Intro to Probability Theory

$$P(a)$$

Intro to Probability Theory

$$P(a = heads)$$

Intro to Probability Theory

$$P(a = heads) = 0.5$$

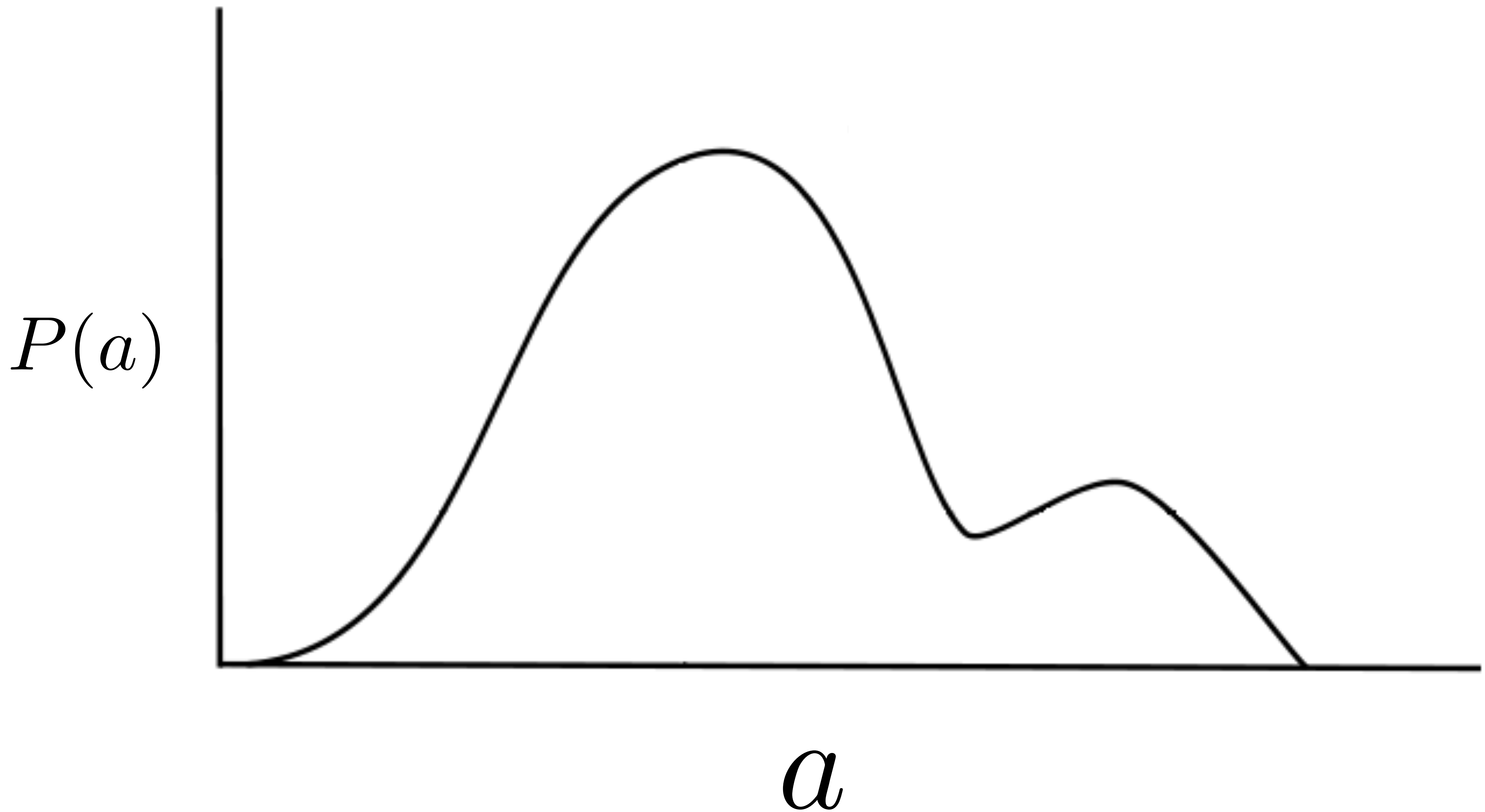
Intro to Probability Theory

$$P(a = heads) = 0.5$$

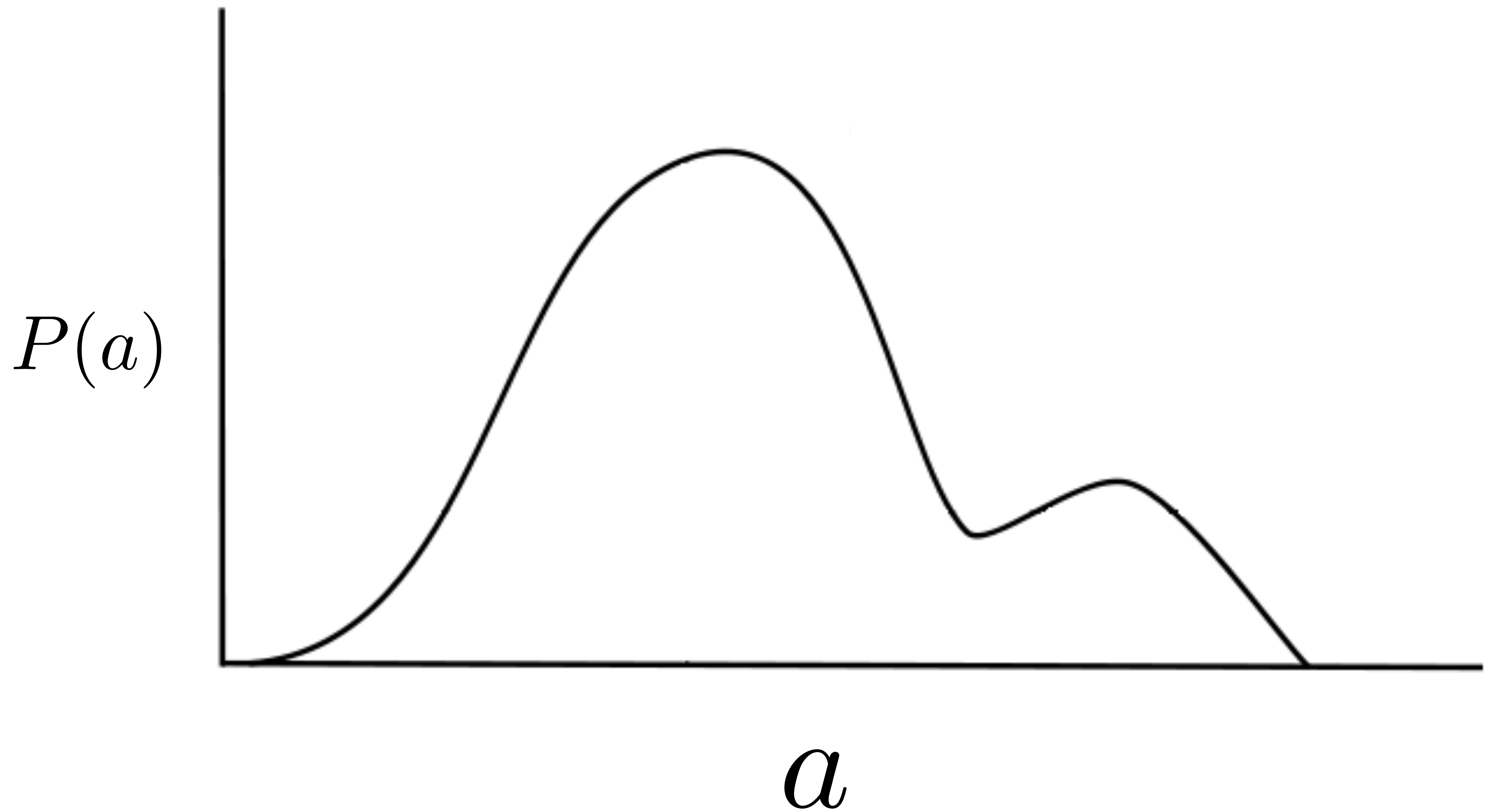
$$= \delta(a = 0.5)$$

Probabilities are **<u>all</u>** distributions.

