华东师范大学期末试卷（B）

2006—2007 学年第二学期

软件工程数学参考答案

一、

（1）	$\exists x(F(x)\wedge G(x))$	前提引入
（2）	$F(c)\wedge G(c)$	(1) EI
（3）	$\forall x(F(x)\to(G(x)\wedge R(x)))$	前提引入
（4）	$F(c)\to(G(c)\wedge R(c))$	(3)UI
（5）	$F(c)$	(2)化简
（6）	$G(c)\wedge R(c)$	(4)(5)
（7）	$\exists x(G(x)\wedge R(x))$	(6)EG

二、

Proof: We first show that (1) implies (2). Assume that aRb. We will prove that $[a]=[b]$ by showing $[a]\subseteq[b]$ and $[b]\subseteq[a]$. Suppose $c\in[a]$. Then aRc. Since aRb and R is symmetric, we know that bRa. Furthermore, since R is transitive and bRa and aRc, it follows that bRc. Hence, $c\in[b]$. This show that $[a]\subseteq[b]$.

The proof that $[b]\subseteq[a]$ is similar.

Second, we will show that (2) implies (3). Suppose that $[a]\cap[b]\neq\Phi$. Assume that $[a]=[b]$. It follows that $[a]\cap[b]\neq\Phi$ since $[a]$ is nonempty.

Next, we will show that (3) implies (1). Suppose that $[a]\cap[b]\neq\Phi$. Then there is an element c with $c\in[a]$ and $c\in[b]$. In other words, aRc and bRc. By the symmetric property, cRb. Then by transitivity, since aRc and cRb, we have aRb.

三、

（a） $C_{10}^1C_9^1C_8^1C_7^1 = 5040$

（b） $C_{10}^1C_{10}^1C_{10}^1C_5^1 = 5000$

（c） $C_4^1 * 9 = 36$

四、

（a） $P(7,7) = 5040$

（b） $P(5,5) = 120$

五、

（a） $C(22,17) = 26334$

（b） $C(25,20) - C(17,12) = 46942$

六、

Solution： Let n be a positive integer. Consider the n integers 1, 11, 111, ⋯, 11⋯1 (where the last integer in this list is the integer with $n+1$ 1s in its decimal expansion). Note that there are n possible remainders when an integer is divide by n. Since there are $n+1$ integers in this list, by the pigeonhole principle there must be two with the same remainder when divided by n. The difference of these two integers has a decimal expansion consisting entirely of 0s and 1s and is divisible by n.

七、

$C(100, \frac{100+k}{2})$

八、

（a） $a_n = a_{n-1} + a_{n-2} + 2^{n-2} \qquad (n > 1)$

（b） $a_0 = 0, a_1 = 0$

九、

Solution：

a）相应的齐次递推关系的特征方程为 $r = 2$，

所以相应的齐次递推关系的通解为：$a_n^{(p)} = \alpha_1 2^n$，而 $2n^2 = 2n^2 \cdot 1^n$，又 1 不是相应的齐次递推关系的特征方程的特征根，所以原递推关系的特征为 $a_n^{(p)} = b_2 n^2 + b_1 n2 + b_0$，代入原递推关系，得

$b_2 n^2 + b_1 n + b_0 = 2b_2(n-1)^2 + 2b_1(n-1) + 2b_0 + 2n^2$，

即 $b_2 n^2 + b_1 n + b_0 = (2b_2 + 2)n^2 + (-4b_2 + b_1)n + 2b_2 - 2b_1 + 2b_0$

故可得：

$$\begin{cases} b_2 = 2b_2 + 2 \\ b_1 = -4b_2 + 2b_1 \\ b_0 = -2b_2 - 2b_1 + 2b_0 \end{cases},$$

则：$b_2 = -2, b_1 = -8, b_0 = -12$，

所以，原递推关系的特解为：$a_n = -2n^2 - 8n - 12$，则原递推关系的通解为：

$$a_n = \alpha_1 2^n - 2n^2 - 8n - 12$$

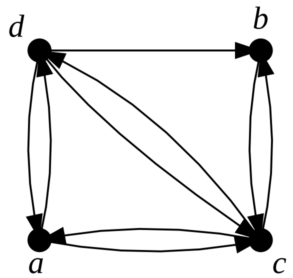
b) 由于 $a_1 = 4$，所以 $\alpha_1 2 - 2 - 8 - 12 = 4$，得到 $\alpha_1 = 13$，

所以，原递推关系在初始条件下的解为：

$$a_n = 13 \cdot 2^n - 2n^2 - 8n - 12$$

十、

（a）



（b） 不同构，Figure(a) 中有两个四度顶点相连，而 Figure(b)中没有

（c） **Solutions:**

For an undirected graph, the entries in a row of the adjancency matrix is the degree of the vertex - the number of loops at the vertex.

For a directed graph, the entries in a row of the adjancency matrix is the out-degree of the vertex.

十一、

（1） 3 种

（2）

a) m 与 n 都必须为偶数时，$K_{m,n}$ 才有 Euler 回路。

b) (1) m 与 n 都为偶数时，有 Euler Path

(2) 另一种情况是 $\begin{cases} m = 2 \\ n\text{为奇数} \end{cases}$ 或 $\begin{cases} m\text{为奇数} \\ n = 2 \end{cases}$，有 Euler Path。

十二、
Proof：

假设每个连通分支含有r_i个区域，e_i条边和v_i个顶点$(i=1,2,...,k)$，那么，根据Euler定理，有$r_i = e_i - v_i + 2$，

$$\sum_{i=1}^{k} r_i = \sum_{i=1}^{k} e_i - \sum_{i=1}^{k} v_i + 2k$$

$$\sum_{i=1}^{k} (r_i - 1) + 1 + k - 1 = e - v + 2k ,$$

即$r + k - 1 = e - v + 2k$，
那么：$r = e - v + k + 1$