Referential Transparency and Equational Reasoning

Referential Transparency

An expression is called *referentially transparent* if it can be replaced with its corresponding value without changing the program's behavior. This requires that the expression is *pure*, that is to say the expression value must be the same for the same inputs and its evaluation must have no *side effects*.

tl;dr

referentially transparent functions are *pure* and have no *side effects*

Pure Functions

Math

```
y(m, x, b) = m * x + b
five = y(1, 2, 3)
```

Scala

```scala
def y(m : Int, x : Int, b : Int): Int = m * x + b
val five = y(1, 2, 3)
```

JavaScript

```javascript
const y = (m: number, x: number, b: number): number => m * x + b;
const five = y(1, 2, 3);
```

Pure Functions: Substitution

Math

```
y(m, x, b) = m * x + b
five = 5
```

Scala

```
def y(m : Int, x : Int, b : Int): Int = m * x + b
val five = 5
```

JavaScript

```
const y = (m: number, x: number, b: number): number ⇒ m * x + b
const five = 5
```

More Pure Functions: Substitution?

JavaScript

```
import { List } from "immutable"

const sum = (nums: List<number>): number ⇒ {
  let total = 0;
  for(let num of nums){
    total += num;
  }
  return total;
}
```

More Pure Functions: Substitution?

Scala

```scala
def sum(nums: List[Int]): Int = {
  var i, total = 0
  while(i < nums.size){
    total = total + nums(i)
    i = i + 1
  }
  return total;
}
```

Impure Functions: Mutation

Scala

```scala
var count: Int = 0
def increment(): Unit = count = count + 1
def getCount(): Int = count

val zero = getCount()
increment()
zero == getCount() //false
```

JavaScript

```typescript
let count: number = 0
const increment = (): void  => { count++ }
const getCount = (): number => count

const zero: number = getCount()
increment()
zero === getCount() //false
```

Impure Functions: Reading values outside the program

Scala

```scala
def isTodayOdd(): Boolean =
  new java.util.Date().getDay() % 2 == 1
```

JavaScript

```javascript
const isTodayDayOdd = (): boolean =>
  (new Date().getDay() + 1) % 2 === 0
```

More Impure Functions: IO

Scala

```scala
def getText(fileName: String): String =
  scala.io.Source.fromFile(fileName).getLines.mkString
```

JavaScript

```typescript
const getText = (fileName: string): string ⇒
  fs.readFileSync(fileName).toString()
```

What makes a function impure?

Side effecting functions:

- Perform I/O (disk, network, console)
- Get values from *outside of the program* (dates, random numbers)
- Mutating values beyond its scope

Functions without these things make us feel safe!

Side Effects

All I do all day is make side-effects! How do I get anything done if I can't do that!

Side Effects: Pure Functions

```
y = m * x + b
```

Side Effects: Side Effects

```
y = m * x + b
y = (e * m) * (e * x) + (e * b)
```

Side Effects: Factor out Side Effects

```
y = m * x + b
y = (e * m) * (e * x) + (e * b)
y = e(m * x + b)
```

Side Effects: Bind More Side Effects

```
y = m * x + b
y = (e * m) * (e * x) + (e * b)
y = e(m * x + b)
y = e(m * x + b) + e(5)
```

Side Effects: Bind More Side Effects

```
y = m * x + b
y = (e * m) * (e * x) + (e * b)
y = e(m * x + b)
y = e(m * x + b) + e(5)
y = e(m * x + b + 5)
```

Side Effects: Map in Pure Functions

```
y = m * x + b
y = (e * m) * (e * x) + (e * b)
y = e(m * x + b)
y = e(m * x + b) + e(5)
y = e(m * x + b + 5)
y = e(m * x + b + 5) + 10
```

Side Effects: Map in Pure Functions

```
y = m * x + b
y = (e * m) * (e * x) + (e * b)
y = e(m * x + b)
y = e(m * x + b) + e(5)
y = e(m * x + b + 5)
y = e(m * x + b + 5) + 10
y = e(m * x + b + 5 + 10)
```

Side Effects: Essence of FP

```
y = m * x + b
y = (e * m) * (e * x) + (e * b)
y = e(m * x + b)
y = e(m * x + b) + e(5)
y = e(m * x + b + 5)
y = e(m * x + b + 5) + 10
y = e(m * x + b + 5 + 10)
```

FP is Essentially

- binding side effecting contexts together (*flatMap*)
- mapping pure functions into side effecting contexts (*map*)

What does this look like in practice?

