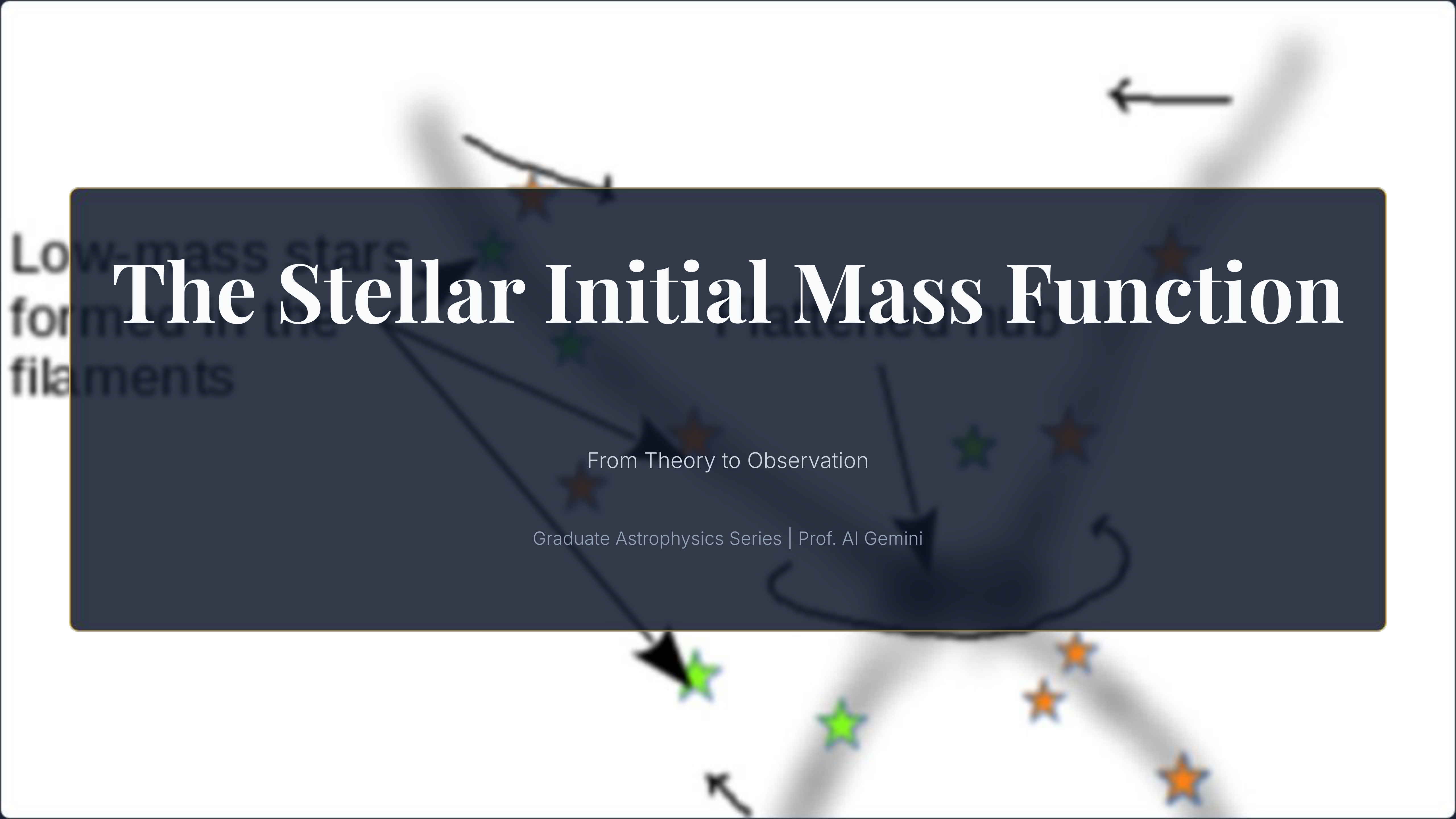
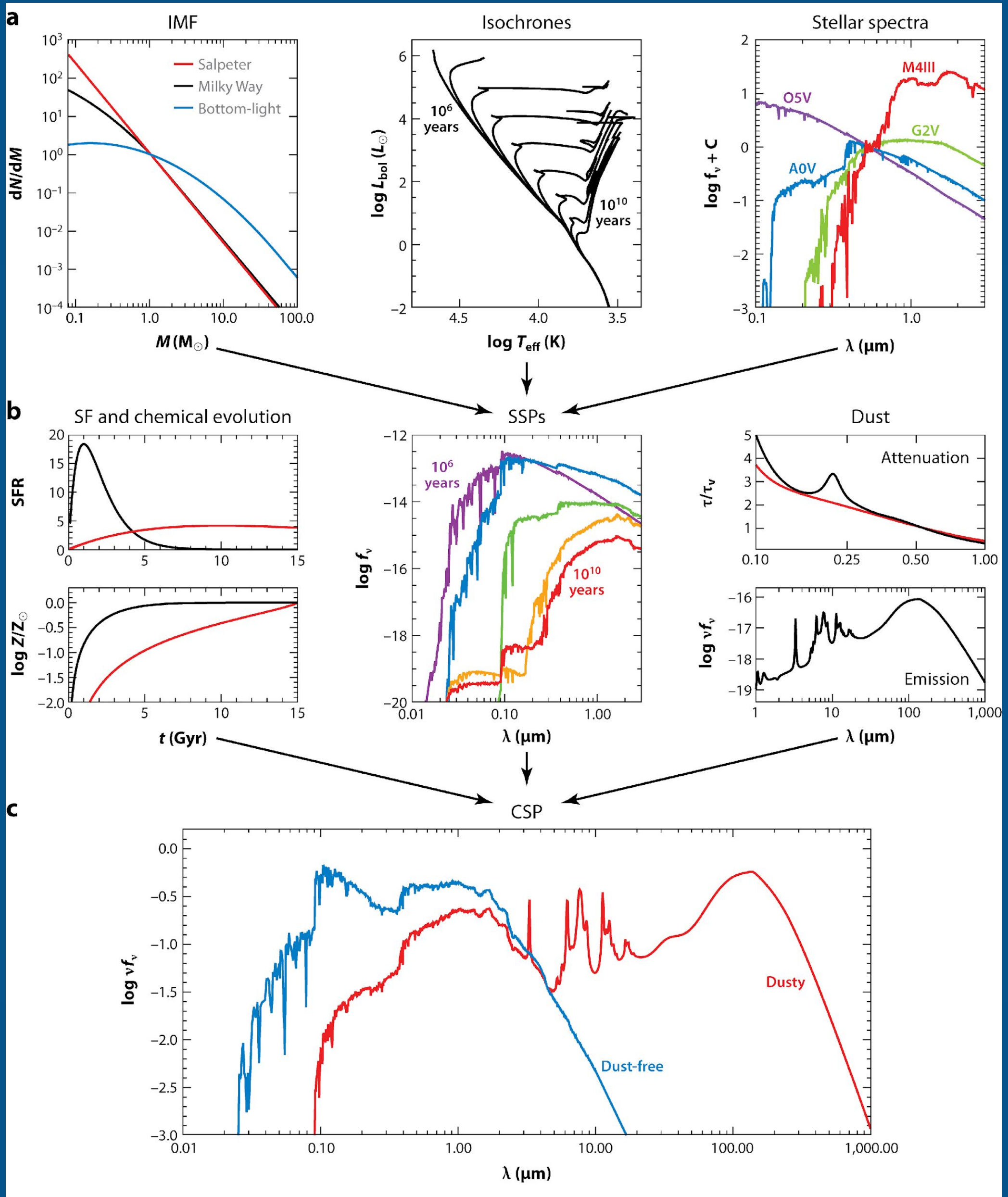


