CS 251 Homework 4

Dwijen Chawra (dchawra)

October 26, 2023

Problem 0.

Problem 1:

Resources used: None

Collaborators: None

Problem 2:

Resources used: None

Collaborators: None

Problem 3:

Resources used: None

Collaborators: None

Problem 4:

Resources used: None

Collaborators: None

Problem 5:

Resources used: None

Collaborators: None

Problem 1.

(a) Simplifying the original functions to their Big O notation:

(i) $\log(2^{4n}) = 4n = O(n)$

(ii) $log^2 4n^2 = (2 + 2logn)^2 = O(log^2 n)$

(iii) $2^{10 \log n} = n^{10} = O(n^{10})$

(iv) $\log_{10}(n^{12}) = 12 \log_1 0n = O(log_{10} n)$

(v) $4^{\log_2 n^2} = n^4 = O(n^4)$

(vi) $4^n = O(4^n)$

(vii) $\sqrt{8n} = O(\sqrt{n})$

(viii) $\binom{n}{2 = \frac{n(n-1)}{2} = O(n^2)}$

(ix) $n^2 / \log n = O(n^2 / \log n)$

(x) $4^{2n} = O(16^n)$

(xi) $n^2 + 64n^{1.89} = O(n^2)$

The functions ordered are as follows:

(i) $\log_{10}(n^{12})$

(ii) $log^2 4n^2$

(iii) $\sqrt{8n}$

(iv) $\log(2^{4n})$

(v) $n^2 / \log n$

(vi) $\binom{n}{2}$ and $n^2 + 64n^{1.89}$

(vii) $4^{\log n^2}$

(viii) $2^{10 \log n}$

(ix) 4^n

(x) 4^{2n}

(b) For i: $f(n) = (\log 4n^2)^2$ vs. $g(n) = \log_{10} n^{12}$

f(n) simplifies to $(\log 4 + logn^2)^2 = (2 + 2\log n)^2 + 4logn^2 + 8logn + 4$
g(n) simplifies to $\frac{12\log n}{\log 10}$

By raising the lower terms to the highest power for f(n) we get

$f(n) = 16\log^2 n$ vs. $g(n) = \frac{12\log n}{\log 10}$

Using C = 1, and k = 1, we get $16\log^2 n \leq \frac{12\log n}{\log 10}$ for all $n \geq 1$.

This example is invalid, and as n approaches infinity, g(n) is never greater than f(n). The fraction term in g(n) approaches 0 as n approaches infinity. Therefore g(n) cannot be a bound for f(n).

For ii)

$\binom{f(n)=n}{2}$ vs. $g(n) = n^2/log_2 n$

Using C = 1, and k = 8, we get $\frac{n(n-1)}{2} \leq \frac{n^2}{log_2 n}$ for all $n \geq 8$.

If we compare the behavior at infinity, the log term in g causes the function to approach 0, which means that g(n) is never greater than f(n) for all $n \geq 8$. This means that g(n) is not a Big O bound for f(n).

For iii)

$f(n) = 4^n$ vs. $g(n) = 4^{2n}$

Using C = 1, and k = 1, we get $4^n \leq 4^{2n}$ for all $n \geq 1$.

Therefore, $f(n) = O(g(n))$.

Problem 2.

(a)

$$\begin{aligned}
&\sum_{i=1}^{n^2}\sum_{j=1}^{i}\sum_{k=1}^{\lceil n/8\rceil} 1 && (1)\\
&= \sum_{i=1}^{n^2}\sum_{j=1}^{i}\lceil n/8\rceil && (2)\\
&= \sum_{i=1}^{n^2}\lceil n/8\rceil \sum_{j=1}^{i} 1 && (3)\\
&= \sum_{i=1}^{n^2}\lceil n/8\rceil i && (4)\\
\text{Let } n^2 = k & && (5)\\
&= \sum_{i=1}^{k}\lceil n/8\rceil i && (6)\\
&= \lceil n/8\rceil \sum_{i=1}^{k} i && (7)\\
&= \lceil n/8\rceil \frac{k(k+1)}{2} && (8)\\
&= \lceil n/8\rceil \frac{n^2(n^2+1)}{2} && (9)\\
&= \frac{n^2(n^2+1)}{2}\lceil n/8\rceil && (10)\\
&= \frac{n^4+n^2}{2}\lceil n/8\rceil && (11)
\end{aligned}$$