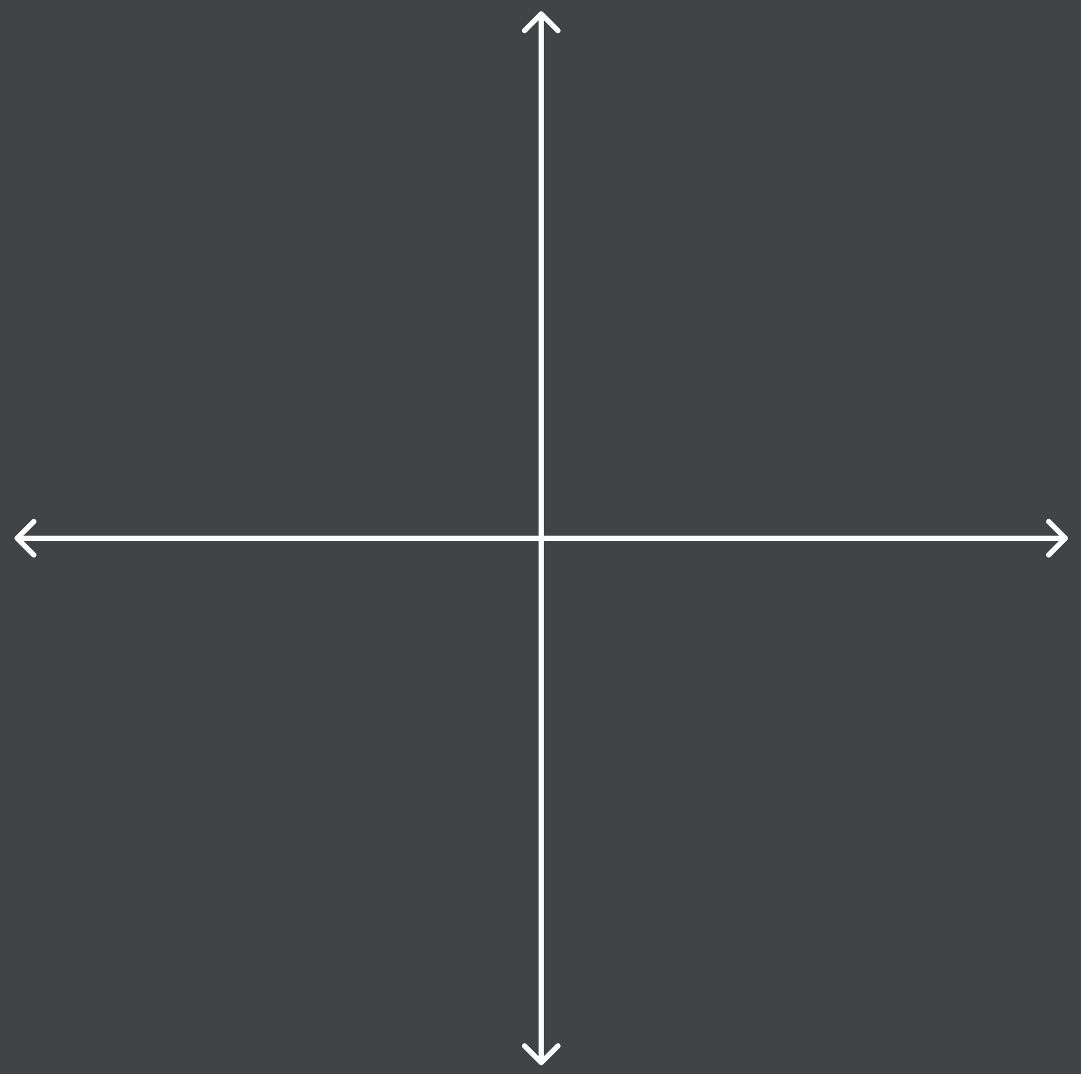
DEMO TIME

Equational Reasoning

Equational reasoning is the notion that we can understand what to expect from a function simply by looking at a function's types and their associated properties.

