Several possible choices for the book-keeping:

$$\prod_{p=i+1}^{j-1} (-1)^{n_p} \mapsto \begin{cases} \displaystyle\prod_{p=i+1}^{j-1} 1 - 2n_p & \text{linear} \\[2em] \displaystyle\prod_{p=i+1}^{j-1} e^{i\pi n_p} & \text{exponential} \\[2em] 1 & \text{none} \end{cases}$$

There's no obvious best choice, so experiment!

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No book-keeping works best!