CS 251 Homework 2

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Problem 0.

Problem 1:

Resources used: None

Collaborators: None

Problem 2:

Resources used: None

Collaborators: None

Problem 3:

Resources used: None

Collaborators: None

Problem 4:

Resources used: None

Collaborators: None

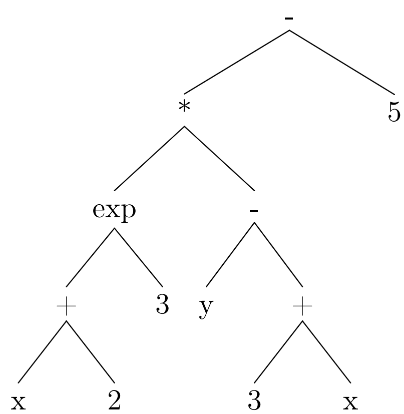
Problem 5:

Resources used: None

Collaborators: None

Problem 1.

(a) Represent the expression $((x+2)^3)*(y-(3+x))-5$ using a binary tree



(b) $((x+2)^3)*(y-(3+x))-5$

Write this expression in prefix notation using a stack. First, we need to convert the expression to prefix notation.

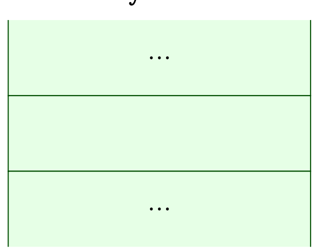
```
- * ^ + x 2 3 - y + 3 x 5
```

Then we reverse it:

```
5 x 3 + y - 3 2 x + ^ * -
```

Then we begin using the stack:

```
5 x 3 + y - 3 2 x + ^ * -     Stack: []
```




```
x 3 + y - 3 2 x + ^ * -       Stack: [5]
```

...
5
...

3 + y - 3 2 x + ^ * -   Stack: [5, x]

...
5
x
...

+ y - 3 2 x + ^ * -   Stack: [5, x, 3]

...
5
x
3
...

Next operator is +, so we pop 3 and x, and push the result of 3 + x

y - 3 2 x + ^ * -   Stack: [5, 3 + x]

...
5
3 + x
...

`- 3 2 x + ^ * -` Stack: [5, 3 + x, y]

...
5
3 + x
y
...

Pop the last two elements, and push the result of y - (3 + x)

`3 2 x + ^ * -` Stack: [5, y - (3 + x)]

...
5
y - (3 + x)
...

`2 x + ^ * -` Stack: [5, y - (3 + x), 3]

...
5
y - (3 + x)
3
...

`x + ^ * -` Stack: [5, y - (3 + x), 3, 2]

...
5
y - (3 + x)
3
2
...

`+ ^ * -` Stack: [5, y - (3 + x), 3, 2, x]

...
5
y - (3 + x)
3
2
x
...

`Pop 2, x, and push the result of 2 + x`

`^ * -                         Stack: [5, y - (3 + x), 3, x + 2]`

...
5
y - (3 + x)
3
x + 2
...

`Pop last two, push (x + 2) ^ 3`

`* -                           Stack: [5, y - (3 + x), (x + 2) ^ 3]`

...
5
y - (3 + x)
(x + 2) ^ 3
...

`Multiply the last two elements`

`-` `Stack: [5, ((x + 2) ^{} 3) * (y - (3 + x))]`

...
5
((x + 2) 3) * (y - (3 + x))
...

`Pop the last two elements, push the result of the subtraction`

`Stack: [((x + 2) ^ 3) * (y - (3 + x)) - 5]`

...
((x + 2) ^ 3) * (y - (3 + x)) - 5
...

Final result: $((x+2)^3) * (y - (3 + x)) - 5$

Problem 2.

Implementing a dequeue using three stacks. One stack will be for storing the front object, and the other stack will be used to store the back elements. The third stack is a temporary stack used to store values when either front or back stacks are empty.

Assuming that the Stack object has `s.isEmpty()`, `s.push(x)`, `s.pop()` implemented.

