Rewrite terms as matrices

Second quantization $\longrightarrow$ 2x2 Matrices $\longrightarrow$ Classical dynamics

Write the matrix element explicitly:

$$\left\langle \mathbf{n}' \middle| \hat{a}_i^\dagger \hat{a}_j \middle| \mathbf{n} \right\rangle = \delta_{n_i, n_i'+1} \delta_{n_j, n_j'-1} \prod_{k \neq i,j} \delta_{n_k, n_k'} \prod_{p=i+1}^{j-1} (-1)^{n_p}$$

raise lower normal order

Each Kronecker delta can be represented as a 2x2 matrix in the occupancy basis

$$\begin{array}{ccc} & |n_i = 1\rangle & |n_i = 0\rangle \\ \begin{array}{c} \langle n_i = 1| \\ \langle n_i = 0| \end{array} & \multicolumn{2}{c}{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}} \end{array}$$

Spin Matrix Mapping

Second quantization → 2x2 Matrices → Classical dynamics

Quantum spin (Pauli matrices)

Classical spin model

$$\mathbf{S}_x/\hbar = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mapsto \sqrt{\sigma^2 - \left(n - \frac{1}{2}\right)^2}\cos(q)$$

$$\mathbf{S}_y/\hbar = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \mapsto \sqrt{\sigma^2 - \left(n - \frac{1}{2}\right)^2}\sin(q)$$

$$\mathbf{S}_z/\hbar = \frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \mapsto n - \frac{1}{2}$$

Meyer and Miller. JCP **71**, 2156 (1979)