Cosmic Ray Counting

Although we (almost) never think about it, we are bombarded by high-energy ionizing radiation every second of our lives. These high energy particles originate in outer space in the "solar wind". Very high in the atmosphere, the dominant particle species are protons or alpha particles, of extremely high energy (10^8 to 10^{20} eV). When these "primary" cosmic ray particles meet an atom in the upper atmosphere, the collision produces a "shower" of energetic hadrons, which then decay into energetic leptons. The most common of these "secondary" cosmic ray reactions are

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \nu_\mu \\ \pi^- &\rightarrow \mu^- + \bar{\nu}_\mu \ , \end{aligned}$$

that is, the decay of pions into muons and neutrinos/antineutrinos.

A muon itself is unstable, and will decay (at rest) into an electron or positron (and associated neutrino or antineutrino) in a little over 2 microseconds. However, because of relativistic time dilation, the muons in high-speed flight from the upper atmosphere survive for much longer (according to our reference frame). Indeed, the lengthening of the muon lifetime in flight is a vivid example of relativistic kinematics in action. The discovery of the cosmic ray muon was one of the first indications that there could be particles other than the ones that make up ordinary matter: protons, neutrons, and electrons.

In this experiment, you will attempt to measure the flux of cosmic rays at about sea level (since Seattle is nearly there), and compare your value to a widely accepted one known since the 1940's. Cosmic rays come out of the sky in all directions, which means there are a couple of ways to talk about the flux. If we restrict the angle to straight down (zenith = 0), then the accepted intensity of high energy (hard) cosmic ray flux is

$$I_v = 0.82 \times 10^{-2} \text{cm}^{-2}\text{s}^{-1}\text{str}^{-1},$$

where the subscript v indicates that this is the vertically directed flux. This says that the number of cosmic ray particles passing through a vertically-facing surface is 0.82×10^{-2} particles per square centimeter per second per steradian. The steradian is a unit of solid angle in the way that a radian is a measure of planar angle. The solid angle intercepted by a sphere is 4π steradians, by the upper half of a hemisphere 2π steradians, and so forth. The steradian measure is necessary because the cosmic rays can come from all angles, not just straight down out of the sky.

If we integrate the intensity in the downward direction across a horizontal surface over the solid angle from horizon to horizon in all directions, we get the downward flux:

$$J = \int I(\theta) \cos\theta d\Omega = 1.27 \times 10^{-2} \text{cm}^{-2}\text{s}^{-1},$$

where the $\cos\theta$ term accounts for the fact that the flux across the surface from particles traveling at an angle to the surface normal will not be as large as the flux from particles traveling straight down.

If $I(\theta)$ were a constant, then the integral would give π times the intensity I_v, but it is notably less. This is because the flux intensity varies according to zenith angle. An approximate empirical relationship is

$$I(\theta) \approx I_v \cos^2\theta \ .$$

The muon telescope

Although cosmic rays come in a variety of particle types (muons, electrons, neutrinos, and their antiparticles) with a variety of energies, research has shown that most (about 80%) of the highest energy particles (the so called "hard" cosmic rays) to reach the surface of the earth are muons, both positive and negative. Thus, for the remainder of this write-up, I will use "muons" as a shorthand for "cosmic rays", since that is what we will mostly look at.

To measure the muon flux, we will use a "telescope", which in this context means an array of scintillator "paddle" detectors arranged with one scintillator overlapping another, as shown in Fig. 1. A muon traversing the array will, with some probability, cause a simultaneous pulse to occur in more than one detector, since with their high energy, they will usually not be stopped by a single paddle. Thus, we can use coincidence gating to count muon fly-bys as a way to distinguish these events from background radiation and photomultiplier noise.

Because the expected muon count rate is rather small, it is helpful to use a coincidence level of three or more in order to really suppress accidental coincidences. One of the aspects of this experiment you will explore is the rate of "accidentals" versus true coincidences. In addition, by comparing the rate of coincidences using 3 paddles to the rate using 4 paddles, you will be able to estimate the counting efficiency of the extra paddle, which you can then use to estimate a correction to your overall data set.