Intro to Probability Theory

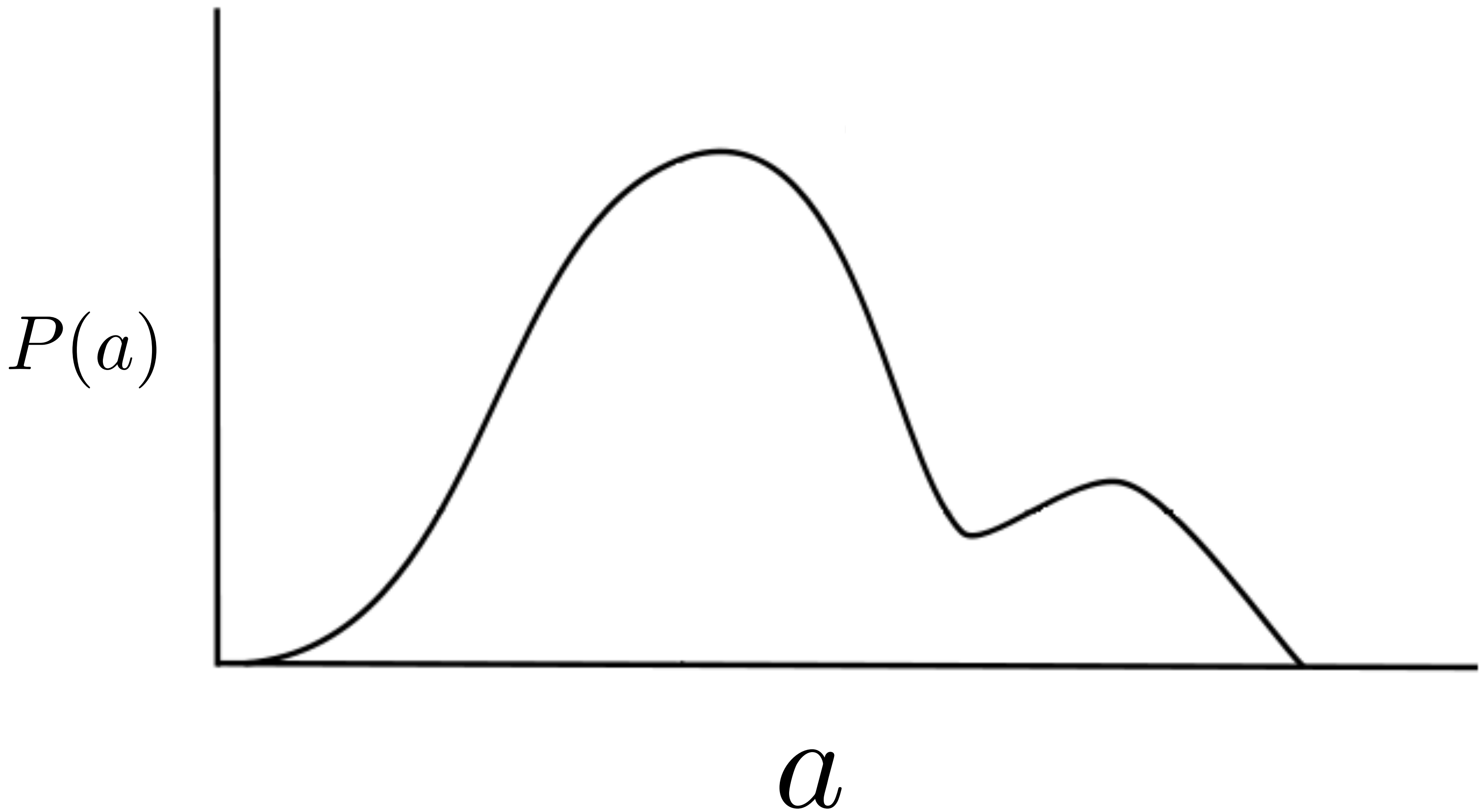


Intro to Probability Theory



$$P(a) \geq 0, \text{for all}\, a$$

Intro to Probability Theory



$$\int P(a)da = 1$$

Intro to Probability Theory

$$P(a = heads) = 0.5$$

$$P(a = tails) = 0.5$$

Intro to Probability Theory

$$\int P(a)da = 1$$

Intro to Probability Theory

$$\int P(a)da = 1$$

$P(a)$ has units of 1/a

Intro to Probability Theory

$$P(a|b)$$

Intro to Probability Theory

$$\int P(a|b)da = 1$$

Intro to Probability Theory

$$P(a) =$$

Intro to Probability Theory

$$P(a) = \int P(a|b)\, P(b) db$$

Intro to Probability Theory

$$P(a) = \int P(a|b)\, P(b) db$$

"Marginalization"

(also "Expected Value")

