(t)$ gives the absolute birthrate at time t in units of the average birthrate. In what follows we refer to $b(t)$ as the *birthrate*.

Schmidt (1959) first suggested the notation of equation (9), and we have given the above derivation to show the motivation behind this notation. The definition of the field star IMF in terms of all stars ever formed may seem confusing at first. However, in the case of a time-constant IMF (as assumed here), the IMF at any given time has the same shape as the IMF at any other time and therefore the same shape as the IMF of all stars ever formed. Note that the field star IMF as defined here and the IMF of stars in open clusters are similar quantities in that both give the mass distribution of stars at birth. In the case of a field star IMF which varies with time (creation function not separable), the mass distribution of all stars ever formed is *not* the field star IMF and cannot be directly compared to the cluster IMF.

Using the separable creation function, equations (3) and (4) become

$$\phi_{ms}(\log M) = \frac{\xi(\log M)}{T_0}\int_{T_0 - T_{ms}}^{T_0} b(t)dt\,, \quad T_{ms} < T_0\,, \tag{10}$$

$$\phi_{ms}(\log M) = \frac{\xi(\log M)}{T_0}\int_0^{T_0} b(t)dt\,, \quad T_{ms} \geq T_0\,. \tag{11}$$

Stars with main-sequence lifetimes greater than the age of the Galaxy will be found on the main sequence today regardless of when they were formed. For these stars the PDMF and the IMF are identical:

$$\phi_{ms}(\log M) = \xi(\log M)\,, \quad T_{ms} \geq T_0\,. \tag{12}$$

Equations (11) and (12) place the following normalization condition on the birthrate:

$$\int_0^{T_0} b(t)dt = T_0\,. \tag{13}$$

It is important to note the relationship between the IMF and the birthrate, as illustrated by equations (10) and (12). Since the PDMF, age of the Galaxy, and main-sequence lifetimes are assumed known, it can be seen from these equations that *either* the IMF can be determined if the birthrate is known *or* the birthrate can be determined if the IMF is known. In other words, a choice of an IMF combined with an arbitrary birthrate will *not*, in general, satisfy the fundamental constraint presented by the observed numbers of main-sequence stars. This consistency requirement is often overlooked. For example, Neckel (1975) used an IMF derived assuming a constant birthrate (Sandage 1957) and combined this IMF with a birthrate that was a rapidly decreasing function of time (Schmidt 1959).

In order to illustrate the relationship between the IMF, birthrate, and PDMF, we have derived the IMF assuming several analytic functions for the birthrate. Figure 6 shows the birthrates used, which range from sharply decreasing to sharply increasing functions of

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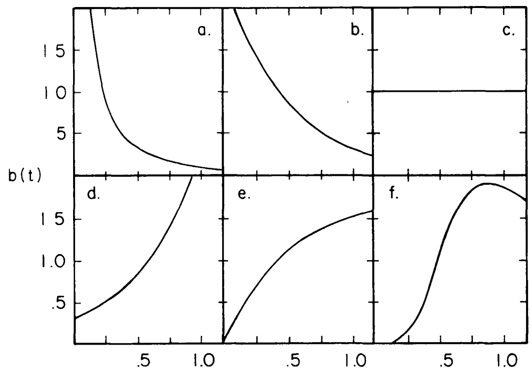

FIG. 6.—Trial birthrates; $b(t)$ gives the ratio of the birthrate at time t to the average birthrate. (*a*) Schmidt $n = 2$; (*b*) decreasing exponential; (*c*) constant; (*d*) increasing exponential; (*e*) asymptotic increasing exponential; (*f*) polynomial.

time. These do not exhaust all possible birthrates, of course, but they do cover a wide range of behavior. Three of these functions have often been used in past studies of the evolution of the solar neighborhood and galaxies, while three of these are novel.

The first birthrate is taken from Schmidt (1959), who assumed that the time dependence of the birthrate of field stars is proportional to some power n of the average gas density. He presented evidence that the value of n was 2. (This evidence is reexamined in § III*c*(ix) and is found not to support a value of $n = 2$ or in fact any simple dependence of the birthrate on gas density.) In this case the birthrate takes the form

$$b_1(t) = b_1(0)\left(1 + \frac{t}{T_0\tau}\right)^{-2},$$

$$\tau = \frac{P}{1 - P},$$

$$b_1(0) = [(1 - P)\tau]^{-1} \tag{14}$$

if a closed model for the solar neighborhood is assumed. The subscript 1 denotes the first of the six birthrates considered here, and P is the present-day ratio of the mass of interstellar gas and dust to the total mass in the solar neighborhood, believed to be between 0.05 and 0.20 (Tinsley 1977*a*). The value $P = 0.1$ was adopted. It should be noted that this birthrate satisfies the normalization requirement, equation (13). The second birthrate considered here is exponentially decreasing:

$$b_2(t) = B_2 \exp(-t/\tau),$$

$$B_2 = \frac{T_0}{\tau[1 - \exp(-T_0/\tau)]}. \tag{15}$$

B_2 is the normalization constant determined from equation (13), and τ is the *e*-folding time. This birthrate is the one most widely found in the literature (e.g., Tinsley 1974*a*). The value of τ is taken as $T_0/2$ here. It should be noted that an exponentially decreasing birthrate is obtained in Schmidt's (1959) formulation when $n = 1$, i.e., when the birthrate is assumed to be directly proportional to the average gas density. In this case the *e*-folding time is fixed by P, the present-day ratio of the mass of interstellar gas and dust to the total mass in the solar neighborhood ($\tau = -T_0/\ln P$). Again, it should be emphasized that there is no firm observational or theoretical evidence supporting such a birthrate, but it is convenient to use in analytic computations. The third birthrate is constant with time:

$$b_3(t) = 1. \tag{16}$$

This is the birthrate used by Salpeter (1955).

The remaining three birthrates are novel in that they increase with time. The fourth is simply an exponentially increasing function of time:

$$b_4(t) = B_4 \exp(t/\tau),$$

$$B_4 = \frac{T_0}{\tau[\exp(T_0/\tau) - 1]}, \tag{17}$$

where τ is taken as $T_0/2$. This birthrate increases without bound as time increases and is in this respect an unphysical choice. This presents no difficulties, however, because the function is used only for times no greater than T_0 where it is well behaved. The fifth birthrate is given by

$$b_5(t) = B_5[1 - \exp(-t/\tau)],$$

$$B_5 = \frac{T_0}{T_0 + \tau[\exp(-T_0/\tau) - 1]}, \tag{18}$$

where τ is $T_0/2$. This birthrate asymptotically approaches B_5 as a maximum value and is referred to as the asymptotic exponential birthrate. The last birthrate has the form

$$b_6(t) = B_6 \frac{(t/\tau)^{m-1}}{1 + (t/\tau)^m},$$

$$B_6 = \frac{mT_0}{t \ln[1 + (T_0/\tau^m)]}, \tag{19}$$

where $m = 4$ and $\tau = 2T_0/3$. This function, referred to as the polynomial birthrate, was selected because it increases rapidly for small t (like t^{m-1}), reaches a maximum, and then decreases. Its behavior is similar to Larson's (1976) models for disk galaxies with infall.

Figure 7 shows the IMFs derived from equations (10) and (12) for the assumed birthrates. The age of the Galaxy has been taken as 12×10^9 years, but the results are insensitive to this choice over the range of 9–15 $\times 10^9$ years. To prevent the diagram from being cluttered, the IMFs resulting from the polynomial and asymptotic exponential birthrates are not shown. These IMFs lie between the IMFs of the constant and increasing exponential birthrates. For stars of low mass ($T_{ms} \geq T_0$) the IMF is identically equal to the PDMF and is the same for all birthrates.

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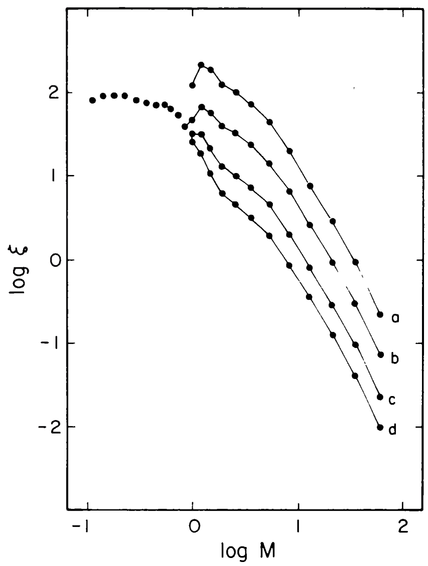

FIG. 7.—The IMFs resulting from the trial birthrates; ξ has units of stars pc^{-2} log M^{-1}. (*a*) Schmidt $n = 2$; (*b*) decreasing exponential; (*c*) constant; (*d*) increasing exponential. The IMFs resulting from the asymptotic exponential and polynomial birthrates lie between curves *c* and *d* and are not shown for clarity.

It is apparent from Figure 7 that the high-mass ends of the IMFs have the same shape regardless of the birthrate. This is a direct result of the fact that the main-sequence lifetimes of massive stars are small compared to the age of the Galaxy. These stars are therefore found in numbers proportional to the present birthrate and not the integrated past birthrate. This can be seen from equation (10) by using a Taylor series to first order:

$$\xi(M) = \frac{\phi_{\mathrm{ms}}(M)}{T_{\mathrm{ms}}} \frac{T_0}{b(T_0)}, \quad T_{\mathrm{ms}} \ll T_0 . \qquad (20)$$

The shape of the IMF is given by the ratio of the PDMF to the main-sequence lifetime, while the normalization is given by the ratio of the age of the disk to the present birthrate.

The IMFs derived from Schmidt's $n = 2$ birthrate and the decreasing exponential birthrate show a discontinuity at the mass where the main-sequence lifetime equals the age of the Galaxy. Several authors, starting with Schmidt (1959), have called attention to the fact that a birthrate which decreases by too large an amount over the age of the disk results in the discontinuity shown in Figure 7. The existence of a discontinuity in the IMF for rapidly decreasing birthrates can be understood as follows: The IMF is the number of stars (per square parsec per log mass) ever formed, while the PDMF is the number of stars (per square parsec per log mass) observed today. For stars with main-sequence lifetimes less than the age of the Galaxy, the PDMF must be increased by some amount to account for stars that have died before the present time. This increase depends on the birthrate. For a birthrate that is a rapidly *decreasing* function of time, a larger fraction of all stars ever formed were born longer than one main-sequence lifetime ago and are not found on the main sequence today. The correction to the PDMF to obtain the IMF will be large. If the birthrate is a rapidly *increasing* function of time (smallest in the past), i.e., a large fraction of all stars ever formed were born *less* than one main-sequence lifetime ago, the PDMF will be only slightly smaller than the IMF and the correction to the PDMF to obtain the IMF will be correspondingly small. Now consider the mass M' where the main-sequence lifetime equals the age of the Galaxy. The IMF for masses smaller than M' is fixed by the PDMF. The IMF for masses greater than M' is determined by increasing the PDMF by some amount determined by the birthrate. For a birthrate that is rapidly decreasing with time, this increase will be so large that the IMF for masses slightly greater than M' will be much greater than the IMF for masses slightly smaller than M', giving rise to the discontinuity.

The existence of a discontinuity in a model often indicates that such a model is incorrect. To astronomers in particular, it is especially unsettling that this discontinuity occurs at the mass where the main-sequence lifetime equals the age of the Galaxy because this implies that the present time is unique. Rather than believe in the physical reality of this discontinuity and invent theories to account for it, it is more reasonable to reject the discontinuity and the birthrate which gives rise to it. Further discussion of this choice is given in § V. Tinsley (1977*a*) has emphasized that the requirement of a continuous IMF gives more quantitative information on the time history of the stellar birthrate than any other available method, and we therefore proceed to examine this constraint in more detail in § III*b* while additional, weaker constraints will be discussed in § III*c*. In addition, the limits on the birthrate imposed by the continuity constraint will provide limits on the form of the IMF, a quantity of fundamental interest.

b) *Continuity Constraint*

The requirement that the IMF be continuous can be cast in a more quantitative form (Tinsley 1977*a*). The task is to choose birthrates that result in a smooth transition between the IMF of low-mass and high-mass stars, considering the uncertainties in the PDMF and main-sequence lifetimes. The IMF of stars with $T_{\mathrm{ms}} \geq T_0$ is fixed by the PDMF. The *shape* of the IMF of stars with short lifetimes ($T_{\mathrm{ms}} \ll T_0$) can be found from the PDMF and main-sequence lifetimes. It can be seen from equation (20) that the constant factor $T_0/b(T_0)$ determines how the IMFs of the low-mass and high-mass stars are joined. If $T_0/b(T_0)$ is too large, the IMF of high-mass stars will exceed the IMF of low-mass stars, giving rise to a discontinuity—the "bump" in the IMF. If $T_0/b(T_0)$ is too small, a discontinuous "dip" will occur.

Approximate limits to $T_0/b(T_0)$ can be set by using Figure 8. The left-hand side of Figure 8 shows the PDMF for stars with lifetimes in excess of the age of the Galaxy, while the right-hand side shows the PDMF

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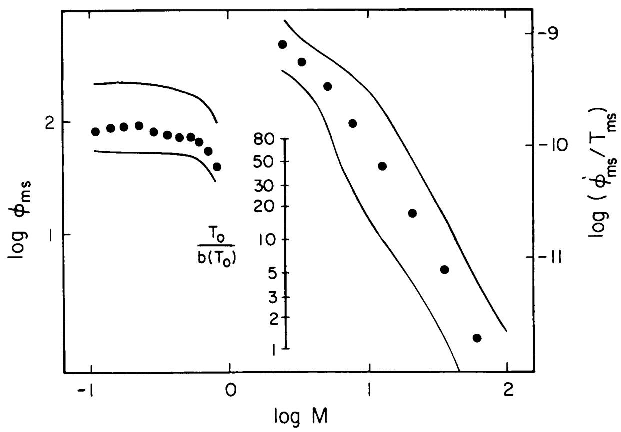

FIG. 8.—Limits on the present birthrates from the continuity constraint. The left-hand side of the figure gives the PDMF ϕ_{ms} for stars with main-sequence lifetimes T_{ms} in excess of the galactic age T_0. For these stars the PDMF equals the IMF. The right-hand side gives the PDMF divided by T_{ms} for stars with $T_{ms} \ll T_0$. For these stars the IMF is given by the product of ϕ_{ms}/T_{ms} and $T_0/b(T_0)$. The scale shows the results of this multiplication for different values of $T_0/b(T_0)$ (in units of 10^9 yr). A smooth transition between the two groups, and thus a continuous IMF, is obtained for 6×10^9 yr $< T_0/b(T_0) < 50 \times 10^9$ yr.

for high-mass stars divided by their main-sequence lifetime. The range of uncertainty in the PDMF (§ II) is also indicated. Superposed on Figure 8 is a scale which gives the value $T_0/b(T_0)$ required to increase the point in Figure 8 at log $M = 0.40$ to the indicated point on the scale. In other words, equation (20) shows that ϕ_{ms}/T_{ms} must be multiplied by $T_0/b(T_0)$ to obtain the IMF. The scale in Figure 8 shows the result of this multiplication, which may be thought of as sliding the right-hand side of Figure 8 vertically with respect to the left-hand side. In addition to the uncertainty in the PDMF indicated in Figure 8, the main-sequence lifetimes and thus the right-hand ordinate are uncertain. In § II*f* the uncertainty in log T_{ms} is estimated to be less than ± 0.18 for $M < 10\ M_\odot$.