Equational Reasoning: Intuition

$$\begin{aligned} y(m, x, b) &= m * x + b \\ \text{where} \quad & m, x, b, y \in \mathbb{R} \end{aligned}$$



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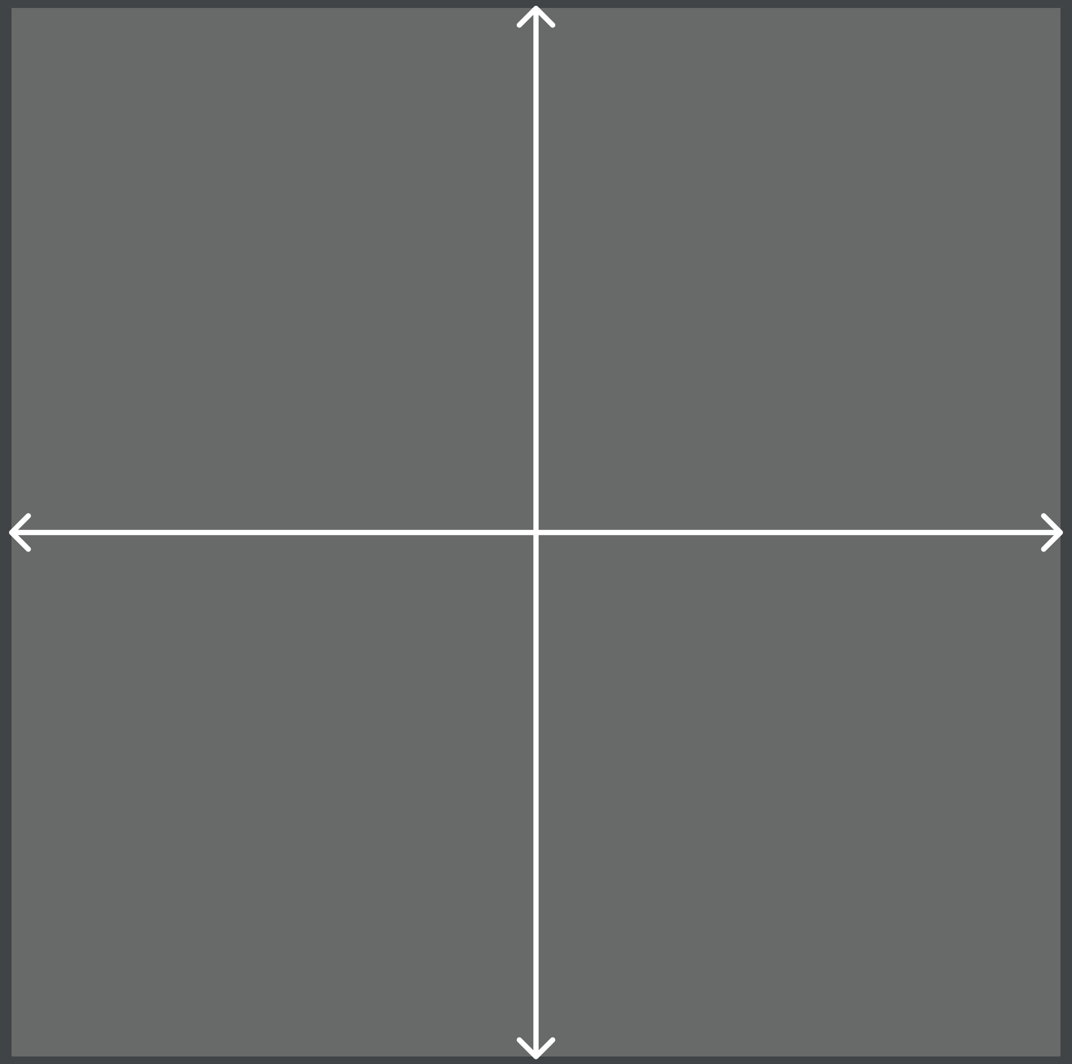
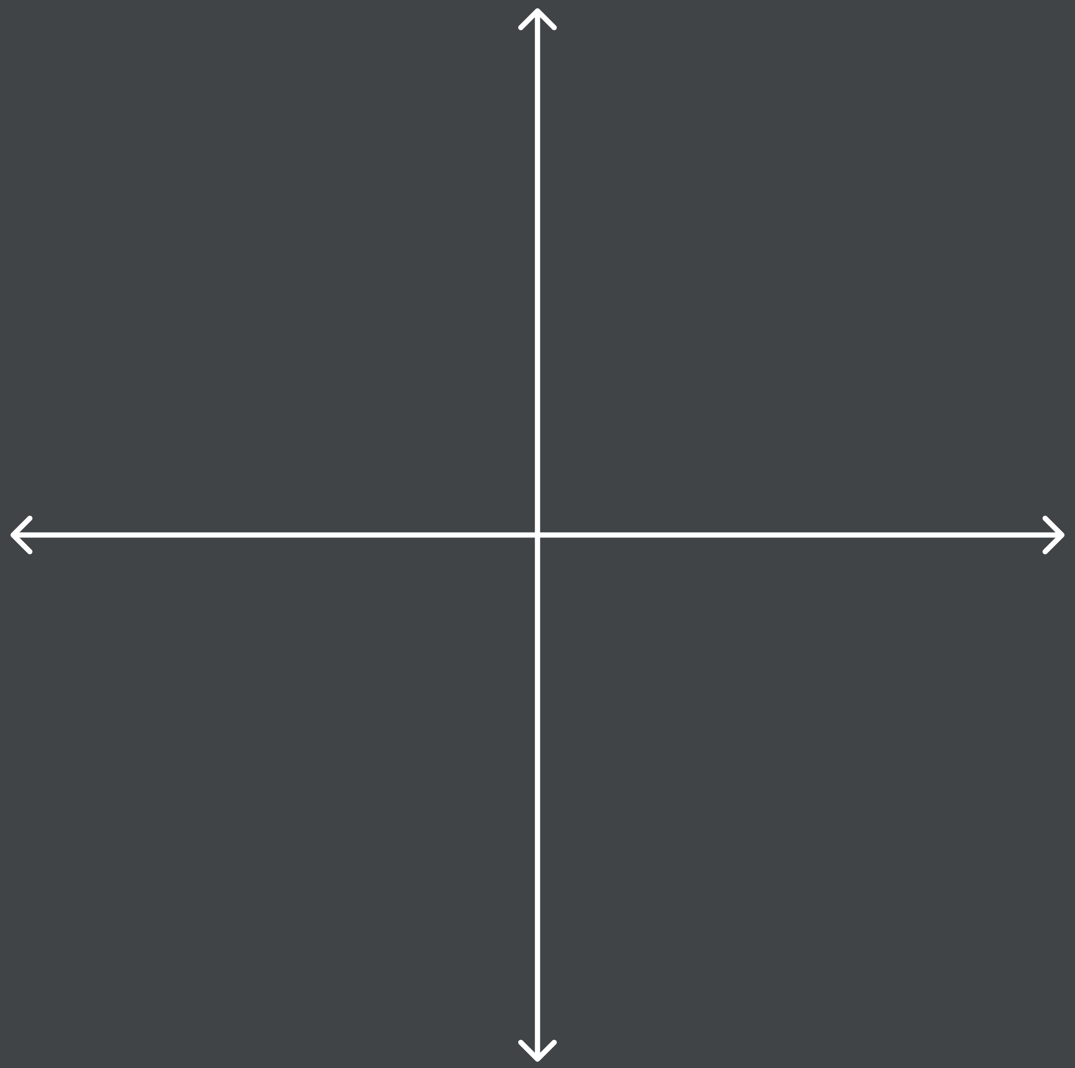