Intro to Probability Theory

$$\int P(a|b)db = 1$$

Intro to Probability Theory

$$\int P(a|b)db = 1$$

has units of b/a

Intro to Probability Theory

$$\cancel{\int P(a|b)\,db = 1}$$

has units of b/a

Intro to Probability Theory

$$P(a, b) = P(a)P(a|b)$$

Intro to Probability Theory

$$P(a, b) = P(a)P(a|b)$$

$$P(b, a) = P(b)P(b|a)$$

Intro to Probability Theory

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$$P(b, a) = P(a, b)$$

Intro to Probability Theory

$$P(a, b) = P(a)P(a|b)$$

$$P(b, a) = P(b)P(b|a)$$

$$P(b, a) = P(a, b)$$

$$P(a|b) = \frac{P(b|a)P(a)}{P(b)}$$

Intro to Probability Theory

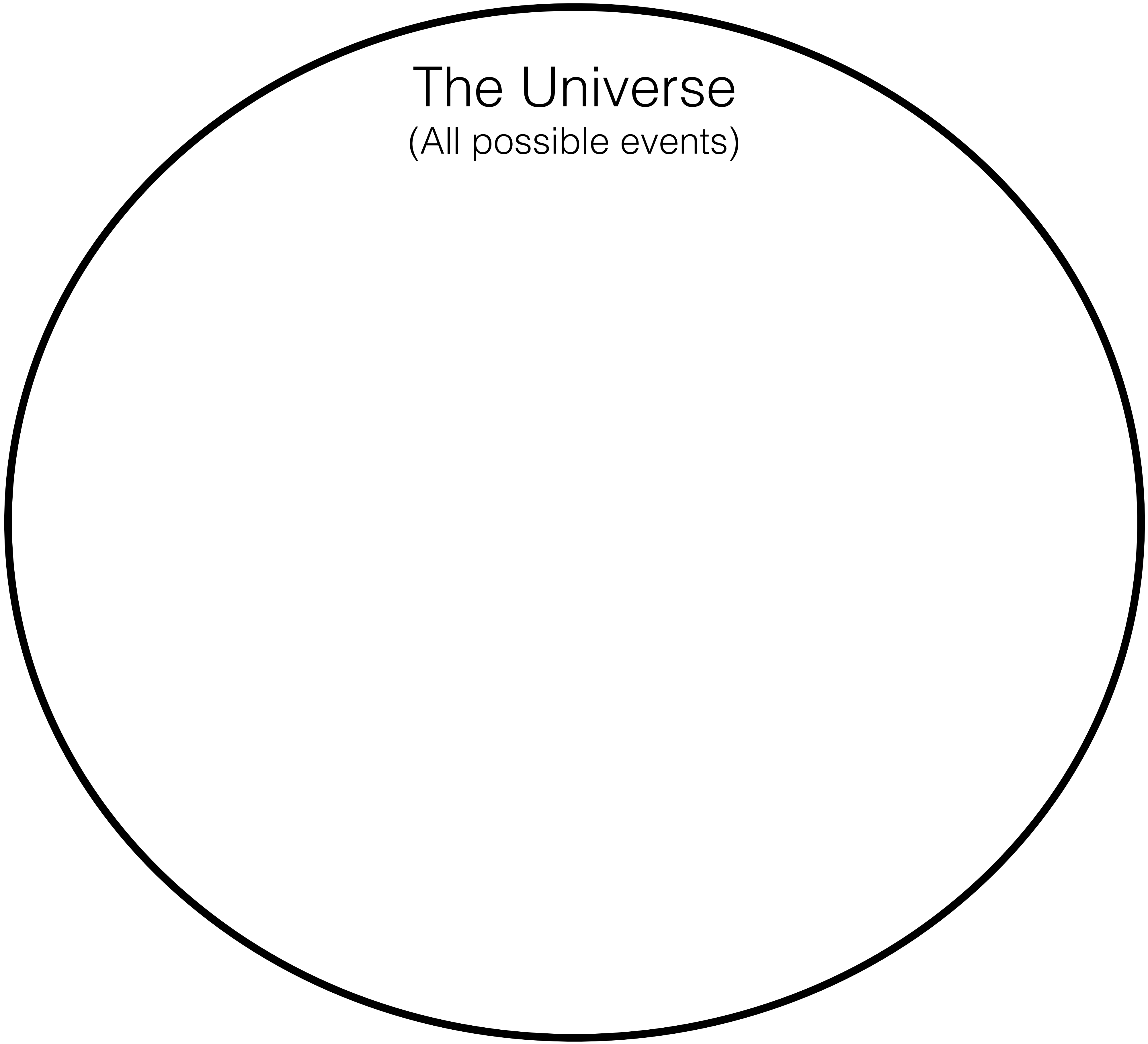
$$P(a,b) = P(a)P(a|b)$$

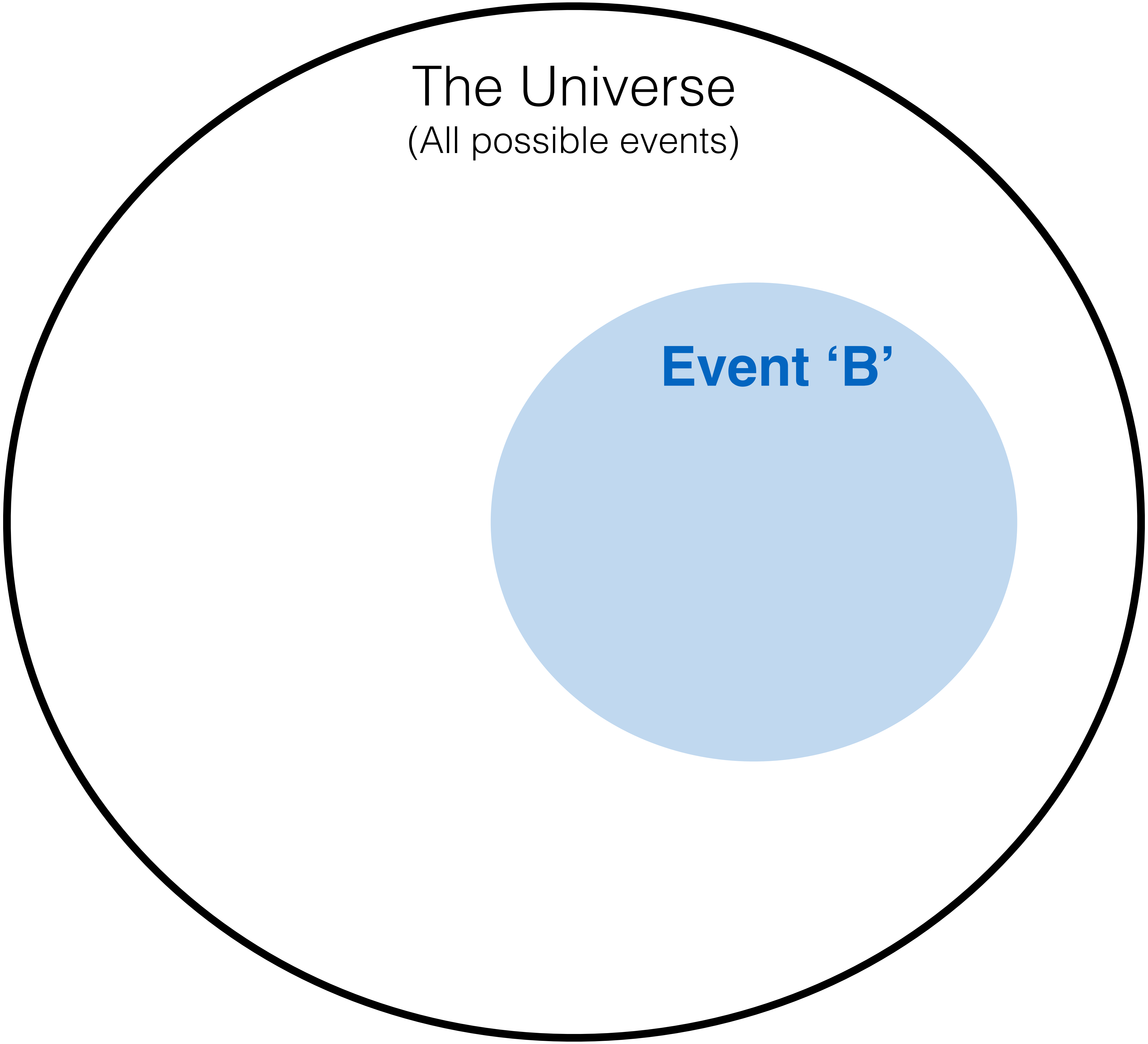
$$P(b,a) = P(b)P(b|a)$$

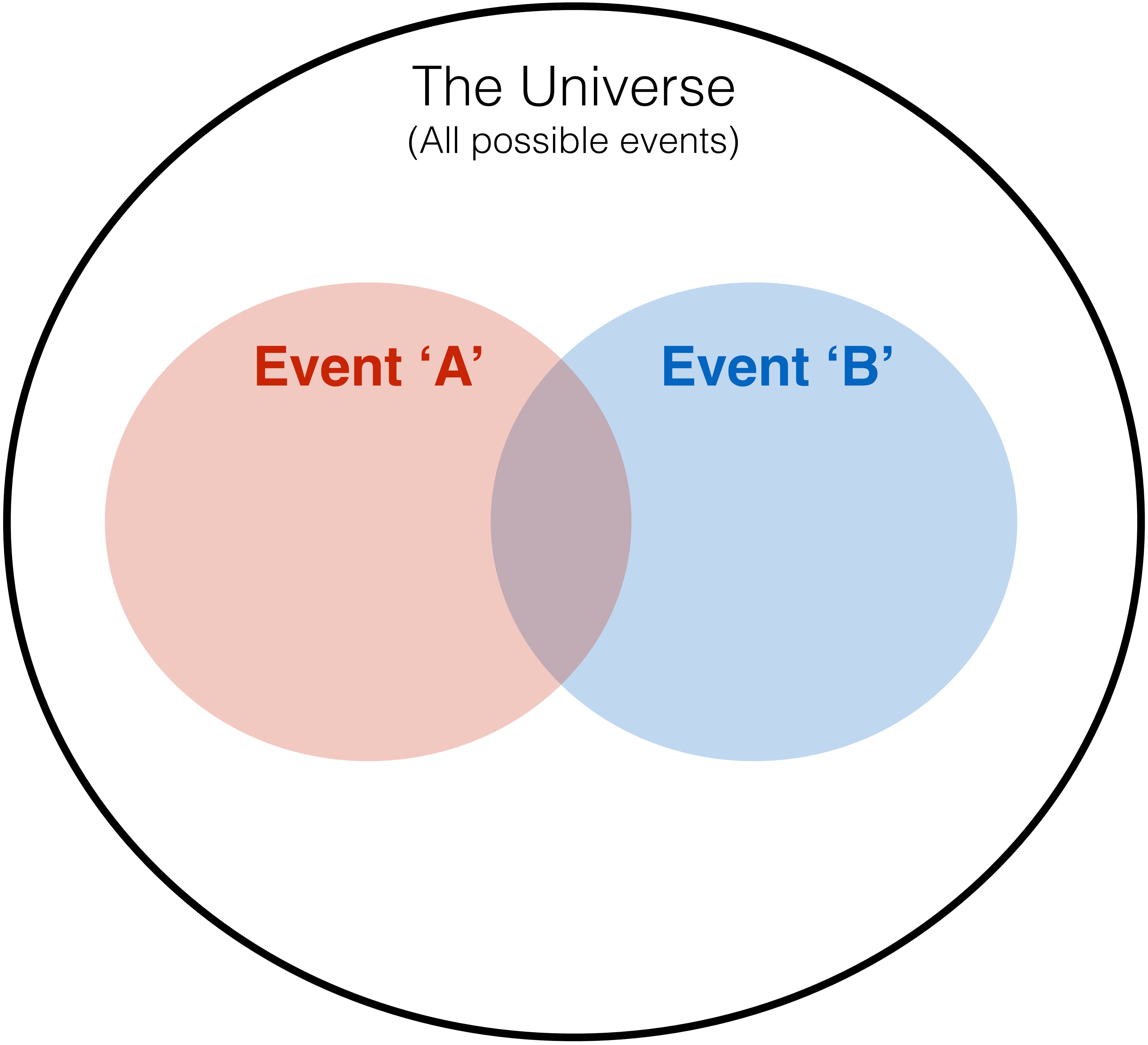
$$P(b,a) = P(a,b)$$

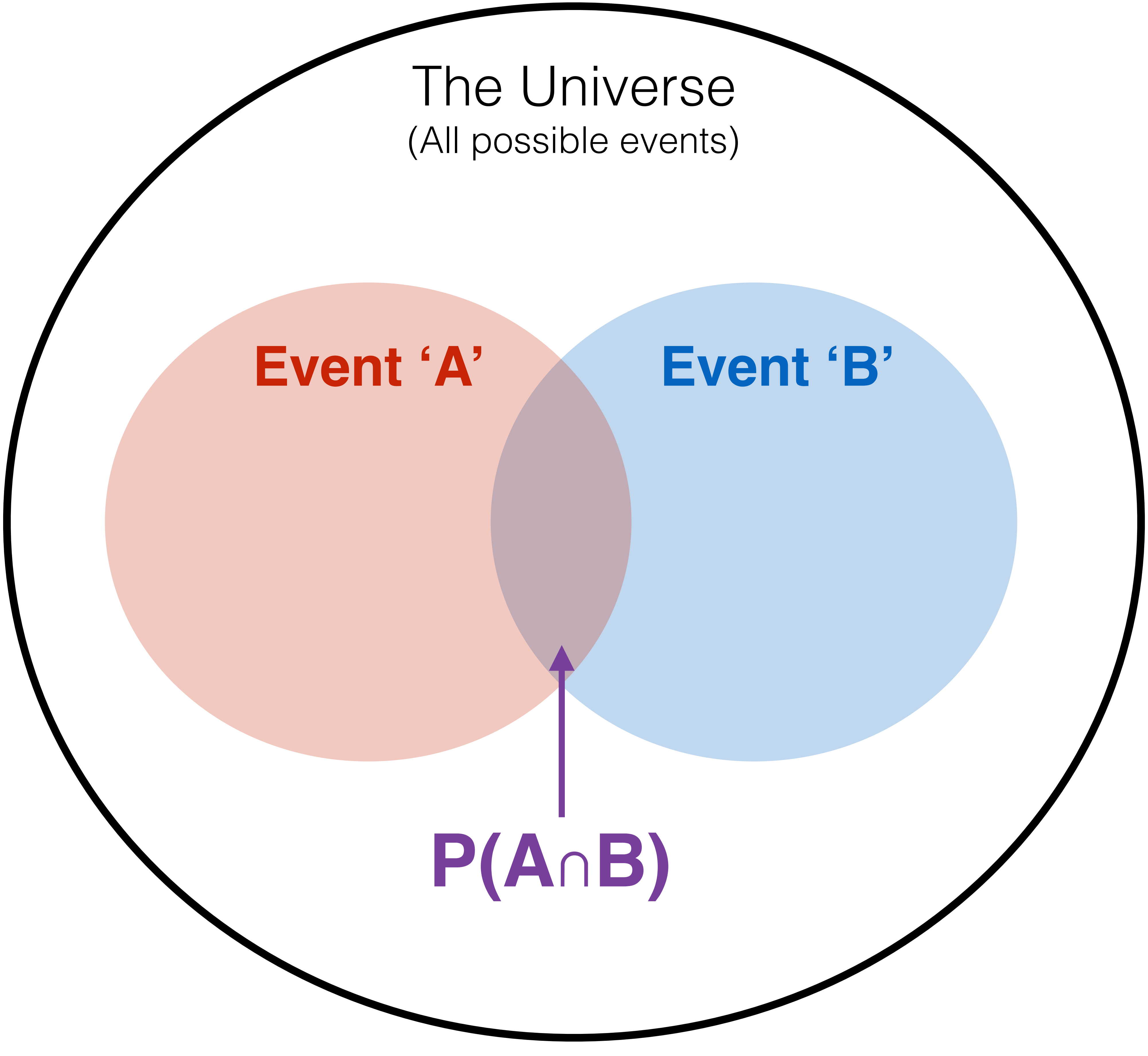
$$\boxed{P(a|b) = \frac{P(b|a)P(a)}{P(b)}}$$

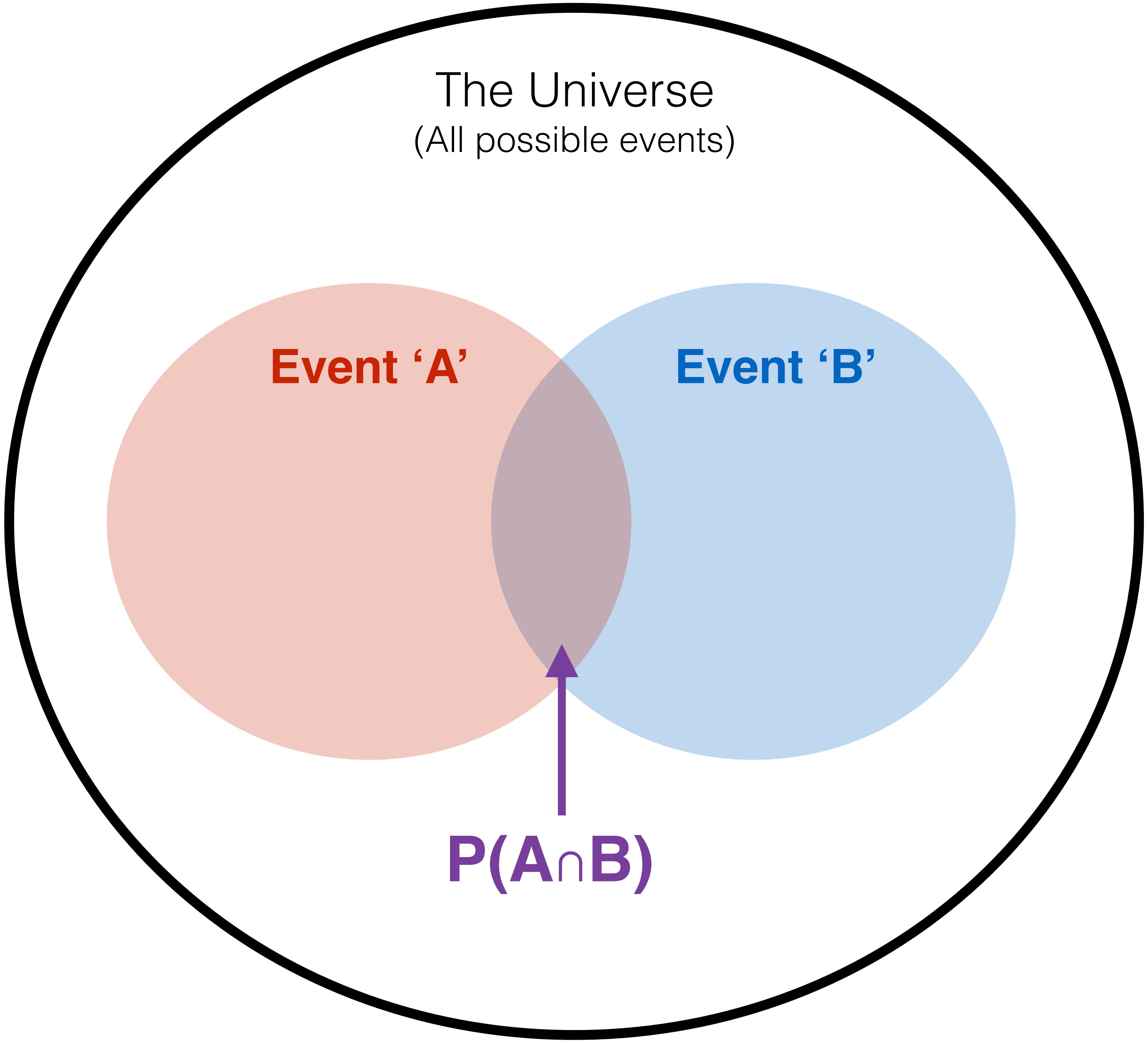
Bayes's Theorem


The Universe
(All possible events)


The Universe
(All possible events)
Event 'B'