Allowing for these two uncertainties, one can see that a smooth fit is obtained for

$$T_0/b(T_0) \lesssim 50 \times 10^9 \text{ yr} . \qquad (21)$$

Recall that in the discussion of uncertainties in the PDMF, the major uncertainty for low-mass stars was in the scale heights, which may be substantially larger than adopted here. If the scale heights of these stars are close to the values adopted here, then the upper limit on $T_0/b(T_0)$ is reduced. Tinsley (1977*a*) finds an upper limit on this quantity (which she calls H) of 80×10^9 yr, and favors 20×10^9 yr as a more realistic upper bound, in good agreement with our value.

Just as a birthrate that decreases too much over the age of the Galaxy gives a discontinuous IMF, a birthrate that increases too much also gives a discontinuity. From Figure 8 it can be seen that we require

$$T_0/b(T_0) \gtrsim 6 \times 10^9 \text{ yr} . \qquad (22)$$

Tinsley (1977*a*) required that the birthrate be a decreasing (or constant) function of time, which implies that $T_0/b(T_0) \geq T_0$. In contrast, we find that an increasing birthrate can satisfy the continuity constraint on the IMF, and is not inconsistent with other observational constraints or theoretical considerations discussed below. Table 5 gives the value of the present birthrate $b(T_0)$ corresponding to the upper and lower limits on $T_0/b(T_0)$ for three values of the age of the Galaxy. The maximum allowable range of $b(t)$ is

TABLE 5

PRESENT RELATIVE BIRTHRATE

AGE OF THE GALAXY T_0 (yr)	PRESENT RELATIVE BIRTHRATE $b(T_0)$	
	$\frac{T_0}{b(T_0)} = 6 \times 10^9$	$\frac{T_0}{b(T_0)} = 50 \times 10^9$
9×10^9..........	1.5	0.18
12×10^9..........	2.0	0.24
15×10^9..........	2.5	0.30

$$0.18 \lesssim b(T_0) \lesssim 2.5 . \qquad (23)$$

In other words, the average past birthrate may be at most about 5 times larger or 3 times smaller than the present birthrate. We emphasize that these limits are extremes, and we favor a birthrate whose past average differs from the present value by less than a factor of 2.

c) *Additional Constraints*

Having discussed the continuity constraint, we now critically examine other lines of observational evidence relating to the history of the birthrate. This examination is important because various authors have presented apparent evidence for a rapidly decreasing birthrate, which is inconsistent with the continuity constraint. It will be seen from the following discussion, however, that all other constraints are either consistent with the continuity constraint or plagued by so many uncertainties as to render them indeterminate, thereby strengthening the validity of the IMFs derived in § IV below. An earlier summary of some of the evidence pertaining to the history of the birthrate has been given by Tinsley (1977*a*).

i) *Initial Mass Functions of Stellar Birth Sites*

Star formation occurs in several domains: open clusters; OB, T, and R associations; low-mass aggregates identified by Aveni and Hunter (1969); and perhaps as isolated field stars (Aveni and Hunter 1967). Upon dissipation of stellar groups (which on the average occurs in times small compared to the age of the Galaxy), their members become field stars. The field star IMF could therefore be determined from the average of the IMFs of these stellar birth sites, weighted by their relative contributions. If the IMF were known in this manner, the history of the birthrate could be found.

The IMF of a group of stars is found by converting the luminosity function below the main-sequence

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turnoff to a mass function. Unfortunately, only open clusters have IMFs which are reasonably well determined over a large range of masses. There is no information on the luminosity function or frequency of occurrence of stars born in the field or of low-mass aggregates. The association luminosity function can be determined for only the brightest stars (see § IV*c* for Ori OB1) for two reasons: (1) Accurate proper motions and radial velocities are required to distinguish association members from field stars, and a prohibitive amount of time would be necessary to obtain such data for faint stars. (2) Associations are so young that the fainter members are still contracting to the main sequence. Converting the luminosity function to a mass function for these stars is not possible because the masses of pre–main-sequence stars are not known.

In § II*a* it was shown that the birthrate, IMF, and PDMF are uniquely related. Since the PDMF is observationally determined, the IMF can be found if the history of the birthrate is known or the history of the birthrate can be determined if the IMF is known. The former approach has been described in § III*a* and is applied in § IV, while the latter approach was used by Ebert, von Hoerner, and Temesváry (1960). For lack of knowledge of other stellar birth sites, they assumed that the field star IMF has the same form as the cluster IMF.

In order to calculate the birthrate when the IMF is known, define a quantity R

$$R = T_0\phi_{\rm ms}(\log M)/\xi_{\rm CL}(\log M)\,, \qquad (24)$$

where $\xi_{\rm CL}$ is the cluster IMF, assumed identical in shape to the field star IMF. The birthrate is then given by

$$b(T_0 - T_{\rm ms}) = \frac{dR}{dT_{\rm ms}}\,, \quad T_{\rm ms} < T_0\,. \qquad (25)$$

In principle, this method is equivalent to assuming a birthrate and deriving the IMF; i.e., given the same constraint, both methods give identical results. In practice, however, this approach is more difficult to apply because the differentiation necessary in equation (25) makes the results sensitive to small errors in the input data and also makes it more difficult to ascertain the effects of the observational uncertainties on the results.

Using this method, Ebert *et al.* found a birthrate which moderately decreased over the first few 10^9 years of galactic history and was approximately constant afterward. This birthrate could be approximated by an exponentially decreasing function of time. The decreasing birthrate at early times appears to be a result of their adopted (mass, main-sequence lifetime)-relation. Although they do not give sufficiently detailed information to reconstruct their relation, it appears that they adopted a relation near 1 $M_\odot$ less steep than the relation of the present paper, which is based on (we hope) more accurate theoretical main-sequence lifetime calculations. In other words, Ebert *et al.* used a smaller $dT_{\rm ms}$ in equation (25), resulting in an increase in the birthrate for early times. We have solved equation (25) for different power-law IMFs consistent with the average cluster IMFs determined by Taff (1974), Piskunov (1976), and Burki (1977), and find a birthrate which is constant at early times, increases gradually, and then increases very rapidly within the last 10^9 years or so, depending on the adopted power law and age of the disk. However, the results are so sensitive to small errors in main-sequence lifetimes that we place no weight on them. The important point is that the conclusion of Ebert *et al.* that the cluster IMF implies a decreasing birthrate is incorrect.

ii) *Stellar Age Distributions*

The distribution of stellar ages would offer a direct means of determining the history of the birthrate since a larger or smaller birthrate in the past would be reflected by larger or smaller numbers of old stars relative to the number of young stars. Ages of stars can be inferred from comparison of the positions of stars in the H-R diagram with theoretical isochrones (Dixon 1970; Cayrel de Strobel 1974; Clegg and Bell 1973), from an adopted calibration of the relation between velocity dispersion and age (Mayor and Martinet 1977), or from an adopted calibration of the Ca II emission strength and age (Wielen 1974). All of the papers just mentioned find a birthrate which is roughly constant or has even increased somewhat. Tinsley (1974*b*) demonstrated that the age distribution does not directly reflect the time dependence of the birthrate because stellar lifetimes vary with mass; i.e., a flat age distribution does not imply a constant birthrate. Tinsley derived the relationship between the birthrate and age distribution for main-sequence stars and concluded that the age distributions of Clegg and Bell (1973) and Cayrel de Strobel (1974) require a decreasing birthrate. However, Clegg (1977, private communication) has pointed out that the observed age distributions are for subgiants and not main-sequence stars. Accounting for the effects of subgiant lifetimes shows that the observed age distributions actually imply an *increasing* birthrate. Clegg has also emphasized that the parallax errors are substantial and should be accounted for in any attempt to obtain age distributions from theoretical isochrones. Considering this, possible selection effects and uncertainties in the relation of velocity dispersion and Ca II strength with age, this method can only be considered a consistency check on the birthrate. At present, all that can be said is that stellar age distributions are consistent with a roughly constant birthrate.

Tinsley (1978) has shown that the distribution of initial masses of planetary nebula progenitors depends on the past history of the birthrate. With a decreasing birthrate the distribution drops off more rapidly with increasing mass because of the smaller contribution from more massive progenitors which were recently formed. For this reason Tinsley points out that the kinematics of planetary nebulae could in principle be used to constrain the age distribution of planetary nebula progenitors, and hence the birthrate history. Although the available data may not be sufficient to distinguish between the two birthrates examined by

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Tinsley (constant and ratio of average birthrate to present birthrate equal to 3), it appears that the kinematics and also spatial distribution of planetary nebulae and white dwarfs do rule out a birthrate which has increased by more than a factor of 2 or 3, or else the mean initial mass would be fairly large. If the identification by Grieg (1971) and Cudworth (1974) of two kinematic classes of planetary nebulae is correct, then a rapidly decreasing birthrate is also ruled out since in that case nearly all the progenitors would have initial masses around 1 $M_\odot$.

iii) *Radioactive Nuclides*

Fowler and Hoyle (1960) and Fowler (1972) investigated models for *r*-process nucleosynthesis in which the rate of nucleosynthesis in the solar neighborhood was an exponential function of time with time constant T_R, occurring over a duration Δ, terminated by a "spike" at a time δ before the formation of meteorites in the early solar system. The meteoritic isotopic ratios $^{232}Th/^{238}U$, $^{235}U/^{238}U$, and $^{244}Pu/^{238}U$ then allow a determination of the best values of the parameters T_R, Δ, and δ. Using "standard values" of the isotopic ratios and production ratios, Fowler (1972) obtained $T_R \approx 3 \times 10^9$ yr, but various uncertainties in the input data allow T_R to range from negative (increasing rate of nucleosynthesis), to zero (no nucleosynthesis previous to the spike), to infinity (constant rate of nucleosynthesis). Furthermore, the time scale for the rate of star formation, which is what is of interest here, cannot be inferred from T_R without assuming a relation between birthrate and gas mass. As discussed below, there is no firm evidence that any such relation exists. A more detailed discussion of the problems involved can be found in Tinsley (1976, 1977*a*). It appears that *r*-process chronologies cannot presently provide any information on the history of the birthrate.

iv) *Yields of Nucleosynthesis*

In a recent paper Arnett (1978) has estimated the yields of nucleosynthesis in massive stars ($>10\ M_\odot$). As a check on the consistency of these stellar evolution models, Arnett combined his estimated yields per star of given mass with the present birthrate of massive stars given by ORT to derive the present rate of nucleosynthesis. He finds that the present rate of nucleosynthesis must be only about 10% of the past average rate in order to account for the present galactic metal abundance, and he notes that this is consistent with models of the chemical evolution of the solar neighborhood in which the present rate of star formation is about this fraction of the average rate (e.g., star formation proportional to the square of the gas density.) Because the continuity constraint and other constraints discussed in this section do not allow such a large decrease in the birthrate, it is important to examine the uncertainties in Arnett's estimate. As discussed in § II*f*, the PDMF and therefore the present stellar birthrate of ORT lie roughly on the lower uncertainty limit of those of the present work. Using the birthrate calculated in § IV and Arnett's Table 4, we find a present rate of nucleosynthesis larger than Arnett's. This rate requires the average birthrate to be only about a factor of 3 larger than the present birthrate, which is entirely consistent with the other constraints. We note that some models of chemical evolution in the solar neighborhood call for an initial metallicity substantially greater than zero to explain the paucity of metal-poor stars (e.g., Tinsley 1974*a*); if such models are correct, the average birthrate must have been even smaller in order to be consistent with the observed metallicity in the solar neighborhood. Considering the above uncertainties and the additional uncertainties in Arnett's stellar models, we conclude that this constraint is consistent with the continuity constraint.

v) H II *Regions*

H II regions mark the sites of recent star formation, so observations of H II regions can be used to estimate the present birthrate. This approach has been applied by Mezger and Smith (1976) and Smith, Biermann, and Mezger (1978) to observations of radio H II regions. They express the present birthrate (in units of the mass of stars formed in the entire Galaxy per year) as

$$dM_*/dt = N_T\langle M\rangle/(\langle N_c\rangle\tau_{\mathrm{RH\,II}})\,. \qquad (26)$$

N_T is the total number of Lyman continuum photons emitted by stars ionizing radio H II regions in the Galaxy, $\langle N_c\rangle/\langle M\rangle$ is the ratio of the number of Lyman continuum photons emitted by the ionizing star to the total mass of stars formed concurrently with the ionizing star, and $\tau_{\mathrm{RH\,II}}$ is the length of time an H II region is detectable at radio frequencies. From radio observations Smith *et al.* estimate $N_T = 5.5 \times 10^{52}$ photons s^{-1}. Mezger, Smith, and Churchwell (1974) and Smith *et al.* used radio observations to determine the number of Lyman continuum photons emitted by the ionizing star of an H II region and combined this with the Salpeter (1955) IMF to find $\langle N_c\rangle/\langle M\rangle = 2.2 \times 10^{46}$ photons s^{-1} $M_\odot^{-1}$. To find the lifetime of an H II region as a detectable radio source, Mezger and Smith compared the number of radio H II regions and optically visible O stars in the solar neighborhood and found that the ionizing stars of radio H II regions constitute 20 ± 5% of all O stars. From this they inferred the lifetime of a radio H II region to be 20% of an O-star lifetime. Adopting 3.2×10^6 yr as the lifetime of an average O star in an H II region, they concluded that the lifetime of a radio H II region is 6.5×10^5 yr. Smith *et al.* revised this figure to $5 \pm 0.5 \times 10^5$ yr.

