Fermions are *very* non-classical

1. Exchange Symmetry

$$|\psi(x_1, x_2)|^2 = |\psi(x_2, x_1)|^2$$

$$\psi(x_1, x_2) = +\psi(x_2, x_1) \quad \text{bosons}$$

$$\psi(x_1, x_2) = -\psi(x_2, x_1) \quad \text{fermions}$$

6. Fermionic Exchange ⇒ Pauli Exclusion Principle

7. Statistics: Bose-Einstein vs. Fermi-Dirac

$$Q = \sum_{n=0}^{\infty} e^{-\beta\epsilon n} \approx \int_0^{\infty} \mathrm{d}n\, e^{-\beta\epsilon n} \quad \text{bosons}$$

$$Q = 1 + e^{-\beta\epsilon} \quad \text{fermions}$$

Second Quantization Review

Raising/Lowering Operators

$$\hat{c}^\dagger |n\rangle = \sqrt{n+1}\, |n+1\rangle$$

$$\hat{c}\, |n\rangle = \sqrt{n}\, |n-1\rangle$$

$$\hat{N}\, |n\rangle \equiv \hat{c}^\dagger \hat{c}\, |n\rangle = n\, |n\rangle$$

$$\hat{c}^\dagger |n_{\max}\rangle = 0$$

Bosonic form gives the “ladder operator” solution to the harmonic oscillator