The Stellar Initial Mass Function

From Theory to Observation

Graduate Astrophysics Series | Prof. AI Gemini



Conroy C. 2013.
Annu. Rev. Astron. Astrophys. 51:393–455

Figure 1

from

Conroy (2013)

The Fundamental Scaling Law

The Initial Mass Function (IMF) ξm describes the distribution of mass for a population of newly formed stars.

$$dN = \xi m dm \propto m^{-\alpha} dm$$

Where α represents the slope of the power law.

Salpeter, E. E. (1955), ApJ, 121, 161

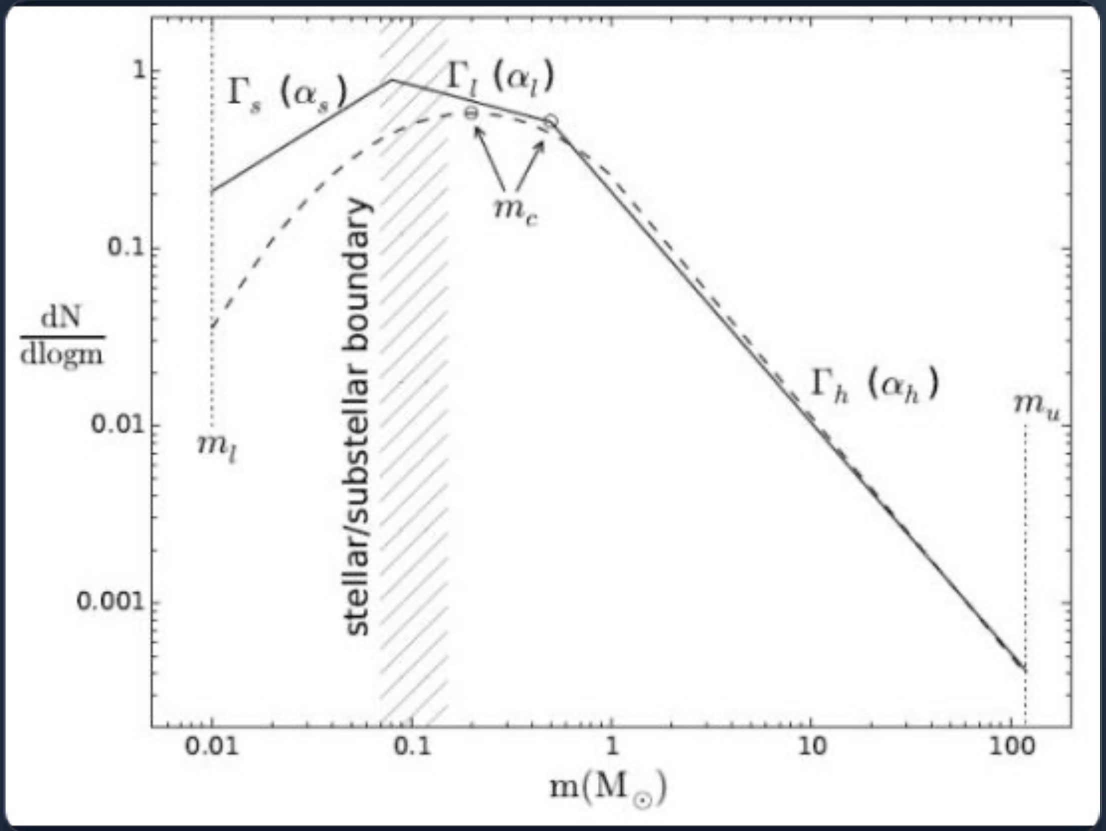
Historical Context: Salpeter (1955)

The First Derivation

Edwin Salpeter's seminal work established the first empirical derivation of the IMF using stars in the Solar neighborhood.

$$\xi m \approx 0.03 \frac{m}{M_{\odot}}^{-2.35}$$

- Salpeter Slope: $\alpha = 2.35 \quad m > 0.4M_{\odot}$
- Derived from the present-day luminosity function (PDLF).



