# The Stellar Initial Mass Function

From Theory to Observation

Graduate Astrophysics Series | Prof. AI Gemini

# The Fundamental Scaling Law

The Initial Mass Function (IMF) $\xi m$ describes the distribution of mass for a population of newly formed stars.

$$dN = \xi m dm \propto m^{-\alpha} dm$$

Where $\alpha$ represents the slope of the power law.

Salpeter, E. E. (1955), ApJ, 121, 161

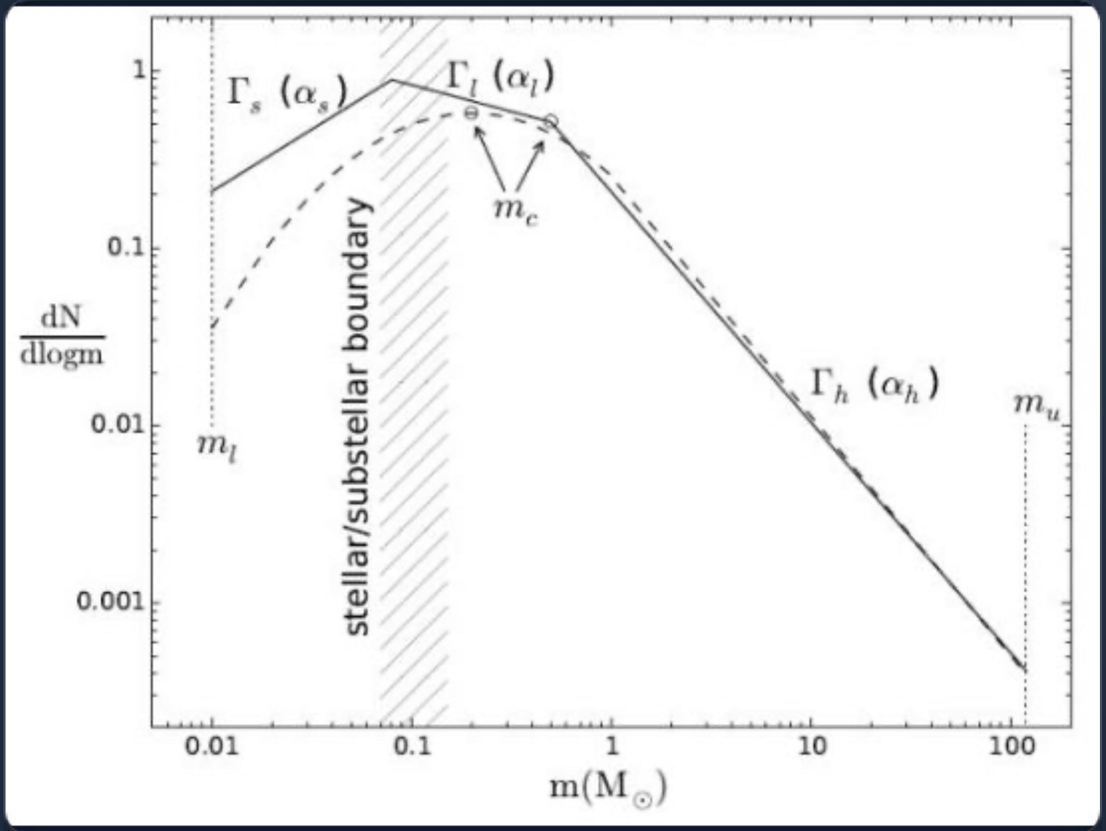
# Historical Context: Salpeter (1955)

## The First Derivation

Edwin Salpeter's seminal work established the first empirical derivation of the IMF using stars in the Solar neighborhood.

$$\xi m \approx 0.03 \frac{m}{M_\odot}^{-2.35}$$

- Salpeter Slope: $\alpha = 2.35 \quad m > 0.4 M_\odot$
- Derived from the present-day luminosity function (PDLF).




*Salpeter, E. E. (1955), ApJ, 121, 161*

# Modern Parametrizations

## Kroupa (2001)

A multi-segment power law that accounts for the flattening at lower masses.

$$\alpha = \begin{cases} 0.3 & \text{for } m < 0.08 \\ 1.3 & \text{for } 0.08 \leq m < 0.5 \\ 2.3 & \text{for } m \geq 0.5 \end{cases}$$

## Chabrier (2003)

Uses a log-normal distribution for low masses and a power law for the high-mass tail.

$$\xi(m) \propto \frac{1}{m} \exp\left(-\frac{(\log m - \log m_c)^2}{2\sigma^2}\right)$$