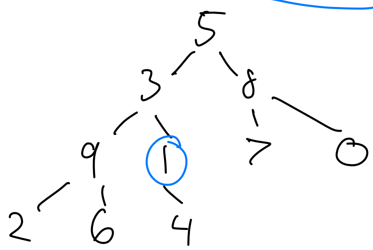
```
Dequeue {
    Stack front
    Stack back
    Stack temp

    //allows us to split the stack into two parts when one is empty
    int frontSize
    int backSize

    // if both front and back are empty, then the dequeue is empty
    d.isEmpty() {
        return front.isEmpty() && back.isEmpty()
    }

    // just push to the front stack
    d.addFront(x) {
        front.push(x)
        frontSize++
    }

    For remove front, if its not empty, I just pop the value on the top.
    If its empty, in order to reduce the number of times I need to swap,
        I split the back stack in half and move half to front.
    I use temp as space to store the top of the stack (new back)
    Move the rest of back to front.
    Then move temp back to back.
    Then I pop the value on the top of front.

    d.removeFront() {
```

```
    if (frontSize == 0) {
        // this is where we need to split the back stack and move half to front
        newBackSize = int(backSize / 2)

        for i in range(newBackSize):
            temp.push(back.pop())
            //this shifted half of back
            //to temp (will become back later)

        newFrontSize = backSize - newBackSize

        for i in range(newFrontSize):
            front.push(back.pop()) //shifting the next half to front

        //now we move temp into back

        for i in range(newBackSize):
            back.push(temp.pop())

        frontSize = newFrontSize
        backSize = newBackSize
    }
}

// just push to the back stack
d.addBack(x) {
    back.push(x)
    backSize++
}

Same logic as removeFront, but we do it for the back stack

d.removeBack() {
    if (backSize == 0) {
        // this is where we need to split the front stack and move half to back
        newFrontSize = int(frontSize / 2)

        for i in range(newFrontSize):
            temp.push(front.pop())
            //this shifted half of front
```

```
                //to temp (will become front later)

            newBackSize = frontSize - newFrontSize

            for i in range(newBackSize):
                back.push(front.pop()) //shifting the next half to back

            //now we move temp into front

            for i in range(newFrontSize):
                front.push(temp.pop())

            frontSize = newFrontSize
            backSize = newBackSize
        }
    }
}
```

