Look again at the book-keeping part:

$$\prod_{p=i+1}^{j-1} (-1)^{n_p} \mapsto \prod_{p=i+1}^{j-1} 1 - 2n_p$$

1. If there are many states, it is likely that a near-zero value will contribute to the semiclassical product.

2. The dynamics will depend on the order in which the states are listed.

Several possible choices for the book-keeping:

$$\prod_{p=i+1}^{j-1} (-1)^{n_p} \mapsto \begin{cases} \prod_{p=i+1}^{j-1} 1 - 2n_p & \text{linear} \\ \\ \prod_{p=i+1}^{j-1} e^{i\pi n_p} & \text{exponential} \\ \\ 1 & \text{none} \end{cases}$$

There's no obvious best choice, so experiment!