Typically $m_c \approx 0.2 M_\odot$ and $\sigma \approx 0.55$

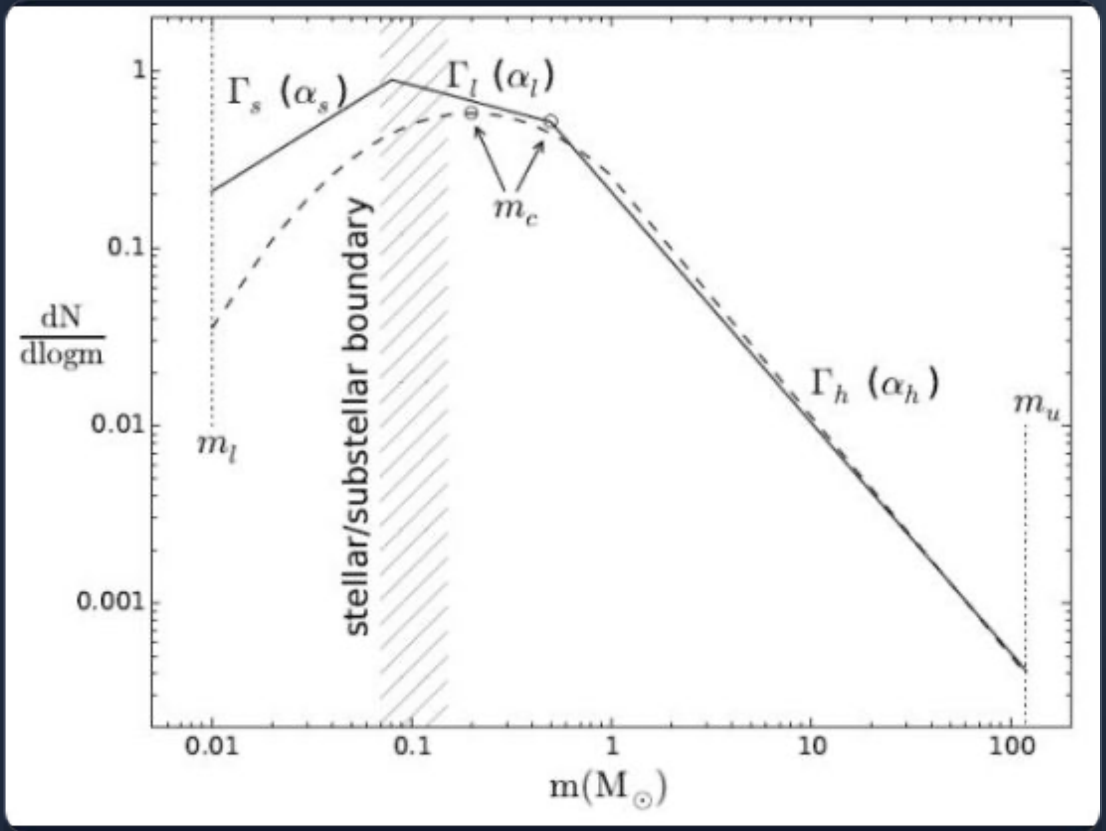
Kroupa, P. (2001), MNRAS, 322; Chabrier, G. (2003), PASP, 115

# The Universality Consensus

## Bastian, Covey & Meyer (2010)

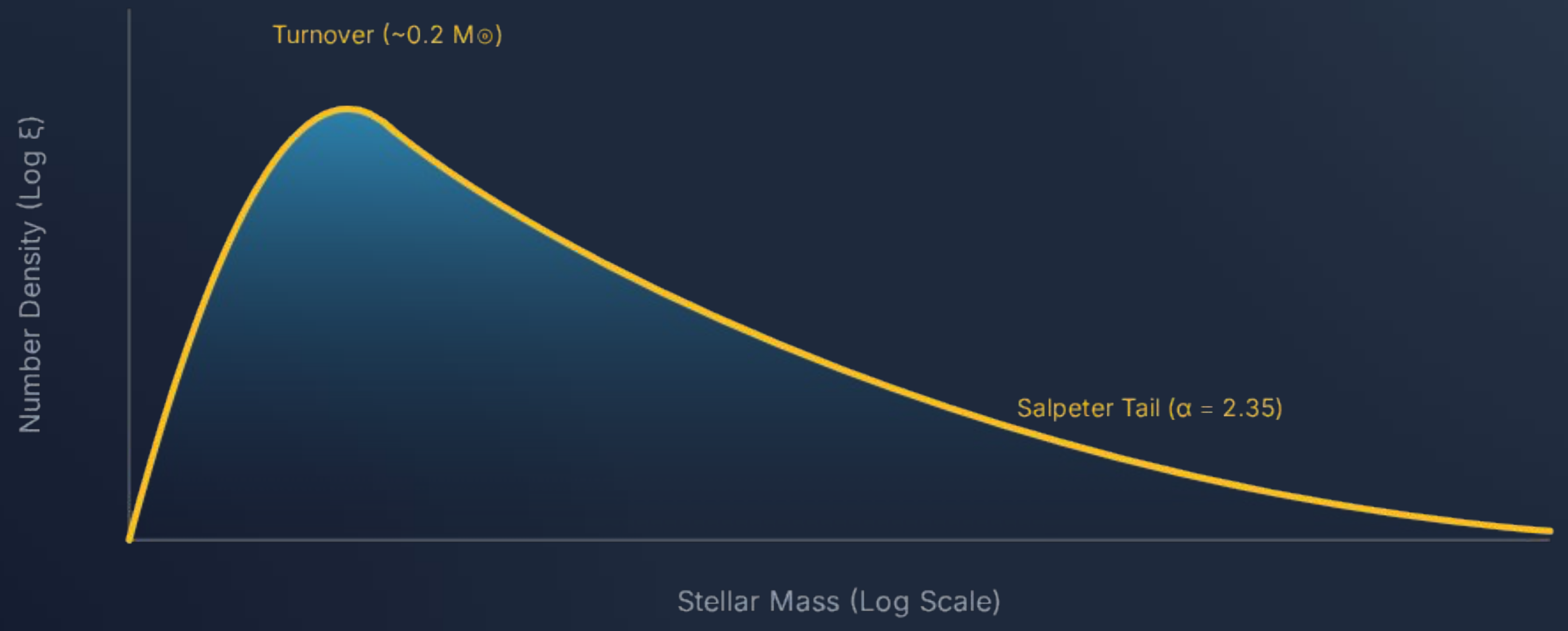
In a comprehensive Annual Review, Bastian et al. assessed IMF measurements across diverse environments, from the Galactic Center to the field.

