Calculating the Left Current

$$\hat{I}_L \propto -\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \sum_{k\in L} \hat{a}_k^\dagger \hat{a}_k \right\rangle$$

$$\propto \boxed{-\left\langle \sum_{k\in L} t_k(\hat{a}_0^\dagger \hat{a}_k - \hat{a}_k^\dagger \hat{a}_0) \right\rangle}$$

1. Map occupation to classical quantity
2. Take time derivative of classical quantity

1. Take quantum time derivative
2. Map resulting operator to a classical quantity

Calculating the Left Current

$$\hat{I}_L \propto -\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \sum_{k\in L} \hat{a}_k^\dagger \hat{a}_k \right\rangle$$

$$\propto -\left\langle \sum_{k\in L} t_k(\hat{a}_0^\dagger \hat{a}_k - \hat{a}_k^\dagger \hat{a}_0) \right\rangle$$

1. Map occupation to classical quantity
2. Take time derivative of classical quantity

1. Take quantum time derivative
2. Map resulting operator to a classical quantity

Formally equivalent for our mapping!