Salpeter, E. E. (1955), ApJ, 121, 161

Modern Parametrizations

Kroupa (2001)

A multi-segment power law that accounts for the flattening at lower masses.

$$\alpha = \begin{cases} 0.3 & \text{for } m < 0.08 \\ 1.3 & \text{for } 0.08 \leq m < 0.5 \\ 2.3 & \text{for } m \geq 0.5 \end{cases}$$

Chabrier (2003)

Uses a log-normal distribution for low masses and a power law for the high-mass tail.

$$\xi m \propto \frac{1}{m} \exp \frac{- \log m - \log m_c{}^2}{2\sigma^2}$$

Typically and . $m_c \approx 0.2 M_\odot$ $\sigma \approx 0.55$

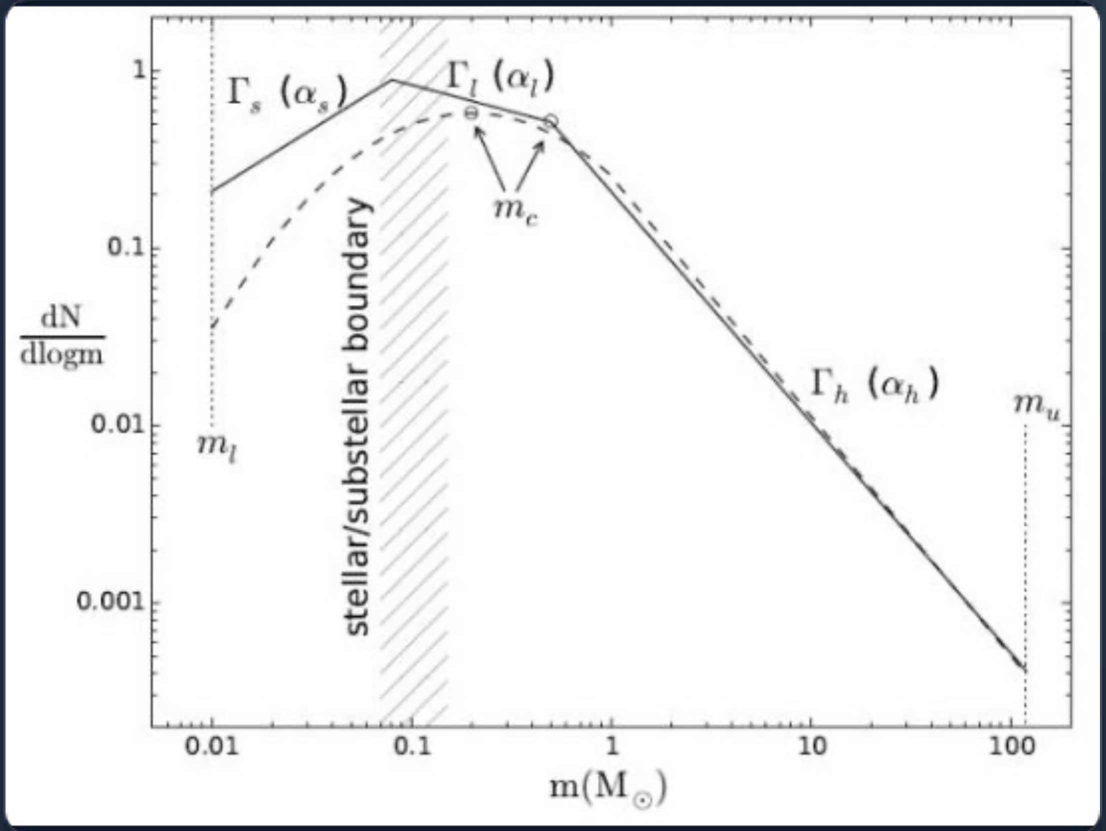
Kroupa, P. (2001), MNRAS, 322; Chabrier, G. (2003), PASP, 115

The Universality Consensus

Bastian, Covey & Meyer (2010)