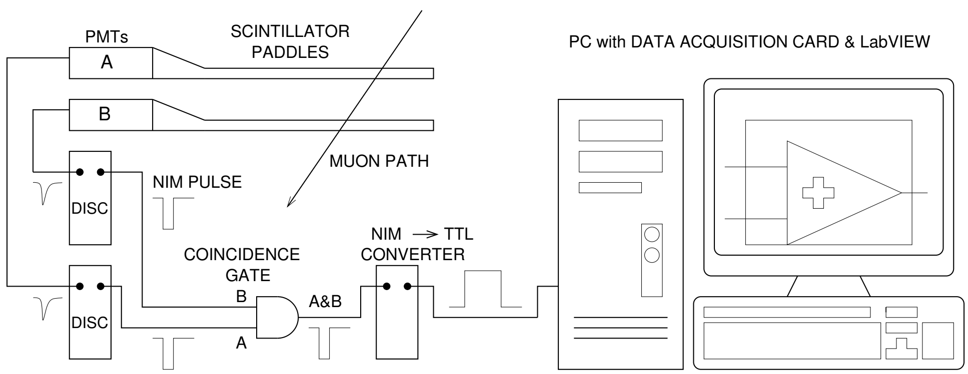

Figure 1: Schematic of the muon telescope apparatus. Although there are only two scintillator paddles shown in the figure, the actual setup uses between three and four paddles. The AND gate is a 4-fold NIM logic unit. Data are collected with the LabVIEW Interval Counter software.

Some details about the scintillator paddles: The scintillator material is Bicron BC-408 plastic scintillator cut into a sheet that is 1 cm thick, with an area of 12 inches by 6 inches. One end of the sheet is glued to a Lucite light pipe, which is then attached to the face of a 2 inch diameter PMT with optical grease. The PMTs are Electron Tubes model 9266KB. The paddle assembly is wrapped in aluminum foil, and then wrapped with heavy black paper and tape. A picture of an unwrapped paddle is shown in Fig. 2

Each paddle assembly rests on a shelf in a large wooden box. The shelves can be moved so that the vertical extent of the paddle array can be varied. The detectors are arranged so that the PMTs alternate which side of the paddle they are on—this minimizes the effect of a muon causing a coincidence by passing through multiple PMTs, rather than multiple scintillator paddles!

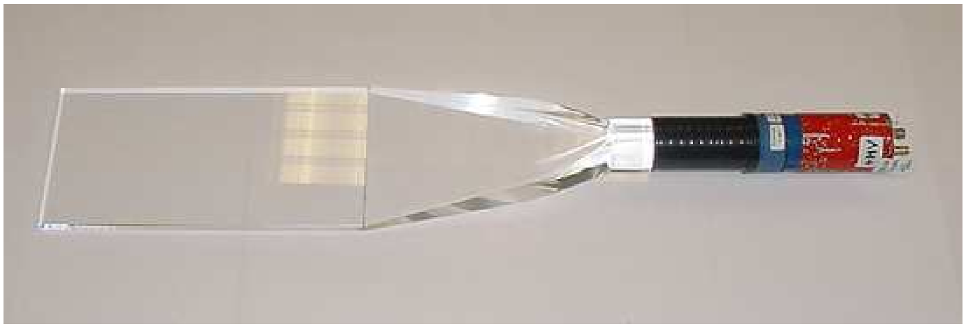

Figure 2: Scintillator paddle detector used in the muon telescope apparatus, before being wrapped with aluminum foil and black paper..

Setting up the telescope

The paddle array box will already be set up at your station with the paddles in place. You will need to connect the cables and set up the electronics.

First, note the tags attached to each PMT base indicating a voltage. This voltage has been chosen by the lab technician to produce a pulse of sufficient height when a ^{22}Na source is brought nearby. The variations in voltage from one paddle to the next are due to differences in the PMT manufacturing. (It is hard to make PMTs all exactly alike.)

