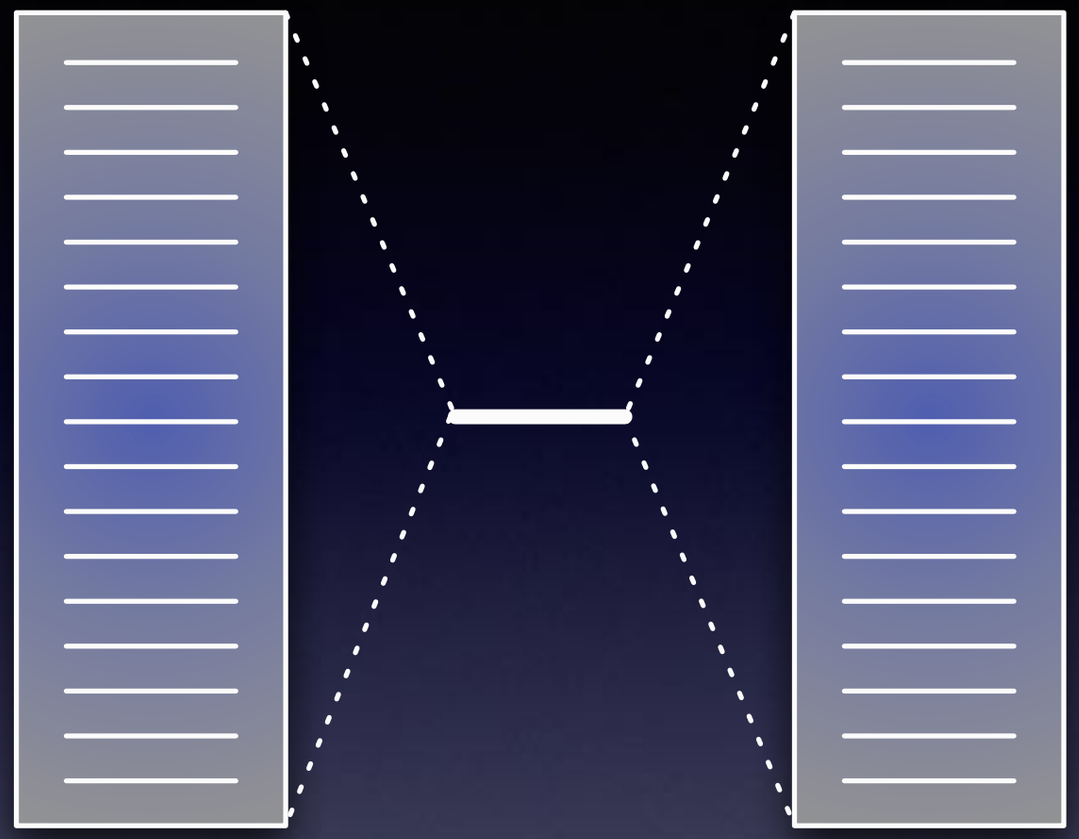
Several possible choices for the book-keeping:

$$\prod_{p=i+1}^{j-1} (-1)^{n_p} \mapsto \begin{cases} \displaystyle\prod_{p=i+1}^{j-1} 1 - 2n_p & \text{linear} \\ \displaystyle\prod_{p=i+1}^{j-1} e^{i\pi n_p} & \text{exponential} \\ 1 & \text{none} \end{cases}$$

No book-keeping works best!

Landauer Model



$$\hat{H} = \sum_{k\in L} \epsilon_k \hat{a}_k^\dagger \hat{a}_k + \epsilon_0 \hat{a}_0^\dagger \hat{a}_0 + \sum_{k\in R} \epsilon_k \hat{a}_k^\dagger \hat{a}_k$$
$$+ \sum_{k\in L} t_k \left(\hat{a}_0^\dagger \hat{a}_k + \hat{a}_k^\dagger \hat{a}_0 \right) + \sum_{k\in R} t_k \left(\hat{a}_0^\dagger \hat{a}_k + \hat{a}_k^\dagger \hat{a}_0 \right)$$