

We'll be starting shortly!

To help us run the workshop smoothly, please kindly:

- Switch off screen sharing and mute your microphone
- Submit all questions using the Q&A function
- If you have an urgent request, please use the “Raise Hand” function

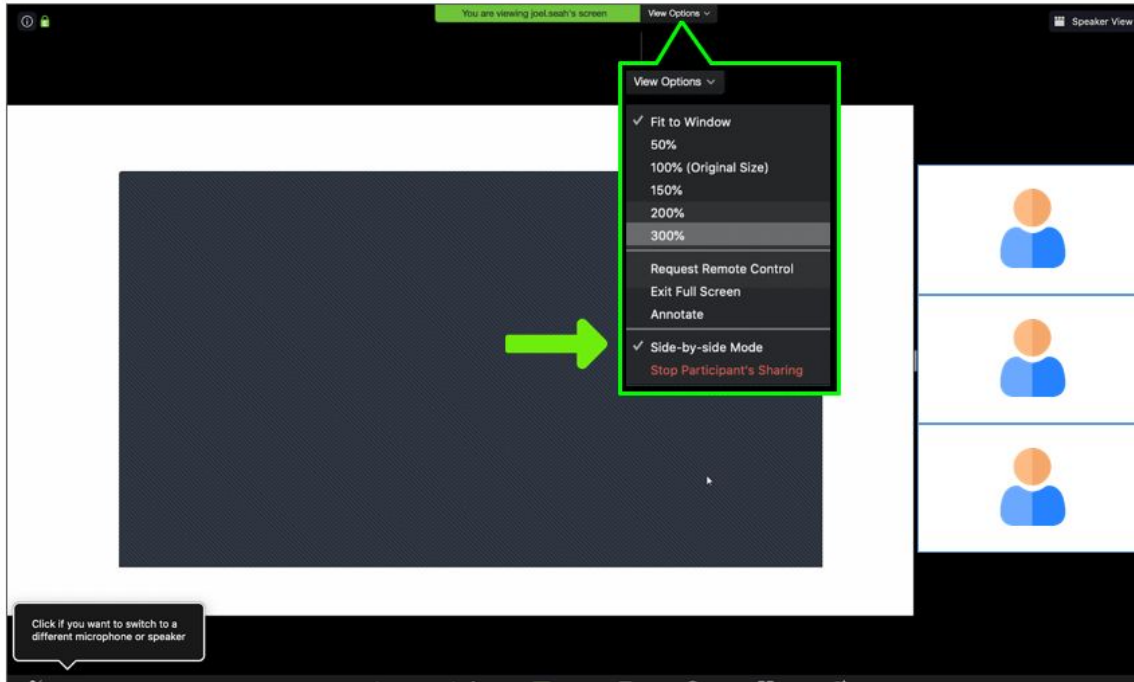
Thank you!



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Using Zoom: People & Slides



Side-By-Side Mode

- When sharing screen (slide share)
- With small thumbnails of people on the sidebar

STEPS:

1. View Options
2. Side-By-Side Mode



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DP: Speaker Notes

Overall Talking Points: What, Why, How

- introduce concepts
- contextualize concepts with examples
- up to **YOU** to practice! (especially for DP, this is the only way to master)

1. Overview Dynamic Programming(DP): What and Why

a. Definition:

- i. **“Solving Problems** by incrementally solving for and **re-using** solutions of its **subproblems**”
- ii. A category of algorithms that solves certain types of problems
- iii. Two Defining Properties:
 1. **Optimal** Substructure: $F(\text{final}) = G(F(\text{small})...)$
 2. **Overlapping** Subproblems: Can get **significant speed-up** from brute force

b. Motivation:

- i. When applicable, often lead to performant solutions(correct solutions, no TLE: ‘time limit exceeded’)
- ii. goto reason #i, more weapons in your algo toolbox :D

c. Example: Fibonacci Sequence

2. Solving Problems with DP

a. Problem Solving Approaches (your choice, but take note of the difference):

- i. Top-Down: Recursion (exhaustive search + memoization)
- ii. Bottom-Up: Tabulation (building a table)
- iii. Top-Down vs Bottom-Up:
 1. Most the problems could be solved by either



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Introduction to Dynamic Programming

