Mapping Multiple States

$$\left\langle \mathbf{n}' \middle| \hat{a}_i^\dagger \hat{a}_j \middle| \mathbf{n} \right\rangle = \delta_{n_i, n_i'+1} \; \delta_{n_j, n_j'-1} \prod_{p=i+1}^{j-1} (-1)^{n_p} \prod_{q \neq i,j} \delta_{n_q, n_q'}$$

This is the direct product of 2x2 matrices for each dof!

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raising lowering book-keeping

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