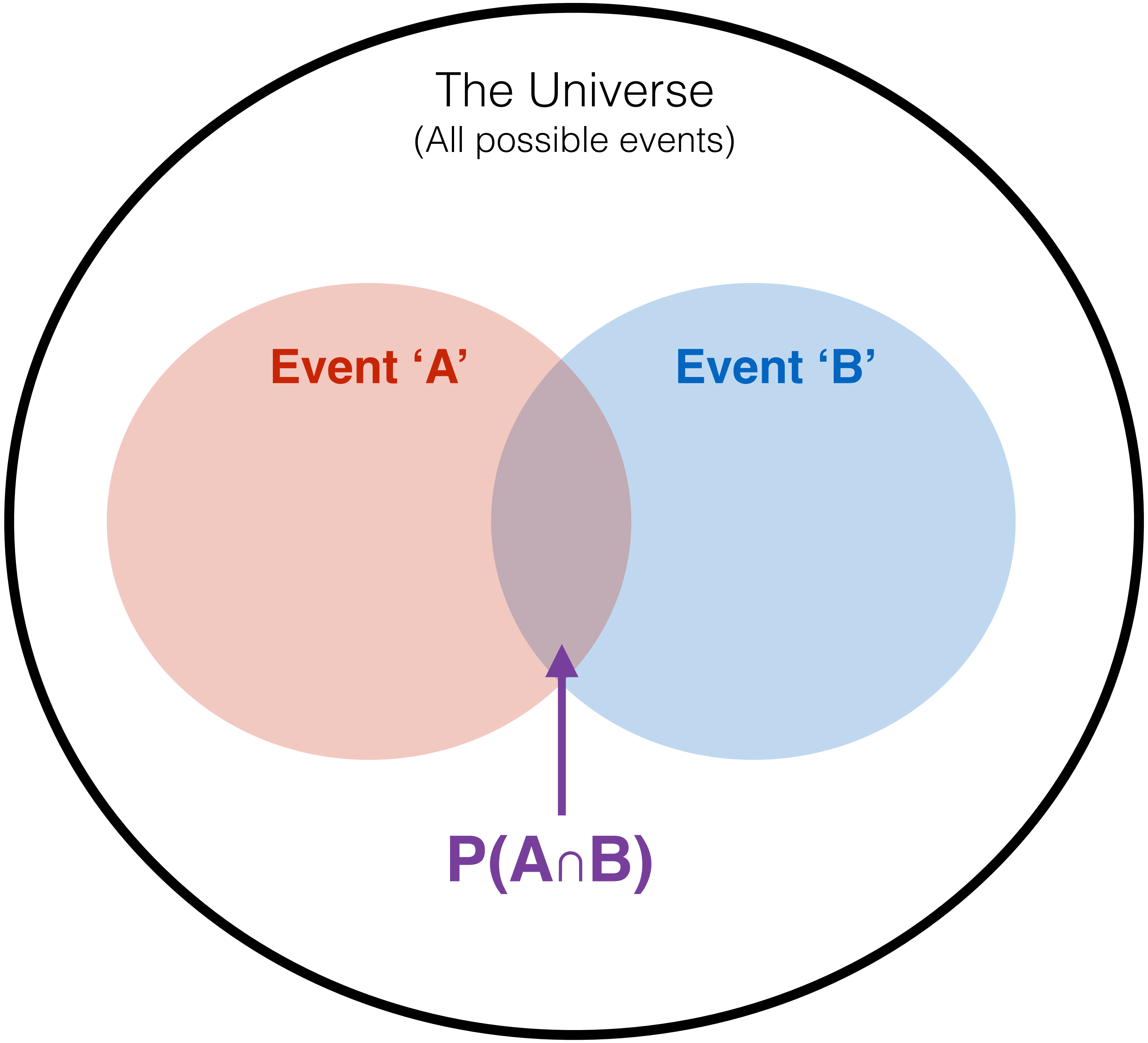

The Universe
(All possible events)
Event ‘A’
Event ‘B’


The Universe
(All possible events)
Event 'A'
Event 'B'
P(A∩B)



1. $P(A|B) = P(A \cap B) / P(B)$

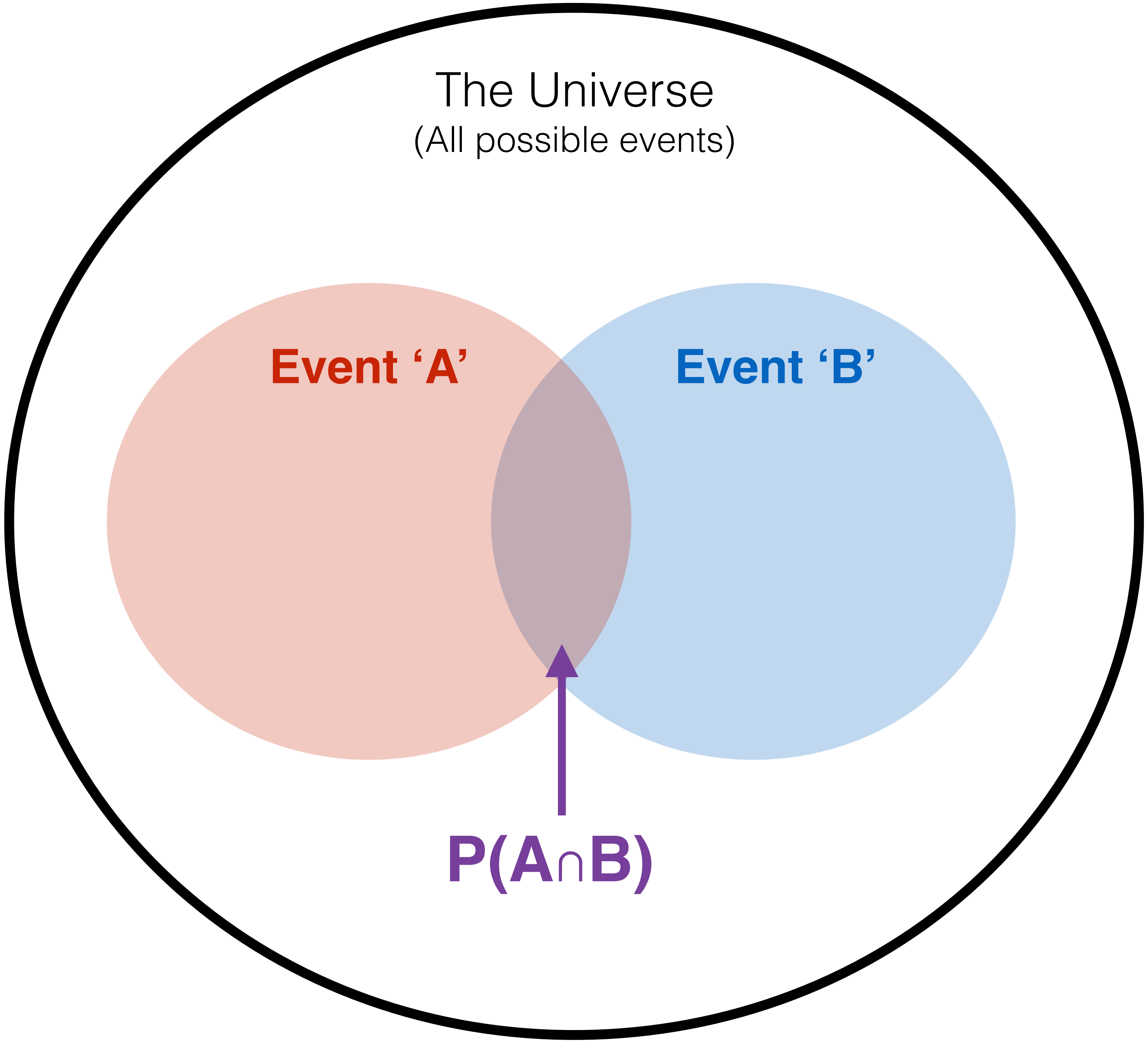
P(B) is our new 'Universe'



1. P(A|B) = P(A∩B) / P(B)

P(B) is our new ‘Universe’