Liang Qiao, Rocket Academy

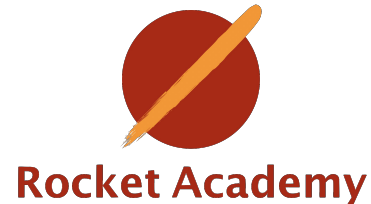
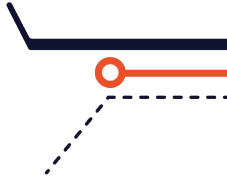
Slides: <https://tinyurl.com/y7pf7vkn>
Register: <https://rocketacademy.co/scl-slides>



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Overview

1. What is dynamic programming (DP)
2. Why/When to use dynamic programming (DP)
3. Problem solving approaches of DP
4. Problem solving strategy



Goal: Be able to solve basic DP problems

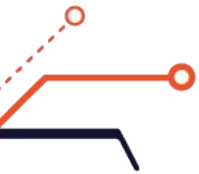


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What is Dynamic Programming?

1. “**Solving Problems** by incrementally solving for and **re-using** solutions of its **subproblems**”
2. **Two** Defining Properties:
 - a. **Optimal** Substructure: $F(\text{final}) = G(F(\text{small})\dots)$
 - b. **Overlapping** Subproblems

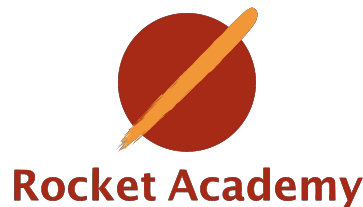
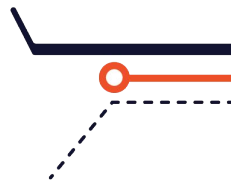


Introducing Fibonacci Sequence

$$f_n = f_{n-1} + f_{n-2}$$

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 ...

Question: how to write program to produce fib(n)
where **n is the index**?



Introducing Fibonacci Sequence

```
def fib(n):  
    if n == 0 or n == 1:  
        return n  
    return fib(n-1) + fib(n-2)
```

What's the time complexity?
(you could answer in chat)



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Naive Fibonacci: time complexity

Recurrence:

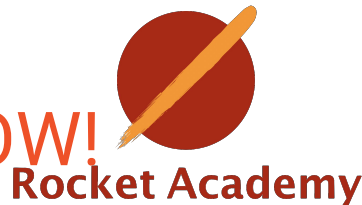
$$T(n) = T(n-1) + T(n-2)$$

$$\geq 2 * T(n-2) \geq 2 * 2 * T(n-2-2)$$

$$\geq 2 * 2 * 2 * T(n-2-2-2) \text{ so on and so forth ...}$$

$$\geq 2^{(n/2)}$$

Exponential is **NOT Good! Too SLOW!**



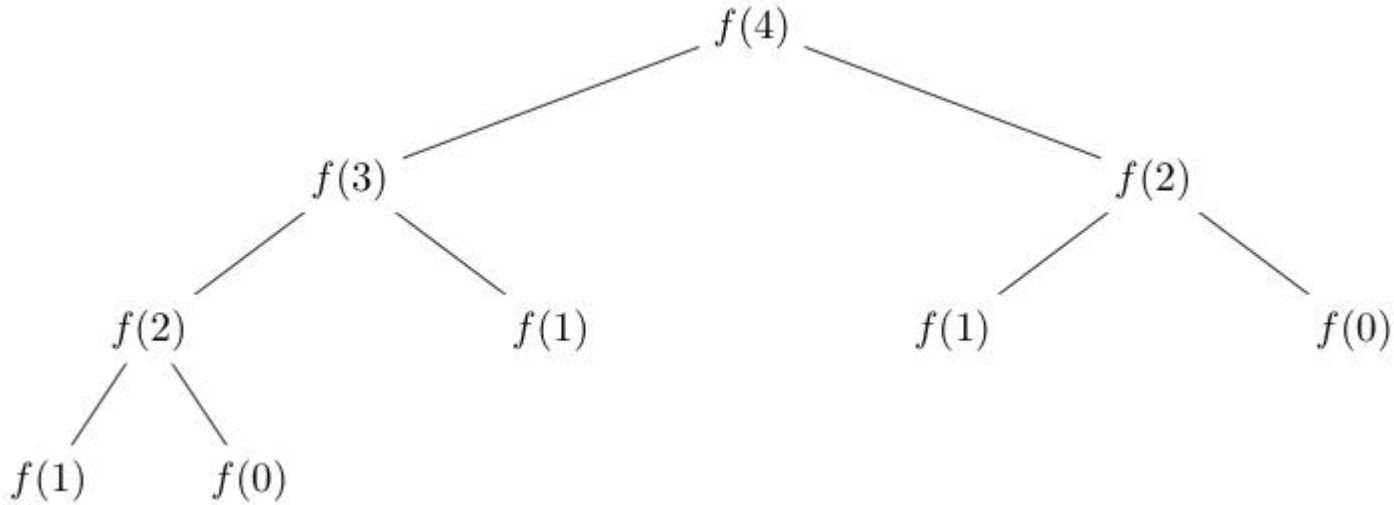
Why Dynamic Programming?