Obtain SHV cables from the cable rack. (Use the long ones.) Since you have two high voltage supplies at your station, you must match, as well as you can, pairs of detectors to be plugged into each high voltage supply. For example, if your labels say 780V, 930V, 950V, and 960V, you would plug the 780 and 930 volt units into one supply, and the 950 and 960 units into the other.

Obtain 50ΩBNC cables from the cable rack. ***Important: the cables must all be the same length.*** (Remember, you are setting up coincidence counting.) Attach each cable appropriately, and double check to make sure the right HV cable goes to the right HV supply.

Start with one supply and paddle detector combination. Plug the signal cables into the two scope channels, appropriately terminated with 50Ω. Turn on the scope, turn the channel sensitivity all the way up (lowest V/div setting), and chose the appropriate trigger conditions. Slowly bring up the HV supply, following the usual protocol, while watching the scope. You are looking to make sure there are no light leaks or other problems. If you think you do see a problem, ask the instructor or TA for help.

Set the HV supply to the higher of the two settings recommended for your PMT pair. You will probably see a lot of noise pulses, and maybe some real event pulses too.

Obtain a ^{22}Na gamma source and put in in the holder. Carefully place the source somewhere between the two paddles that you are setting up, so that both are exposed to the source. Adjust the scope sensitivities so that you can comfortably see the pulses from both paddles. You may need to fiddle with the trigger settings somewhat. Estimate, as best you can from the scope screen, the peak height corresponding to the "bright line" of the 0.511 MeV annihilation photons for each detector in your pair. (No need to draw the pulse, just record its properties.)

When you are satisfied that your first pair of paddles is working OK, leave the HV on for them, and move on to the second pair of paddles. Repeat the procedure you just used: look at their outputs on the scope while bringing up the HV, check for problems, and check and record the response to the gamma source.

Feed the signal from each PMT into a discriminator channel. Use the source to set the width from each to 40 ns. After setting the width, set the discriminator level to the value you recorded for the "bright-line" peak height, or a bit less (i.e., 5–10 mV). Use the test point on the discriminator to dial in these numbers.

After setting the widths and the thresholds, rout the discriminator outputs into a channel in the 4-fold logic unit. Use a NIM counter/timer unit to measure 2-fold coincidence rates between the top two paddles, the middle two paddles and the bottom two paddles, over a reasonable counting period (10–20 seconds at most). You should see roughly the same count rates in each pair (with the appropriate pins set in the logic unit). If one pair seems excessively low, you my need to tweak the discriminator level and/or voltage to the PMT. Ask for help if you don't think it is working properly.

It is interesting to see what happens when you select 3-fold coincidence. You should see the count rate drop a lot, even though you still have the source nearby.

For your report. Explain why a 3-fold coincidence gives such a low count rate even with a ^{22}Na source in between two panels.

Characterizing the muon telescope

If all is well, you should be able to collect data in order to address two questions:

- What number of muons pass through the scintillators of the telescope? In other words, what is the *acceptance* of the telescope?
- What percentage of these muons does the telescope actually count? In other words, what is the *efficiency* of the telescope?

Acceptance of the telescope

The acceptance of the telescope depends on the surface area of the scintillator paddles, the solid angle subtended by surfaces of the paddle array, and on the angular dependence of the muon flux. Clearly a larger paddle area will see more muons, and it is not hard to believe that the counting rate would be directly proportional to this area, all else being equal.