- **Key Conclusion:** There is "no clear evidence" for systematic variation in the IMF.
- **Scatter Explanation:** Observed variations are consistent with stochastic sampling effects, dynamical evolution, and observational uncertainties (e.g., binaries).
- **Universality:** The IMF implies a robust formation mechanism independent of metallicity or density.




Bastian, N., Covey, K. R., & Meyer, M. R. (2010), ARA&A, 48, 339

# The Shape of the Stellar IMF




The distribution rises to a peak around 0.2-0.5$M_\odot$ before following the classic Salpeter power-law decline.

*Miller, G. E., & Scalo, J. M. (1979), ApJS, 41, 513*

# Origin Theories: Gravoturbulent Fragmentation

## Jeans Mass

The thermal Jeans mass defines the scale where gravity overcomes thermal pressure. The IMF peak is often associated with the characteristic Jeans mass in molecular clouds.

## Turbulence

Supersonic turbulence creates local density enhancements (shocks). The mass function of these "cores" may map directly onto the stellar IMF (Core Accretion Model).

## Feedback

For high-mass stars, radiation pressure and outflows may limit accretion, potentially setting the upper mass limit ( $M_{max}$ ).

*Jeans (1902); Padoan & Nordlund (2002); Hennebelle & Chabrier (2008)*

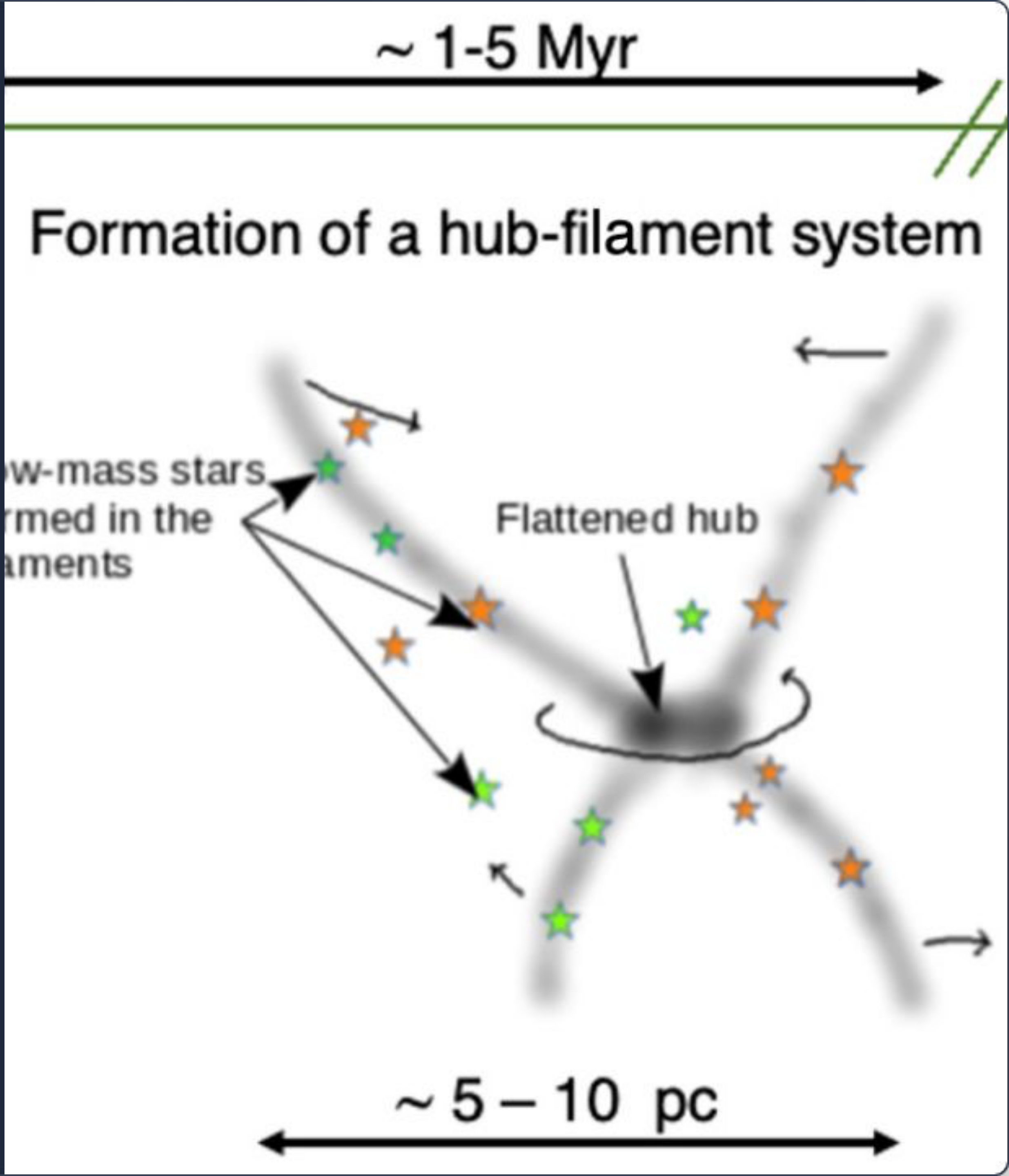
# Competitive Accretion

**The Dynamical View:** Unlike core accretion, this model suggests stars compete for gas in a common potential well.

Stars form in clusters. Massive stars are not born massive; they become massive because they sit at the center of the potential well and accrete gas more efficiently than their siblings.

This model naturally predicts mass segregation and the power-law tail of the IMF.

*Bate et al. (2012); Bonnell et al. (2004)*



# From PDMF to IMF

Deriving the Initial Mass Function from Observations of the Present Day Mass Function

# Defining the Mass Functions

## Initial Mass Function (IMF)

$\xi m$

The distribution of stellar masses formed in a single star-formation event **at birth** (t=0). It represents the intrinsic outcome of the star formation process.