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Speeding up Fibonacci Sequence



Speeding up Fibonacci Sequence

```
def solve_fib(n):  
    memo = {}  
  
    def fib(n):  
        if n in memo:  
            return memo[n]  
        if n == 0 or n == 1:  
            return n  
        memo[n] = fib(n-1) + fib(n-2)  
        return memo[n]  
  
    return fib(n)
```

Now it's linear time
complexity $O(n)$!



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Dynamic Programming Fibonacci: time complexity $O(n)$!

$$T(n) = T(n-1) + T(n-2)$$

$$\geq T(n-1) + O(1) \quad \leftarrow \text{Why?}$$

$$\geq [T(n-2) + T(n-3)] + O(1)$$

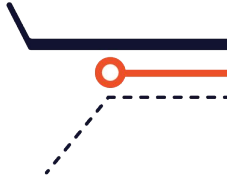
$$\geq T(n-2) + O(1) * 2$$

$$\geq T(n-3) + O(1) * 3$$

$$\geq \dots$$

$$\geq T(n-n) + O(1) * n$$

$$\geq O(n) \quad \leftarrow \text{Fast Enough!}$$



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Speeding up Fibonacci Sequence: Iterative approach

```
def solve_fib(n):  
    if n == 0 or n == 1:  
        return n  
    fib = [0] * (n+1)  
    fib[1] = 1  
    for i in range(2, n+1):  
        fib[i] = fib[i-1]+fib[i-2]  
  
    return fib[n]
```

Why linear time
complexity $O(n)$?



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Speeding up Fibonacci Sequence: Iterative approach

```
def solve_fib(n):  
    if n == 0 or n == 1:  
        return n  
    fib_n2, fib_n1 = 0, 1  
    for i in range(n-1):  
        fib = fib_n2 + fib_n1  
        fib_n2, fib_n1 = fib_n1, fib  
  
    return fib
```

Why linear time
complexity $O(n)$?



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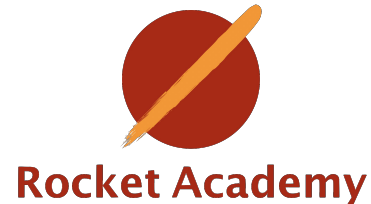
Recall: What is Dynamic Programming

1. “**Solving Problems** by incrementally solving for and **re-using** solutions of its **subproblems**”
2. **Two** Defining Properties:
 - a. **Optimal** Substructure: $F(\text{final}) = G(F(\text{small})...)$
 - b. **Overlapping** Subproblems



Problem Solving with Dynamic Programming

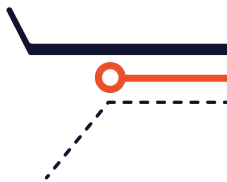
1. Problem Solving Approaches:
 - a. Top-down: Recursion + Memoization
 - b. Bottom-up: Tabulation
2. Problem Solving Strategy



Speeding up Fibonacci Sequence

```
def fib_topdown(n):  
    memo = {}  
  
    def fib(n):  
        if n in memo:  
            return memo[n]  
        if n == 0 or n == 1:  
            return n  
        memo[n] = fib(n-1) + fib(n-2)  
        return memo[n]  
  
    return fib(n)
```

```
def fib_bottomup(n):  
    if n == 0 or n == 1:  
        return n  
  
    fib_n2, fib_n1 = 0, 1  
    for i in range(n-1):  
        fib = fib_n2 + fib_n1  
        fib_n2, fib_n1 = fib_n1, fib  
  
    return fib
```