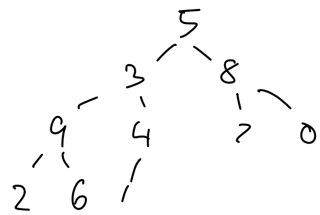
$\boxed{3A}$

5 3 8 9 1 7 0 2 6 4

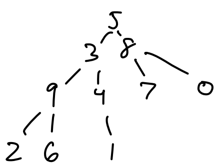


Sink

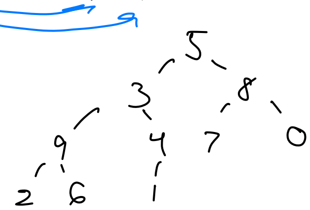
5 3 8 9 4 7 0 2 6 1



5 3 8 9 4 7 0 2 6 1

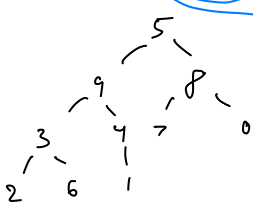


5 3 8 9 4 7 0 2 6 1



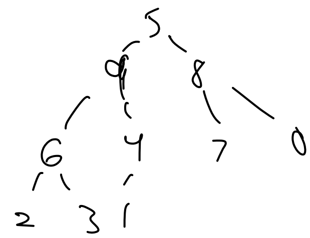
Sink

5 9 8 3 4 7 0 2 6 1



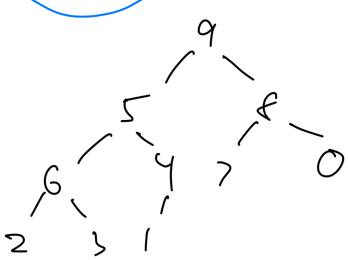
Sink again

5 9 8 6 4 7 0 2 3 1



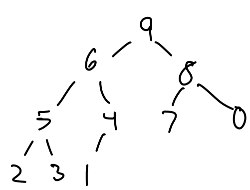
Sink

9 5 8 6 4 7 0 2 3 1



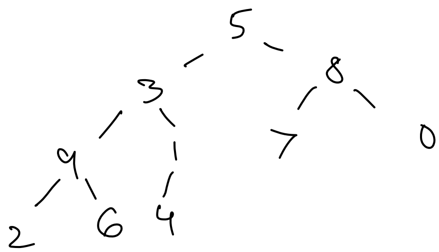
Sink

9 6 8 5 4 7 0 2 3 1



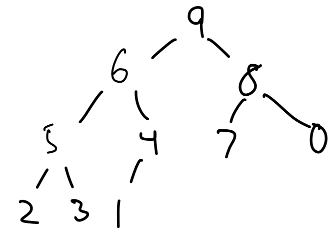
3 B

5 3 8 9 1 7 0 2 6 4

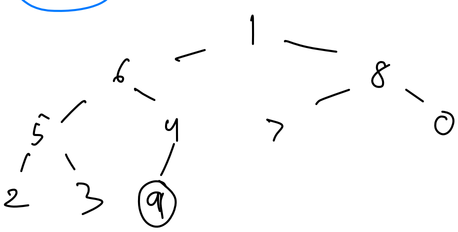


→ Build heap →

9 6 8 5 4 7 0 2 3 1

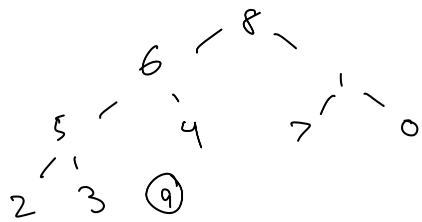


1 6 8 5 4 7 0 2 3 9



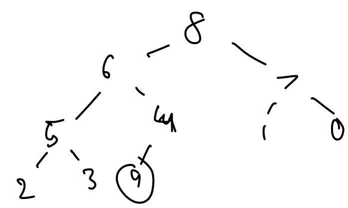
→

8 6 1 5 4 7 0 2 3 9

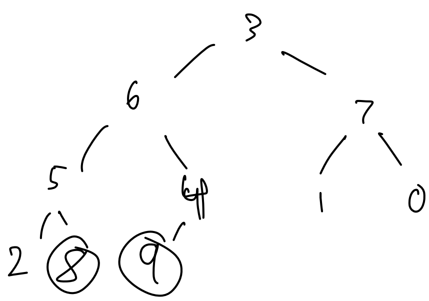


→

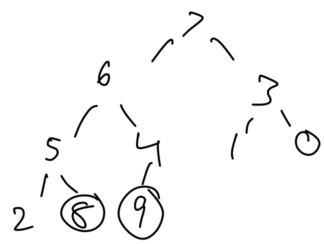
8 6 7 5 4 1 0 2 3 9



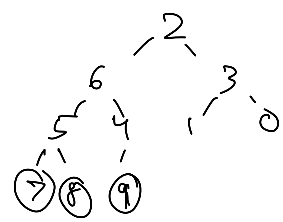
3 6 7 5 4 1 0 2 8 9



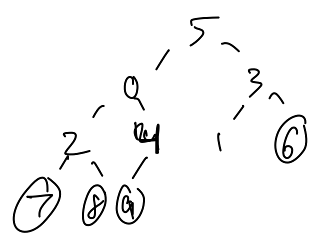
7 6 3 5 4 1 0 2 8 9



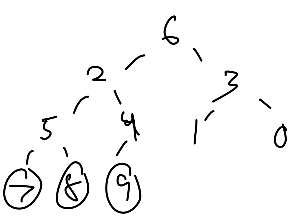
2 6 3 5 4 1 0 7 8 9



5 0 3 2 4 1 6 7 8 9



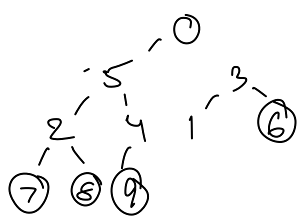
6 2 3 5 4 1 0 7 8 9



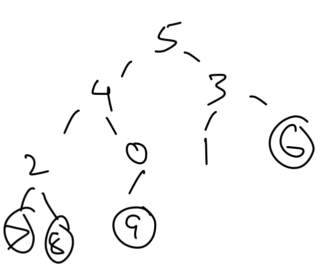
→ 0 → 6 swap

* forgot →

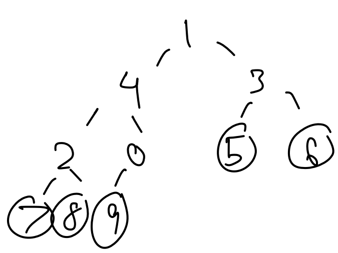
0 5 3 2 4 1 6 7 8 9



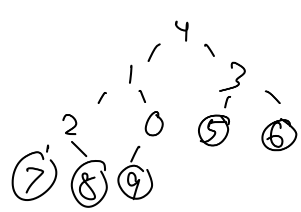
5 4 3 2 0 1 6 7 8 9



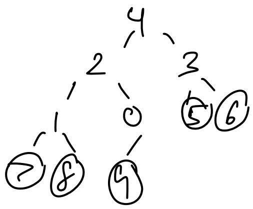
1 4 3 2 0 5 6 7 8 9



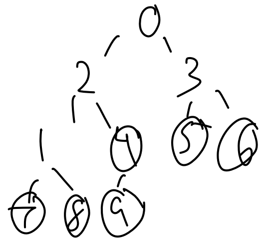
4 1 3 2 0 5 6 7 8 9



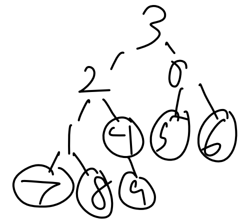
4 2 3 1 0 5 6 7 8 9



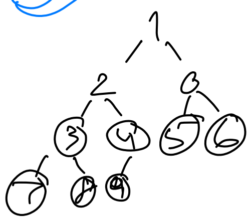
0 2 3 1 4 5 6 7 8 9



