Mapping Multiple States

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mapsto 1 - n \qquad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \mapsto \sqrt{n - n^2 + \lambda}\; e^{-iq}$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \mapsto \sqrt{n - n^2 + \lambda}\; e^{iq} \qquad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mapsto n$$

So we obtain the mapping:

$$\hat{a}_i^\dagger \hat{a}_j \mapsto \left(\sqrt{n_i - n_i^2 + \lambda}\; e^{-iq_i}\right)\left(\sqrt{n_j - n_j^2 + \lambda}\; e^{iq_j}\right) \prod_{p=i+1}^{j-1} 1 - 2n_p$$

raising lowering book-keeping

Miller & White, JCP 84, 5059 (1986)

Look again at the book-keeping part:

$$\prod_{p=i+1}^{j-1} (-1)^{n_p} \mapsto \prod_{p=i+1}^{j-1} 1 - 2n_p$$

1. If there are many states, it is likely that a near-zero value will contribute to the semiclassical product.
2. The dynamics will depend on the order in which the states are listed.