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Problem Solving Approaches

Top-down

More intuitive,
because of **recursion + memo**

vs



Bottom-up

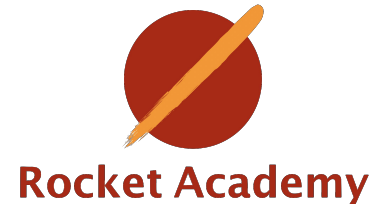
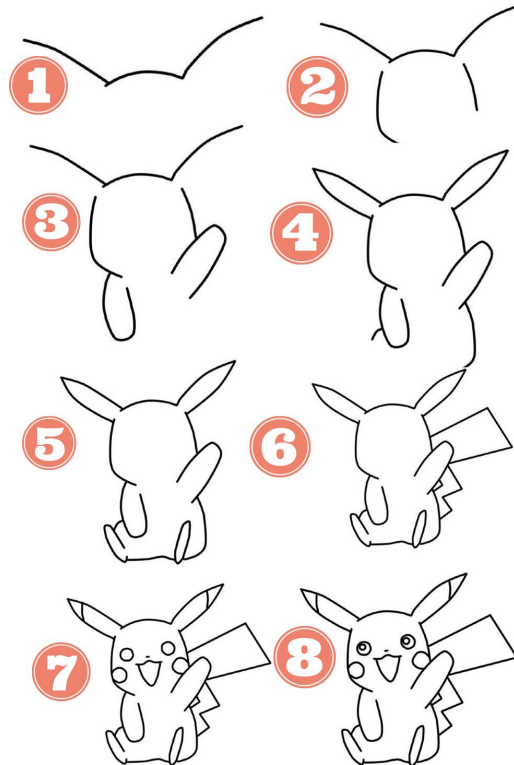
More efficient, and
easier to optimize for
space complexity



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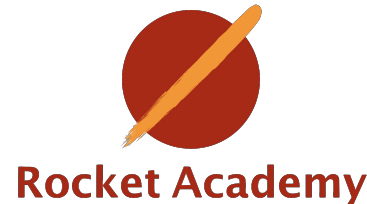


Problem Solving Strategy: steps to follow



Problem Solving Strategy: top down

- i. Step 1: Recognize DP Problems
 - 1. weak signal: problems asking for min / max / total number / if solution exists
 - 2. strong signal: **two defining properties of DP**
- ii. Step 2: Find the brute force solution (recursion)
- iii. Step 3: Apply memoization (save the result for **reuse**)



Problem Solving Strategy: bottom up

- i. Step 1: Recognize DP Problems
 - 1. weak signal: problems asking for min / max / total number / if solution exists
 - 2. strong signal: **two defining properties of DP**
- ii. Step 2: Define Subproblem: $F(\text{small})$
- iii. Step 3: Define Recurrence: $F(\text{big}) = G(F(\text{small})\dots)$
 - 1. If works:
 - a. Define $F(\text{initialize smallest})$
 - b. Incrementally solve for $F(\text{bigger})$
 - c. Output $F(\text{final})$
 - 2. Else: go back to Step 2 and revise



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Longest Increasing Subsequence

Given input: n numbers: x_1, x_2, \dots, x_n

Task: find the length of **longest** increasing subsequence.

e.g

0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15

Ans: 6

0, 8, 4, 12, **2**, 10, **6**, 14, 1, **9**, 5, 13, 3, **11**, 7, 15



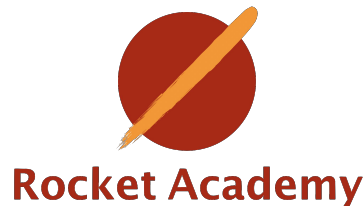
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Top-down: Longest Increasing Subsequence

arr	0	8	4	12	2	10	6	14	1	9	5	13
LIS	1	2	2	3	?							

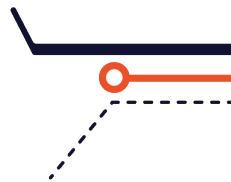
- i. Step 2: Find the brute force solution (recursion)
- ii. Step 3: Apply memoization (save the result for **reuse**)



Top-down: Longest Increasing Subsequence

```
def solve_lis(arr, i, prev):  
    if i == len(arr):  
        return 0  
  
    excl_i = solve_lis(arr, i + 1, prev)  
    incl_i = 0  
    if arr[i] > prev:  
        incl_i = 1 + solve_lis(arr, i + 1, arr[i])  
  
    return max(incl_i, excl_i)
```

time complexity?



