

Short-Time Behavior

Assume initial condition in which state 0 is occupied.
Match the $t=0$ derivatives of the exact quantum result:

Quantum	Semiclassical
$\dot{\hat{N}}_0(0) = 0$	$\dot{n}_0(0) = 0$
$\ddot{\hat{N}}_0(0) = -2k^2$	$\ddot{n}_0(0) = -4k^2\lambda$

$$\lambda = \frac{1}{2}$$

$$\sqrt{n - n^2 + \lambda} \implies n \in \left[\frac{1}{2} - \sqrt{\frac{1}{4} + \lambda}, \frac{1}{2} + \sqrt{\frac{1}{4} + \lambda} \right]$$