Combining these values yields a present birthrate of 5 $M_\odot$ yr^{-1} in the Galaxy, with an uncertainty of a factor of 3 due to uncertainties in $\tau_{\mathrm{RH\,II}}$ and N_T. It is difficult to estimate the uncertainty in $\langle N_c\rangle/\langle M\rangle$. Using the IMFs of the present paper, we find $\langle N_c\rangle/\langle M\rangle$ smaller by a factor of 1.5 and take this as a measure of the uncertainty. This does not include any uncertainty in the Lyman continuum flux of OB stars. Mezger and Smith attach a factor of 1.25 uncertainty in the fraction of all O stars in radio H II regions. The

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uncertainty in the average O-star lifetime contributes another factor of 1.5. The combined uncertainty in the present total galactic birthrate derived by this method is therefore about a factor of 4. The average galactic birthrate is just the stellar mass, $2 \times 10^{11}\ M_{\odot}$, divided by the age of the disk, $9\text{–}15 \times 10^9$ yr. Comparing dM_*/dt with the average rate and allowing for the uncertainties, we obtain

$$b(T_0) \equiv \frac{B(T_0)}{\langle B(t) \rangle} = 0.3\,(+1.24,\ -0.24)\,. \quad (27)$$

Although the "best" value of 0.3 is just consistent with the continuity constraint, the uncertainties allow a time-averaged birthrate which is about 15 times smaller or 1.5 times larger than the present birthrate. Therefore, while this method provides an upper limit on $b(T_0)$, the uncertainties render it much less useful than the continuity constraint in estimating a lower limit on $b(T_0)$.

vi) *Formation Rate of Supernovae*

The rate of supernova explosions in the Galaxy contains information on the present death rate of stars. Combined with the mass range of stars which make supernovae and the IMF, the present supernova rate can be used to calculate the present rate of formation of all stars. Comparison with the average birthrate then gives some idea of the time dependence of the birthrate. Such a calculation was carried out by Truran and Cameron (1971, see also Truran 1973), who found a rate of about $2\ M_{\odot}\ \text{yr}^{-1}$ in the entire Galaxy. The rate of supernova explosions calculated from the IMFs of the present paper (§ IV) is given in Figure 9. We have assumed that all stars more massive than some mass M_L (see Tinsley 1977*b* for a review) become supernovae, and vary M_L between 4 and 10 $M_{\odot}$. Since these stars have lifetimes much less than the age of the Galaxy, the present death rate of these stars equals the present birthrate, which can be calculated for the solar neighborhood from equation (37) below. To calculate the supernova rate in the Galaxy, the rate in the solar neighborhood (per square parsec) must be multiplied by the effective area of the Galaxy (which takes into account both the geometrical area of the Galaxy and the ratio of the average rate in the Galaxy to the rate in the solar neighborhood). The distribution of pulsars found by Taylor and Manchester (1977) can be used to calculate the effective area, assuming that pulsars and supernovae occur in stars of similar mass range. They find that the local pulsar birthrate is 3×10^{-5} to $1 \times 10^{-4}\ \text{yr}^{-1}\ \text{kpc}^{-2}$ for mean electron densities of 0.02 and 0.03 cm^{-3}, respectively. The corresponding pulsar birthrates in the Galaxy are 0.167 and 0.025 yr^{-1}. The effective area of the Galaxy then lies between 800 and 1600 kpc^2. A value of 1000 kpc^2 was adopted. Note that the present total birthrate corresponding to each supernova rate must be computed using an IMF which is consistently derived from the adopted form of $b(t)$. The extreme two curves in Figure 9 were calculated using birthrates which satisfy the extremes of the continuity constraint

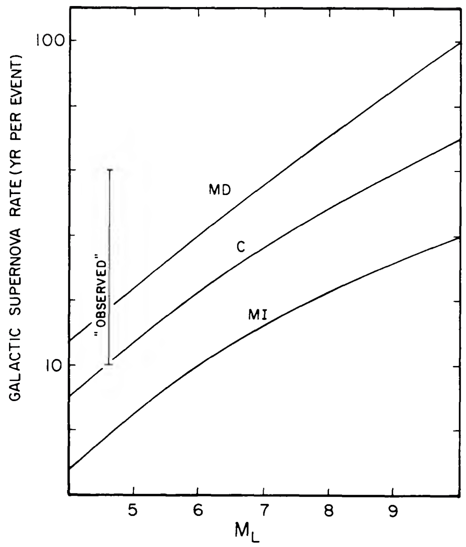

FIG. 9.—Calculated supernova rate in the Galaxy (years per supernova) versus the lower mass limit for supernovae M_L. The three curves are for three different birthrates and corresponding IMFs. The maximum increasing (MI) and maximum decreasing (MD) birthrates are the two which satisfy the extremes of the continuity constraint. The constant (C) birthrate is intermediate. Tammann's (1977) observationally estimated supernova rate for the Galaxy is also indicated.

(eq. [23]), while the center curve was calculated using a constant birthrate.

Tammann (1977) states that the observed Type II supernova rates in Sb galaxies are about 9 times higher than in Sc galaxies and suggests an interpolated overall rate of 20 (+20, −10) yr per event for our Galaxy, which is believed to be intermediate in type between Sb and Sc. Given the uncertainty in this interpolation and the uncertainty in M_L, we see from Figure 9 that the observed supernova rates do not give a useful constraint on the present birthrate.

vii) *Formation Rates of White Dwarfs and Planetary Nebulae*

A similar, but more direct, procedure can be followed using white dwarfs and planetary nebulae. Both observational (Weidemann 1977) and theoretical (Wood and Cahn 1977; Hills 1978) arguments yield a white dwarf formation rate of about $2\text{–}3 \times 10^{-12}\ \text{pc}^{-3}\ \text{yr}^{-1}$. The planetary nebula formation rate is less certain, but most estimates (e.g., Alloin, Cruz-González, and Peimbert 1976; Cahn and Wyatt 1976) are consistent with the rate for white dwarfs. Smith's (1976) revision of the planetary nebula distance scale gives a larger rate of about $1 \times 10^{-11}\ \text{pc}^{-3}\ \text{yr}^{-1}$. Adopting a scale height of 300 pc for both types of objects from their observed z distribution (Blaauw 1965) and the scale height of G dwarfs, which are expected to be the progenitors of most of these objects, we find the formation rates to lie in the range $1\text{–}6 \times 10^{-9}\ \text{pc}^{-2}\ \text{yr}^{-1}$. The present formation rate reflects only an average birthrate which is weighted toward a time in the past corresponding to the mean main-sequence lifetime of the progenitors (see eq. [1]

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of Tinsley 1978). The distribution of initial masses of progenitors has been calculated by Tinsley (1978). For a ratio of past average birthrate to present birthrate of about 3 the distribution is sharply peaked at $\sim 1\ M_{\odot}$, while for a constant birthrate the distribution decreases less rapidly above $1\ M_{\odot}$. In any case the white dwarf–planetary nebula formation rate must reflect the birthrate averaged over some time interval between about 5 and 10×10^9 yr ago. The *present* birthrate consistent with the continuity constraint, derived in § IV below, is $4\text{–}10 \times 10^{-9}\ \mathrm{pc}^{-2}\ \mathrm{yr}^{-1}$. Therefore, the white dwarf and planetary nebula formation rates are consistent with a birthrate which is roughly constant with time. The present approach and our conclusions are different than those of Tinsley (1978), who calculated the planetary nebula rate for two different forms of birthrate, because her adopted normalization gives different present birthrates in each case.

viii) *Chemical Evolution Constraints*

Smith, Biermann, and Mezger (1978) have parametrized the time dependence of the birthrate as $B(t) \propto m_{\mathrm{gas}}{}^n$, where m_{gas} is the column density of gas and n is a parameter. Using this birthrate, Smith *et al.* sought the value of n which best reproduces the following three quantities: (1) the present "observed" rate of star formation as inferred from local H II regions (§ III*c*[v]); (2) the increase in metal and helium abundances over the age of the disk, $Z + \Delta Y$; and (3) the present deuterium abundance of 2.5×10^{-5}, assuming a primordial abundance of 5×10^{-5}. Although Smith *et al.* suggest that $n = \frac{1}{2}$ can fit the adopted constraints, it is clear than $n = 0$ gives a better fit. Smith *et al.* dismiss $n = 0$ on the grounds that a constant birthrate is physically unrealistic and the model colors are too blue with such a birthrate. The first point will be discussed in § III*c*(ix) below. The colors depend not only on the birthrate, but on the slope of the IMF at high masses (which Smith *et al.* took as -1.35) and on the upper limit to stellar masses. A steeper IMF, as found in the present paper, would result in redder colors. Unfortunately, the number of parameters besides n which must be specified is large and includes the lower and upper limits to the masses of stars, the slope of the IMF, and the age of the disk T_0. The results also depend on the particular model adopted for the solar neighborhood. Also, the rate of star formation inferred from H II regions is uncertain by a factor of 3, and one cannot really say what the primordial deuterium abundance was without major cosmological assumptions. We therefore feel that the problem is overdetermined and cannot presently provide a useful constraint on the history of the birthrate.

ix) *Lack of Evidence for a Dependence of the Birthrate on Mean Gas Density*

Mathis (1959), Schmidt (1959), and Salpeter (1959) introduced the idea that the birthrate of field stars might be proportional to some power of the average gas density, $B(t) \propto \langle\rho\rangle^n$. Schmidt presented several arguments that $n \approx 2$. Such a parametrization of the birthrate has become prevalent in the literature, and it is therefore important to reexamine Schmidt's arguments in light of more recent work.

Some of the arguments have already been discussed. The requirement that the IMF not exhibit a "bump" shows that the birthrate could not decrease as rapidly as would be the case for $n = 2$. The bump can also be seen in Figure 1 of Schmidt (1959). Schmidt also compared the initial luminosity function (ILF) predicted by his models for various values of n with the luminosity functions of open clusters. Although he states that the agreement for $n = 1$ and 2 is satisfactory, it is more correct to say that none of the ILFs are in good agreement with the cluster luminosity functions. The method is basically a graphical version of the method discussed in § III*c*(i) above, where we found no evidence for a rapidly decreasing birthrate. Schmidt suggested that the present abundance of helium could be accounted for by his models if n were at least 2, assuming that the initial helium abundance of the Galaxy was zero. Currently it is widely held that helium is cosmologically produced; therefore, this constraint would now favor lower values of n. However, even for $n = 0$, the fractional abundance of helium by mass would have increased by 0.12 over the age of the Galaxy in Schmidt's models, which is too large an increase to be compatible with the observed limits on the helium abundance (Trimble 1975). This overproduction results from the fact that Schmidt overestimated the amount of helium released to the interstellar medium by dying stars by about a factor of 2 compared to more modern values.

The idea that $B(t) \propto \langle\rho\rangle^n$, with $n \approx 2$, had received the strongest apparent support from several independent studies of correlations between gas densities and densities of luminous stars of H II regions in galaxies. A useful summary of this work is given in Table 1 of Madore (1977). It is now clear that $n \approx 2$ does *not* follow from the correlation analyses for a number of reasons. First, the correlations are based on neutral hydrogen, while it is known, at least in our Galaxy, that stars form preferentially in molecular clouds. Second, many of the data reflect only local correlations between gas and star densities, not the spatial and time averages relevant to the proposed form of the birthrate. Some additional difficulties with the correlation approach were discussed by Talbot (1971). For example, counts of H II regions and young stars may not be similarly correlated with gas density if the number of stars exciting the H II regions depends on gas density.

The most serious objection to the value $n \approx 2$ usually inferred from correlation analyses has been given by Madore (1977), who points out that the theoretical relationship predicts a correlation only between gas density and the rate of star formation, *not* between gas density and the number of stars formed, which is the observed quantity. Madore then demonstrates by simulations of star formation with various rates that the values of n inferred from correlation analyses are much larger than the actual values of n in the simulation, and that an "actual n" in the range

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0.25–1.0 agrees with the observed correlations. The "actual n" deduced still depends on the assumption that the number of stars formed in a region is proportional to the mass of that region and refers to the local gas density. In any case, the conclusion that correlation analyses can grossly overestimate the value of n [assuming that $B(t)$ does indeed depend on $\langle\rho\rangle^n$] seems firm.

A similar correlation analysis can be carried out for our own Galaxy. It has become apparent in recent years that the surface density of objects related to star formation (molecular hydrogen, H II regions, CO clouds, etc.), averaged over annuli concentric with the galactic center, shows a pronounced minimum at galactocentric radii between about 1 and 4 kpc, and a maximum at about 6 kpc (see Burton and Gordon 1978). Since this feature exists in the total gas surface density, a birthrate which depends on gas density should also exhibit this feature. Smith *et al.* have estimated the birthrate as a function of galactocentric radius from H II regions, using the method described in § III*c*(v), and find that it shows the pronounced minimum at 1–4 kpc. With $B(t) \propto \rho_{\rm gas}{}^n$, Smith *et al.* show that, as expected, the case $n = 0$ gives no minimum at 1–4 kpc but $n \approx 0.5$ does. The fit is insensitive to n as long as $n \gtrsim 0.5$. The correlation therefore supports the idea that the birthrate depends on the gas density. A value of $n \approx 0.5$ will give a birthrate whose decrease is just small enough that the continuity constraint is not violated, and is consistent with Madore's (1977) simulations of star formation.

Black and Kellman (1974) point out that, assuming the supernova rate to be proportional to the birthrate, the fact that the supernova rate in Sc galaxies appears to be several times larger than in Sb galaxies implies a significantly larger mean gas density in Sc than in Sb galaxies if $B(t) \propto \langle\rho\rangle^n$. Since the mean H I density does not show systematic variations from Sb to Sc galaxies, Black and Kellman conclude that gas density is not the primary factor controlling the birthrate.

Schmidt noted that the distribution of young objects perpendicular to the galactic plane could be used to determine the exponent n because, if $n > 1$, young objects should form a disk thinner than that of the gas by a factor $n^{1/2}$ if the gas distribution is Gaussian. Schmidt adopted the dispersions of neutral hydrogen, Cepheids, and young galactic clusters as 144, 80, and 58 pc, respectively, and thus concluded that $n = 2$–3. One problem with this interpretation is that, if star formation is proportional to the average gas density, then the above argument should be based on the z dispersion of the gas out of which stars condense. Stars in our Galaxy are not born in regions of neutral hydrogen but in regions of dense molecular clouds. Burton (1976) states that the layer of CO in the Galaxy is substantially thinner in the z direction than the neutral hydrogen, lowering the deduced value of n. A more recent and detailed comparison of vertical and radial distributions of gas and young stellar objects has been given by Guibert, Lequeux, and Viallefond (1978). Assuming that the z distributions of young objects do in fact offer a consistent method of testing the hypothesis that $B \propto \rho^n$, their paper demonstrates the sensitivity of the derived value of n to the assumed relation between the abundances of CO and H_2 in our Galaxy. Guibert *et al.* found that only one of their three trial models for the CO/H_2 ratio gave results consistent with the hypothesized birthrate if $1.3 < n < 2.0$.

One problem with this approach, as in some of the correlation studies discussed earlier, is that it assumes that the *number* of stars formed is proportional to some power of the gas density (e.g., eq. [2] of Guibert *et al.*) whereas the assumption ostensibly being tested involves the rate of star formation, not the number of stars. The simulations of Madore (1977) clearly demonstrate that this inconsistency can seriously affect the derived values of the exponent n.

Another general problem with this approach is that the statement that the dispersion in z of these constituents indicates a particular value of n contains the implicit assumption that star formation is indeed proportional to some power of the average gas density. If the rate of star formation does not depend on the average gas density but on some other physical quantity, as discussed below, or if z dispersions are controlled by other processes, then relationships between the z dispersions of various galactic constituents may not be physically meaningful in this context. These comments also apply to Schmidt's evidence based on the hydrogen distribution in the plane. For example, Talbot (1978) has pointed out that even if the scale height of young stars actually is smaller than that of H I gas, this may show only that star formation occurs in material which has a smaller velocity dispersion than the H I gas. This interpretation is physically reasonable because stars form from massive, dense clouds and because there are two processes which tend to give a larger velocity dispersion to the H I gas than to molecular clouds: (1) H I clouds are less massive, and inelastic cloud collisions can lead to a quasi-Maxwellian velocity distribution (Penston *et al.* 1969; Pumphrey and Scalo 1979); (2) supernova remnants can more easily affect the velocities of lower-mass H I clouds than of massive molecular clouds.

d) *Theoretical Models for the Birthrate History*

There are many suggestions, but no general agreement, concerning the dominant processes which control the rate of star formation. Since each of these processes may lead to a different time dependence of the stellar birthrate and because it is likely that more than one process contributes, we should expect no simple expression for the birthrate history. Nevertheless, it is still of interest to enumerate some of the theoretical possibilities. Excellent reviews of the consequences of various star formation processes on the stellar birthrate have been given recently by Larson (1977) and Talbot (1978), and so the present discussion will be brief. Our purpose is to show that there are a number of proposals which are consistent with a roughly constant birthrate.