The solid angle question is a bit more complicated. For a muon to make a coincidence between the top and bottom paddle, it must pass through both. Certainly a muon coming straight down will do this. But a muon coming in at an angle will also do this, if the angle is not too steep. The steepest angle would be a path from one edge of the top detector to the opposite edge of the bottom detector. If the top and bottom paddle are far apart, the solid angle will be smaller than if they are close together. However, calculating this quantity is tricky, for a couple of reasons. First, not every element of the detector sees the same solid angle. A small element of area dA in the center of the bottom paddle would register a count if the muon went through it and *any* spot on the top paddle, covering the whole surface A of that paddle. If you imagine a point on the bottom paddle, and rays intercepting this point and all points on the top paddle, you will get the solid angle for that portion of the array. But another element of the top paddle dA' located, say, near the edge, would see a different solid angle by the same construction. Moreover, the directions of the two solid angles, with respect to straight up (zenith angle $\theta = 0$) would also be different. It is known that the muon flux varies with zenith angle proportional to (about) $\cos^2\theta$. Thus, a good calculation of the acceptance of the telescope requires integrating the flux as a function of θ over the solid angle $\Omega(x, y)$ at each point (x, y) on the surface of the paddle, and then integrating this function over that surface A. At best, this is tedious; at worst, it requires a computer.

So, we will estimate the flux by making a couple of simplifying assumptions:

1. Assume that each small element of the paddle array sees the *same solid angle*, which is equal to the solid angle seen by the center of the paddle array.
2. Assume that the rectangular (approx 6"×12") paddles can be modeled as circular paddles having the same area.

The purpose of this second assumption is that it is easy to calculate what the flux per unit area would be for this cylindrical geometry. The solid angle would be that subtended by a cone. If the cone has zenith angle θ between the axis and the rim, then it is easy to show that

$$\int_{\text{cone}} d\Omega = \int_0^{2\pi} \int_0^{\theta} \sin\theta' d\theta' d\phi = 2\pi(1 - \cos\theta)\ . \tag{1}$$

Likewise, to find the flux, we need to first integrate $I(\theta) \approx I_v \cos^2\theta$ over the solid angle. Hence,

$$J(\theta) \approx 2\pi \int_0^{\theta} \left(I_v \cos^2\theta\right) \cos\theta \sin\theta d\theta = \left(\frac{\pi}{2}\right) I_v(1 - \cos^4\theta)\ . \tag{2}$$

As an exercise, you should verify that $J(\theta = \pi/2)$ gives the expected result of 0.0127 particles per cm^2 per second.

Use a ruler or tape measure to find the separation of the scintillator paddles. Also measure the area of the paddles, and check their alignment in the box by eye. You want to insure that they overlap as much as possible.

For your report. From your paddle dimensions, calculate the effective angle θ seen by cone whose area is the same as the area of one paddle and whose height is the same as the separation between the top and bottom paddle. Use this value, and the results above to calculate an estimate of the expected counting rate of hard muons. Comment on whether you think this estimate would be higher or lower than a more careful calculation. (Assume 100% efficiency of your detector paddles).

Efficiency of the paddles

Once you have your paddle array set up (HV on, discriminators set, test of coincidence between paddle pairs complete), remove the ^{22}Na source from the vicinity.

Connect the output of the logic to a gate generator or NIM/TTL converter, and feed the pulse into the computer data acquisition system. Run the LabVIEW "Interval Counter" program.

Assume that the paddles are labeled as follows, A for the top paddle, proceeding down to D for the bottom paddle. Use the NIM counter timer to perform the following tests:

1. Count with 2-fold coincidence using paddles A and D only.

2. Count with 3-fold coincidence using A, B, and D.
3. Count with 3-fold coincidence using A, C, and D.
4. Count with 4-fold coincidence using A, B, C, and D.

Make sure to collect enough counts so that you can estimate the counting rate with each configuration to better than 5% uncertainty, based on Poisson statistics. (How many counts is this?)

For your report. From your results of the above measurements, calculate the efficiency (and the uncertainty on that efficiency) for paddles B and C. Your efficiency estimates should be consistent with all of your measurements.

Describe the method and reasoning you used to find the efficiency of your paddles. Here is a definition of *efficiency*. The efficiency of a detector is the *detected* number of events divided by the *actual* number of events. In other words, if a detector counts 50 particles during a time when there were 100 particle hitting it that it *could* count, then the efficiency is 50% or 0.5. Here is something to think about: the efficiency is the probability of registering a count, given that there is a valid count to register. If we have two detectors arranged to count in a coincident manner, a particle may be detected by both detectors, by the first but not the second, by the second but not the first, or by neither. Only in the first case will a count be recorded, since coincidence is required.