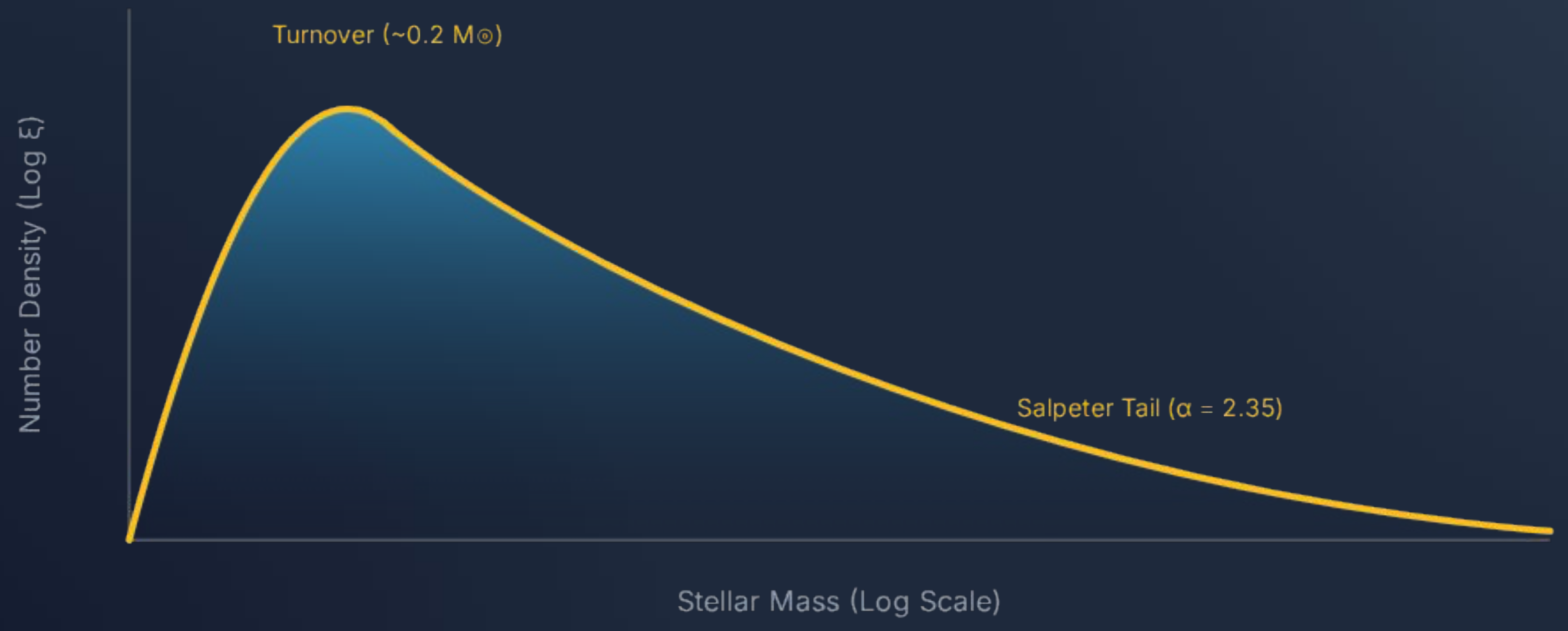
In a comprehensive Annual Review, Bastian et al. assessed IMF measurements across diverse environments, from the Galactic Center to the field.

- **Key Conclusion:** There is "no clear evidence" for systematic variation in the IMF.
- **Scatter Explanation:** Observed variations are consistent with stochastic sampling effects, dynamical evolution, and observational uncertainties (e.g., binaries).
- **Universality:** The IMF implies a robust formation mechanism independent of metallicity or density.



Bastian, N., Covey, K. R., & Meyer, M. R. (2010), ARA&A, 48, 339

The Shape of the Stellar IMF



The distribution rises to a peak around 0.2-0.5$M_\odot$ before following the classic Salpeter power-law decline.

Miller, G. E., & Scalo, J. M. (1979), ApJS, 41, 513

Origin Theories: Gravoturbulent Fragmentation

Jeans Mass

The thermal Jeans mass defines the scale where gravity overcomes thermal pressure. The IMF peak is often associated with the characteristic Jeans mass in molecular clouds.

Turbulence

Supersonic turbulence creates local density enhancements (shocks). The mass function of these "cores" may map directly onto the stellar IMF (Core Accretion Model).

Feedback

For high-mass stars, radiation pressure and outflows may limit accretion, potentially setting the upper mass limit ( ). M_{max}

Jeans (1902); Padoan & Nordlund (2002); Hennebelle & Chabrier (2008)