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The most common assumption for the birthrate history is that $b(t)$ depends on some power n of the gas volume or surface density. We have already seen in § IIIc(ix) that observational constraints require n to be small if such a relation holds at all. The physical motivation for the assumed gas density dependence seems extremely weak to us. At the beginning of the evolution of the galactic disk most of the matter was in the form of gas and the birthrate had some finite value, while at some time in the future the gas will be completely locked up in low-mass stars and stellar remnants, at which time the star formation rate must drop to zero. It is usually assumed that the birthrate will be a monotonically decreasing function of time in this interval, hence the assumed dependence of birthrate on gas density. We see no reason that the birthrate should be a monotonic function of time. For example, if star formation requires some threshold value of the mean interstellar density, as is the case if gravity must overcome galactic tidal forces (Goldreich and Lynden-Bell 1965), the birthrate may have a time dependence (either decreasing or increasing) which is physically unrelated to the average gas density until this density drops below the threshold value. Furthermore, the intuitive notion that the rate of star formation should depend on the mass or density of available gas is not generally valid. The birthrate might depend on the *local* gas density through the free-fall time, Jeans mass, or cooling rate, but such a relation would not necessarily apply to the larger regions of space (length scale ~ 1 kpc) relevant to the global birthrate.

It may also be invalid to assume that the gas density is a monotonically decreasing function of time. For example, Larson's (1976) model for disk galaxies assumes a *local* birthrate proportional to the square of the gas density, yet because of infall, the average birthrate predicted by the model for galactocentric distances similar to the Sun's first increases, then later decreases with time, similar to the form shown in Figure 6f. A galactic model in which the global star formation rate in the disk is controlled by the rate of infall from the halo was originally suggested by Larson (1972). The mass consumption rates derived in § IVb below are consistent with limits on the infall rate, but whether infall actually occurs at the present time is uncertain.

One physical mechanism which may predict a strong dependence of birthrate on gas density is cloud-cloud collisions (see Field and Saslaw 1965; Talbot 1978). *If* the number density of clouds is proportional to the average gas density, this model gives $b(t) \propto \langle \rho \rangle^2$. Although there is observational evidence that cloud collisions can initiate star formation (Loren 1977), the observational constraints on $b(t)$ given earlier suggest either that cloud collisions are not the dominant process controlling the birthrate or that the number of clouds does not depend much on the average gas density.

Larson (1977) has pointed out that several mechanisms for initiating star formation should depend on the *total* mass or density, not just that of the gas. Examples are the dissipation-settling process described by Talbot and Arnett (1975) and compression due to spiral density waves, whose frequency depends on the mass interior to the point of interest and hence is expected to be roughly constant with time. In the case of density waves, the birthrate would also depend on the time dependence of the wave amplitude. Although this dependence is unknown, it is possible that the amplitude is controlled by external disturbances such as encounters with other galaxies. Larson and Tinsley (1978) have presented convincing evidence that bursts of star formation in other galaxies are the result of galactic encounters.

There are observational and theoretical reasons for thinking that compression due to supernova explosions and OB stars may cause star formation (see Woodward 1978 for references). Kaufman (1978) has derived an expression for the birthrate for supernova-induced star formation which is an exponential function of time. The sign of the time constant depends on the product of the number of stars formed per supernova and the fraction of all stars which become supernovae. If this product is less than (greater than) unity, the birthrate decreases (increases) with time. In the models investigated by Kaufman the supernova-induced birthrate is periodically "revived" by a spiral density wave so that even though her preferred values for the time constant are quite small ($\lesssim 10^8$ yr), the *total* birthrate changes only by modest amounts (less than about a factor of 3), consistent with the limits on the birthrate found in the present paper.

It is seen that, contrary to the popular view, most theories for the initiation of star formation predict birthrates which are expected to be constant or to change by relatively small amounts over the history of the disk.

IV. INITIAL MASS FUNCTION

a) *Calculation and Results*

From the discussion in § III we have placed observational constraints on the ratio of the present to average birthrate $b(T_0)$ (see Table 5). In this section IMFs consistent with these constraints are derived. However, in order to derive the IMF, the birthrate for all past times must be known, not just the ratio of present to average birthrate. Because such information is lacking, the approach used here is to assume analytical functions for the birthrate and require these functions to be consistent with the continuity constraint. We have verified that the resulting IMFs are not sensitive to the particular analytical function adopted but depend primarily on $b(T_0)$.

Three birthrates are considered: (1) the birthrate which increases by the largest amount over the age of the disk and still satisfies the continuity constraint, termed the maximum increasing (MI) birthrate; (2) a constant birthrate; and (3) the birthrate which decreases by the largest amount and still satisfies the continuity constraint, termed the maximum decreasing (MD) birthrate. We repeat that unless the observational errors have conspired against us the constant birthrate should give the best representation. For the

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TABLE 6

CONSTANTS FOR THE MAXIMUM INCREASING AND DECREASING BIRTHRATES

BIRTHRATE T_0 (10^9 yr)	MI 9	MI 12	MI 15	MD 9	MD 12	MD 15
τ(10^9 yr)......	+10.3	+7.5	+6.7	−3.2	−5.0	−7.3
B............	0.63	0.41	0.27	3.0	2.6	2.4

MI and MD birthrates, an exponential function is assumed:

$$b(t) = B \exp(t/\tau)\,, \tag{28a}$$

where τ is the time constant and B is the normalization, determined from equation (13) to be

$$B = \frac{T_0}{\tau[\exp(T_0/\tau) - 1]}\,. \tag{28b}$$

Table 6 gives the values of τ, which are determined from the requirement that the present birthrate $b(T_0)$ agrees with that from the continuity constraint on the IMF (Table 5). In this and what follows, the age of the Galaxy has been taken as 9, 12, and 15 $\times$ 10^9 yr.

We emphasize that the exponential function is merely a convenient parametrization of the birthrate and that no physical significance should be attached. The exponential function adopted for the MI birthrate increases without bound. This causes no difficulties because this function is used only to describe the past behavior of the birthrate. It is clear that if the true birthrate has been increasing with time, it must reach a maximum and decline as the interstellar medium is consumed.

The IMFs resulting from these birthrates have been derived using equations (10) and (12) and are shown in Figure 10. The IMFs for the MI and MD birthrates are each independent of the age of the Galaxy. For these birthrates the PDMF was adjusted to the extremes of the estimated uncertainties. The adjustment was necessary to maintain consistency since the MI and MD birthrates were derived from the extremes found for $T_0/b(T_0)$ (eqs. [21] and [22]), which in turn were found by pushing the PDMF to the limits of the uncertainties. In the case of the MD birthrate, log ϕ_{ms} was therefore increased by 0.40 for log $M \leq -0.08$ and decreased by 0.21 for log $M \geq 0.40$. Similarly, in the case of the MI birthrate, log ϕ_{ms} was decreased by 0.15 for log $M \leq -0.08$ and increased by 0.21 for log $M \geq 0.40$.

In order to facilitate the use of these IMFs, an analytic function (parabola) has been fitted to log ξ:

$$\log \xi(\log M) = A_0 + A_1 \log M + A_2(\log M)^2\,. \tag{29}$$

This relation is shown as the solid line in Figure 10, and the coefficients are given in Table 7. An equivalent expression is the Gaussian function

$$\xi(\log M) = C_0 \exp[-C_1(\log M - C_2)^2]\,, \tag{30}$$

where

$$C_0 = \exp\{\ln(10)[A_0 - A_1^2/(4A_2)]\}\,,$$
$$C_1 = -A_2 \ln(10)\,,$$
$$C_2 = -A_1/(2A_2)\,.$$

These coefficients are also given in Table 7. The IMF can thus be represented by a half-Gaussian in terms of log M or a half-lognormal in terms of M.

In many applications the slope of the IMF (defined as $d \log \xi / d \log M$) is the quantity of interest. From Figure 10 it can be seen that results for the different birthrates have similar shapes; i.e., the slope of the IMF at a given mass is largely independent of the birthrate. When the constant birthrate with $T_0 = 12 \times 10^9$ yr is taken as a representative case, equation (29) can be used to give a useful formula for the slope of the IMF:

$$d \log \xi(\log M)/d \log M = -(1 + \log M)\,. \tag{31}$$

The slope varies from 0 at 0.1 $M_\odot$, to −1.0 at 1 $M_\odot$, to −1.9 at 10 $M_\odot$, to −2.6 at 50 $M_\odot$. Although the slope of the IMF is insensitive to the birthrate history, the actual value of the IMF does depend on the birthrate. Therefore, the importance of using an IMF and its corresponding birthrate in any application involving the solar neighborhood cannot be overemphasized.

Another analytic representation for the IMFs, which contains the slope of the IMF explicitly and is often more convenient in analytical calculations than equation (29), is a power-law formula

$$\xi(\log M) = D_0 M^{D_1}\,. \tag{32}$$

D_1 is the slope of the IMF. Three separate power laws have been fitted to the IMFs, with the transitions arbitrarily taken at 1 and 10 $M_\odot$. The coefficients are given in Table 7. Since the IMFs are similar in shape, D_1 was taken to be independent of the birthrate history.

The IMFs of the present paper give the number of stars per unit logarithmic mass, while some other investigations have used the IMF per unit mass, denoted $\xi(M)$. The two are related by

$$\xi(M) = \xi(\log M)\frac{d \log M}{dM}\,. \tag{33}$$

From this expression it can be seen that if the slope of $\xi(\log M)$ is D_1, the slope of $\xi(M)$ is $D_1 - 1$.

b) *Derived Quantities*

Several useful quantities can now be calculated from the IMF. The first is the cumulative number distribution of the IMF,

$$N(>M) = \int_M^{M_{max}} \xi(\log M) d \log M\,, \tag{34}$$

where M_{max} is the maximum mass of stars. The actual value of M_{max} is probably greater than ~100 $M_\odot$ (see

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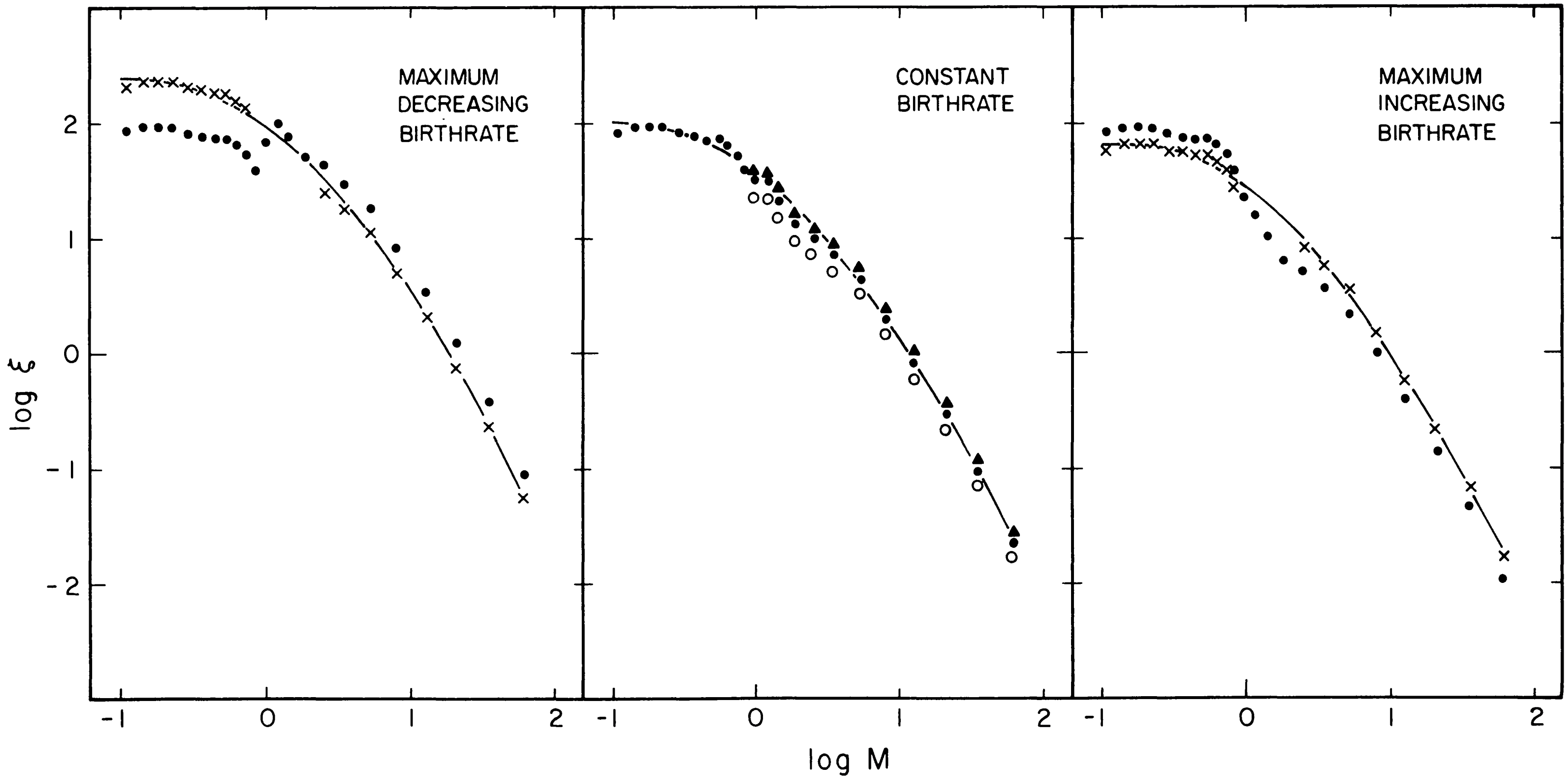

FIG. 10.—IMFs derived from the continuity constraint. For the MI and MD birthrates, the filled circles give the IMF derived from the adopted PDMF while the crosses give the IMF calculated from the PDMF adjusted by the extremes of the estimated uncertainty. This procedure was necessary because the limits on the present birthrate derived from Fig. 8 took these uncertainties into account. The IMFs for the MI and MD birthrates are independent of T_0 for 9×10^9 yr $\leq T_0 \leq 15 \times 10^9$ yr. For the constant birthrate, the IMF depends slightly on T_0. The open circles, filled circles, and filled triangles are for $T_0 =$ 9, 12, and 15×10^9 yr, respectively. For all birthrates, the solid line is an analytic fit to the IMF. For the constant birthrate, only the analytic fit for $T_0 = 12 \times 10^9$ yr is shown.

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TABLE 7

PARAMETERS FOR ANALYTIC FITS TO THE IMF

BIRTHRATE T_0 (10^9 yr)	MI 9, 12, 15	CONSTANT 9	CONSTANT 12	CONSTANT 15	MD 9, 12, 15
Quadratic Fit to log ξ(log M)					
A_0	1.43	1.47	1.53	1.59	1.96
A_1	−0.88	−1.02	−0.96	−0.87	−0.92
A_2	−0.50	−0.44	−0.47	−0.50	−0.49
Half-Gaussian Fit to ξ(log M)					
C_0	66.2	113.3	106.0	93.2	242.4
C_1	1.15	1.02	1.09	1.14	1.14
C_2	−0.88	−1.15	−1.02	−0.88	−0.93
Three-Segment Power-Law Fit to ξ(log M)					
$0.1 \leq M/M_\odot \leq 1$:					
D_0	32	36	42	42	100
D_1	−0.4	−0.4	−0.4	−0.4	−0.4
$1 \leq M/M_\odot \leq 10$:					
D_0	32	36	42	42	100
D_1	−1.5	−1.5	−1.5	−1.5	−1.5
$10 \leq M/M_\odot$:					
D_0	180	200	240	240	560
D_1	−2.3	−2.3	−2.3	−2.3	−2.3

Scalo 1979), but because of lack of knowledge concerning the luminosity function for very massive stars, we arbitrarily take $M_{\max} = 62\ M_\odot$ for the purposes of these calculations. The derived quantities are insensitive to this choice. The value of $N(>M)$ at any mass M gives the number of stars in the IMF, i.e., all stars ever born (per square parsec) more massive than M. Since $0.1\ M_\odot$ is the minimum mass of main-sequence stars used in the present investigation, $N(>0.1)$ gives the total number of stars in the IMF per square parsec. For reasons detailed in § II*b*, this choice of the minimum mass of main-sequence stars is the result of the lack of knowledge of stellar properties for $M_v > 16$. It should be emphasized that the IMFs of the present paper and any derived quantities therefore refer only to main-sequence stars more massive than $0.1\ M_\odot$, although objects with (initial) masses less than $0.1\ M_\odot$ may contribute significantly to the total mass of the solar neighborhood. However, if our presently derived IMF is extrapolated to smaller masses, it can be seen that the contribution of lower-mass objects is not significant. Of course, it is conceivable that the IMF steepens again for masses less than $0.1\ M_\odot$. The lack of a serious discrepancy between the observed mass in the solar neighborhood and the Oort limit (§ II*g*) also provides (admittedly weak) evidence that such objects do not make a large contribution.

