$$\mathbf{S}_x/\hbar = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mapsto \sqrt{\sigma^2 - \left(n - \frac{1}{2}\right)^2}\cos(q)$$

$$\mathbf{S}_y/\hbar = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \mapsto \sqrt{\sigma^2 - \left(n - \frac{1}{2}\right)^2}\sin(q)$$

$$\mathbf{S}_z/\hbar = \frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \mapsto n - \frac{1}{2}$$

$$\frac{1}{2}\mathbf{I}_2 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mapsto \frac{1}{2}$$

$$m \in \left\{\pm\frac{1}{2}\right\} \to n \in \{0, 1\}$$

$$\frac{1}{2}\mathbf{I}_2 - \mathbf{S}_z/\hbar = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mapsto 1 - n$$

$$\mathbf{S}_x/\hbar + i\mathbf{S}_y/\hbar = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \mapsto \sqrt{\sigma^2 - \left(n - \frac{1}{2}\right)^2}\ e^{iq}$$

$$\mathbf{S}_x/\hbar - i\mathbf{S}_y/\hbar = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \mapsto \sqrt{\sigma^2 - \left(n - \frac{1}{2}\right)^2}\ e^{-iq}$$

$$\frac{1}{2}\mathbf{I}_2 + \mathbf{S}_z/\hbar = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mapsto n$$