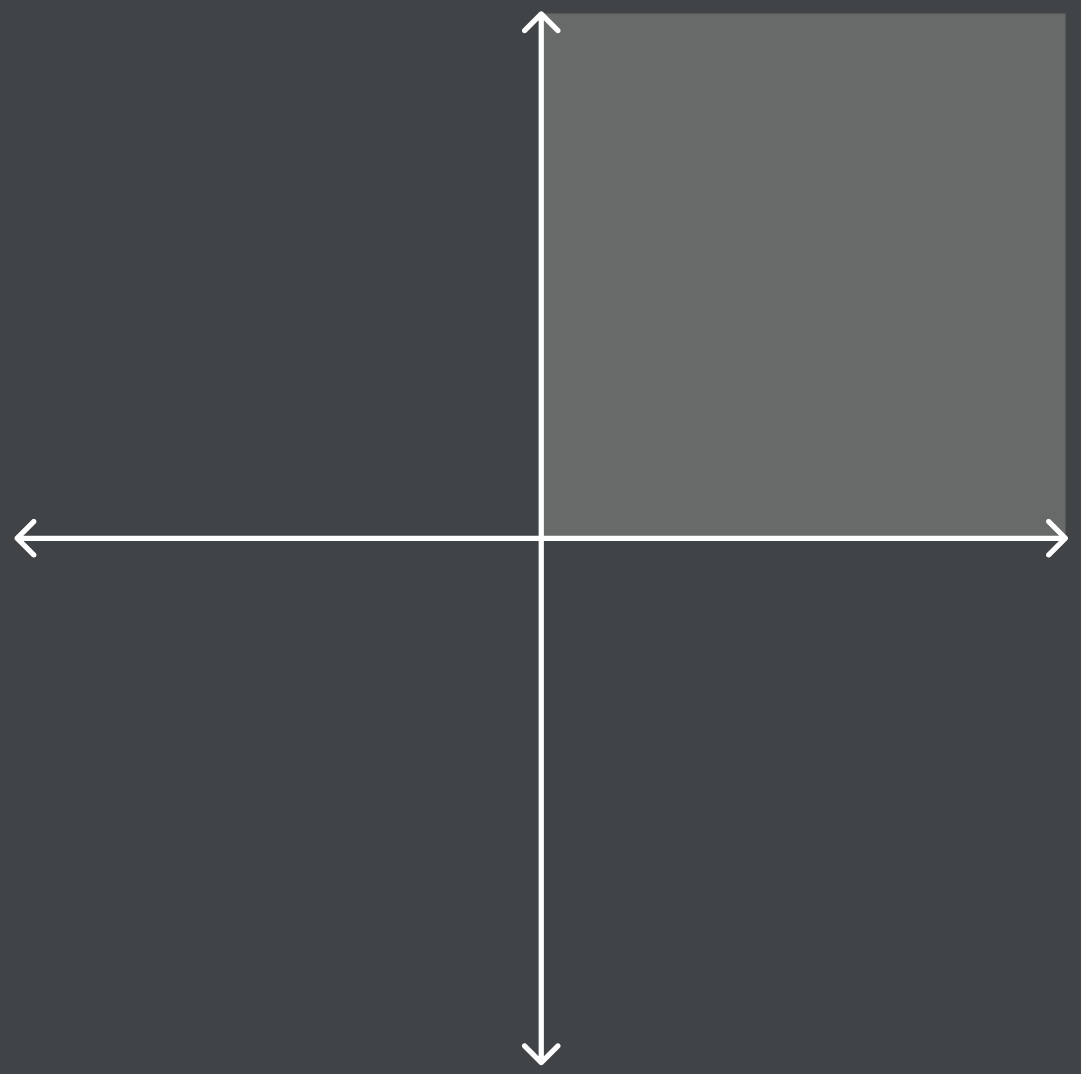
Equational Reasoning: Intuition

$$\begin{aligned} y(m, x, b) &= m * x + b \\ \text{where} \quad & m, x, b, y \in \mathbb{N} \end{aligned}$$