Table 8 gives $N(>0.1)$ for the different birthrates. The value of $N(>0.1)$ for the MD birthrate is much larger than that of the other birthrates since the MD birthrate was derived by pushing the PDMF for low-mass stars to the upper extreme of the uncertainty limits. The fractional number of stars more massive than a mass M, $F_n(>M)$, is

$$F_n(>M) = N(>M)/N(>0.1)\,. \tag{35}$$

Since the *shapes* of the IMFs for the different birthrates are similar, to a good approximation $F_n(>M)$ is independent of the birthrate and age of the Galaxy and is given in Table 9.

It is often useful to know the present birthrate of stars in units of stars $\mathrm{yr}^{-1}\ \mathrm{pc}^{-2}$. The birthrate $b(t)$ gives the absolute birthrate at a time t in terms of the average absolute birthrate. The average birthrate per unit mass is simply the IMF (all stars ever born) divided by the age of the Galaxy. So the present birthrate of stars in stars $\mathrm{yr}^{-1}\ \mathrm{pc}^{-2}\ \log M^{-1}$ is

$$\frac{b(T_0)}{T_0}\,\xi(\log M)\,. \tag{36}$$

TABLE 8

TOTAL NUMBER OF STARS AND TOTAL AMOUNT OF MASS IN THE INITIAL MASS FUNCTION

Birthrate	N (>0.1)*	$\mathfrak{M}$ (>0.1)†
Maximum increasing	62.4	42.9
Constant $T_0 = 9 \times 10^9$	82.2	46.7
Constant $T_0 = 12 \times 10^9$	85.4	54.7
Constant $T_0 = 15 \times 10^9$	88.6	62.3
Maximum decreasing	218.8	144.1

* Total number of stars ever formed (pc^{-2}).

† Total mass in stars ever formed ($M_\odot\ \mathrm{pc}^{-2}$).

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TABLE 9

VALUES OF F_n AND F_m

$\log M/M_{\odot}$	$\log F_n(>M)$	$\log F_m(>M)$
+1.6	−4.03	−2.15
+1.4	−3.42	−1.69
+1.2	−2.93	−1.35
+1.0	−2.49	−1.07
+0.8	−2.09	−0.84
+0.6	−1.73	−0.65
+0.4	−1.40	−0.49
+0.2	−1.11	−0.35
0.0	−0.85	−0.24
−0.2	−0.62	−0.16
−0.4	−0.42	−0.10
−0.6	−0.25	−0.05
−0.8	−0.11	−0.02
−1.0	0.00	0.00

NOTE.—$F_n(>M)$ gives the fraction of stars in the IMF (all stars ever formed) more massive than M. $F_m(>M)$ gives the fraction of mass in the IMF contained in stars more massive than M.

The present birthrate of stars more massive than some mass M, $B_n(>M)$ (in stars $\mathrm{yr^{-1}\ pc^{-2}}$), is just

$$B_n(>M) = \frac{b(T_0)}{T_0}\int_M^{M_{\max}} \xi(\log M)d\log M$$
$$= \frac{b(T_0)}{T_0} N(>0.1)F_n(>M)$$
$$= G_nF_n(>M)\,, \tag{37}$$

where G_n gives the total present birthrate of stars. $B_n(>M)$ can be found using the values of G_n in Table 10. $B_n(>M)$ is often used in interpretations of the galactic supernova rate, as discussed in § III*c*(vi) above.

The cumulative mass distribution of the IMF is

$$\mathfrak{M}(>M) = \int_M^{M_{\max}} M\xi(\log M)d\log M\,. \tag{38}$$

The value of $\mathfrak{M}(>M)$ at any mass M gives the amount of mass in the IMF contained in stars more massive than M (in $M_{\odot}\ \mathrm{pc^{-2}}$). $\mathfrak{M}(>0.1)$ gives the total amount of mass contained in the IMF and is given in Table 8.

TABLE 10

TOTAL PRESENT BIRTHRATE, TOTAL PRESENT MASS CONSUMPTION RATE, AND TIME SCALE FOR CONSUMPTION OF THE INTERSTELLAR GAS

Birthrate	G_n*	G_m†	τ_g‡
Maximum increasing	1×10^{-8}	7×10^{-9}	1×10^{9}
Constant $T_0=9\times10^9$	9×10^{-9}	5×10^{-9}	2×10^{9}
Constant $T_0=12\times10^9$	7×10^{-9}	5×10^{-9}	2×10^{9}
Constant $T_0=15\times10^9$	6×10^{-9}	4×10^{-9}	2×10^{9}
Maximum decreasing	4×10^{-9}	3×10^{-9}	3×10^{9}

* Total present birthrate (stars $\mathrm{yr^{-1}\ pc^{-2}}$).

† Total present mass consumption rate ($M_{\odot}\ \mathrm{yr^{-1}\ pc^{-2}}$).

‡ Time scale for consumption of gas (yr).

The fraction of mass in stars more massive than M, $F_m(>M)$, is

$$F_m(>M) = \mathfrak{M}(>M)/\mathfrak{M}(>0.1)\,. \tag{39}$$

Since the shapes of the IMFs for the different birthrates are similar, $F_m(>M)$ is approximately independent of the birthrate and age of the Galaxy and is given in Table 9.

It is also useful to know the present birthrate of stars in terms of how much mass is being converted to stars. To distinguish this from the birthrate $B_n(>M)$ in units of stars $\mathrm{yr^{-1}\ pc^{-2}}$, the *present mass consumption rate* $B_m(>M)$ is defined as the amount of matter being turned into stars more massive than M at the present time (in $M_{\odot}\ \mathrm{pc^{-2}\ yr^{-1}}$) and is given by

$$B_m(>M) = \frac{b(T_0)}{T_0}\int_M^{M_{\max}} M\xi(\log M)d\log M$$
$$= \frac{b(T_0)}{T_0} M(>0.1)F_m(>M)$$
$$= G_mF_m(>M)\,. \tag{40}$$

G_m gives the total amount of mass being converted to stars at the present time (Table 10) and varies between 3 and $7\times10^{-9}\ M_{\odot}\ \mathrm{pc^{-2}\ yr^{-1}}$. Tinsley (1977*a*) finds a range of 1–16 $\times\ 10^{-9}\ M_{\odot}\ \mathrm{pc^{-2}\ yr^{-1}}$. The means of these determinations are in agreement, but the range found by Tinsley is larger for several reasons. Both Tinsley and the present investigation find a roughly similar range for $b(T_0)/T_0$. Since $b(T_0)$ gives the present birthrate in units of the average birthrate, the average birthrate must be known in order to obtain the present mass consumption rate from the ratio $b(T_0)/T_0$. It is in the evaluation of the average rate that the present method differs from that of Tinsley. Tinsley uses the total amount of matter in the solar neighborhood derived from dynamical arguments (the "Oort limit") to derive the average mass consumption rate. As discussed in § II*g*, the dynamically derived value for the total mass in the solar neighborhood exceeds the amount of mass that can be accounted for by observed stars and gas, although we have argued that this difference is probably not significant. However, if this discrepancy is real and if the missing mass is not in the form of (presently undetected) stars and gas, then Tinsley's method will not give the average mass consumption rate for field stars. The present investigation uses the amount of matter in observed stars to find the average field star mass consumption rate. Further, we find that the value of $b(T_0)$ is related to the value of the average birthrate; namely, the larger the $b(T_0)$, the smaller the average rate. The result is that the range in the present birthrate is smaller than the range in $b(T_0)$. Another reason that the range found by Tinsley is larger is that her method requires knowledge of the average mass fraction of material which formed stars that is eventually returned to the interstellar medium, and uncertainties in this fraction increase the range in the values of the present mass consumption rate over that of the present method, which does not require this fraction.

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The present mass consumption rate G_m can be used to find the amount of time necessary to consume the interstellar gas in the solar neighborhood τ_g. Tinsley (1977*a*) states that there is 6–9 $M_\odot$ pc^{-2} of interstellar material in the solar neighborhood. Dividing this value by the total present birthrate gives the time for consumption of the interstellar gas (see Table 10). These times range from 1 to 3 $\times$ 10^9 years, a smaller range than found by Tinsley (0.6 to 10 $\times$ 10^9 yr) for the same reasons as explained above. Such small values of τ_g are consistent with three general models for the evolution of our Galaxy. (1) Gas may be replenished by infall. A galactic model in which the star formation rate is controlled by the infall rate was suggested by Larson (1972). Estimates for the local infall rate are around 2 $\times$ 10^{-9} $M_\odot$ pc^{-2} yr^{-1}, only a factor of 1.5–3 times smaller than the mass consumption rates given in Table 10. (2) Gas in the solar neighborhood may be replenished by radial flows. (3) The birthrate may fluctuate on relatively small time scales with the present birthrate larger than average. Such a situation could arise if galactic encounters caused bursts of star formation (Larson and Tinsley 1978). The gas depletion time scale derived here is also similar to some of Kaufman's (1978) values for supernova-induced star formation "revived" by density waves.

c) *Comparison with Other Work*

In Figure 11 we compare the IMFs of the present investigation with those of previous authors. The pioneering study was made by Salpeter (1955), who derived the IMF assuming a constant birthrate. He found that the IMF follows a power law of slope -1.35 from 0.4 to 10 $M_\odot$, with a steeper slope for larger masses. This value for the slope of the IMF has been used throughout the literature. Comparison with the results of the present investigation as well as with other determinations shows that the Salpeter slope is roughly applicable from about 2 to 10 $M_\odot$ but that it does not hold outside this range: the upper IMF is steeper, while the lower IMF is flatter. Audouze and Tinsley (1976) have also cautioned against the use of the Salpeter slope outside this range. The differences between the present results and those of Salpeter are due mainly to improved main-sequence lifetimes and because Salpeter considered star densities in the galactic plane and not integrated perpendicular to the plane. Sandage (1957) derived the initial luminosity function assuming a constant birthrate, and Warner (1961) converted it to an IMF. Schmidt (1959) relaxed the assumption of a constant birthrate (see also Mathis 1959; Salpeter 1959) by assuming that the rate of star formation varies as some power n of the average interstellar gas density (§ III*c*[ix]). Schmidt's IMF for a constant ($n = 0$) birthrate (derived from his tabulated ILF and his mass-luminosity relation) is shown in Figure 11. His IMFs for $n = 1$ and 2 exhibit the "bump", or discontinuity, at about 1 $M_\odot$. Limber's (1960) IMF is the mean of Sandage's (1957) IMF, Sandage's (1957) cluster IMF, and van den Bergh's (1957) cluster IMF. The IMF of Hartmann (1970) is a combination of the cluster IMF (a power law of slope -1.2 fitted to the Pleiades, Hyades, and NGC 2264) from 1 to 8 $M_\odot$ and essentially an IMF derived assuming a constant birthrate for higher masses. Audouze and Tinsley (1976) give the slope of the IMF for stars more massive than 3 $M_\odot$ and less massive than 1 $M_\odot$. Their slopes for the upper IMF are taken from ORT, while their lower IMF is derived from Wielen's (1974) luminosity function assuming a decreasing birthrate. As discussed in § II*f*, ORT have probably underestimated the number of high-mass stars, although the slope of their PDMF for $M > 25$ $M_\odot$ (and thus the slope of the IMF) agrees with others (see Fig. 5). The plateau (slope -0.6) in the IMF of Audouze and Tinsley at about 25 $M_\odot$ is apparently due to the transition of ORT from one source of data (counts of OB stars by Richstone and Davidson 1972) to another (McCuskey's 1966 luminosity function), and should not be viewed as real since it appears in no other determinations. In conclusion, it is seen from Figure 11 that the IMFs of various authors are in qualitative agreement, especially considering differences in the input data and improvement in main-sequence lifetimes over the years. However, there are significant quantitative differences.

A significant fraction of, if not all, field stars are formed in open clusters and associations which subsequently dissipate (see Miller and Scalo 1978 and references therein). Accordingly, it is of interest to compare the cluster and association IMFs with the field star IMF. Figure 11 shows the average cluster IMF of 62 open clusters (average slope -1.7) as determined by Taff (1974), which is roughly representative of other recent investigations (Piskunov 1976; Burki 1977; see Scalo 1979 for a comparison). (The same slope is obtained using our [absolute magnitude, mass]-relation and Taff's cluster luminosity function.) Within the uncertainties, the cluster IMF agrees with the field star IMF for $M > 1$ $M_\odot$. However, several authors have noted that at least some clusters are deficient in stars below 1 $M_\odot$ relative to the field IMF. A reexamination of the problem is necessary to determine the cause and extent of this deficiency. A more detailed review of the cluster IMF is given by Scalo (1979).

