## Mapping Multiple States

$$\left\langle \mathbf{n}' \middle| \hat{a}_i^{\dagger} \hat{a}_j \middle| \mathbf{n} \right\rangle = \delta_{n_i, n'_i + 1} \ \delta_{n_j, n'_j - 1} \prod_{p = i + 1}^{j - 1} (-1)^{n_p} \prod_{q \neq i, j} \delta_{n_q, n'_q}$$

raising lowering book-keeping

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For example, for degree of freedom i, the matrix is:

$$|n_i = 1\rangle \qquad |n_i = 0\rangle$$
 $\langle n_i = 1| \qquad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ 
 $\langle n_i = 0| \qquad 0$