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Top-down: Longest Increasing Subsequence

```
def solve(arr):  
    memo = {}  
    def solve_lis(arr, i, prev):  
        if i == len(arr):  
            return 0  
        if (i, prev) in memo:  
            return memo[(i, prev)]  
  
        excl_i = solve_lis(arr, i + 1, prev)  
        incl_i = 0  
        if arr[i] > prev:  
            incl_i = 1 + solve_lis(arr, i + 1, arr[i])  
  
        memo[(i, prev)] = max(incl_i, excl_i)  
        return memo[(i, prev)]  
    return solve_lis(arr, 0, 0)
```

time complexity?



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Bottom-up: Longest Increasing Subsequence

arr	0	8	4	12	2	10	6	14	1	9	5	13
LIS	1	2	2	3	?							

i. Step 2: Define Subproblem:

$LIS(i)$ = length of the longest increase subsequence

ii. Step 3: Define Recurrence: $F(\text{big}) = G(F(\text{small}))...$

How to express $LIS(i)$ with $LIS(i-1)$ or $LIS(\text{smallers})$?

1. If works:

- Define F (initialize smallest)
- Incrementally solve for F (bigger)
- Output F (final)

2. Else: go back to Step 2 and revise



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Bottom-up: Longest Increasing Subsequence

arr	0	8	4	12	2	10	6	14	1	9	5	13
LIS	1	2	2	3	?							

i. Step 2: Define Subproblem:

**LIS(i) = length of the longest increase subsequence,
that end with arr(i)**

ii. Step 3: Define Recurrence: $F(\text{big}) = G(F(\text{small})\dots)$

How to express LIS(i) with LIS(i-1) or LIS(smaller)?

LIS(n) = max(1+LIS(i) for i from 1..n-1)

1. If works:

- Define F(initialize smallest)
- Incrementally solve for F(bigger)
- Output F(final)

2. Else: go back to Step 2 and revise

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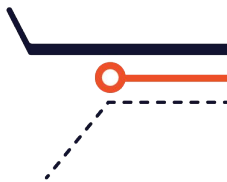
Bottom-up: Longest Increasing Subsequence

```
def solve_lis(arr):  
    n = len(arr)  
    lis = [1]*n
```

```
    for i in range(1, n):  
        for j in range(0, i):  
            if arr[i] > arr[j]:  
                lis[i] = max(lis[i], lis[j]+1)
```

```
    return max(lis)
```

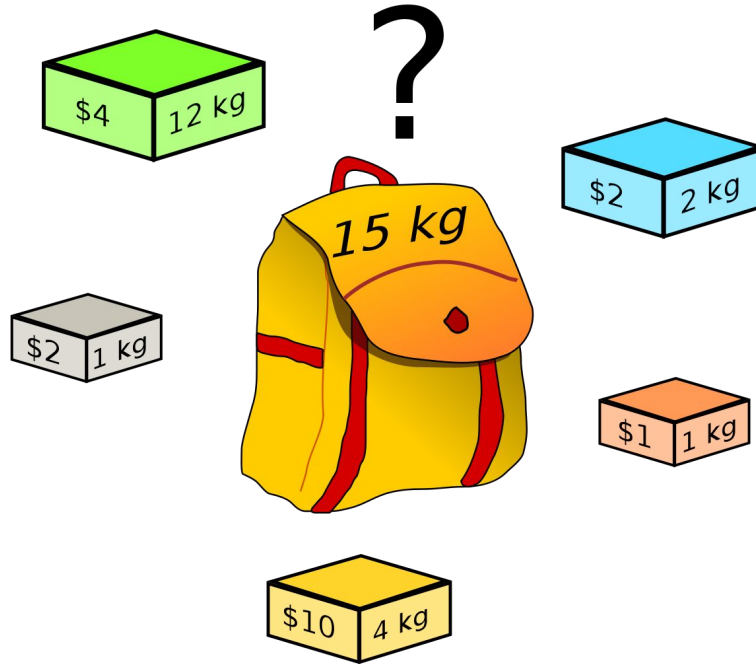
time complexity?