Because of formidable observational difficulties (see § II), only the high-mass end of the association IMF can be determined. In Figure 11*c* we show the IMF of the most extensively observed association, Ori OB1. This IMF was determined from the luminosity function given in Warren and Hesser (1978, Tables 5–11) using the (absolute magnitude, mass)-relation given in Table 1 of the present paper. For $M \geq 2.5$ $M_\odot$, the Ori OB1 association IMF is in good agreement with the field star IMF. Warren and Hesser state that incompleteness becomes significant for lower masses, thus explaining the rapid decline relative to the field. The published luminosity functions for other OB and also R associations contain too few stars for meaningful comparison. The IMF of T Tauri associations cannot be determined from the luminosity function

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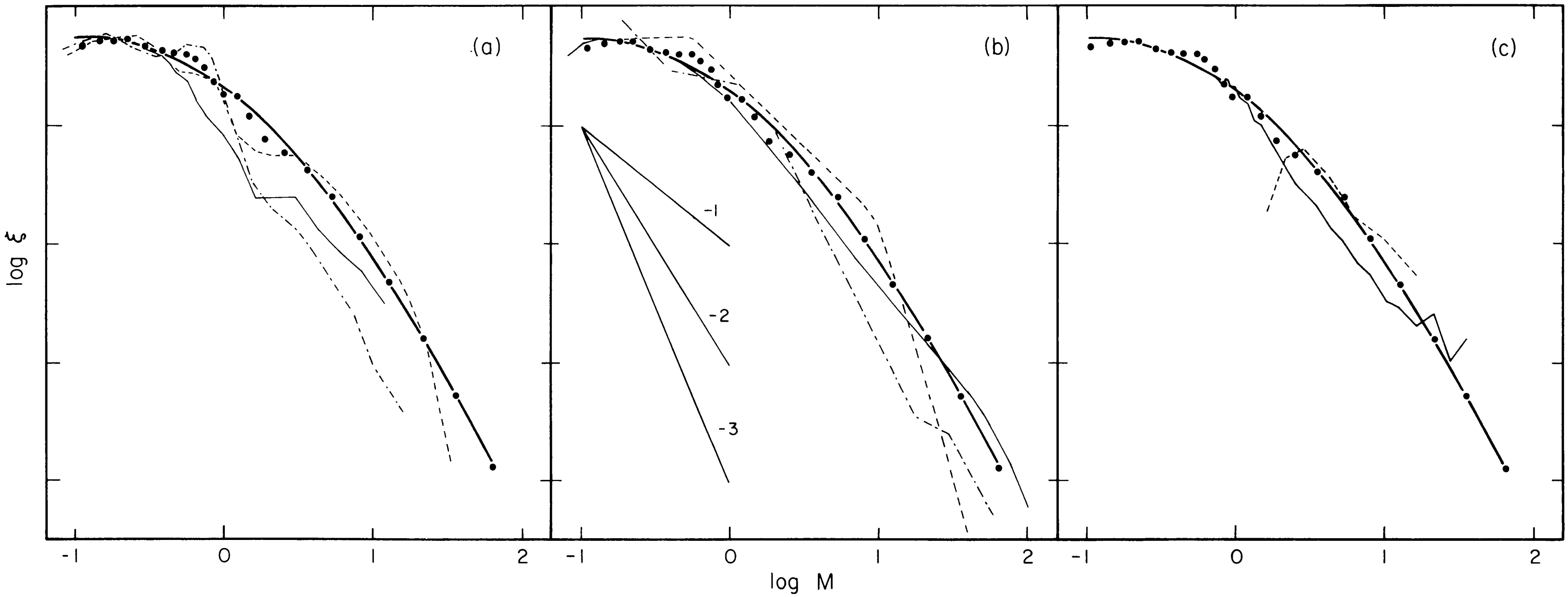

FIG. 11.—Comparison of published empirical IMFs. The filled circles and thick line are the IMF of the present paper for a constant birthrate with $T_0 = 12 \times 10^9$ yr. (*a*) *Thin solid line*, Salpeter (1955); *dashed line*, Sandage (1957) as given in Warner (1961); *dot-dashed line*, Schmidt (1959) constant ($n = 0$) birthrate. (*b*) *Thin solid line*, Limber, (1960); *dashed line*, Hartmann (1970); *dot-dashed line*, Audouze and Tinsley (1976). (*c*) *Thin solid line*, mean cluster IMF of Taff (1974); *dashed line*, IMF of Ori OB1 from data in Warren and Hesser (1978). The IMFs are arbitrarily normalized to agree at the low-mass end except for that of Ori OB1, where the luminosity function is incomplete for $M \leq 2.5\ M_\odot$. The ticks on the ordinate show one unit in log ξ. Approximate slopes of any section of these IMFs may be estimated by a comparison with the three straight lines of indicated slope in Fig. 11*b*.

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since the (absolute magnitude, mass)-relation for these stars is extremely uncertain.

We conclude that there is no evidence for departures from the field star IMF in local clusters for $M \gtrsim 1\ M_{\odot}$ and in associations for $M \gtrsim 2.5\ M_{\odot}$.

d) *Comparison with Theory*

A primary goal in establishing the observed form of the IMF is to provide a constraint on theories of star formation. Such an approach has proved fruitful in areas like comparative ecology (e.g., May 1975), where the observed distribution of the total populations of species in a closed environment can be used to infer how these species divide up resources and how the species interact. In general studies of size distributions (e.g., aerosol physics), a lognormal distribution signifies the operation of several independent mass alteration processes. For the problem at hand there are essentially only two general types of theories to be considered. First, the IMF (or at least some portion of it) may reflect the initial distribution of protostellar fragment masses, with fragments evolving independently of each other. We refer to this type of interpretation as a *fragmentation model.* The second possibility is that the form of the IMF may be a signature of certain interactions of protostellar fragments with ambient gas and with each other. This type of interpretation is referred to as a *fragment interaction model.* In §§ IV*d*(i) and IV*d*(ii) we briefly list the predicted form of the IMF in several variants of each type of model. Our description of the theories is brief and is intended only to give the reader the basic assumptions on which each theory is based; no attempt is made at detailed analysis or criticism. The reader is referred to Silk (1979) for a more detailed discussion of some of these mechanisms.

i) *Fragmentation Models*

Geometrical and probabilistic models.—Belserene (1970) calculated the mass function which would result if a homogeneous cloud completed its fragmentation all in one step, in such a way that no particular size would be favored, with smaller fragments forming freely between the larger ones. The IMF slope is -3. Auluck and Kothari (1954) considered the random fragmentation of a volume, modeled as a homogeneous parallelepiped subdivided by planes parallel to its sides, and found a mass function

$$\xi(M) \propto \left(\frac{M}{M_0}\right)^{2/3} \exp\left[-3(M/M_0)^{1/3}\right], \qquad (41)$$

where M_0 is the average fragment mass and the formula is only valid for $M > M_0$. This result was compared with the mass function in the Hyades cluster by Kushwaha and Kothari (1960). This model also views the fragmentation as occurring in one step. Auluck and Kothari (1965) generalized the derivation to include smaller masses. Kruszewski (1961) assumed that the density perturbations in a cloud have a white-noise spectrum, with a distribution of linear sizes l in one dimension (number per unit length) proportional to l^{-3}. Assuming that the three dimensions of a fragment are independent, Kruszewski finds a mass function essentially proportional to M^{-2}. Another random fragmentation model, which, however, gives a very poor fit to the stellar IMF, has been discussed by Kiang (1966).

Reddish (1962, 1966) considered the distribution of gravitational potential energies of fragments for a given power-law distribution of radii. Assuming equipartition of gravitational energy of fragments in equal ranges of fragment size, Reddish finds a mass function with slope $-4/3$. Fowler and Hoyle (1963) considered a hierarchical fragmentation process in which all fragments at step n have the same mass M_n, and they assumed that the mass of fragments of step n that survive without further fragmentation to become stars of mass M_n is the same at each step ("equipartition of mass"). The resulting mass function, if approximated by a power law, has a slope which varies from about -1.1 at $10^{-1}\ M_{\odot}$ to -1.4 at $10\ M_{\odot}$, steepening thereafter.

Larson (1972) presented a model for random fragmentation which considered the process to occur in stages, starting with a cloud of mass M_0, and ending with fragments of a minimum size or equivalently a total number of fragmentation stages N. At each stage the fragments are supposed to split into two equal subcondensations, each with half of the initial mass. Larson assumes that the minimum Jeans mass is reduced by a factor of 2 at each stage and assigns a fragmentation probability p to the process; p is a free parameter. Since Larson's model yields a Gaussian distribution, it can be made to agree with the IMF of the present work by a suitable choice of the parameters, for example, for $M_0 = 10^3$, $p = 0.62$ and $N = 22$ and for $M_0 = 10^4$, $p = 0.69$ and $N = 24$. However, the slope of the IMF is very sensitive to p. For example, if $M_0 = 10^4$ and $N = 24$, a decrease in p from 0.69 to 0.5 changes $d \log \xi / d \log M$ from -1.9 to -0.5 at $M = 10\ M_{\odot}$. Since one expects at least modest variations of p (and other parameters) from cloud to cloud, it is difficult for this theory to account for the observed degree of uniformity of cluster mass spectra, as found by Taff (1974).

Physical Mechanisms.—Larson (1978) has presented three-dimensional hydrodynamic collapse calculations which suggest a hierarchical model in which "accretion domains" of various sizes occur. A massive subcondensation accretes some fraction of the surrounding gas which depends on the effective viscosity, and the leftover gas tends to arrange itself in accreting subcondensations of a smaller scale. Larson gives a simple geometrical construction to illustrate that if the same fraction of gas is accreted at each level of the hierarchy, the resulting mass spectrum should be a power law. For a very small accretion efficiency the slope of the resulting IMF is -1.

Reddish (1975), following a similar idea by Reddish and Wickramasinghe (1969), suggested that instabilities associated with a changing mean molecular weight will occur when molecular hydrogen forms via

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grain surface reactions. Reddish argues that the rate of H_2 formation depends sensitively on grain temperature, which should fluctuate on a length scale corresponding to unit grain optical depth, leading to a cellular structure. The predicted fragment IMF has a slope between -1.0 and -1.5, depending on the assumed central condensation of the individual fragments. This mechanism has been criticized by Silk (1979).

Arny (1971) has pointed out that if the interiors of clouds are turbulent, one expects the critical mass for gravitational instability to be controlled by the turbulent velocity rather than by the temperature. Assuming a turbulent spectrum of the form

$$\rho v^{\alpha} = l^{\beta} f(r) , \qquad (42)$$

where ρ is the density of a region of size l, α and β are constants, and $f(r)$ gives the radial dependence of the spectrum, and that the fragments satisfy $v^2 = GM/l$, Arny shows that the mass function will be a power law of slope $n = -c(1 + 1/2\alpha)$, where $c = 3(3 + 1/2\alpha + \beta)^{-1}$. For constant angular momentum density, ρvl is constant, $\alpha = 1$, $\beta = -1$, and $n = -9/5$. For constant kinetic energy density, ρv^2 is constant, $\alpha = 2$, $\beta = 0$, and $n = -3/2$. A Kolmogorov spectrum also gives $n = -3/2$.

Takebe, Unno, and Hatanaka (1962) assumed that the stellar mass spectrum is set simply by the spatial distribution of the local Jeans mass throughout the cloud, which is controlled by the cloud's temperature and density distribution. Takebe *et al.* calculate the cloud structure which will give the observed stellar mass spectrum. The required density distribution resembles a polytrope of index 5 with a gradient in turbulent velocity. It is difficult to imagine that all clouds have structures similar enough to account for the degree of uniformity in the observed mass spectrum of various clusters.

Silk (1977) has derived the form of the IMF which would result from the following continuous hierarchical fragmentation scheme: The dominant fragment mass in a region of the cloud is equal to the Jeans mass corresponding to fragment optical depth unity, since larger masses contract adiabatically and do not fragment further while smaller masses cannot contract at all. The energy input from the contracting fragments heats up the rest of the gas, leading to a larger dominant mass as fragmentation proceeds since the Jeans mass increases with temperature. For lower-mass stars the main energy input should be radiation and the calculated IMF is a power law with index -1.3 to -1.6 for any reasonable grain opacity. Silk suggests that dissipation of turbulence will be a more effective heat input at larger masses and that the mass spectrum should steepen, with an index of about -2.9. The critical mass separating these two regimes is uncertain, but Silk estimates a value of around $0.6\ M_{\odot}$.

Mass spectra resulting from some plausible physical mechanisms have not yet been derived. For example, Oppenheimer (1977) has shown that isentropic instabilities can occur in dense interstellar clouds, possibly leading to the formation of protostars and maser condensations. Oppenheimer suggests a spectrum of sizes which increases toward small sizes. Mestel (1972 and references therein) has long advocated the view that the masses of fragments should be controlled by magnetic fields, and Nakano (1973) has examined a similar fragmentation model in which the fragment mass gradually decreases as the field leaks out. Since the rate of field relaxation depends on the ion density, a magnetic model might lead to an interesting dependence of the mass function of metallicity. On the other hand, recent determinations of electron densities in clouds (e.g., Snyder *et al.* 1977; Wootten 1978) suggest that the magnetic field may not be so important in this regard.

Figure 12 compares the various fragment IMFs described above with each other and with the field star mass function. All IMFs are arbitrarily normalized at $M = M_{\odot}$. The predicted spectra of Belserene (1970, $n = -1$) and Arny (1971, $n \approx -1.5$ to -1.8) are not shown for the sake of clarity. The dark solid line represents the field star IMF of the present paper for a constant birthrate and $T_0 = 12 \times 10^9$ yr. Agreement or disagreement of theoretical models with the observations over the entire spectrum should not necessarily be taken as evidence for or

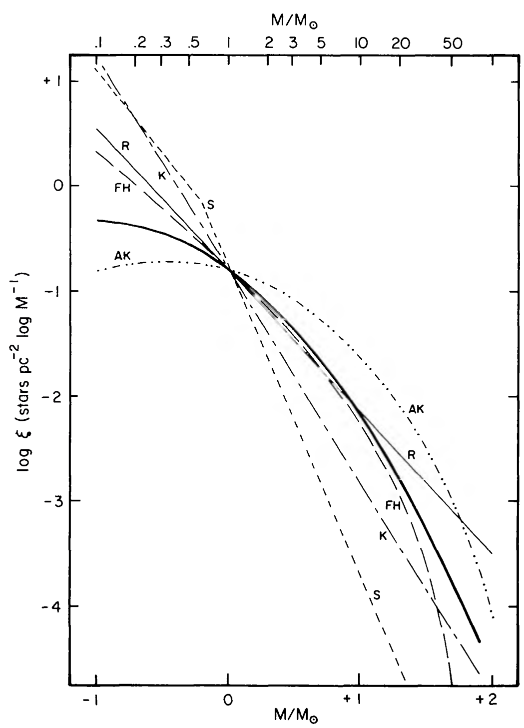

FIG. 12.—Predicted mass functions according to various fragmentation hypotheses (*light lines*) are compared with the IMF of field stars (*dark solid line*). All the mass functions are normalized at 1 $M_{\odot}$. The theoretical curves are from Reddish (1966, 1975, R), Fowler and Hoyle (1963, FH), Auluck and Kothari (1965, AK), Kruszewski (1961, K), and Silk (1977, S). The Auluck and Kothari curve can be shifted arbitrarily. Larson's (1972) prediction can be made to fit the solid line exactly with a suitable choice of parameters.

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against a particular model, since more than one process may be important and/or different mass ranges may be controlled by different processes; also, many of the theories are so nonphysical that any agreement with observations is probably coincidental.

Considering the simplicity of the theoretical models, it is rather surprising that most of them crudely resemble the observations. However, all the predictions except that of Auluck and Kothari (1965, "random fragmentation") are far too steep at the low-mass end. Auluck and Kothari's spectrum can be shifted by changing the adopted mean mass (1 $M_\odot$ in the figure). Larson's (1972) model is not shown since it can be made to agree with the observations by adjusting parameters. None of the other theoretical spectra can be altered significantly by varying parameters, although the mass at the slope change in Silk's (1977) model, taken as 0.6 $M_\odot$, is uncertain. We conclude that the main deficiency of most fragmentation models is their inability to account for the flattening of the IMF at low masses.

ii) *Fragment Interaction Models*

It seems likely that protostellar fragments within a cloud do not evolve independently. Nakano (1966) showed that under reasonable conditions the mean collision time scale for fragments can be smaller than the fragment free-fall time and that strong radiative cooling should result in coalescence during fragment collisions. Recent observations of large density fluctuations in clouds (e.g., Mahoney, McCutcheon, and Shuter 1976; Clark, Giguere, and Crutcher 1977; Clark and Johnson 1978) also suggest the importance of fragment interactions. Nakano (1966) numerically calculated the mass function of coalescing fragments and found that for a given assumed collision rate the mass function approached an asymptotic form similar to that observed in some open clusters and to the IMF derived in the present work. Silk and Takahashi (1979) presented approximate analytic solutions to the coalescence equations which agree well with Nakano's results and which show explicitly the dependence of the form of the mass function on collision rates. Both of these works neglected the influence of the fragment velocity distribution and protostellar collapse and assumed essentially a δ function for the initial distribution of fragment masses.