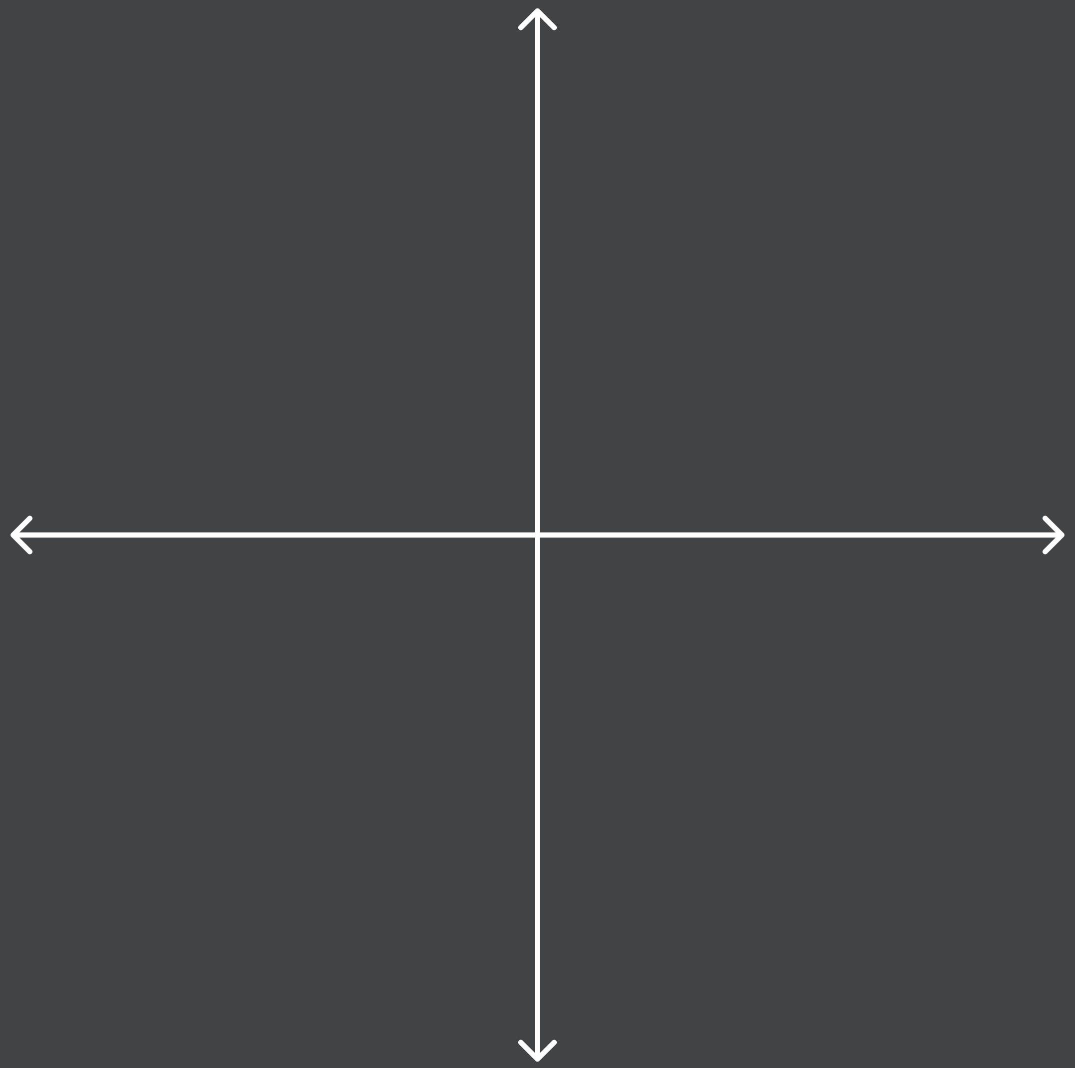
Equational Reasoning: Intuition

$$\begin{aligned} y(m, x, b) &= m * x + b \\ \text{where} \quad & m, x, b, y \in \mathbb{N} \end{aligned}$$



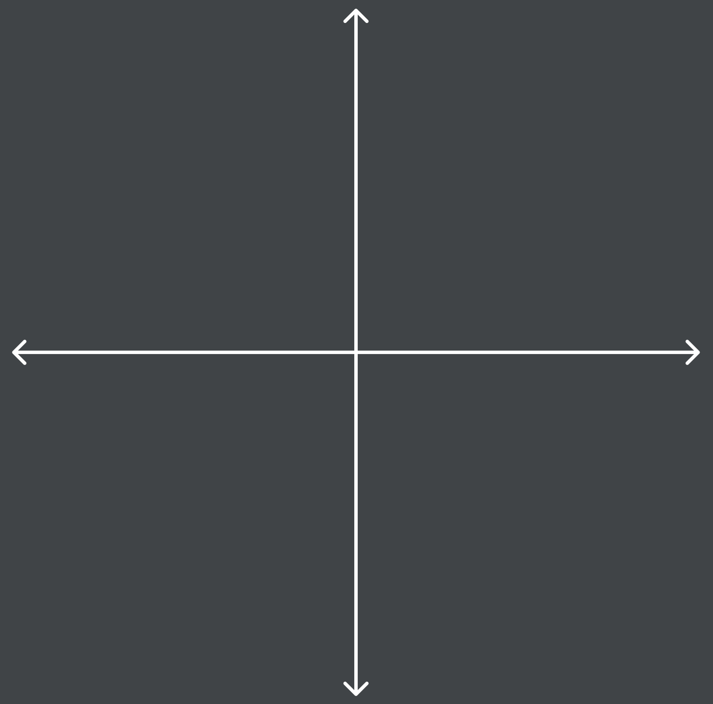
Equational Reasoning: Intuition

$$\begin{aligned} y(m, x, b) &= \ldots \\ \text{where} \quad & m, x, b, y \in \mathbb{N} \end{aligned}$$