## Present Day Mass Function (PDMF)

$\Phi m$

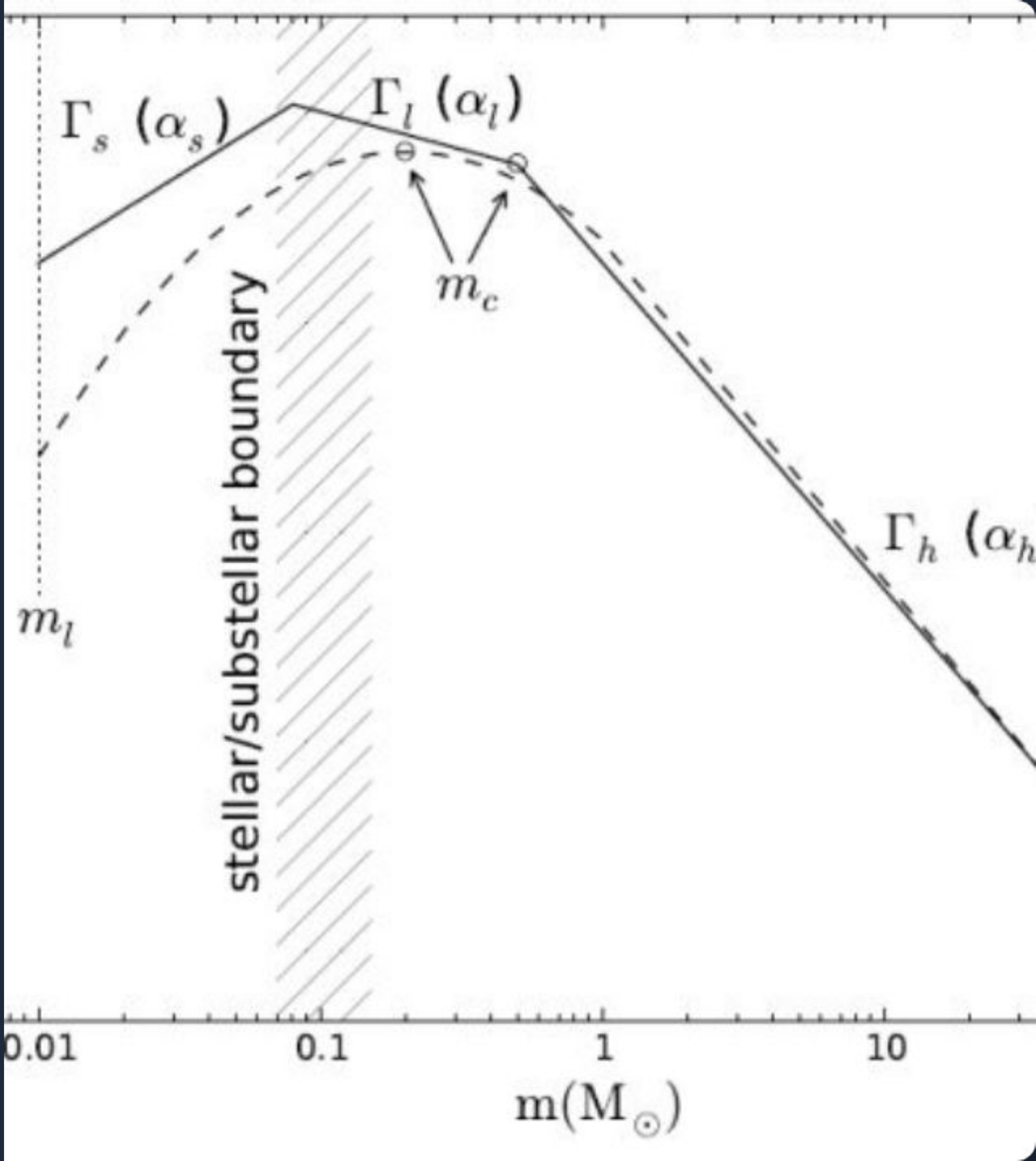
The distribution of stellar masses observed **today** (t=now) in the Main Sequence. This is what we actually count in the sky.

# The "Missing" Stars

The IMF and PDMF are not identical because stars evolve.

**Massive stars** burn fuel quickly and die, disappearing from the Main Sequence. Observing the PDMF today misses these dead stars.

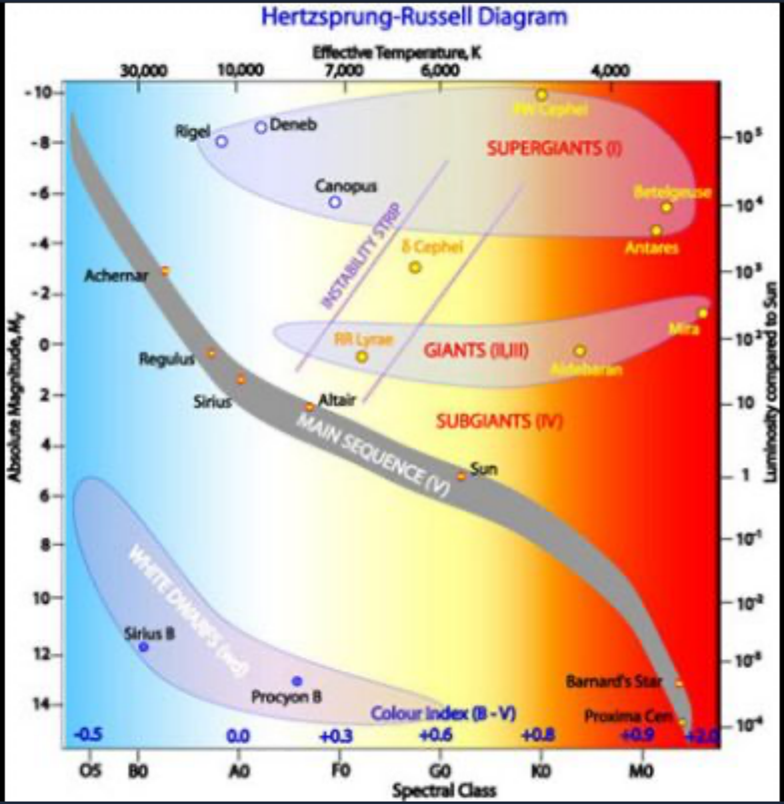
**Low-mass stars** live longer than the age of the Universe. For them, what we see today (PDMF) is effectively what was born (IMF).



# Step 1: Observation (Star Counts)

We cannot "weigh" most stars directly. We observe them photometrically.

- **Observation:** Apparent Magnitudes ($m_V$) and Colors ($B - V$).
- **Tool:** The Color-Magnitude Diagram (CMD) or Hertzsprung-Russell (HR) Diagram.
- **Goal:** Identify Main Sequence stars and correct for distance (modulus) and extinction to get Absolute Magnitude ($M_V$).