Arny and Weissman (1973) took into account the velocity distribution and collapse in *N*-body simulations designed primarily to answer criticisms of gravitational fragmentation by Layzer (1963), but the number of bodies is too small for one to infer a resulting mass function. Pumphrey and Scalo (1979) have performed similar simulations with 2000 bodies, in which interaction between fragments and ambient cloud gas is accounted for. The important result for the present work is that the high-mass end of the predicted mass functions is independent of the initial fragment mass distribution if the initial fragment spectrum has no preferred mass, as in a power-law spectrum (see Scalo 1979, Fig. 4), while the flattening of the low-mass end due to accretion of ambient gas depends largely on the ambient gas density. It is reasonable to assume that the higher star densities in clusters relative to associations reflect higher initial gas densities for the subclouds out of which clusters have condensed. Therefore, such a fragment interaction model can account for both the uniformity of the upper portion of the mass functions of clusters and field stars and the variation of the low-mass end among clusters and between clusters and field stars (most of which were probably formed in associations). Although these results make fragment interaction theories appealing, much more quantitative theoretical work and many more observational comparisons are required before any stronger statement can be made.

V. DISCUSSION AND SUMMARY

a) Is the IMF Time Independent?

A fundamental assumption of this paper is that the IMF is constant in time, and we now attempt to argue that there is no compelling evidence to the contrary, and even that some weak evidence in favor of this assumption exists. We reiterate that we are concerned with the IMF only in the solar neighborhood; evidence for and against variations in the IMF in other galaxies is reviewed by Scalo (1979).

It is often stated that from a theoretical point of view it would be naive to expect a constant IMF because the process of star formation may depend on a number of variables—in particular, metal abundance. On the other hand, many specific idealized models for the fragmentation of interstellar clouds, as discussed in § IV*d* and reviewed by Silk (1979), predict IMFs whose form is *not* sensitive to such quantities. The fragment interaction models discussed above make the same prediction. Furthermore, it appears that the mean metal abundance has not increased significantly over most of the history of the galactic disk, although spatial fluctuations probably do occur (see Audouze and Tinsley 1976). There is consequently no apparent reason for assuming that the spatially averaged local IMF has varied with time on the basis of metal abundance.

Several authors (e.g., Schmidt 1963; Truran and Cameron 1971; Biermann and Tinsley 1974) have suggested that the IMF was enhanced in high-mass stars in the past, in order to account for the small number of metal-poor dwarfs in the solar neighborhood relative to the predictions of simple chemical evolution models. However, so many other mechanisms exist which could account for this discrepancy (for references see Trimble 1975; Audouze and Tinsley 1976) that this indirect argument for a time variation of the IMF is considerably weakened.

The apparent differences between the field star IMF and some open cluster IMFs, as mentioned in § IV*c* and discussed in more detail by Scalo (1979), have occasionally been interpreted in terms of a time-dependent IMF. The age distribution of open clusters (Wielen 1971) shows that most clusters have ages of

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about 10^8 yr or less so that the mean cluster mass function reflects essentially present-day conditions in the solar neighborhood, whereas low-mass field stars have a considerably larger mean age. This result has led several authors to suggest that the differences between cluster and field star IMFs reflect a change in the IMF during the last few 10^9 yr, with the fraction of low-mass stars being larger in the past. We feel that this interpretation is incorrect because there is no tendency for the deficiency of low-mass stars to appear only in the young clusters. For example, the low-mass deficiency occurs not only in the (young) clusters NGC 6193 and the Pleiades, but also in the old cluster M67; it does not occur in the Hyades, which is intermediate in age. A more likely explanation is that the low-mass end of the mass function depends on initial conditions, or equivalently that the mass function of associations (where most field stars are born [Miller and Scalo 1978]) is different from that of some clusters for $M \lesssim 1\ M_\odot$, a possibility which was noted by van den Bergh (1972). A theoretical model which accounts for this behavior is given by Pumphrey and Scalo (1979) as discussed earlier.

Besides the weakness of previous arguments against a time-dependent IMF, we also point out some positive support for a constant IMF. First, Reeves and Johns (1976) have inferred that the abundances and production ratios of long-lived radio activities are consistent with a constant IMF *and* a constant gas depletion lifetime (which depends on the birthrate) over the last few 10^9 years. Second, an often overlooked paper by Dixon (1970) sought evidence for temporal variations in the IMF from variations in the fraction of stars of different masses in two different age groups. For example, if the IMF was flatter in the past, one should see more higher-mass stars in the older group. By using only two age groups, Dixon's work is somewhat less prone to the uncertainties involved in estimating age distributions from space motions and the color-magnitude diagram. Although the results are still uncertain, it may be significant that Dixon found no variations in this fraction.

Two of the subsidiary constraints on the birthrate history discussed in § III*c* also yield an important, albeit uncertain, consistency check on the assumption that the IMF is independent of time. The comparison of the present white dwarf–planetary nebula formation rate with the present total stellar birthrate indicates that the birthrate of stars in the range 1–2 $M_\odot$ has *not* changed much over the life of the galactic disk. The yields of nucleosynthesis from Arnett's (1978) calculations suggest that the birthrate of massive stars has also been fairly uniform, although this conclusion is model dependent. Taken together, these two results imply that the ratio of the birthrate of low-mass and high-mass stars has not varied strongly over the history of the galactic disk. One does not expect this result if the mass spectrum of newly formed stars has changed drastically during the last 10×10^9 years or so. As far as we know, this is the only unambiguous argument for or against the constancy of the local IMF over a large mass range.

b) Is the IMF Continuous?

Another fundamental assumption we have made in constraining the time history of the birthrate is that the IMF is continuous. Several papers (e.g., Quirk and Tinsley 1973; Ostriker and Thuan 1975; Smith, Biermann, and Mezger 1978) have speculated that the discontinuity in the IMF which results from adopting a rapidly decreasing or increasing birthrate is actually a physical discontinuity resulting from different mechanisms for star formation above and below about 1 $M_\odot$. While it is entirely possible that different processes of star formation control the high- and low-mass ends of the IMF, we find it difficult to believe that one set of processes cuts off at some upper mass limit while the other set cuts off at some lower mass limit *and* that both limits are roughly equal, as required to produce the discontinuity. It would seem more natural to expect that if there are two processes at work, there is some intermediate mass at which they contribute about equally to the star formation rate, providing for a continuous mass function. Similar situations are encountered in many studies of mass distribution functions. An important example occurs in terrestrial cloud microphysics: the mass spectrum of droplets in a cloud is controlled by nucleation and condensation at the low-mass end but by coalescence at the high-mass end, yet the observed *and* theoretical mass spectra are continuous and usually monotonic.

Eggen (1976) has suggested that stars brighter than $M_v = -1$ ($M \gtrsim 5\ M_\odot$) and those fainter than $M_v = 0$ ($M \lesssim 4\ M_\odot$) are formed by independent processes, and that stars of intermediate luminosity may not be produced at all in the solar neighborhood. His two lines of evidence are apparent gaps in the color-luminosity diagram of a few clusters, and his finding that no stars in the Bright Star Catalog with $-1.0 < M_v < +0.5$ and ages less than about 10^7 yr exist. The gaps, assuming that they are not fluctuations, would produce a dip in the field star IMF in a small mass range if field stars were all formed in such clusters (they are not), not a discontinuity of the type implied by a rapidly decreasing birthrate. Indeed, the dip would be at a significantly higher mass than the mass corresponding to the discontinuity. Additionally, the gaps certainly are *not* apparent in the color-magnitude diagram of most clusters. We give little weight to Eggen's conclusion concerning the lack of stars in the range $-1.0 < M_v < +0.5$ with ages less than 10^7 yr in the solar neighborhood since it rests almost entirely on his adopted zero-age main sequence (ZAMS) in this luminosity range, which is much different from the ZAMS determined by other authors and which strongly disagrees with the theoretical ZAMS. We also note that if Eggen's ZAMS and his claim about the lack of young stars in the "special" luminosity range were correct, then at larger and smaller luminosities the lower limit to the observed luminosity distribution of Bright Star Catalog stars (Eggen 1977, Fig. 8) should agree with his ZAMS, yet Eggen does not present data for such stars.

Another observation which might be interpreted in

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terms of a discontinuous IMF is the absence of OB stars in some T Tauri associations (Herbig 1962) and the lack of infrared sources in the Taurus cloud complex. Although this interpretation may be the correct explanation, it is equally probable that massive stars are simply the last to form, as suggested by Herbig (1962) and supported by the comparison of color-magnitude diagrams of young clusters with theoretical isochrones (e.g., Iben and Talbot 1966).

c) *Summary*

In this paper we have examined the observational and theoretical considerations pertaining to the history of the stellar birthrate and the IMF in the solar neighborhood. Our major conclusions are as follows:

1. Independent determinations of the present-day luminosity function using different methods and for different volumes of space show that there exists a meaningful average luminosity function for the disk population in the solar neighborhood. Within the observational uncertainties this luminosity function smoothly increases with increasing magnitude to $M_v \approx 15$. Rough estimates of the uncertainties, especially for high-luminosity stars, have been given.

2. The conversion of the present-day luminosity function into the present-day mass function of main-sequence stars (PDMF) introduces uncertainties which we have attempted to estimate. These include the mass-luminosity relation, the distribution of stars perpendicular to the galactic plane, the calibration of mass and spectral type, correction for main-sequence brightening, and correction for non–main-sequence stars. Corrections which we are presently unable to estimate quantitatively include the effect of mass loss in massive stars on the apparent luminosity function, the fraction of OB stars hidden within dense interstellar clouds, the effect of unresolved multiple star systems, and the effect of variations in chemical composition. Although we suspect that the last two effects are relatively small, they all deserve further investigation.

3. We have emphasized that the determination of the IMF from the PDMF is inextricably tied to the adopted history of the stellar birthrate. The primary constraint on the birthrate history, the continuity constraint, is the requirement that the IMF not exhibit an unphysical discontinuity at the mass where the main-sequence lifetime equals the age of the disk. Maximizing all observational uncertainties, we find that the average past birthrate may be at most about 5 times larger or 3 times smaller than the present birthrate according to this constraint, and we favor a variation of less than a factor of 2.

4. Consideration of 12 secondary constraints on the birthrate shows that these constraints either are consistent with the continuity constraint with a much larger uncertainty or else are indeterminate.

5. There is no evidence that the birthrate depends on the square of the mean gas density. If the birthrate depends on some power of the mean gas density, the exponent must be small ($\lesssim 0.5$).

6. Most theoretical scenarios for the birthrate history are consistent with a roughly constant birthrate.

7. The allowed range of birthrate histories consistent with the continuity constraint yields IMFs which are smooth and can be well approximated by a half-Gaussian distribution in log M from 0.1 $M_\odot$ to about 50 $M_\odot$. Within the observational uncertainties, the cluster IMF agrees with our derived field star IMF for 1 $M_\odot < M < 10$ $M_\odot$, but some clusters are deficient in low-mass stars. The IMF of the Ori OB1 association agrees well with the field star IMF for $M \geq 2.5$ $M_\odot$.

8. The main deficiency of theoretical pure fragmentation models for the IMF is their inability to account for the flattening of the observed IMF at low masses or the low-mass turnovers seen in some open cluster IMFs. Fragment interaction models are more successful in accounting for these features.

9. The present mass consumption rate is calculated as 3–7 $\times$ 10^{-9} $M_\odot$ pc^{-2} yr^{-1}. The corresponding time scales for gas depletion suggest that star formation is balanced by infall and/or that the birthrate is controlled by stochastic factors.

10. Two basic assumptions used in this paper are that the IMF is independent of time and a continuous function of mass. Our discussion of both observational and theoretical considerations related to these questions indicates that the assumptions are not unreasonable.

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Note added in manuscript, 1979 February 18.—After this work was completed, we received a preprint from J. Lequeux on the IMF for stars more massive than 2 $M_\odot$. Using several recent catalogs, he calculated the surface density of stars as a function of spectral type and luminosity class. He used theoretical evolutionary tracks to assign masses and calculated the PDMF. Comparison with the PDMF of the present paper shows that Lequeux's PDMF is systematically smaller by about 0.4 in the log and quite similar in shape. For log $M > 0.8$ this is within our uncertainty limits, while for smaller masses the disagreement is somewhat larger than our uncertainty estimate. Lequeux cites evidence that roughly half of all O stars ($M \gtrsim 15$ $M_\odot$) are not on the main sequence; i.e., they are evolved stars of lower mass. In our check on the upper PDMF using counts of stars versus spectral class (§ II*f*), we assumed that all O stars were main-sequence stars, and this assumption may not be justified. However, this factor of 2 is substantially less than

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the total uncertainty and therefore does not alter any conclusions. The IMF as defined by Lequeux is ϕ_{ms}/T_{ms} in the notation of the present paper. For these short-lived stars, this is equivalent to the present birthrate $\xi b(T_0)/T_0$. Since the (mass, main-sequence lifetime)-relation used by Lequeux is similar to ours, the IMFs of his study and ours have essentially the same relationship as the PDMFs: Lequeux's IMF is below ours and similar in shape. Lequeux finds a slope of -2 for $M > 2$, while the power-law representation of our IMF has slope -1.5 for $1 \leq M \leq 10$ and -2.3 for $M > 10$.

APPENDIX

MAIN-SEQUENCE BRIGHTENING CORRECTION

In § II of the text it was noted that the decrease in absolute magnitude of a star during its main-sequence lifetime has an effect on the PDMF and IMF, and that to a good approximation the use of the (mass, mean absolute magnitude)-relation rather than the (mass, zero-age absolute magnitude)-relation in the derivation of the PDMF corrects for main-sequence brightening. In this Appendix the effects of main-sequence brightening are first examined in detail because the effects of brightening are not negligible and because no complete discussion exists in the literature. With this background, the approximation employed is then justified.

In deriving the PDMF, the main-sequence luminosity function (observed distribution of present absolute magnitudes) was converted to a mass distribution via a (mass, absolute magnitude)-relation. This assumes a unique relation between mass and absolute magnitude on the main sequence. However, the present absolute magnitude of a main-sequence star is determined by its age in addition to its mass, so at any given absolute magnitude there is a contribution from stars over a range in mass. Let $\phi_{ms}(M_v)$ denote the present-day main-sequence luminosity function (stars pc^{-2} mag^{-1}). Then the IMF uncorrected for brightening ξ^u was derived from (see eq. [10])

$$\xi^u(\log M) = \left|\frac{dM_v}{d\log M}\right|\phi_{ms}(M_v)T_0 \Big/ \int_{T_0-T_{ms}(M)}^{T_0} b(t)dt\,. \tag{A1}$$

The first term on the right-hand side is the slope of the (mass, absolute visual magnitude)-relation and converts the luminosity function to a mass distribution.