2. P(B|A) = P(B∩A) / P(A)

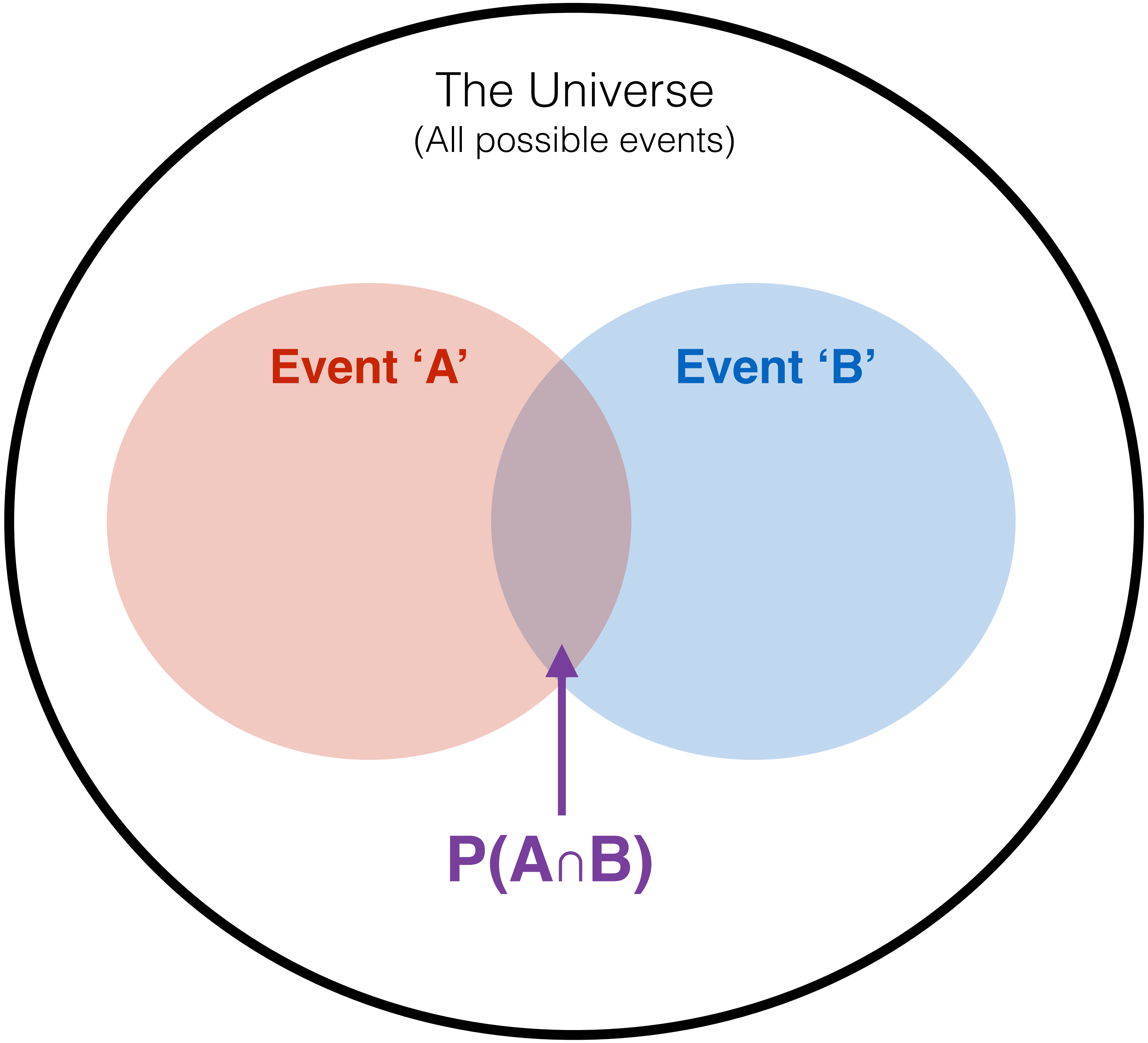


1. $P(A|B) = P(A \cap B) \,/\, P(B)$

P(B) is our new ‘Universe’

2. $P(B|A) = P(B \cap A) \,/\, P(A)$

3. $P(B \cap A) = P(A \cap B)$



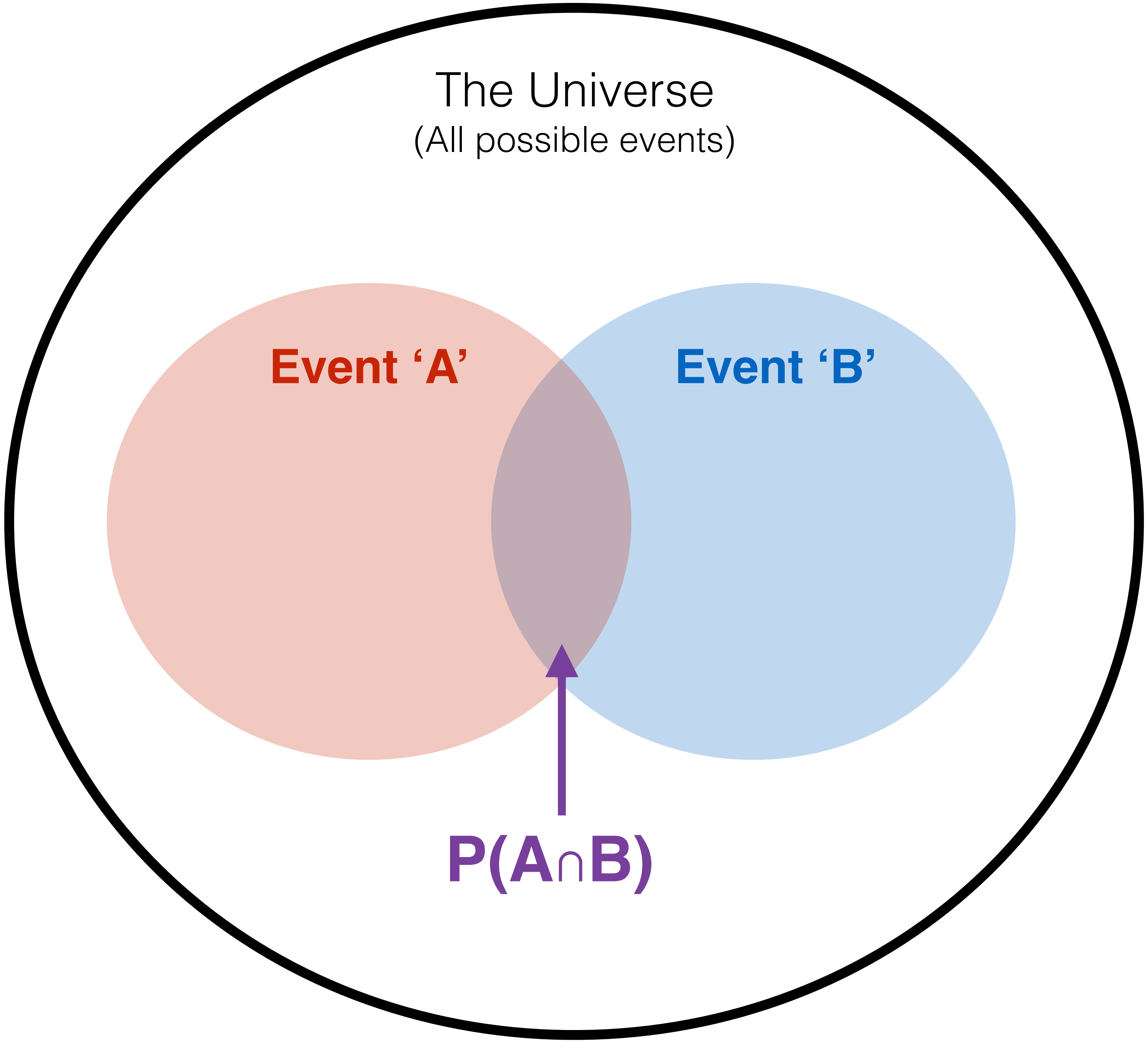
1. $P(A|B) = P(A \cap B) / P(B)$

P(B) is our new 'Universe'

2. $P(B|A) = P(B \cap A) / P(A)$

3. $P(B \cap A) = P(A \cap B)$

4. $P(A|B) = P(A \cap B) / P(B)$
$= P(B|A)\, P(A) / P(B)$



1. P(A|B) = P(A∩B) / P(B)

P(B) is our new 'Universe'

2. P(B|A) = P(B∩A) / P(A)

3. P(B∩A) = P(A∩B)

4. P(A|B) = P(A∩B) / P(B)
= P(B|A) P(A) / P(B)

5. P(A|B) = P(B|A) P(A) / P(B)

The Monty Hall Problem

(1963 - present)

