Action-angle variables

Just like momentum and position;
different “basis” for Hamiltonian dynamics

$$\dot{x} = \frac{\partial H}{\partial p} \qquad \dot{q} = \frac{\partial H}{\partial n}$$

$$\dot{p} = -\frac{\partial H}{\partial x} \qquad \dot{n} = -\frac{\partial H}{\partial q}$$

Term in second-quantized Hamiltonian

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Make 2x2 matrix for each degree of freedom involved

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Classical spin analogy for matrix

$$\hat{a}_i^\dagger \hat{a}_j \mapsto \sqrt{(n_i - n_i^2 + \sigma^2 - 1/4)(n_j - n_j^2 + \sigma^2 - 1/4)}\; e^{i(q_j - q_i)}$$

$$\hat{a}_i^\dagger \hat{a}_i \mapsto n_i$$