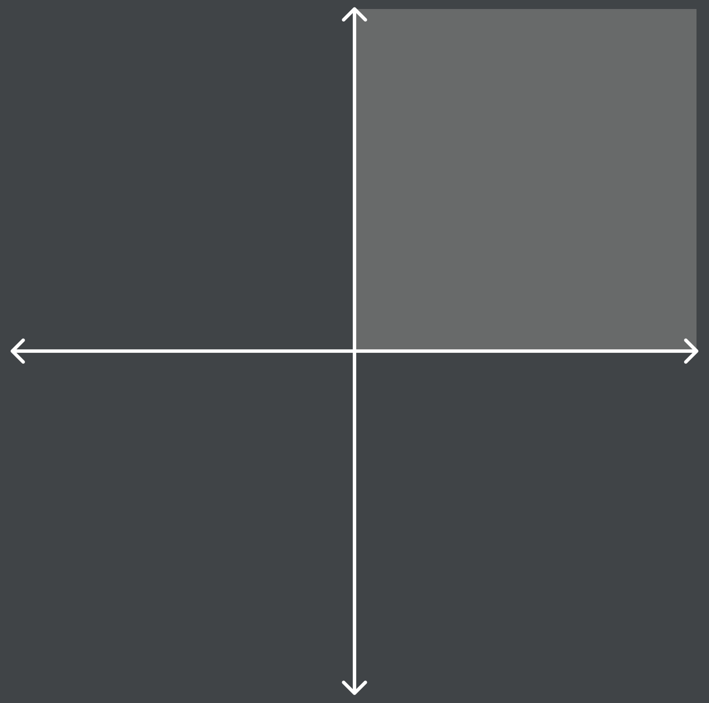
Equational Reasoning: Intuition

$$\begin{aligned}
y(m, x, b) &= \dots \\
\text{where} \quad & m, x, b, y \in \mathbb{N} \\
\\
r(n) &= \dots \\
& n \in \mathbb{N} \\
& r \in \mathbb{R} \\
\\
n(r) &= \dots \\
& n \in \mathbb{N} \\
& r \in \mathbb{R}
\end{aligned}$$



Equational Reasoning: Intuition

$$\begin{aligned} y(m, x, b) &= \ldots \\ \text{where} \quad & m, x, b, y \in \mathbb{N} \\ \\ r(n) &= \ldots \\ & n \in \mathbb{N} \\ & r \in \mathbb{R} \\ \\ n(r) &= \ldots \\ & n \in \mathbb{N} \\ & r \in \mathbb{R} \end{aligned}$$



Equational Reasoning: Practice

Scala

```
def addNaturalNumbers(a: Int, b: Int): Either[Throwable,Int] = ???
```

JavaScript

```
const addNaturalNumbers = (a: number, b: number): Either<Error,number> ⇒ ...
```

Equational Reasoning: Practice

Scala

```scala
case class NaturalNumber private (n: Int)
object NaturalNumber{
  def create(n: Int): Either[Throwable, NaturalNumber] = ???
}

def addNaturalNumbers(a: NaturalNumber, b: NaturalNumber): NaturalNumber = ???
```

JavaScript

```javascript
opaque type NaturalNumber = number
const createNaturalNumber = (n: number): Either<Error, NaturalNumber> ⇒

const addNaturalNumbers = (a: NaturalNumber, b: NaturalNumber): NaturalNumber ⇒
```

Equational Reasoning: Practice

Scala

```scala
def createUser(username: String, email: String): Either[Throwable, User]

def createUser(username: Username, email: Email): User
```

JavaScript

```javascript
const createUser = (username: string, email: string): Either<Throwable,User> ⇒ ...

const createUser = (username: Username, email: Email): User ⇒ ...
```

Equational Reasoning: Theory

Imagine each function below must be pure. How many possible implementations can each function have?

Scala

```scala
def foo[A](a: A): A = ???
```

JavaScript

```javascript
const foo = <A>(a: A): A ⇒ ???
```

Equational Reasoning: Theory

Imagine each function below must be pure. How many possible implementations can each function have?

Scala

```
def foo[A](a: A, a2: A): A = ???
```

JavaScript

```
const foo = <A>(a: A, a2: A): A ⇒ ???
```

Equational Reasoning: Theory

Imagine each function below must be pure. How many possible implementations can each function have?