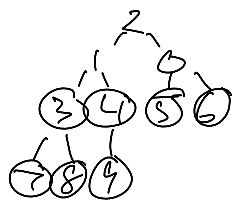
3 2 0 1 4 5 6 7 8 9



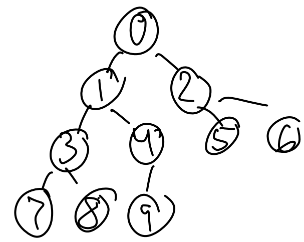
1 2 0 3 4 5 6 7 8 9



2 1 0 3 4 5 6 7 8 9



0 1 2 3 4 5 6 7 8 9



Problem 4.

Runtime analysis: Radix sort works on each digit of the number. For each digit, it uses counting sort to sort the numbers. Counting sort has a runtime of O(n + k), where k is the range of the numbers. Since the range of the numbers is 0 to 9, k is a constant. Therefore, the runtime of counting sort is O(n). Since radix sort uses counting sort for each digit, the runtime of radix sort is O(d * n), where d is the number of digits, but d is also constant, so the runtime of radix sort is O(n).

```
algorithm CountingSort(A, k)
    let C be an array of length k + 1

    fill C with 0s

    let n be the length of array A
    for (i = 0, i < n, 1)
        C[A[i]] = C[A[i]] + 1
    end for

    for (i = 1, i <= k, 1)
        C[i] = C[i] + C[i - 1]
    end for

    let B be an array of size n

    for (i = n - 1, i >= 0, -1)
        B[C[A[i]] - 1] = A[i]
        C[A[i]] = C[A[i]] - 1
    end for

    return B
end algorithm


algorithm RadixSort(A)

    let d be the dimension of
    the tuples in A
```

```
    for i from d to 1
        CountingSort(A, d) using the
        ith dimension as the key
    end for

end algorithm
```

Problem 5.

My approach for this was flawed at first until I found quickselect. First, I calculate the total number of elements in the list. (O(n)) Then, I find the index where the median should be (Kth smallest element), which is the middle index of the list. (O(1)) Then, I use quickselect to find the Kth smallest element. (O(n)).