Accidental coincidences

the efficiency analysis assumes that only real muons get detected by our array. But, as you will probably already have noticed, each paddle individually records a large number of counts per unit time. Only a small fraction of these are high-energy muons; most can be ascribed to PMT noise and low-level background sources. But with the low count rate for muons in this setup, the question of accidental coincidences becomes important.

As derived in the tutorial on counting statistics, the rate of accidental coincidences can be easily estimated, given the assumption that the coincidences are truly random. For two detectors, the rate of 2-fold coincidences is

$$R_2 = 2T r_A r_B \ , \tag{3}$$

where T is the pulse width, and r_A and r_B are the rates of detectors A and B, respectively. For 3-fold coincidences the rate is

$$R_3 = 3T^2 r_A r_B r_C \ . \tag{4}$$

To test these relationships, we need to force any real coincidences out of the experiment. We can do this by adding in sufficient delay to the discriminator signals so that array is no longer in good synchronization.

Choose three paddles in your array (A, B, and C). Between the output of the discriminator and the input to the logic unit, add sufficient cable delay to move the second two paddles out of time with each other and with the first paddle. (for 40 ns pulse widths, 100 ns is probably sufficient.)

Use the pins on the logic unit to set up counting of the singles rates for each of the three detectors, and then count doubles rates (R_2) for any two detector pairs i.e., A+B, A+C, or B+C, and the finally count the triples rate R_3. You should see that the rate of random coincidences, especially for the triple, is much lower than when they are in time. This is a good way to show that you really are counting cosmic rays by the coincidence method!

For your report. Use your results to test the theoretical accidental rate predictions. Since these are statistical processes, your numbers will not match exactly, but you should be able to define a reasonable uncertainty based on Poisson statistics.

The long count

Reset correct timing in your paddle array in order to make a good coincidence count with 3 paddles. (You may use 4, if you find that their efficiency is very good.)

Start data collection with the Interval Counting program and let it run for a while. After about 5 minutes, stop data collection and look at the histograms for the following settings:

(a) Count distributions for interval times equal to 10 times the mean counts per minute.

(b) Count distributions for interval times equal to 20 times the mean counts per minute.

(c) Interval distributions for intervals of 1, 2, 5, 20, and 100 count intervals, using the "Interleaving" summing method on the interval distribution calculation.

After printing these histograms, restart the data collection, ***but do not clear the previous data.*** You want to see how the histograms build up over time. Repeat the above data collection after an additional 5 minutes has passed, and continue repeating the "5 minute interval/histogram grab" until the end of the lab period.

While the data are being collected, you can explore the settings on the program. What is the difference between "Scaling" and "Interleaving"? How can you explain the evolution of the histograms as you change the interval number? What is the effect of changing the bin number on the interval distribution?

Ideally, you should have a final data set corresponding to 20 minutes or more of counting data.

For your report. Discuss the relationships among the various histograms that you see. In particular, note how the mean and variance change, as the data sets get larger, and as you vary how you look at the same data set. Also how do the means and variances of the two different types of distributions (interval vs. count) compare with each other?

For your report. Calculate the flux measured by your telescope, and compare it to the ranges expected from your earlier calculation. Don't forget to calculate an uncertainty in the number. (It will probably be somewhat less.) From your efficiency measurements, make a correction to your data, by assuming that the efficiency of the outer two paddles is the same as the geometric mean of the efficiency of the inner two paddles. This will give you an estimate of the efficiency of the telescope as a whole. How much error do you get by ignoring the rate of accidental coincidences?

Optional: Simulation of random data

If you click on the "Show features" button, you will reveal a tab that will allow you to simulate data according to different interval distribution functions. Try this feature out. It is especially interesting to study the interval distribution evolution for the "uniform" function. Extra credit will be awarded according to how much you study and discuss.

Prepared by D. Pengra
`counting_telescope.tex -- Updated 30 July 2008`