Scala

```scala
def foo(a: Int): Int = ???
```

JavaScript

```javascript
const foo = (a: number): number ⇒ ...
```

Equational Reasoning: Theory

Forget purity for a moment...

Scala

```scala
def foo(a: Int): Unit = ???
```

JavaScript

```javascript
const foo = (a: number): Void ⇒ ...
```

Equational Reasoning: Composition

Scala

```scala
def makePoints(xs: List[Int]): List[Points] = ...
def plot(points: List[Points]): Chart = ...
def toSVG(chart: Chart): Chart = ...
def render(xs: List[Int]): Chart = toSVG(plot(makePoints(xs)))
```

JavaScript

```javascript
const makePoints = (xs: List<number>): Points ⇒
const plot = (points: Points): Chart ⇒
const toSVG = (chart: Chart): SVG ⇒
const render = (xs: List<number>): SVG ⇒ toSVG(plot(makePoints(xs)))
```

Equational Reasoning: Composition with Side Effects

Scala

```scala
def makePoints(xs: List[Int]): IO[List[Points]] = ???
def plot(points: List[Points]): IO[Chart] = ???
def toSVG(chart: Chart): IO[SVG] = ???
def render(xs: List[Int]): IO[SVG] = for {
  points  ← makePoints(xs)
  chart   ← plot(points)
  svg     ← toSVG(chart)
} yield svg

def render2(xs: List[Int]): IO[SVG] =
  makePoints(xs)
    .flatMap(plot)
    .flatMap(toSVG)
```

Equational Reasoning: Composition with Side Effects

JavaScript

```
const makePoints = (xs: List<number>): IO<Points> ⇒
const plot = (points: Points): IO<Chart> ⇒
const toSVG = (chart: Chart): IO<SVG> ⇒
const render = (xs: List<number>): IO<SVG> ⇒
  makePoints(xs)
    .chain(plot)
    .chain(toSVG)
```

Equational Reasoning: Theory

Adding properties...

Scala

```
def foo[A](a: A, a2: A, combiner: (A,A) ⇒ A): A = ???
foo(1, 2, (a, b) ⇒ a + b)
```

JavaScript

```
foo("a","b", ((a,b) ⇒ a + b))
```

Equational Reasoning: Practice

You can work these concepts into your day by not using curly braces.

Scala

```
def render(xs: List[Int]): IO[SVG] =
  makePoints(xs)
    .flatMap(plot)
    .flatMap(toSVG)
```

JavaScript

```
const render = (xs: List<number>): IO<SVG> ⇒
  makePoints(xs)
    .chain(plot)
    .chain(toSVG)
```

Summary

- *Referentially transparent* functions are *pure* and have no *side effects*
- *pure* functions make us feel "safe" because they are predictable and do not mutate external state
- *pure* functions are easy to reason about because they have clear boundaries (domain/range)
- *pure* functions *compose*, which allows you to build large programs (functions) by combining small components (functions)
- functional programming seeks to "factor out" side effects from pure code, so that you can make referentially transparent functions

Exercises: find the last element of a list

for now assume the list isn't empty

Example:

```
scala> last(List(1, 1, 2, 3, 5, 8))
res0: Int = 8*
```

Exercises: find the last element of a list

for now assume the list isn't empty

Example:

```
scala> penultimate(List(1, 1, 2, 3, 5, 8))
res0: Int = 5
```

Exercises: Reverse a list.

for now assume the list isn't empty

Example:

```
scala> reverse(List(1, 1, 2, 3, 5, 8))
res0: List[Int] = List(8, 5, 3, 2, 1, 1)
```

Exercises: Flatten a nested list structure.

for now assume the list isn't empty

Example:

```
scala> flatten(List(List(1, 1), 2, List(3, List(5, 8))))
res0: List[Any] = List(1, 1, 2, 3, 5, 8)
```

Exercises: Eliminate consecutive duplicates of list elements.

for now assume the list isn't empty

Example:

```
scala> compress(List('a, 'a, 'a, 'a, 'b, 'c, 'c, 'a, 'a, 'd, 'e, 'e, 'e, 'e))
res0: List[Symbol] = List('a, 'b, 'c, 'a, 'd, 'e)
```

Next time?

- Reader monad?
- ADTs and Optics?
- Parametric Polymorphism, Higher Kinded Types, and Type Classes?