Spin Matrix Mapping

Second quantization → 2x2 Matrices → Classical dynamics

Quantum spin
(Pauli matrices)

Classical spin model

$$\mathbf{S}_x/\hbar = \tfrac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mapsto \sqrt{\sigma^2 - \left(n - \frac{1}{2}\right)^2}\cos(q)$$

$$\mathbf{S}_y/\hbar = \tfrac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \mapsto \sqrt{\sigma^2 - \left(n - \frac{1}{2}\right)^2}\sin(q)$$

$$\mathbf{S}_z/\hbar = \tfrac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \mapsto n - \frac{1}{2}$$

Meyer and Miller. JCP **71**, 2156 (1979)

Action-angle variables

Just like momentum and position;
different “basis” for Hamiltonian dynamics

$$\dot{x} = \frac{\partial H}{\partial p} \qquad \dot{q} = \frac{\partial H}{\partial n}$$

$$\dot{p} = -\frac{\partial H}{\partial x} \qquad \dot{n} = -\frac{\partial H}{\partial q}$$