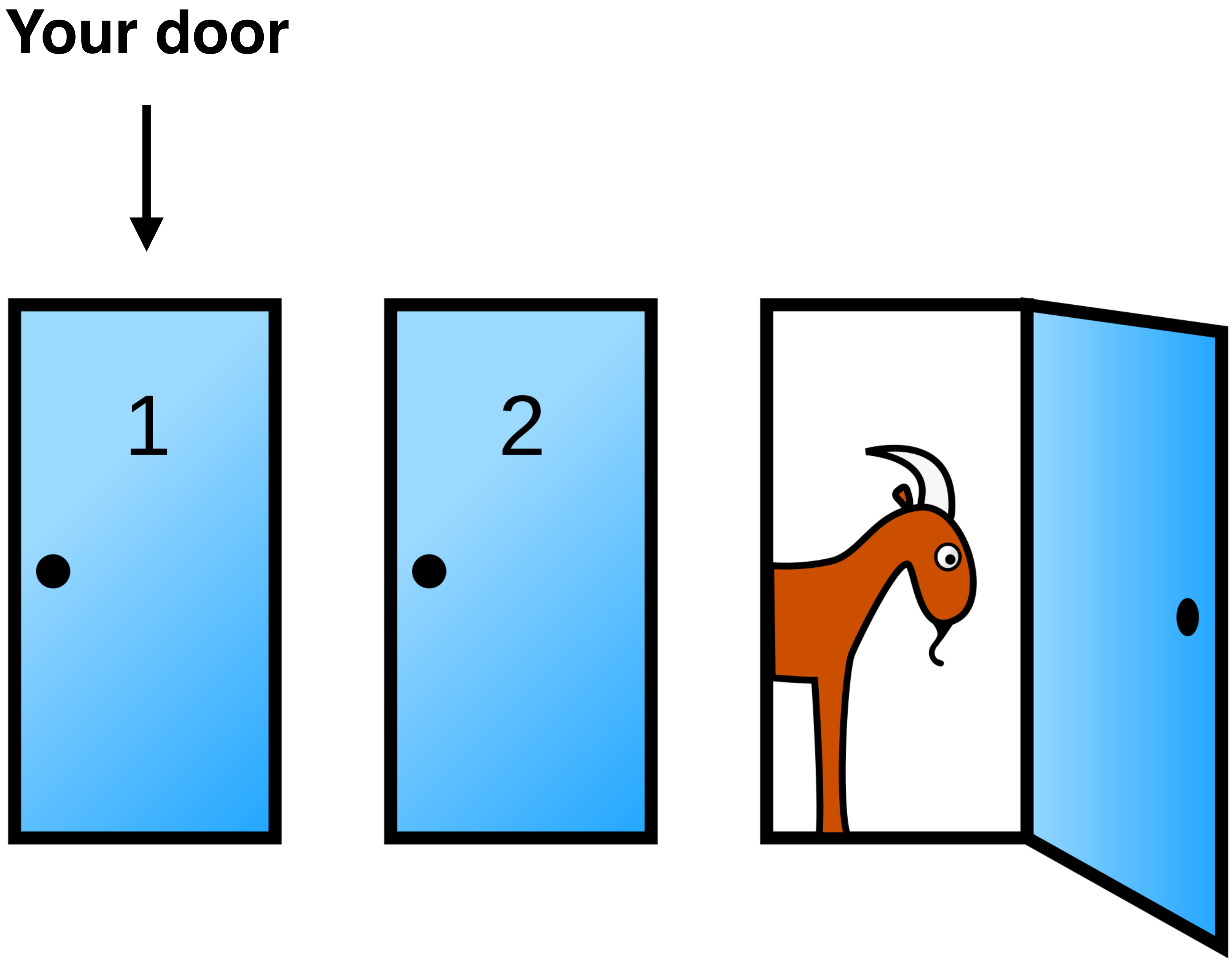
Rules

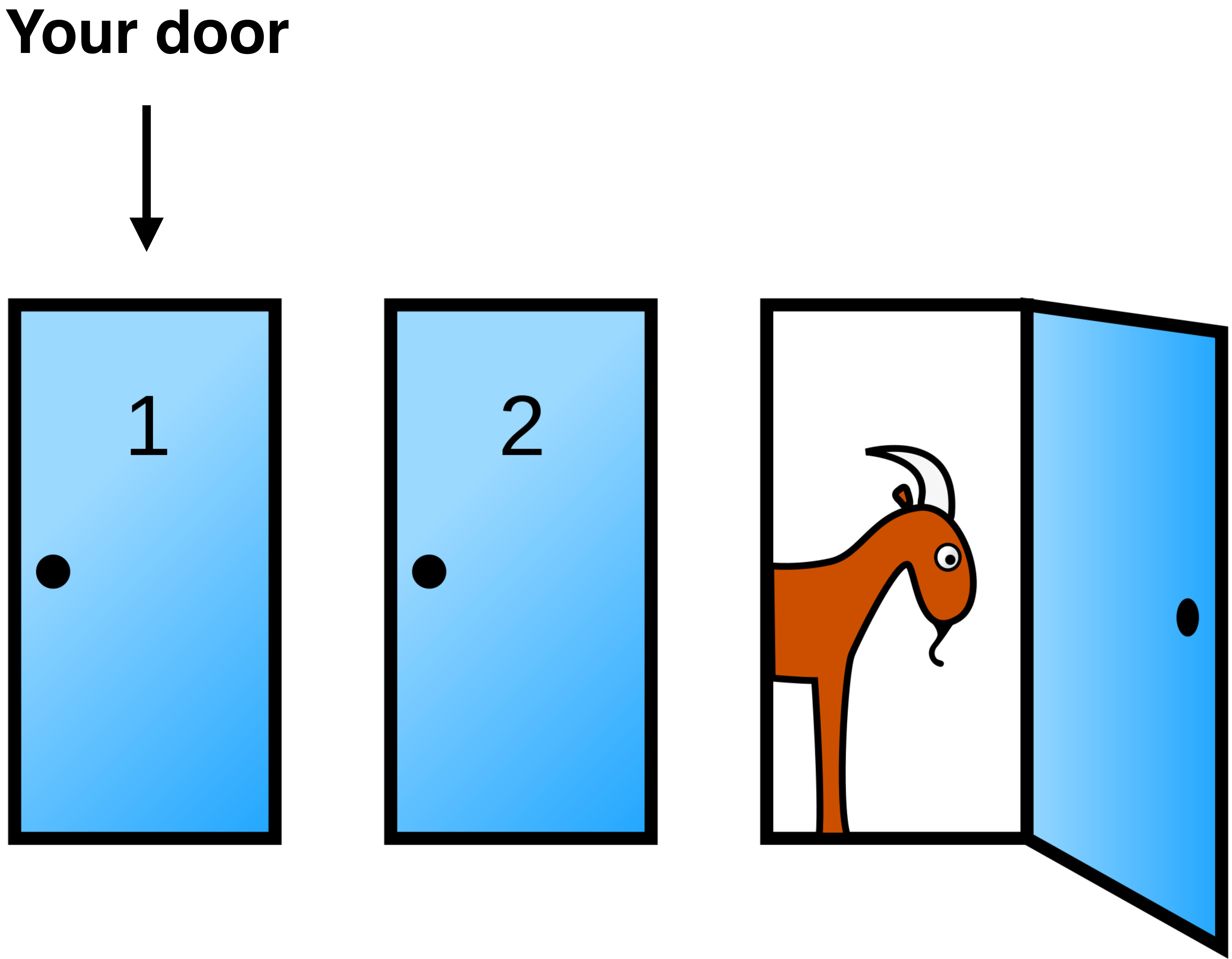
3 doors

3 prizes — 1 behind each door
(goat, hair extensions, new car)

Rules

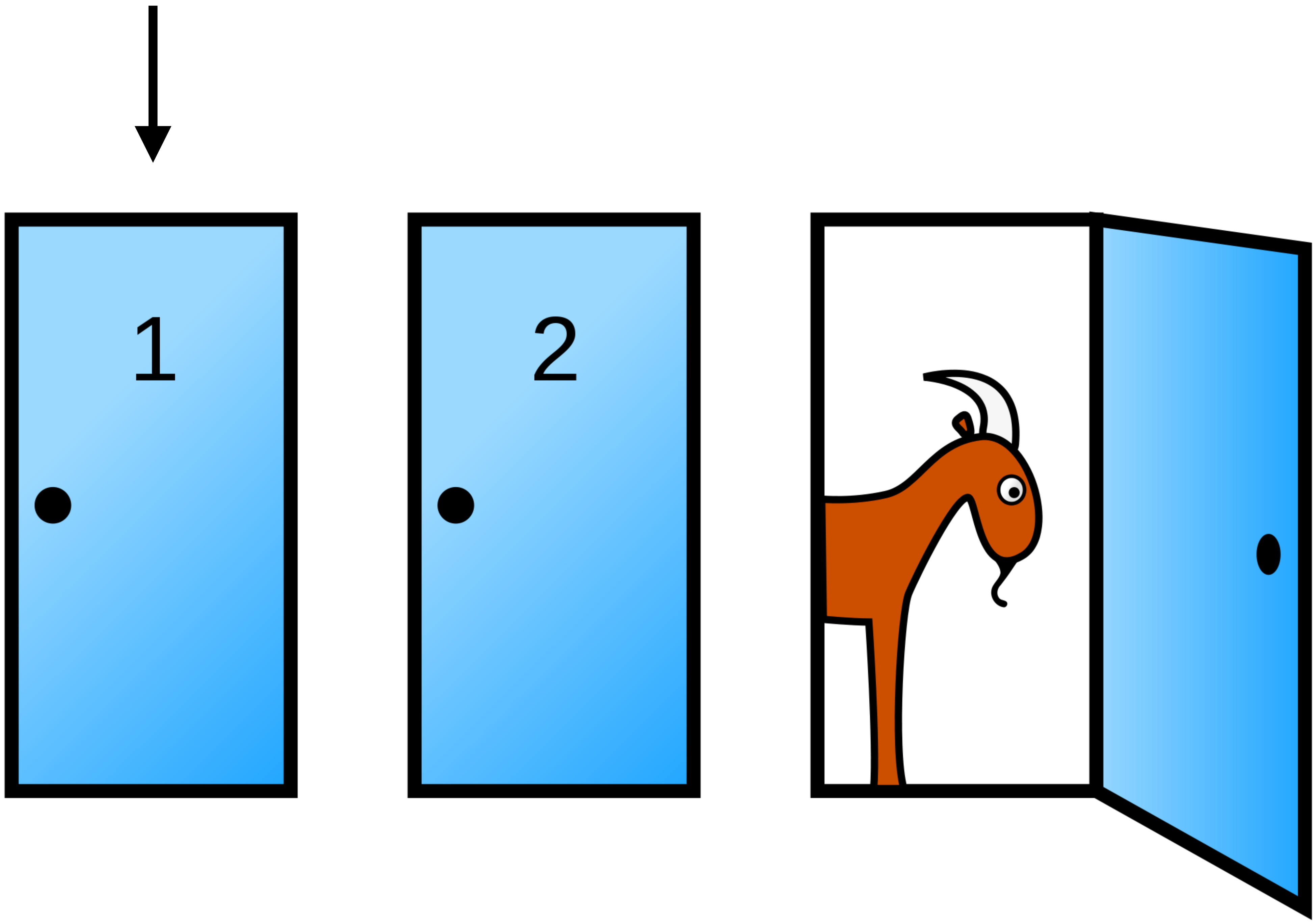
You select **door 1**.
Monty opens one of the remaining doors…