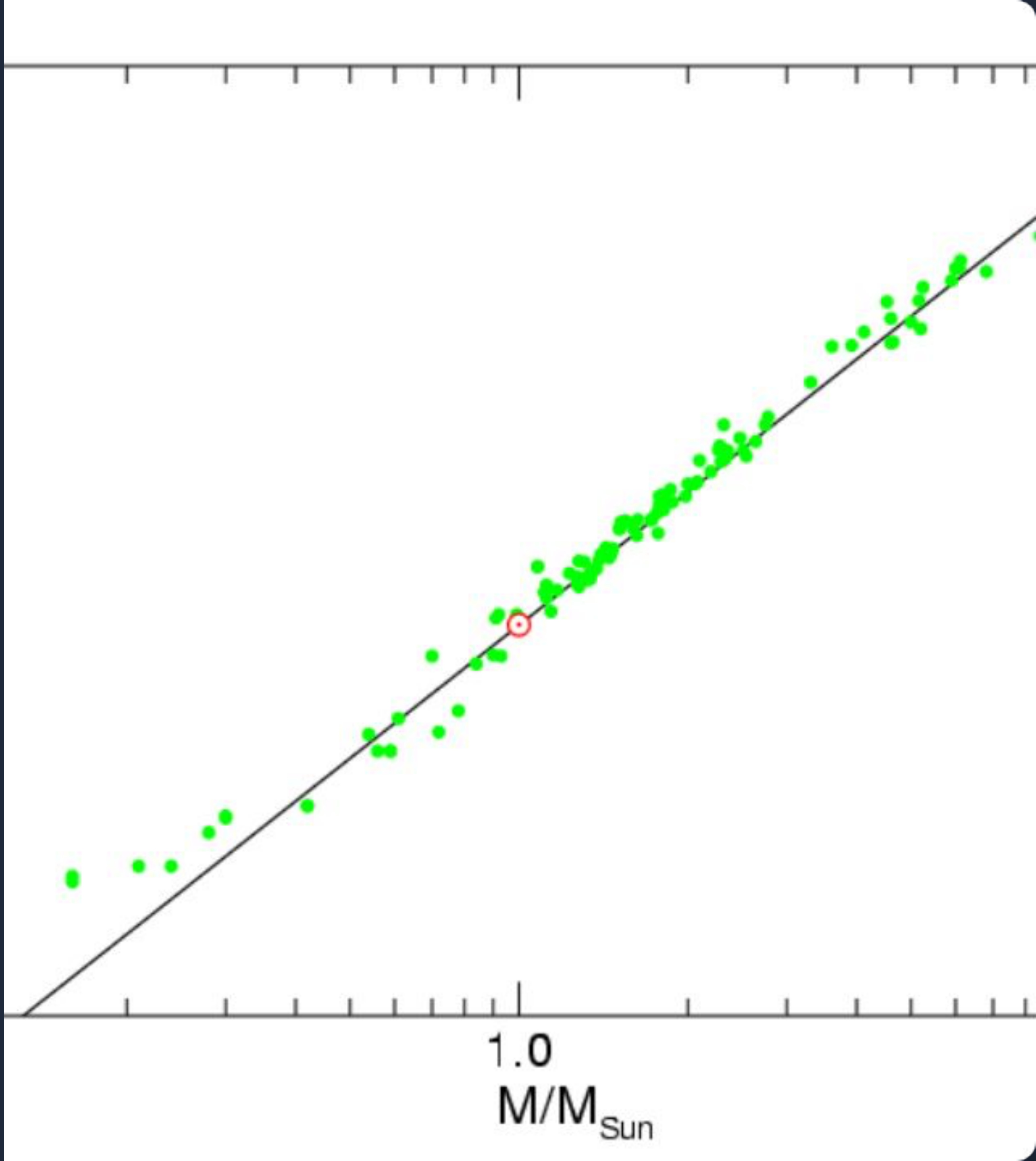
# Step 2: Light to Mass

To convert the observed Luminosity Function ($\Phi M_V$) into a Mass Function, we use the **Mass-Luminosity Relation (MLR)**.

For Main Sequence stars:

$$L \propto M^{3.5}$$

This empirical relation allows us to assign a mass to every observed star, yielding the PDMF.



# The Evolutionary Divide

We must define a critical turn-off mass, , $M_{to}$ corresponding to the age of the stellar population ( ). $T_0$

## Mass $< M_{to}$

Lifetime $\tau_m > T_0$

These stars have not yet evolved off the Main Sequence. Number is conserved.

**PDMF IMF**

## Mass $> M_{to}$

Lifetime $\tau_m < T_0$

Many of these stars have already died. The observed number is a fraction of the total formed.

