一、填空题

1. $16x-14y-11z-65=0$

2. $\frac{\partial z}{\partial x}=f_1yx^{y-1}+f_2y^x\ln y$

3. $df(1,1)=10dx+2dy$

4.$\int_1^{\sqrt{2}}dx\int_0^{\sqrt{2-x^2}}f(x,y)dy+\int_0^1dx\int_0^{x^2}f(x,y)dy$

5. $6x+e^y-\cos z$

二、选择题

6. D

7. A

8. D

9. C

10. C

三、计算题

11. 原点到$P(x,y,z)$的距离为$d=\sqrt{x^2+y^2+z^2}$..........................2分

只需求$x^2+y^2+z^2$的最小值，建立拉格朗日函数：

$L=x^2+y^2+z^2+\lambda_1(x+2y-1)+\lambda_2(x^2+2y^2+z^2-1)$................4分

解方程组

$$\begin{cases} L_x = 2x + \lambda_1 + 2x\lambda_2 = 0 \\ L_y = 2y + 2\lambda_1 + 4y\lambda_2 = 0 \\ L_z = 2z + 2z\lambda_2 = 0 \\ L_{\lambda_1} = x + 2y - 1 \\ L_{\lambda_2} = x^2 + 2y^2 + z^2 - 1 = 0 \text{.......................................6分} \end{cases}$$

得

$$x = 1, y = 0, z = 0 \text{..7分}$$

或

$$x = -\frac{1}{3}, y = \frac{2}{3}, z = 0 \text{.......................................8分}$$

故最小距离为$d = \sqrt{\frac{1}{9} + \frac{4}{9} + 0} = \frac{\sqrt{5}}{3}$............10分

12. 解：因为被积函数关于x, y均为偶函数，且积分区域关于x, y轴均对称，因此$\int\int_D f(x,y)d\sigma = 4\int\int_{D_1} f(x,y)d\sigma$,其中$D_1$为$D$在第一象限的部分，...4分

那么$\int\int_{D_1} f(x,y)d\sigma = \int\int_{x+y\le 1} x^2 d\sigma + \int\int_{1\le x+y\le 2} \frac{1}{\sqrt{x^2+y^2}} d\sigma$..6分

$= \int_0^1\int_0^{1-x} x^2 dydx + \int_0^1\int_{1-x}^{2-x} \frac{1}{\sqrt{x^2+y^2}} dydx + \int_1^2\int_0^{2-x} \frac{1}{\sqrt{x^2+y^2}} dydx = \frac{1}{12} + \sqrt{2}\ln(\sqrt{2}+1)$,....8分

所以原积分为$\int\int_D f(x,y)d\sigma = \frac{1}{3} + 4\sqrt{2}\ln(\sqrt{2}+1)$...10分

13. 令$x = r\cos\theta, y = r\sin\theta$, ...3分

则$\iiint\limits_{\Omega} (x^2+y^2)dxdydz = \int_2^8\int_0^{2\pi}\int_0^{\sqrt{2z}} r^3 drd\theta dz = 336\pi$..10分

14. 令$x'=x-1,y'=y-1,z'=z-1$, 则原式转化为$\int\int_{S'}(x'+y'+z'+4)^2dS'$,..........................4分

其中$S'=x'^2+y'^2+z'^2=1$, 由对称性可知原式$=\int\int_{S'}(x'^2+y'^2+z'^2+16)dS'=\int\int_{S'}17dS'=$ 68π..10分

15. 根据题意可知$z'_x=2x, z'_y=2y$, ...2分

所以$I=\int\int\limits_S(2x+z)dydz+zdxdy=\int\int\limits_S(2x+z)(-z'_x)+zdxdy=\int\int\limits_S((-4x^2-2xz+z)dxdy=$ $\int\int\limits_S(-4x^2-2x(x^2+y^2)+x^2+y^2)dxdy$...5分

令$x=r\cos\theta, y=r\sin\theta$,那么原式为$=\int_0^{2\pi}\int_0^1(-4r^2\cos^2\theta-2r^3\cos\theta+r^2)rdrd\theta=-\frac{\pi}{2}$.........10分

16.(1)由平面上曲线积分和路径无关的条件可知$\frac{\partial Q}{\partial x}=\frac{\partial(2xy)}{\partial y}=2x$, 所以$Q(x,y)=x^2+C(y)$, 其中$C(y)$待定，因为积分与路径无关，因此取$(0,0)\to(t,0)\to(t,1)$, 则$\int_{(0,0)}^{(t,1)}2xydx+Q(x,y)dy=$ $\int_0^1(t^2+C(y))dy=t^2+\int_0^1dy$, 同理取$(0,0)\to(0,t)\to(1,t)$,则$\int_{(0,0)}^{(1,t)}2xydx+Q(x,y)dy=\int_0^tC(y)dy+$ $\int_0^1 2txdx=\int_0^tC(y)dy+t$, ...3分

根据题意$\int_{(0,0)}^{(t,1)}2xydx+Q(x,y)dy=\int_{(0,0)}^{(1,t)}2xydx+Q(x,y)dy$可知$t^2+\int_0^1C(y)dy=\int_0^tC(y)dy+t$, 同时对$t$求导可得$2t=C(t)+1$, 所以$C(t)=2t-1$, 故有$Q(x,y)=x^2+2y-1$........................6分

(2)根据题意$u_x=2xy, u_y=x^2+2y-1$, 可知$u=x^2u+y^2-y+C$, 再由初始条件可知$C=1$, 因此$u=x^2y+y^2-y+1$...4分

17.(1)根据题意令$x=r\cos\theta, y=r\sin\theta$, 则$F(u,v)=\int\int_{D_{uv}}=\int_{\sqrt{\frac{1}{v}}}^{\sqrt{v}}\int_0^u\frac{f(r^2)}{r}rd\theta dr$, 因此由变限积分求导可知$\frac{\partial F}{\partial u}=\int_{\sqrt{\frac{1}{v}}}^{\sqrt{v}}f(r^2)dr$, 而$\frac{\partial F}{\partial v}=u\left[\frac{1}{2\sqrt{v}}f(v)+\frac{1}{2}v^{-\frac{3}{2}}f(\frac{1}{v})\right]$..4分

(2). 根据题意原式转化为$F(\frac{\pi}{2},4)=\int_{\frac{1}{2}}^2\int_0^{\frac{\pi}{2}}\frac{1}{1+r^2}d\theta dr=\frac{\pi}{2}(\arctan(2)-\arctan(\frac{1}{2}))$................4分

(3). 根据题意原式可以转化为$F(\frac{\pi}{2},v)=\int_{\sqrt{\frac{1}{v}}}^{\sqrt{v}}\int_0^{\frac{\pi}{2}}e^{-r^2}d\theta dr=\frac{\pi}{2}\lim_{v\to\infty}\int_0^{\infty}e^{-r^2}dr=\frac{\pi}{2}\cdot$ $\frac{\sqrt{\pi}}{2}$..........................3分