

Calculating the Left Current

$$\begin{aligned}\hat{I}_L &\propto -\frac{d}{dt} \left\langle \sum_{k \in L} \hat{a}_k^\dagger \hat{a}_k \right\rangle \\ &\propto - \left\langle \sum_{k \in L} t_k (\hat{a}_0^\dagger \hat{a}_k - \hat{a}_k^\dagger \hat{a}_0) \right\rangle\end{aligned}$$

1. Map occupation to classical quantity
2. Take time derivative of classical quantity

1. Take quantum time derivative
2. Map resulting operator to a classical quantity

Formally equivalent for our mapping!

SMM: Spin Matrix Mapping