Your door
1
2



Monty asks: stay with door 1 or switch to door 2?

Your door



Re-phrase: is it more probable that the car is behind door 1 or door 2? Or doesn't it matter?

How does Bayes's Theorem help?

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Data:

1. You chose door 1.
2. Monty opened door 3 **and** there was no car.

Data:

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Likelihood:

Data:

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Likelihood:

Data:

1. You chose door 1.
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Likelihood:

1. If car behind door 1, probability of Monty choosing door 3 is 1/2.

Data:

1. You chose door 1.
2. Monty opened door 3 **and** there was no car.

Likelihood:

1. If car behind door 1, probability of Monty choosing door 3 is 1/2.
2. If car is behind door 3, Monty has to open door 2. So probability he opens 3 and finds no car is 1.

Data:

1. You chose door 1.
2. Monty opened door 3 **and** there was no car.

Likelihood:

1. If car behind door 1, probability of Monty choosing door 3 is 1/2.
2. If car is behind door 3, Monty has to open door 2. So probability he opens 3 and finds no car is 1.
3. If car is behind door 2, Monty opens door 3 with a probability of 1, and finds no car w/ probability of 1.

Data:

1. You chose door 1.
2. Monty opened door 3 **and** there was no car.

Likelihood:

1. If car behind door 1, probability of Monty choosing door 3 is 1/2.
2. If car is behind door 3, Monty has to open door 2. So probability he opens 3 and finds no car is 1.
3. If car is behind door 2, Monty opens door 3 with a probability of 1, and finds no car w/ probability of 1.

I have data.

I have data.

I have a model.*

* often based on physics

I have data.

I have a model.*

I want to learn about my parameters of my model (and/or physics) based on my data.

*** often based on physics**

Model Fitting

I have data.

I have a model.*

I want to learn about my parameters of my model (and/or physics) based on my data.

* often based on physics

Bayesian Approach to Model Fitting

$$P(\theta|D) = \frac{P(D|\theta)\,P(\theta)}{P(D)}$$

Bayesian Approach to Model Fitting

D = Data
Theta = Model parameters

$$P(\theta|D) = \frac{P(D|\theta)\,P(\theta)}{P(D)}$$

Bayesian Approach to Model Fitting

D = Data
Theta = Model parameters

$$\boxed{P(\theta|D)} = \frac{P(D|\theta)\,P(\theta)}{P(D)}$$

Probability of Theta conditioned on (or given) on Data
“Posterior Probability Distribution Function”
aka
“The Posterior”

Bayesian Approach to Model Fitting

D = Data
Theta = Model parameters

$$P(\theta|D) = \frac{\boxed{P(D|\theta)}\, P(\theta)}{P(D)}$$

Probability of Data conditioned on (or given) Theta
"**Likelihood Function**" or "**Data Model**"
aka
"**The Likelihood**"

Bayesian Approach to Model Fitting

D = Data
Theta = Model parameters

$$P(\theta|D) = \frac{P(D|\theta)\, P(\theta)}{P(D)}$$

Marginal or Prior Probability(ies) for Theta
aka
“The Prior(s)”

Bayesian Approach to Model Fitting

D = Data
Theta = Model parameters

$$P(\theta|D) = \frac{P(D|\theta)\,P(\theta)}{\boxed{P(D)}}$$

Probability of the Data
or
“The Evidence”

Bayesian Approach to Model Fitting

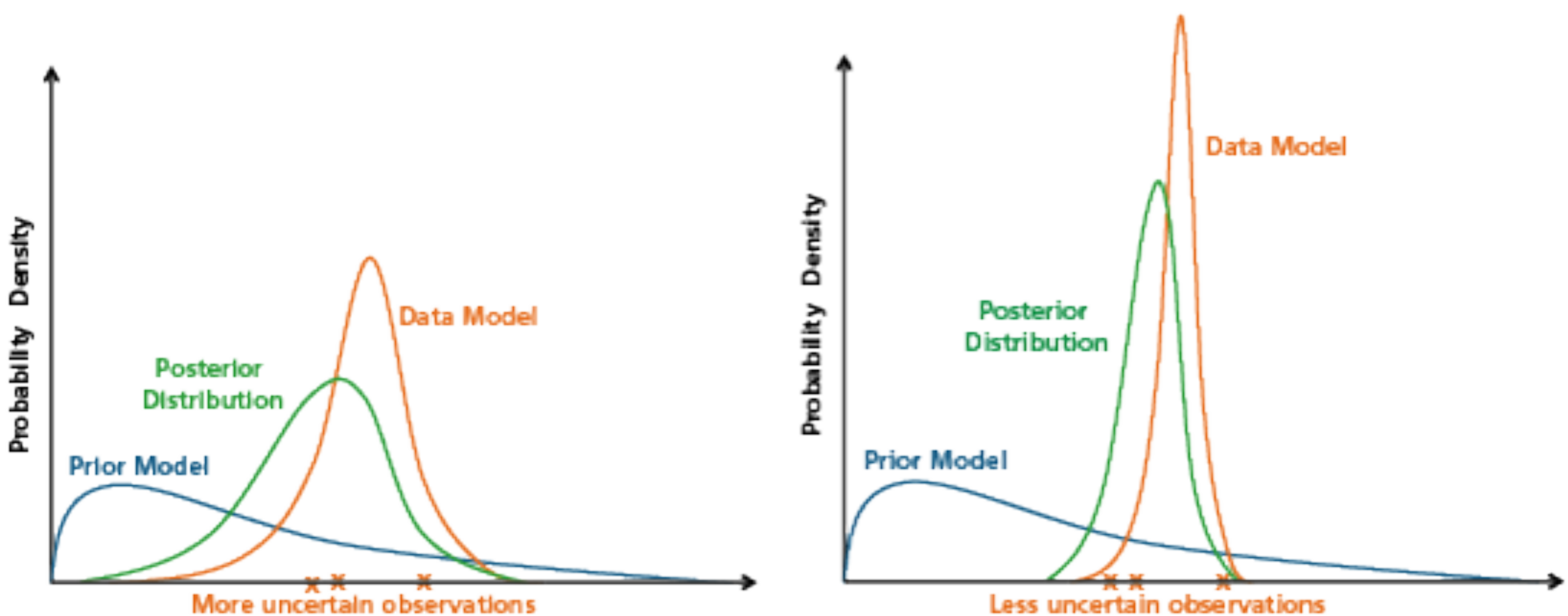
$$\underbrace{P(\theta|D)}_{\text{Posterior}} = \frac{\overbrace{P(D|\theta)}^{\text{Likelihood}}\ \overbrace{P(\theta)}^{\text{Prior}}}{\underbrace{P(D)}_{\text{Evidence}}}$$

D = Data

Theta = Model parameters

Updating Beliefs: Graphically

$$P(\theta|D) = \frac{P(D|\theta)\, P(\theta)}{P(D)}$$




5 DAY FORECAST
The Weather Channel
FRI
90%
60
SAT
59
48
SUN
99%
51
44
MON
78%
38
29
TUE
80
75