

# 1. STRESS FEEDBACK MODEL

Assume cell wall is a smooth surface  $S \subset \mathbf{R}^3$ ,  $\mathbf{n}$  is the exterior normal vector to  $S$ . Define the deformation is  $\mathbf{u}$ , its tangential gradient is denoted by  $\nabla_S \mathbf{u} = \mathbf{n} \times (\nabla \mathbf{u} \times \mathbf{n})$ . Green-Lagrangian strain tensor is defined by  $\mathbb{E} = \frac{1}{2} (\nabla_S \mathbf{u}^\top \nabla_S \mathbf{u} - \mathbb{I})$ . The motified St. Venant-Kirchhoff model is given by

$$(1.1) \quad \mathcal{E} = \int_S \mathbb{E} : (\mathbb{C}) \mathbb{E} + \int_S ((\nabla \mathbf{u} - P\mathbb{I})\mathbf{n})^2,$$

where  $:$  is the double dot product defined by  $\mathbb{A} : \mathbb{B} = \text{tr}(\mathbb{A}^\top \mathbb{B})$ ,  $\mathbb{C} = \mathbb{C}_g + \mathbb{C}_f$ ,  $\mathbb{C}_g, \mathbb{C}_f$  respectively are the stiffness matrixes for the gel and fiber,

$$\mathbb{C}_g = Y_g \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix}, \quad \mathbb{C}_f = \frac{\pi Y_f \rho_0}{16} \begin{pmatrix} 3 + \frac{\rho_2 + 4\rho_1}{\rho_0} & 1 - \frac{\rho_2}{\rho_0} & \frac{2\widetilde{\rho_1} + \widetilde{\rho_2}}{\rho_0} \\ 1 - \frac{\rho_2}{\rho_0} & 3 + \frac{\rho_2 - 4\rho_1}{\rho_0} & \frac{2\widetilde{\rho_1} - \widetilde{\rho_2}}{\rho_0} \\ \frac{2\widetilde{\rho_1} + \widetilde{\rho_2}}{\rho_0} & \frac{2\widetilde{\rho_1} - \widetilde{\rho_2}}{\rho_0} & 1 - \frac{\rho_2}{\rho_0} \end{pmatrix},$$

$P$  is the pressure per unit volume applied to the surface,  $\nu, Y_g, Y_f$  are constants,  $\rho(\theta)$  is  $\pi$ -periodic angular microfibril distribution function,  $\{\widehat{\rho_n}(t)\}_{n=0}^2$  are the Fourier transform coefficients given by

$$\widehat{\rho_n} = \frac{1}{\pi} \int_0^\pi \rho(t, \theta) e^{-2in\theta} d\theta, \quad (\rho_n, \widetilde{\rho_n}) = 2(\text{Re}(\widehat{\rho_n}), -\text{Im}(\widehat{\rho_n})).$$

$\phi(\theta)$  is  $\pi$ -periodic angular microtubule distribution function. We have the following evolution and equilibrium equation for angular microfibril and microtubul distribution.

$$(1.2) \quad \frac{d\rho(\theta)}{dt} = k_\rho \frac{\phi(\theta)}{\int_0^\pi \phi(\theta') d\theta'} - k'_\rho \rho(t, \theta),$$

$$(1.3) \quad \phi(\theta) = \frac{c_0 k_\phi e^{\gamma f(\mathbb{C}\mathbb{E}, \theta)}}{1 + k_\phi \int_0^\pi e^{\gamma f(\mathbb{C}\mathbb{E}, \theta')} d\theta'}, \quad f(\mathbb{C}\mathbb{E}, \theta) = \mathbf{e}_\theta^\top (\mathbb{C}\mathbb{E}) \mathbf{e}_\theta.$$

where  $\mathbb{C} = \mathbb{C}_g + \mathbb{C}_f$ ,  $c_0, \gamma, k_\phi, k_\rho, k'_\rho$  are suitable positive constants. By Fourier transform, we have

$$(1.4) \quad \frac{d\widehat{\rho_n}}{dt} = \frac{k_\rho}{\pi} \frac{\widehat{\phi_n}}{\widehat{\phi_0}} - k'_\rho \widehat{\rho_n}, \quad \widehat{\phi_n} = \frac{1}{\pi} \int_0^\pi \phi(\theta) e^{-2in\theta} d\theta,$$

$$(1.5) \quad \widehat{\phi_n} = \frac{c_0}{\pi} \frac{I_n(2\gamma |\widehat{f_1}|)}{e^{-\gamma f_0}/(\pi k_\phi) + I_0(2\gamma |\widehat{f_1}|)} e^{-2in\theta^*}, \quad \widehat{f_1} = \frac{1}{\pi} \int_0^\pi f(\mathbb{C}\mathbb{E}, \theta) e^{-2i\theta} d\theta,$$

where  $\theta^*$  is the direction of the main stress,  $I_n$  is the following modified Bessel function of the first kind.

$$I_n(x) = \frac{1}{\pi} \int_0^\pi e^{x \cos(\theta)} \cos(n\theta) d\theta \quad \forall x \in \mathbf{R},$$

## 2. WEAK FORMULATION

For a bounded surface domain  $S \subset \mathbf{R}^3$ , let the inner product on  $L^2(S)$  be denoted by

$$(u, v)_S = \int_S uv, \quad \forall u, v \in L^2(S).$$

For  $\Gamma \subset \partial S$ , the sub-space of  $H^1(S)$  with homogeneous boundary conditions on  $\Gamma$  is denoted by

$$H_\Gamma^1(S) = \{v \in H^1(S) : v = 0 \text{ on } \Gamma\}.$$

Moreover, vector-valued and matrix-valued quantities will be denoted by boldface and hollow notations, respectively, such as  $\mathbf{L}^2(S) = (L^2(S))^3$ ,  $\mathbb{L}^2(S) = (L^2(S))^{3 \times 3}$ .

The weak formulation of problem