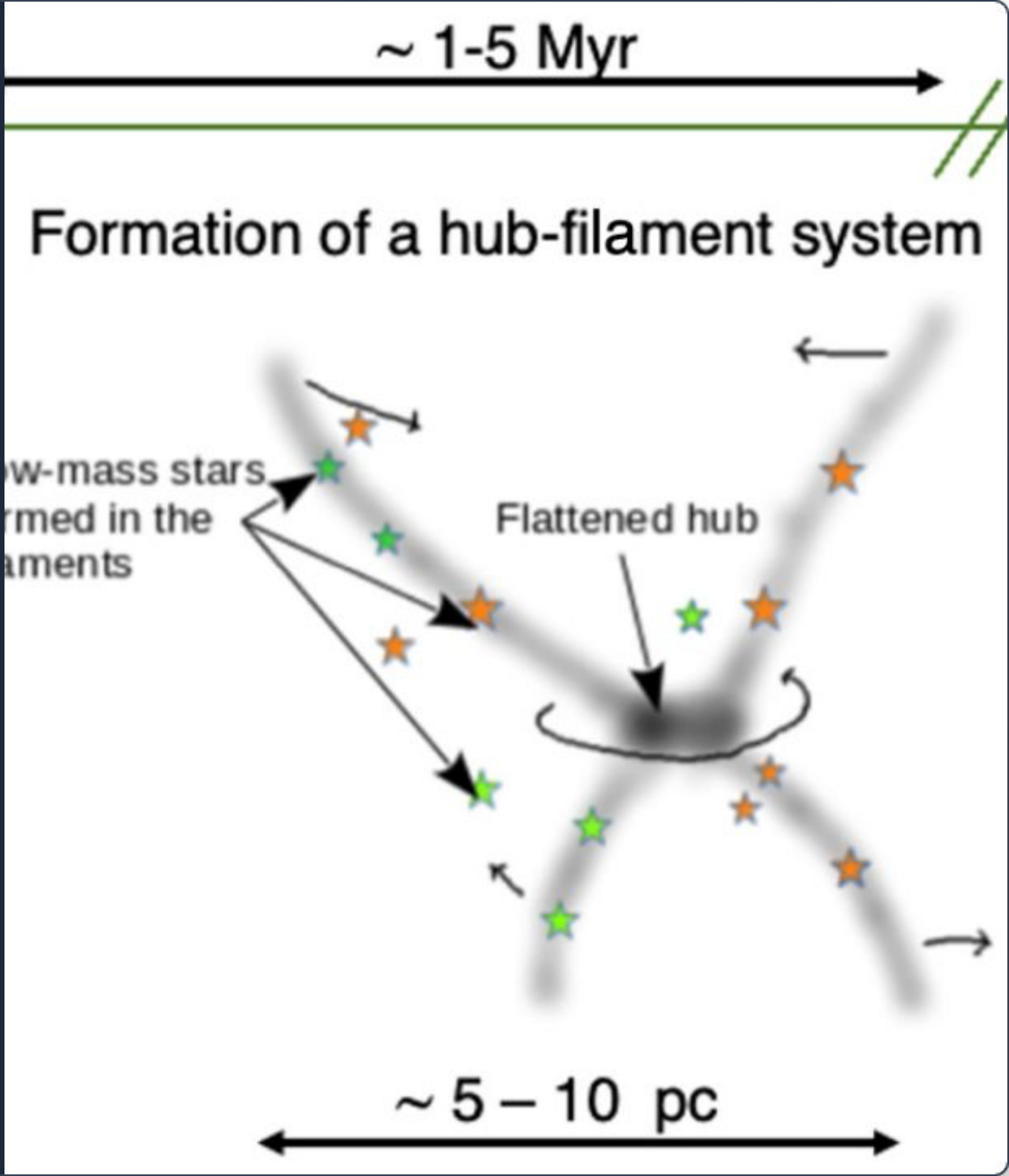
Competitive Accretion

The Dynamical View: Unlike core accretion, this model suggests stars compete for gas in a common potential well.

Stars form in clusters. Massive stars are not born massive; they become massive because they sit at the center of the potential well and accrete gas more efficiently than their siblings.

This model naturally predicts mass segregation and the power-law tail of the IMF.

Bate et al. (2012); Bonnell et al. (2004)



From PDMF to IMF

Deriving the Initial Mass Function from Observations of the Present Day Mass Function

Defining the Mass Functions

Initial Mass Function (IMF)

ξm

The distribution of stellar masses formed in a single star-formation event **at birth** (t=0). It represents the intrinsic outcome of the star formation process.

Present Day Mass Function (PDMF)

Φm

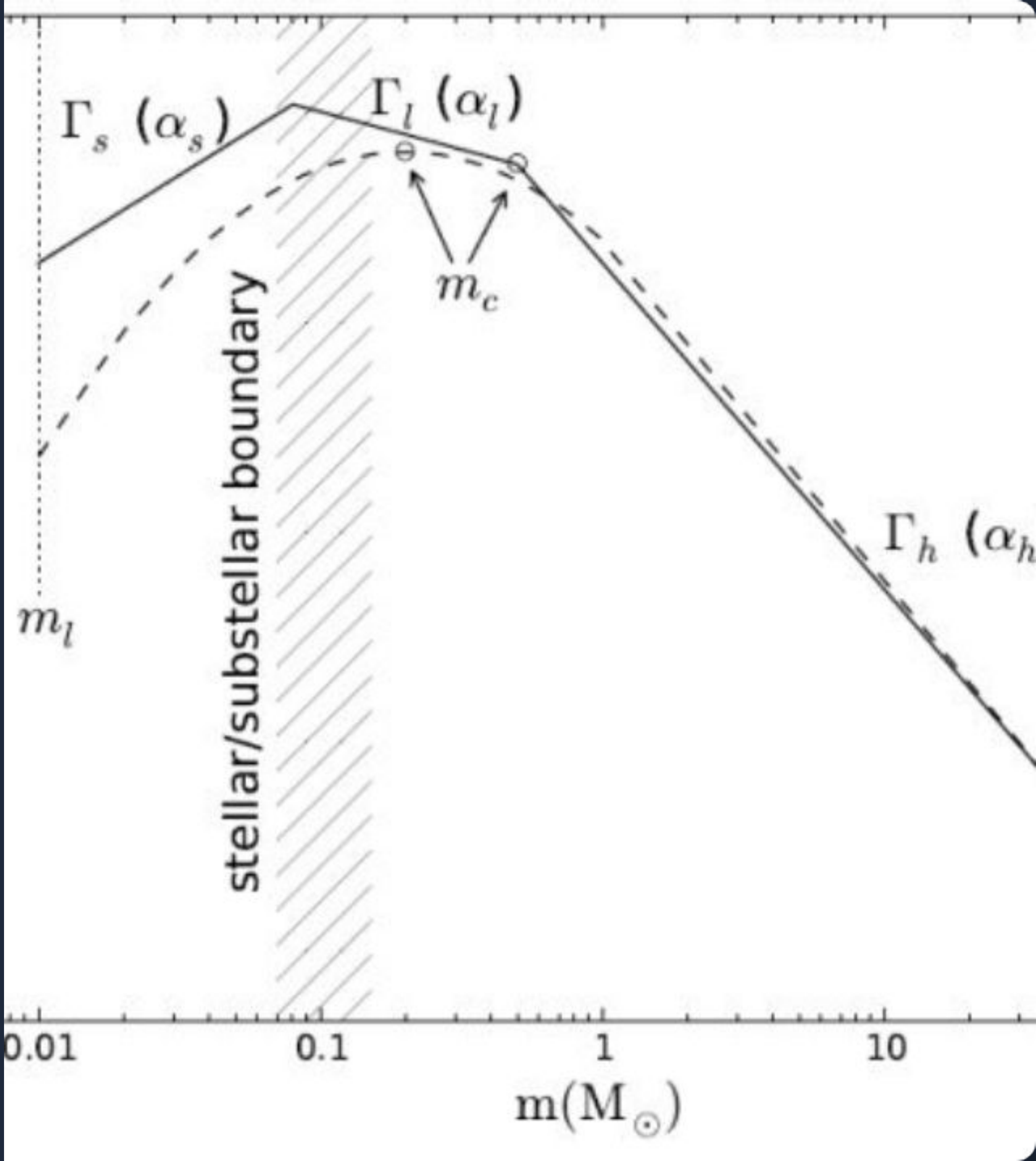
The distribution of stellar masses observed **today** (t=now) in the Main Sequence. This is what we actually count in the sky.

