Semiclassical Initial Value Representation

$$C_{AB}(t) \approx (2\pi)^{-F} \int \mathrm{d}\mathbf{q}_0 \int \mathrm{d}\mathbf{p}_0 \int \mathrm{d}\mathbf{q}_0' \int \mathrm{d}\mathbf{p}_0' \times \underbrace{\left\langle \mathbf{q}_0\mathbf{p}_0 \middle| \hat{A} \middle| \mathbf{q}_0'\mathbf{p}_0' \right\rangle}_{A(x)} \underbrace{\left\langle \mathbf{q}_t'\mathbf{p}_t' \middle| \hat{B} \middle| \mathbf{q}_t\mathbf{p}_t \right\rangle}_{B(x)} \underbrace{\mathcal{C}_t^{\mathrm{HK}} \mathcal{C}_{t'}^{\mathrm{HK}^*} e^{i(S_t - S_t')}}_{C(x)}$$

- *A(x)* requires no trajectory, just matrix element
- *B(x)* requires “classical trajectory” (forces)
- *C(x)* requires “semiclassical trajectory” (Hessian)

Monte Carlo Integration

$$\int \mathrm{d}x\, f(x)g(x) = \left\langle \frac{1}{\rho(x)} f(x)g(x) \right\rangle_{\rho(x)}$$

This average will give us the integral,
but how to do it efficiently?