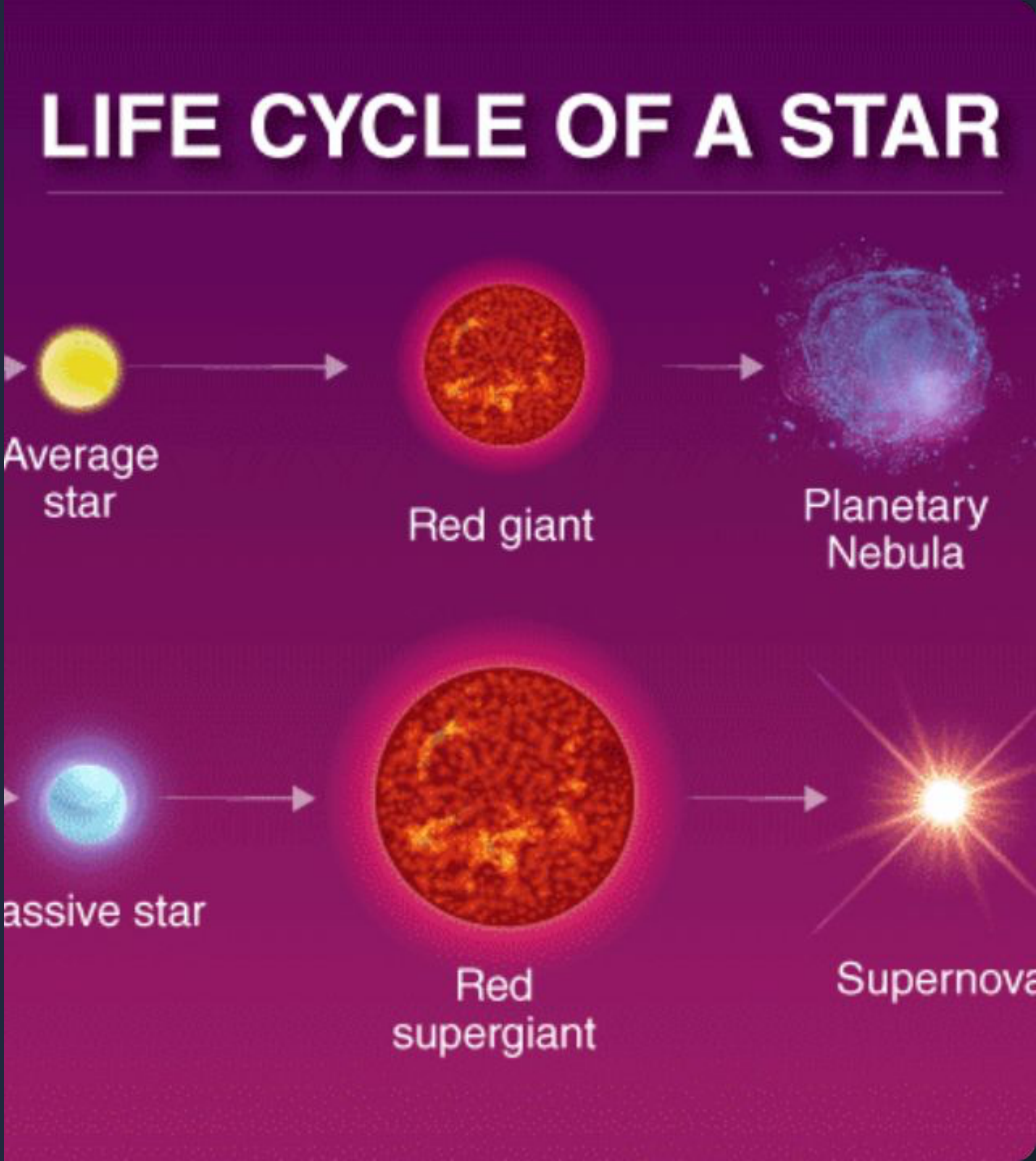
**PDMF IMF**

# Correcting for Lifetimes

For massive stars, the PDMF depends on the **Star Formation Rate (SFR)**, denoted as $bt$.

We are only seeing stars formed recently enough to still be alive (within the last years).

If SFR is constant, the number of observed stars is proportional to their lifetime.