The "Missing" Stars

The IMF and PDMF are not identical because stars evolve.

Massive stars burn fuel quickly and die, disappearing from the Main Sequence. Observing the PDMF today misses these dead stars.

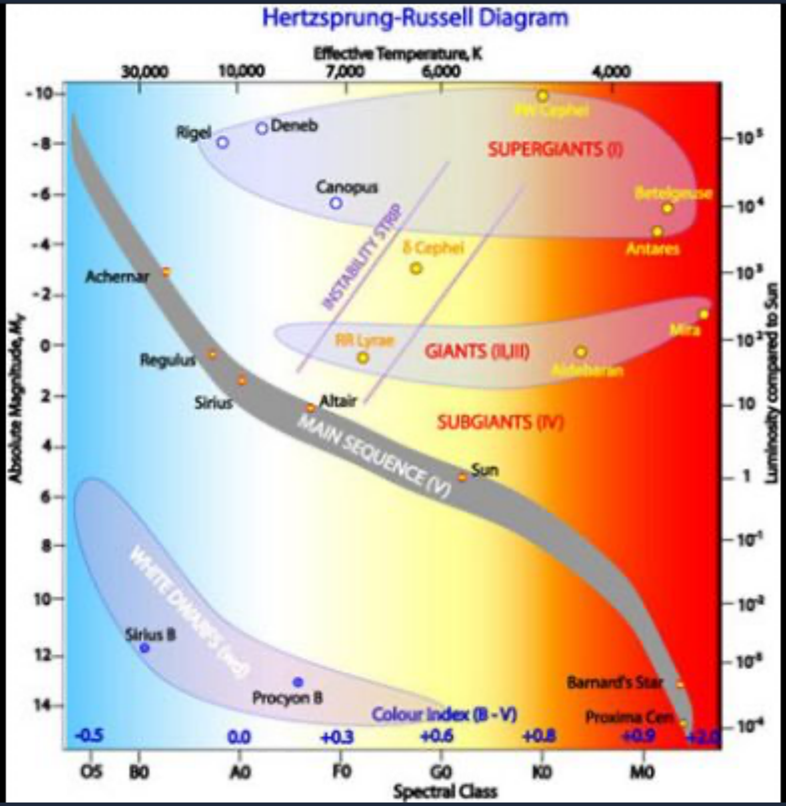
Low-mass stars live longer than the age of the Universe. For them, what we see today (PDMF) is effectively what was born (IMF).



Step 1: Observation (Star Counts)

We cannot "weigh" most stars directly. We observe them photometrically.

- **Observation:** Apparent Magnitudes (m_V) and Colors ($B - V$).
- **Tool:** The Color-Magnitude Diagram (CMD) or Hertzsprung-Russell (HR) Diagram.
- **Goal:** Identify Main Sequence stars and correct for distance (modulus) and extinction to get Absolute Magnitude (M_V).



