Mapping Multiple States

$$\left\langle \mathbf{n}' \middle| \hat{a}_i^\dagger \hat{a}_j \middle| \mathbf{n} \right\rangle = \delta_{n_i, n_i'+1} \; \delta_{n_j, n_j'-1} \prod_{p=i+1}^{j-1} (-1)^{n_p} \prod_{q \neq i,j} \delta_{n_q, n_q'}$$

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This is the direct product of 2x2 matrices for each dof!

For example, for degree of freedom *i*, the matrix is:

$$\begin{array}{ccc} & |n_i = 1\rangle & |n_i = 0\rangle \\ \langle n_i = 1| & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$}} \\ \langle n_i = 0| & & \end{array}$$

Mapping Multiple States

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mapsto 1 - n \qquad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \mapsto \sqrt{n - n^2 + \lambda}\, e^{-iq}$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \mapsto \sqrt{n - n^2 + \lambda}\, e^{iq} \qquad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mapsto n$$

So we obtain the mapping:

$$\hat{a}_i^\dagger \hat{a}_j \mapsto \left(\sqrt{n_i - n_i^2 + \lambda}\, e^{-iq_i}\right)\left(\sqrt{n_j - n_j^2 + \lambda}\, e^{iq_j}\right) \prod_{p=i+1}^{j-1} 1 - 2n_p$$

raising lowering book-keeping

Miller & White, JCP 84, 5059 (1986)