# The Mathematical Relation

For a system with age $T_0$ and Star Formation History : $bt$

$$\Phi m = \xi m \times \int_{T_0 - \tau(m)}^{T_0} btdt$$

If $bt$ is Constant:

$$\Phi m = \xi m \times \tau m$$

Therefore, the IMF is:

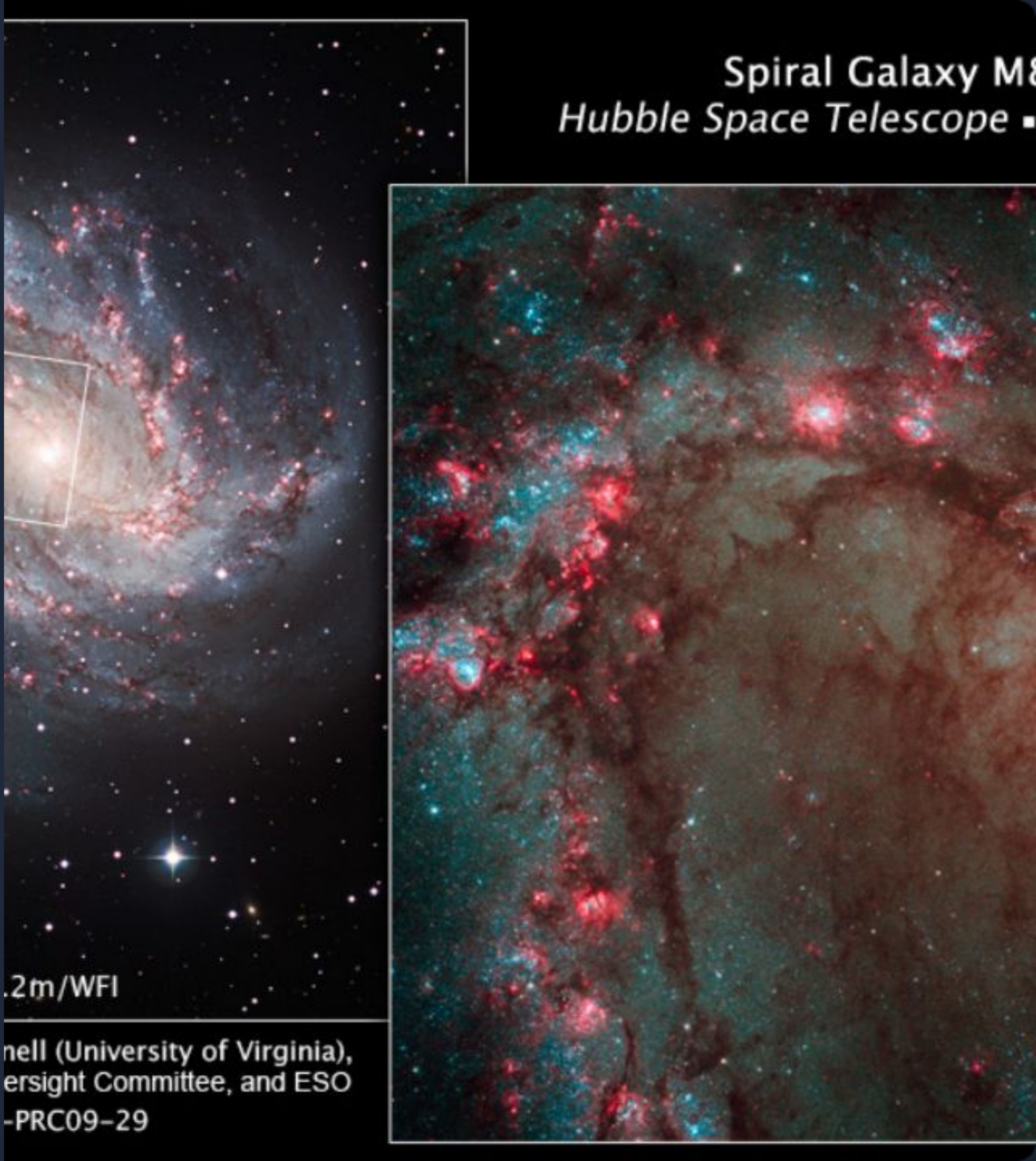
$$\xi m = \frac{\Phi m}{\tau m}$$

# The Role of History

The "Constant SFR" assumption works for galactic disks on average, but not for starburst regions or old globular clusters.

**Starburst:** If all stars formed at once (single burst), massive stars are simply gone. We cannot derive the high-mass IMF directly from the PDMF in old bursts.

**Continuous:** In spiral galaxies, new stars are constantly replacing dead ones, allowing statistical derivation.



# Major Challenges

### Unresolved Binaries

Two stars close together look like one bright star. This makes low-mass stars look like single high-mass stars, flattening the observed slope.

### Extinction

Dust obscures light, making stars look fainter and redder. Incorrect extinction corrections lead to incorrect masses.

### Mass Segregation

In clusters, heavy stars sink to the center. Observing only the core or outskirts gives a biased PDMF.

# Summary

- Observe Photometry ($m_V$, Color)
- Convert to Mass (PDMF) using MLR
- Apply Lifetime Correction ($t/T$) for High Mass
- Result: The Initial Mass Function (IMF)