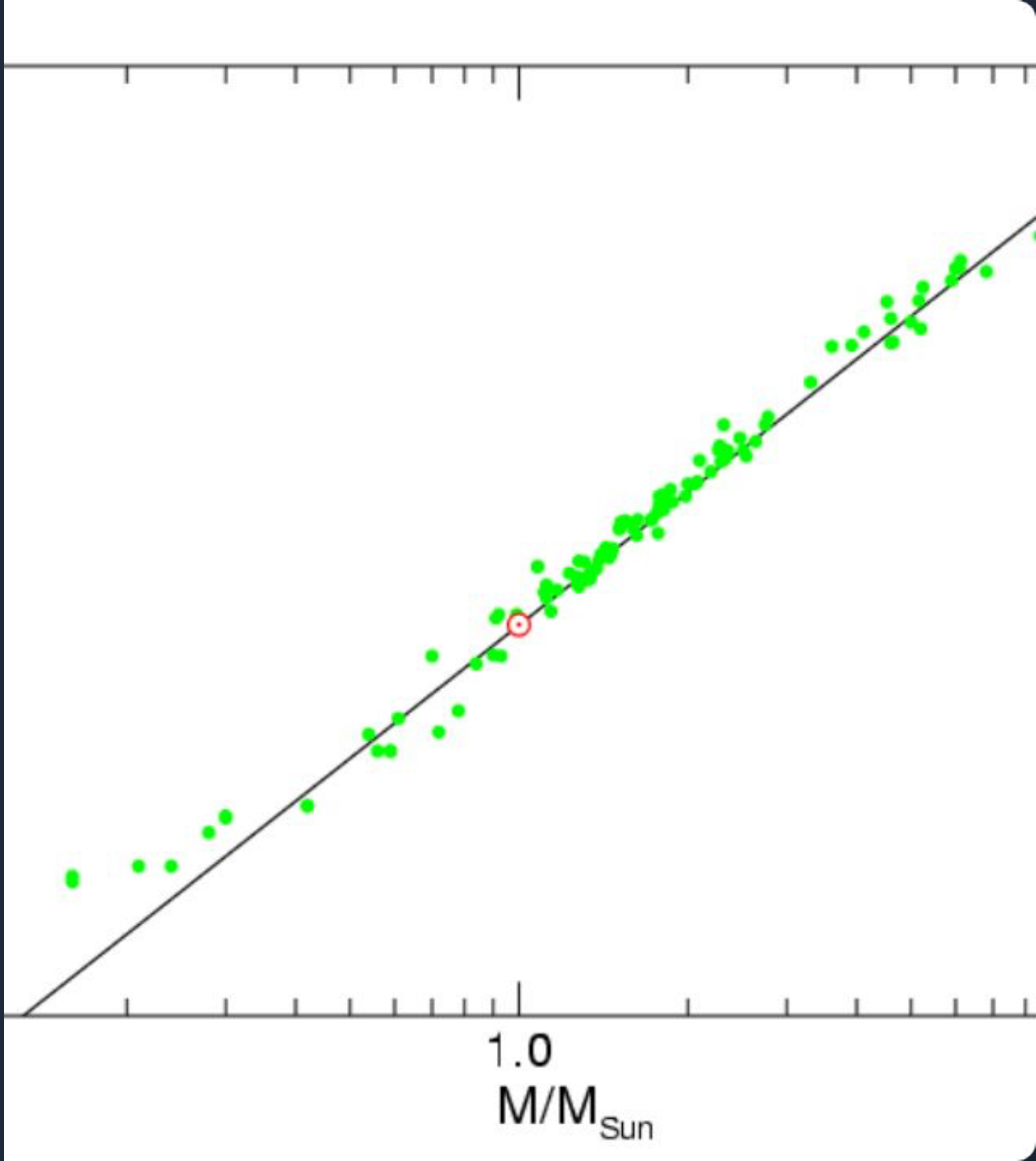
Step 2: Light to Mass

To convert the observed Luminosity Function (ΦM_V) into a Mass Function, we use the **Mass-Luminosity Relation (MLR)**.

For Main Sequence stars:

$$L \propto M^{3.5}$$

This empirical relation allows us to assign a mass to every observed star, yielding the PDMF.



The Evolutionary Divide

We must define a critical turn-off mass, M_{to}, corresponding to the age of the stellar population (T_0).

Mass $< M_{to}$

Lifetime $\tau m > T_0$

These stars have not yet evolved off the Main Sequence. Number is conserved.

PDMF $\simeq$ IMF

Mass $> M_{to}$

Lifetime $\tau m < T_0$

Many of these stars have already died. The observed number is a fraction of the total formed.

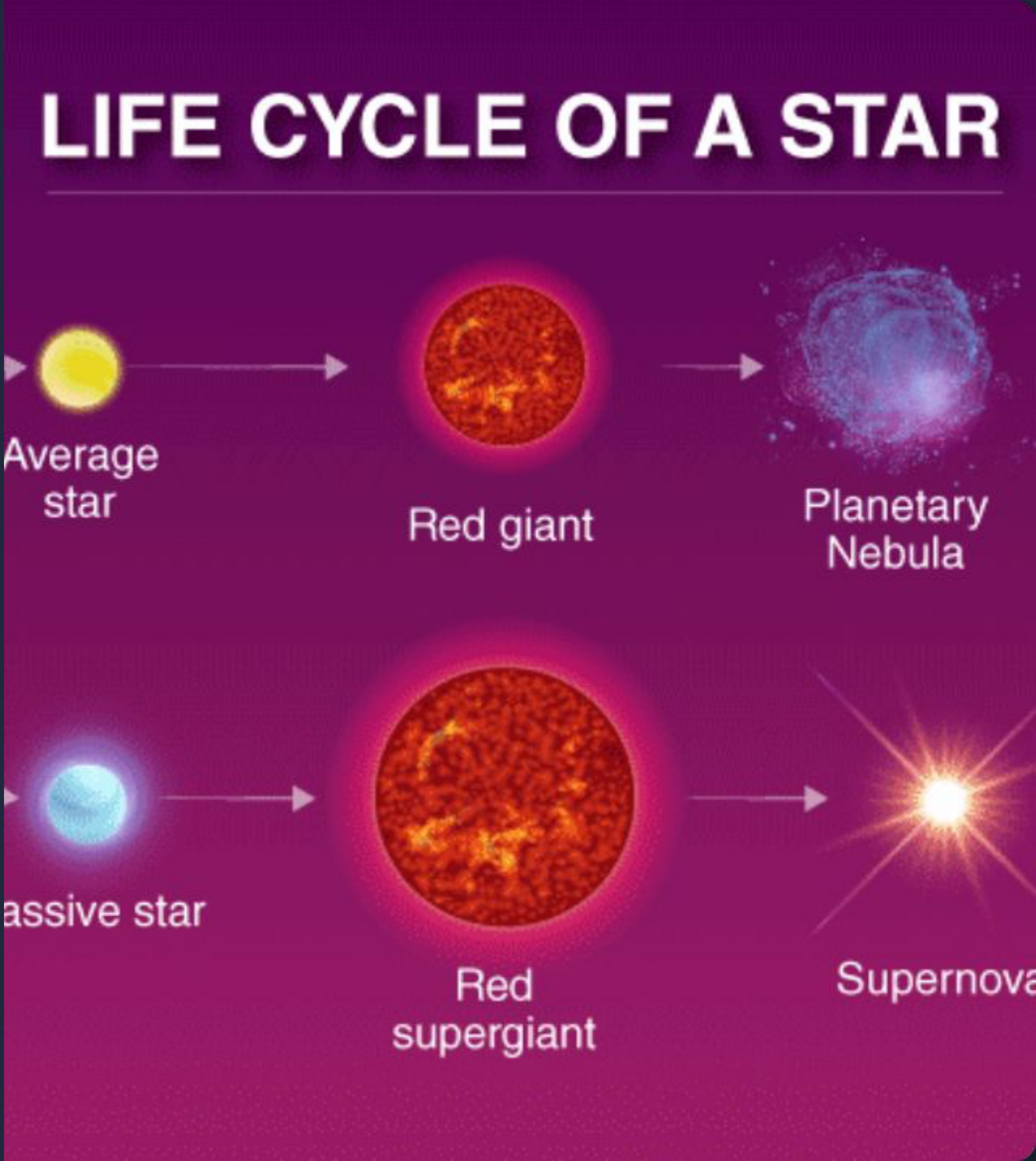
PDMF $\lesssim$ IMF

Correcting for Lifetimes

For massive stars, the PDMF depends on the **Star Formation Rate (SFR)**, denoted as bt.