The total runtime is O(n) + O(1) + O(n) = O(2n) = O(n)

```
def median(list):
    n = len(list)
    k = int(n / 2)
    return partition(list, 0, n - 1, k)

def partition(list, l, r, k):

    if l < r:
        pivotIndex = partition(list, l, r)

        if k == pivotIndex:
            return list[k]
        elif k < pivotIndex:
            return partition(list, l, pivotIndex - 1, k)
        else:
            return partition(list, pivotIndex + 1, r, k)
    else:
        return list[l]

    Dry Run on [38, 12, 31, 1, 34, 15, 11, 48, 24, 23, 3, 47, 4, 32, 35]

    n = 15
    k = 7

    l:  0  r:  14  k:  7
    Partitioning:
    [38, 12, 31, 1, 34, 15, 11, 48, 24, 23, 3, 47, 4, 32, 35]
    [12, 38, 31, 1, 34, 15, 11, 48, 24, 23, 3, 47, 4, 32, 35]
    [12, 31, 38, 1, 34, 15, 11, 48, 24, 23, 3, 47, 4, 32, 35]
    [12, 31, 1, 38, 34, 15, 11, 48, 24, 23, 3, 47, 4, 32, 35]
```

```
[12, 31, 1, 34, 38, 15, 11, 48, 24, 23, 3, 47, 4, 32, 35]
[12, 31, 1, 34, 15, 38, 11, 48, 24, 23, 3, 47, 4, 32, 35]
[12, 31, 1, 34, 15, 11, 38, 48, 24, 23, 3, 47, 4, 32, 35]
[12, 31, 1, 34, 15, 11, 24, 48, 38, 23, 3, 47, 4, 32, 35]
[12, 31, 1, 34, 15, 11, 24, 23, 38, 48, 3, 47, 4, 32, 35]
[12, 31, 1, 34, 15, 11, 24, 23, 3, 48, 38, 47, 4, 32, 35]
[12, 31, 1, 34, 15, 11, 24, 23, 3, 4, 38, 47, 48, 32, 35]
[12, 31, 1, 34, 15, 11, 24, 23, 3, 4, 32, 47, 48, 38, 35]
Position of pivot:  11
Pivot is to the right of k, so we recurse on the left side
l:  0  r:  10  k:  7
Partitioning:
[12, 31, 1, 34, 15, 11, 24, 23, 3, 4, 32, 35, 48, 38, 47]
[12, 31, 1, 34, 15, 11, 24, 23, 3, 4, 32, 35, 48, 38, 47]
[12, 31, 1, 34, 15, 11, 24, 23, 3, 4, 32, 35, 48, 38, 47]
[12, 31, 1, 34, 15, 11, 24, 23, 3, 4, 32, 35, 48, 38, 47]
[12, 31, 1, 15, 34, 11, 24, 23, 3, 4, 32, 35, 48, 38, 47]
[12, 31, 1, 15, 11, 34, 24, 23, 3, 4, 32, 35, 48, 38, 47]
[12, 31, 1, 15, 11, 24, 34, 23, 3, 4, 32, 35, 48, 38, 47]
[12, 31, 1, 15, 11, 24, 23, 34, 3, 4, 32, 35, 48, 38, 47]
[12, 31, 1, 15, 11, 24, 23, 3, 34, 4, 32, 35, 48, 38, 47]
[12, 31, 1, 15, 11, 24, 23, 3, 4, 34, 32, 35, 48, 38, 47]
Position of pivot:  9
Right of k, so we recurse on the left side
l:  0  r:  8  k:  7
Partitioning:
[12, 31, 1, 15, 11, 24, 23, 3, 4, 32, 34, 35, 48, 38, 47]
[1, 31, 12, 15, 11, 24, 23, 3, 4, 32, 34, 35, 48, 38, 47]
[1, 3, 12, 15, 11, 24, 23, 31, 4, 32, 34, 35, 48, 38, 47]
Position of pivot:  2
Left of K, recurse on right side
l:  3  r:  8  k:  4
Partitioning:
[1, 3, 4, 15, 11, 24, 23, 31, 12, 32, 34, 35, 48, 38, 47]
[1, 3, 4, 11, 15, 24, 23, 31, 12, 32, 34, 35, 48, 38, 47]
Position of pivot:  4
Left of K, recurse on right side
l:  5  r:  8  k:  2
Partitioning:
[1, 3, 4, 11, 12, 24, 23, 31, 15, 32, 34, 35, 48, 38, 47]
```

```
Position of pivot:  5
Left of K, recurse on right side
l:  6  r:  8  k:  1
Partitioning:
[1, 3, 4, 11, 12, 15, 23, 31, 24, 32, 34, 35, 48, 38, 47]
[1, 3, 4, 11, 12, 15, 23, 31, 24, 32, 34, 35, 48, 38, 47]
Position of pivot:  7
Right of K, recurse on left side
l:  6  r:  6  k:  1
Partitioning:
[1, 3, 4, 11, 12, 15, 23, 24, 31, 32, 34, 35, 48, 38, 47]
Position of pivot:  6

23
```