The true but as yet unknown IMF, ξ, is given by the same equation when the distribution of initial (or zero-age) absolute magnitudes, $\phi_{ms}(M_v{}^0)$, is used:

$$\xi(\log M) = \left|\frac{dM_v}{d\log M}\right|\phi_{ms}(M_v{}^0)T_0 \Big/ \int_{T_0-T_{ms}(M)}^{T_0} b(t)dt\,. \tag{A2}$$

(Strictly speaking, $dM_v{}^0/d\log M$ should be substituted for $dM_v/d\log M$, but the differences are small and are not considered further.) The ratio of the true IMF to the uncorrected IMF, $\chi(\log M)$, is just

$$\chi(\log M) = \frac{\xi(\log M)}{\xi^u(\log M)} = \frac{\phi_{ms}(M_v{}^0)}{\phi_{ms}(M_v)}\,. \tag{A3}$$

Once the correction factor χ is shown as a function of mass, the true IMF can be found from the uncorrected IMF. It is necessary then to evaluate $\phi_{ms}(M_v{}^0)$ and $\phi_{ms}(M_v)$. The quantity $\phi_{ms}(M_v{}^0)$ is simply (see eq. [A2])

$$\phi_{ms}(M_v{}^0) = \left[\left|\frac{d\log M}{dM_v}\right|\xi(\log M)\right]\int_{T_0-T_{ms}}^{T_0}\frac{b(t)}{T_0}\,dt\,. \tag{A4}$$

The term in brackets gives the number of stars of original magnitude $M_v{}^0$ ever born (per square parsec per magnitude), while the integral gives the fraction of all such stars which are still on the main sequence.

The derivation of $\phi_{ms}(M_v)$ is considerably more complicated. The quantity $\phi_{ms}(M_v)$ gives the number of stars (per square parsec per magnitude) with present magnitude M_v. Since the absolute magnitude of a star is a function of both age τ and mass M, $\phi_{ms}(M_v)$ contains contributions from stars in a range of mass as well as in age. {Note that τ and t are related by $t + \tau = T_0$.) If $M_v{}^*$ is a specific value of the present absolute magnitude and $M_v(\tau, \log M)$ symbolically denotes the (mass, age, absolute magnitude)-relation, then the most massive stars, with $M_v = M_v{}^*$, have mass M_{max} defined by $M_v(\tau = 0, \log M_{max}) = M_v{}^*$. The original absolute magnitude of these stars equals $M_v{}^*$. The least massive stars with $M_v = M_v{}^*$ have mass M_{min} defined by $M_v[\tau = T_{ms}(M_{min}), \log M_{min}] = M_v{}^*$; i.e., their absolute magnitude brightens to $M_v{}^*$ only at the end of their main-sequence lifetime T_{ms}. To find $\phi_{ms}(M_v{}^*)$, it is necessary to integrate over all stars with $M_v = M_v{}^*$ in the range $0 \leq \tau \leq T_{ms}(M_{min})$ and $M_{min} \leq M \leq M_{max}$.

The mass and age of a star with $M_v = M_v{}^*$ are uniquely related, allowing the problem to be reduced to one independent variable. When the present absolute magnitude is fixed at some value $M_v{}^*$, the relation $M_v{}^* = M_v(\tau, \log M)$ defines a unique relation between τ and $\log M$. With age as the independent variable, the mass of a star with $M_v = M_v{}^*$ becomes a function of its age. In what follows, we understand mass to be a function of age without explicitly writing $M(\tau)$.

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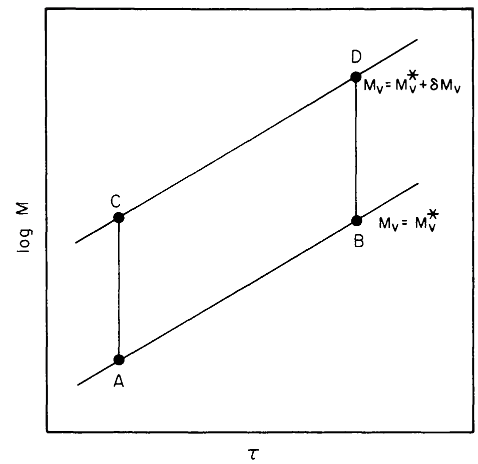

FIG. 13.—Neighborhood of an arbitrary point in (τ, log M)-space used in the derivation of $\phi_{ms}(M_v)$

To find the total number of stars per unit present absolute magnitude as a function of present absolute magnitude, the general expression for the number of stars per unit absolute magnitude in the neighborhood of an arbitrary point in (τ, log M)-space must be found and then integrated over the above range in τ and log M. Figure 13 shows the neighborhood of some arbitrary point (τ, log M). Points on the curve AB have $M_v = M_v{}^*$, and the curve CD is drawn so that points on this curve have $M_v = M_v{}^* + \delta M_v$, where the increment δM_v is to be specified. Points A and C lie at the same age, and likewise for points B and D. The total number of stars in ABCD is given by the product of the creation function $\xi(\log M)b(t)/T_0$ and the area of ABCD. The number of stars per unit absolute magnitude is then $\{\xi(\log M)[b(t)/T_0](\text{area of ABCD})\}/\delta M_v$. The area and δM_v must be explicitly evaluated.

Point A has coordinates (τ, log M). Point B is arbitrarily chosen to be at $\tau + d\tau$. Since B must lie on $M_v = M_v{}^*$, the coordinates of this point are $[\tau + d\tau, \log M + (d \log M/d\tau)d\tau]$, where we have used the relation

$$\frac{\partial M_v}{\partial \tau} = -\frac{\partial M_v}{\partial \log M}\frac{d \log M}{d\tau}\,, \tag{A5}$$

which is derived by differentiating $M_v{}^* = M_v(\tau, \log M)$. Point C lies at age τ and is arbitrarily chosen to have the same mass as point B. Therefore, the absolute magnitude at point C is $M_v{}^* - (\partial M_v/\partial\tau)d\tau$. This fixes the value of δM_v as $-(\partial M_v/\partial\tau)d\tau$. Point D lies at age $\tau + d\tau$ and is required to have $M_v = M_v{}^* + \delta M_v$, so its coordinates must be $[\tau + d\tau, \log M + 2(d \log M/d\tau)d\tau]$. The quadrilateral ABCD is therefore a parallelogram with area $(d \log M/d\tau)(d\tau)^2$. The number of stars per unit present absolute magnitude in ABCD is

$$\frac{-\xi(\log M)b(\tau)(d \log M/d\tau)d\tau}{T_0(\partial M_v/\partial\tau)}\,. \tag{A6}$$

The total number of stars per unit present absolute magnitude with $M_v = M_v{}^*$ is then given by

$$\phi_{ms}(M_v{}^*) = \int_0^{T_{ms}(M_{min})} \frac{-\xi(\log M)b(\tau)(d \log M/d\tau)}{T_0(\partial M_v/\partial\tau)}\,d\tau\,. \tag{A7}$$

We note that this equation is quite general and applies to any stellar property which is a function of mass and age. The generalization to a stellar property which is a function of mass, age, and an additional property (e.g., metal abundance or rotation) is also apparent.

Having found expressions for $\phi_{ms}(M_v{}^0)$ and $\phi_{ms}(M_v)$, we can calculate the correction factor χ as a function of mass once a brightening law is adopted. The following form was chosen:

$$M_v(\tau, \log M) = M_v{}^0(\log M) - 2.5 \log [1 + f_1\tau/T_{ms}(\log M)]\,. \tag{A8}$$

According to this prescription, the luminosity of a star increases linearly with time and brightens by a factor $1 + f_1$ in luminosity [the magnitude decreases by $2.5 \log (1 + f_1)$] by the end of its main-sequence lifetime. The parameter f_1 is referred to as the brightening constant and has a value in the range 0.75–1.5. This formula is a satisfactory representation of theoretical models of main-sequence evolution.

It is important to note that because the brightening depends on the main-sequence lifetime, which in turn depends on mass, equation (A8) cannot be analytically solved for mass as a function of age. This complication is expected to be true for any brightening formula since the rate at which a star brightens is expected to be related to its main-sequence lifetime.

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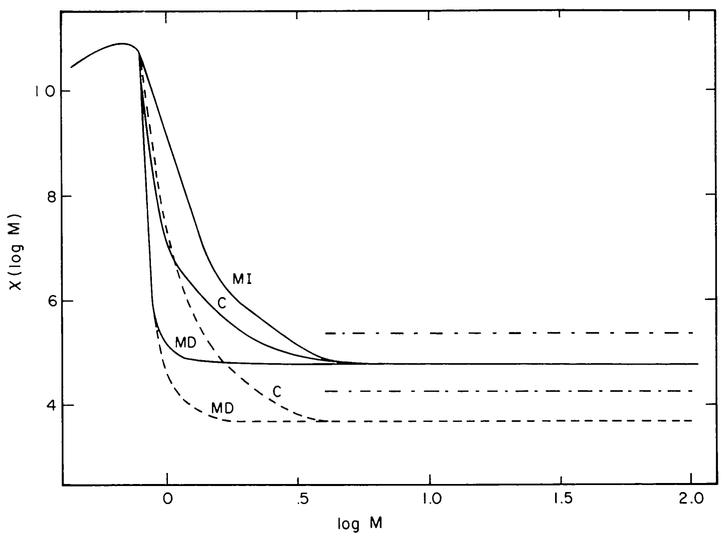

FIG. 14.—Main-sequence brightening correction factor $\chi(\log M)$. The solid lines correspond to a total brightening of 0.75 mag ($f_1 = 1$), for three birthrates: MI, C, and MD. The dashed lines are for a total brightening of 1.0 mag ($f_1 = 1.5$). For high masses, $\chi(\log M)$ is independent of the birthrate and age of the Galaxy and depends primarily on f_1. The dependence of $\chi(\log M)$ on the age of the Galaxy is small at all masses and is not shown. The dot-dashed lines above and below the solid line at high masses show the effect of altering the slope of the IMF at a given mass by $+0.5$ and -0.5, respectively.

For this reason equation (A7) and thus $\chi(\log M)$ must be evaluated by numerical techniques. In addition to numerically integrating equation (A7), a simulation has been used to calculate $\chi(\log M)$. In this simulation N stars are randomly distributed in mass according to $\xi(\log M)$ and in age according to $b(t)$. The zero-age and present absolute magnitudes are calculated for each star. Then the number of stars with M_v^0 within a small range of M_v^* are counted and compared to the number of stars with M_v in the same range. For convenience, power laws were used for the IMF, (mass, zero-age luminosity)- and (mass, main-sequence lifetime)-relations. These are sufficiently accurate over the range of integration. We have verified that numerical integration of equation (A7) agrees with the results of the simulation.

It is noted here that this is actually an iterative procedure: the true IMF must be known in order to calculate $\chi(\log M)$, whereas only the uncorrected IMF is known at first. As shown below, $\chi(\log M)$ depends only weakly on the slope of the IMF. Since the correction factor depends only on the slope of the IMF, one interation is sufficient.

Figure 14 shows $\chi(\log M)$ for a range in parameters. Below 0.8 $M_\odot$, the correction is near unity. As mass increases from 0.8 to 2 $M_\odot$, the correction factor rapidly decreases from unity to about 0.45, with the exact value of the correction factor depending on the birthrate history and the brightening constant f_1. Above 2 $M_\odot$, the correction factor is independent of the birthrate since the range of integration in age is much smaller than the time scale for a change in the birthrate for these massive, short-lived stars. In this mass range, the correction factor depends primarily on the brightening constant f_1. As shown in Figure 14, a change from $f_1 = 1$ to $f_1 = 1.5$ results in a decrease of $\chi(\log M)$ from 0.48 to 0.38. This figure also demonstrates the lack of sensitivity of the correction factor to the slope of the IMF. A large change in the slope (± 0.5) results in a small change in $\chi(\log M)$. The dependence of $\chi(\log M)$ on the age of the Galaxy is small at all masses and is not shown in Figure 14.

The behavior of $\chi(\log M)$ can be understood on the basis of the time scales involved. For $M \lesssim 0.8\ M_\odot$, the main-sequence lifetime $T_{\rm ms}$ exceeds the age of the Galaxy T_0. These stars have not had sufficient time to brighten appreciably; consequently, $\chi(\log M)$ is nearly unity. For high-mass stars ($M > 2\ M_\odot$), $T_{\rm ms} \ll T_0$. For these masses, the average star contributing to a particular absolute magnitude M_v^* has a mass less than that given by the (mass, zero-age absolute magnitude)-relation because of main-sequence brightening. Since the IMF increases with decreasing mass, there are more stars with $M_v = M_v^*$ than stars with $M_v^0 = M_v^*$. Additionally, $T_{\rm ms}$ increases with decreasing mass, so that the average star with $M_v = M_v^*$ resides on the main sequence longer than a star with $M_v^0 = M_v^*$. Both of these effects increase the number of stars observed with $M_v = M_v^*$ over the number with $M_v^0 = M_v^*$; thus $\chi(\log M) < 1$. Since the main-sequence lifetime rapidly decreases with increasing mass, the transition region between the low- and high-mass regimes is relatively sharp, as seen in Figure 14.

To apply the brightening correction to the PDMF and IMFs of this paper, we have used a simple but accurate approximation: the relation between mass and mean observed absolute magnitude has been used instead of the relation between mass and zero-age absolute magnitude. For low-mass, long-lived stars, these two relations are equal since these stars have not had time to brighten appreciably. As mass increases, the mean observed absolute magnitude is less than the zero-age absolute magnitude since the lifetimes are much shorter than the age of the Galaxy. As an illustration of the effect of using the (mass, mean absolute magnitude)-relation, consider some

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absolute magnitude M_v*. Using the correction factor $\chi(\log M)$ explicitly, one would have converted the observed present-day luminosity function $\phi_{ms}(M_v^*)$ to the PDMF $\phi_{ms}(\log M^*)$ by using the (mass, zero-age absolute magnitude)-relation. Then $\phi_{ms}(\log M^*)$ would be multiplied by $\chi(\log M^*)$. Using the approximation, at magnitude M_v^* we assigned $\phi_{ms}(M_v^*)$ to some mass $\log M'$ smaller than before. Thus at the mass $\log M^*$, we are using ϕ_{ms} from an absolute magnitude smaller than M_v^*. Since ϕ_{ms} decreases with decreasing M_v, a smaller value of ϕ_{ms} is assigned to $\log M^*$. The resulting change in $\log \phi_{ms}$ is

$$\Delta \log \phi_{ms}(\log M) = \frac{d \log \phi_{ms}(\log M)}{d \log M} \frac{d \log M}{dM_v} \Delta M_v \,, \tag{A9}$$

where ΔM_v is the difference between the (mass, mean absolute magnitude)- and (mass, zero-age absolute magnitude)-relations. The quantity $\Delta \log \phi_{ms}$ must therefore be nearly equal to the logarithm of the correction factor for this approximation to be valid. To verify this, recall first the behavior of the correction factor. For $M < 0.8\ M_\odot$, χ can be taken as unity ($\log \chi = 0$) to good accuracy. From 0.8 to 2 $M_\odot$, χ decreases sharply from 1 to about 0.45 ($\log \chi \approx 0$ to -0.35) and maintains this value for larger masses. We now demonstrate the behavior of $\Delta \log \phi_{ms}$. For $M < 0.8\ M_\odot$, $\Delta M_v = 0$, and thus $\Delta \log \phi_{ms} = 0$ as desired. For $M > 2\ M_\odot$, $\Delta M_v \approx 0.5$, and calculating $d \log \phi_{ms}/d \log M$ and $d \log M/dM_v$ from data given in § II shows that $\Delta \log \phi_{ms} \approx -0.35$ for $M > 1.5\ M_\odot$. This approximation also gives the steep drop in the correction between 0.8 and 2 $M_\odot$ since ΔM_v must go from 0 to 0.5 over this same range. In summary, the use of the (mass, mean absolute magnitude)-relation in the calculation of the PDMF and IMF gives a good approximation to the IMF derived from the (mass, zero-age absolute magnitude)-relation and the rigorous brightening correction.

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