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0-1 Knapsack Problem



Longest Common Subsequence

0, 1, 2, 9, 6, 1, 9, 5, 11, 7, 15, 7

0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15

Ans: 0, 2, 6, 9, 11, 15.

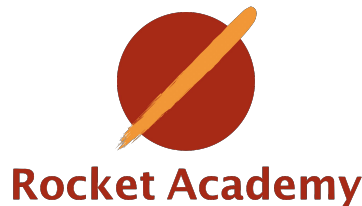
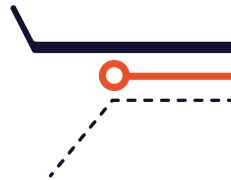


Edit Distance

0, 1, 2, 9, 6, 1, 9, 5, 11, 7, 15, 7

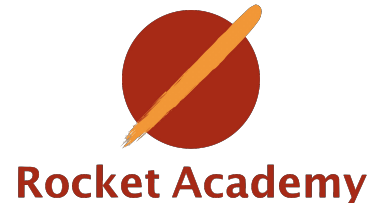
0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15

Ans: 0, 2, 6, 9, 11, 15.



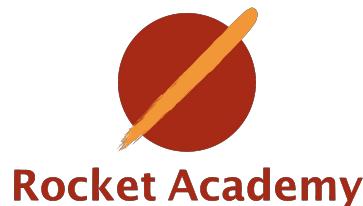
More Examples: LIS, Knapsack, LCS, Edit Distance

1. Longest Increasing Subsequence (LIS)
2. Knapsack problems
3. Longest Common Subsequence (LCS)
4. Edit Distance
5. More on <https://leetcode.com/>



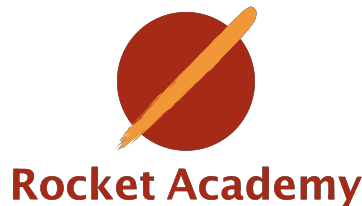
More Learning Resources on DP

1. [Introduction to Algorithm](#) (MIT)
2. [Introduction to Graduate Algorithm](#) (Georgia Tech)
3. [Dynamic Programming](#) (Stanford)

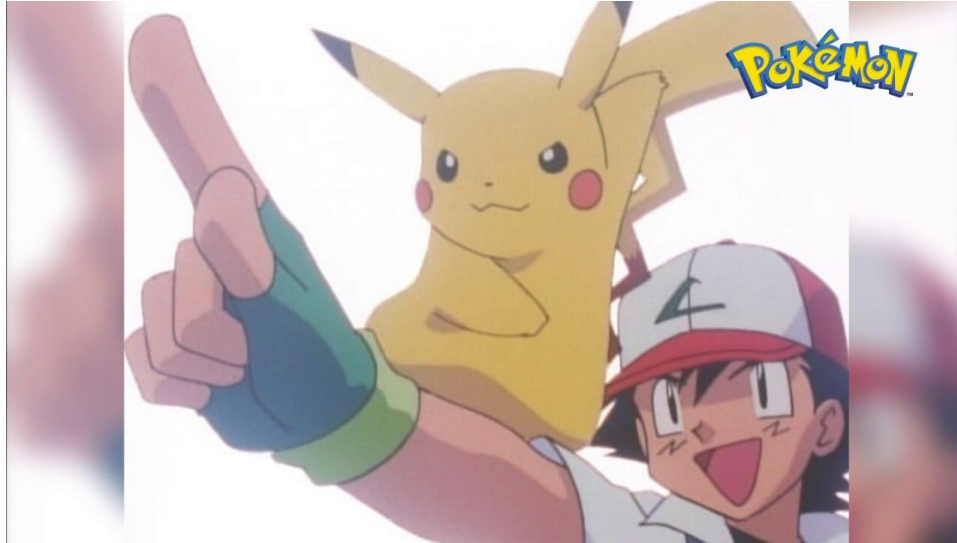


Summary

1. **What is** dynamic programming (DP)
2. **Why/When** to use dynamic programming (DP)
3. DP problem solving **approaches** (topdown vs bottomup)
4. Problem solving strategy (steps i, ii, iii)



Now Practice Practice and Practice!



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Introduction to Dynamic Programming

Liang Qiao, Rocket Academy

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Your Feedback Matters!

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