A Special Integrand

$$\int \mathrm{d}x\, A(x)B(x)C(x)$$

- *A(x)* is cheap
- *B(x)* is somewhat expensive
- *C(x)* is **very** expensive

$$\int \mathrm{d}x\, A(x)B(x)C(x) = N_A \left\langle e^{i\phi_A(x)} B(x)C(x) \right\rangle_{|A(x)|}$$

but what if:

$$|A(x)| \gg 0$$
$$|B(x)| \approx 0$$

$$\int \mathrm{d}x\, A(x)B(x)C(x) = N_{AB} \left\langle e^{i\phi_{AB}(x)} C(x) \right\rangle_{|A(x)B(x)|}$$

Time-Dependent Importance Sampling

$$C_{AB}(t) \approx (2\pi)^{-F} \int \mathrm{d}\mathbf{q}_0 \int \mathrm{d}\mathbf{p}_0 \int \mathrm{d}\mathbf{q}_0' \int \mathrm{d}\mathbf{p}_0'$$

$$\times \underbrace{\left\langle \mathbf{q}_0\mathbf{p}_0 \middle| \hat{A} \middle| \mathbf{q}_0'\mathbf{p}_0' \right\rangle}_{A(x)} \underbrace{\left\langle \mathbf{q}_t'\mathbf{p}_t' \middle| \hat{B} \middle| \mathbf{q}_t\mathbf{p}_t \right\rangle}_{B(x)} \underbrace{\mathcal{C}_t^{\mathrm{HK}} \mathcal{C}_{t'}^{\mathrm{HK}^*} e^{i(S_t - S_t')}}_{C(x)}$$

$$\int \mathrm{d}x\, A(x)B(x)C(x) = N_{AB} \left\langle e^{i\phi_{AB}(x)} C(x) \right\rangle_{|A(x)B(x)|}$$

1. Run classical trajectories to select points
2. Run semiclassical trajectories to accumulate results