We are only seeing stars formed recently enough to still be alive (within the last τm years).

If SFR is constant, the number of observed stars is proportional to their lifetime.



The Mathematical Relation

For a system with age T_0 and Star Formation History : bt

$$\Phi m = \xi m \times \int_{T_0 - \tau(m)}^{T_0} btdt$$

If bt is Constant:

$$\Phi m = \xi m \times \tau m$$

Therefore, the IMF is:

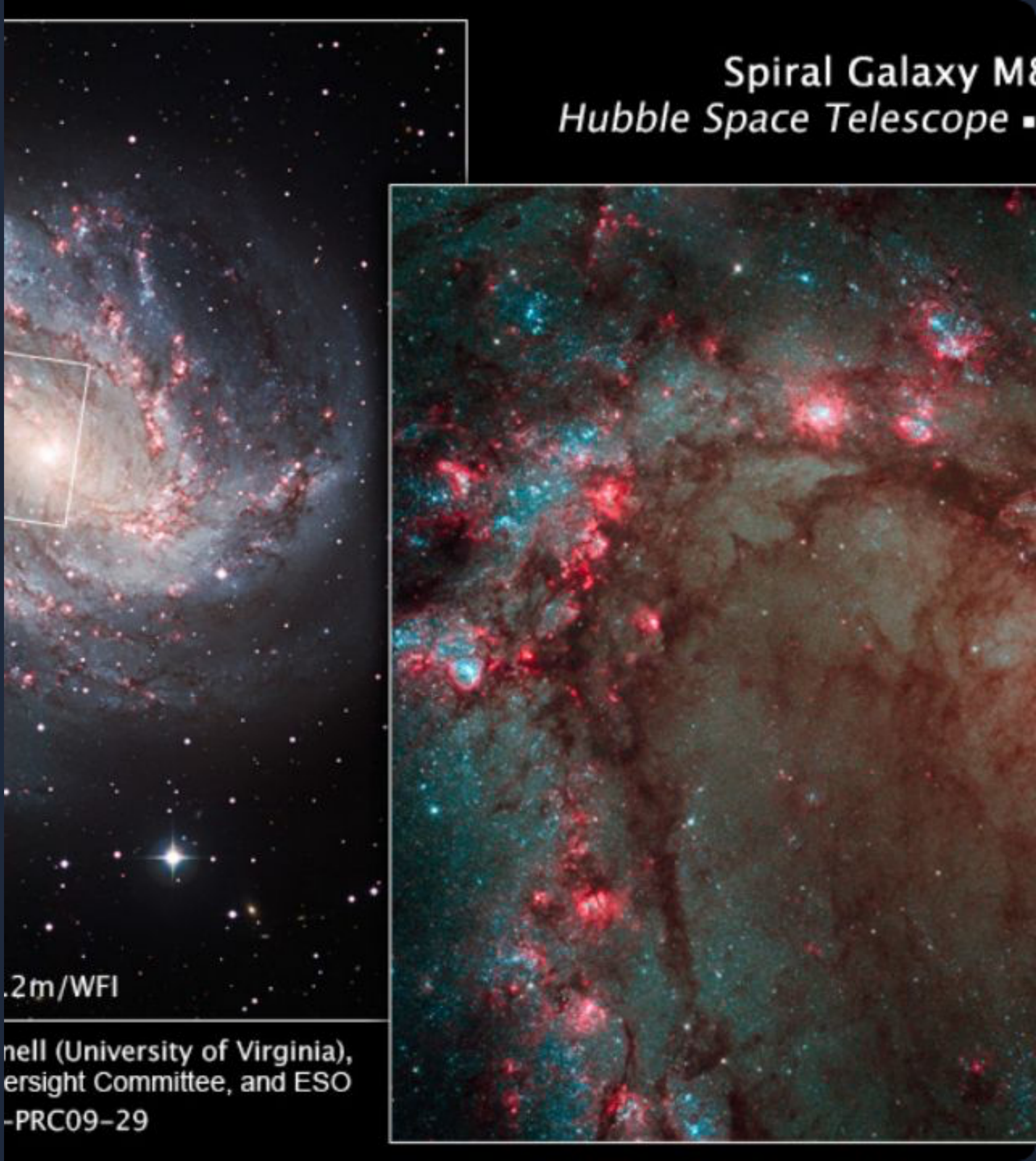
$$\xi m = \frac{\Phi m}{\tau m}$$

The Role of History

The "Constant SFR" assumption works for galactic disks on average, but not for starburst regions or old globular clusters.

Starburst: If all stars formed at once (single burst), massive stars are simply gone. We cannot derive the high-mass IMF directly from the PDMF in old bursts.

Continuous: In spiral galaxies, new stars are constantly replacing dead ones, allowing statistical derivation.



Major Challenges

Unresolved Binaries

Two stars close together look like one bright star. This makes low-mass stars look like single high-mass stars, flattening the observed slope.

Extinction

Dust obscures light, making stars look fainter and redder. Incorrect extinction corrections lead to incorrect masses.

Mass Segregation

In clusters, heavy stars sink to the center. Observing only the core or outskirts gives a biased PDMF.

Summary

- Observe Photometry (m_V, Color)
- Convert to Mass (PDMF) using MLR
- Apply Lifetime Correction (τ/T) for High Mass
- Result: The Initial Mass Function (IMF)