(b)

$$
\begin{aligned}
&\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=j}^{n} 3 && (12)\\
&= \sum_{i=1}^{n}\sum_{j=1}^{i} 3(n-j+1) && (13)\\
&= \sum_{i=1}^{n} 3\sum_{j=1}^{i}(n-j+1) && (14)\\
&= \sum_{i=1}^{n} 3(\sum_{j=1}^{i} n - \sum_{j=1}^{i} j + \sum_{j=1}^{i} 1) && (15)\\
&= \sum_{i=1}^{n} 3(ni - \frac{i(i+1)}{2} + i) && (16)\\
&= 3\sum_{i=1}^{n}(ni - \frac{i(i+1)}{2} + i) && (17)\\
&= 3(\sum_{i=1}^{n} ni + \sum_{i=1}^{n} i - \sum_{i=1}^{n}\frac{i(i+1)}{2}) && (18)\\
&= 3(n\sum_{i=1}^{n} i + \sum_{i=1}^{n} i - \frac{1}{2}\sum_{i=1}^{n} i^2 - \frac{1}{2}\sum_{i=1}^{n} i) && (19)\\
&= 3(\frac{n^2(n+1)}{2} + \frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{12} - \frac{3n(n+1)}{12}) && (20)\\
&= 3(\frac{n^3+n^2}{2} - \frac{1}{2}(\frac{2n^3+3n^2+n}{6} + \frac{n^2+n}{2}) + \frac{n^2+n}{2}) && (21)\\
&= 3(\frac{2n^3+2n^2}{4} - \frac{2n^3+3n^2+n}{12} - \frac{n^2+n}{4} + \frac{2n^2+2n}{4}) && (22)\\
&= 3(\frac{2n^3+4n^2+2n}{4} - \frac{2n^3+3n^2+n}{12} - \frac{n^2+n}{4}) && (23)\\
&= \frac{6n^3+9n^2+3n}{4} - \frac{2n^3+3n^2+n}{4} && (24)\\
&= \frac{2n^3+3n^2+n}{2} && (25)
\end{aligned}
$$

(c) Adding 1 because of Edstem, and assuming n is a power of 4 also according to Ed.

$$
\begin{aligned}
&\sum_{i=1}^{\log_4 n^2+1} 1 && (26)\\
&= \sum_{i=1}^{\log_4 n^2} 1 + 1 && (27)\\
&= \log_4 n^2 + 1 && (28)\\
&= 2\log_4 n + 1 && (29)
\end{aligned}
$$

(d)

$$
\begin{aligned}
&\sum_{i=1}^{n} \sum_{j=1}^{\lfloor logn \rfloor+1} 1 && (30)\\
&= \sum_{i=1}^{n} \lfloor logn \rfloor + 1 && (31)\\
&= n\lfloor logn \rfloor + 1 && (32)
\end{aligned}
$$

3 a

first insertions

0	
1	
2	
3	6003 → 6033 → 1953 → 2023 → 5763
4	
5	
6	
7	
8	3408 → 8958 → 2308 → 1028
9	

2 deletions

6 probes

0	
1	
2	
3	6003 → 1953 → 2023
4	
5	
6	
7	
8	3408 → 8958 → 2308 → 1028
9	

final insertions

2 probes

0	
1	
2	
3	6003 → 1953 → 2023
4	7634
5	
6	
7	
8	3408 → 8958 → 2308 → 1028
9	4959

3b

insert 9

0	8958
1	3408
2	
3	5763
4	2023
5	1953
6	6033
7	6003
8	1028
9	2308

delete 2

5 probes

0	8958
1	3408
2	
3	X
4	2023
5	1953
6	X
7	6003
8	1028
9	2308

insert 2

4 + 3 = 7 probes

0	8958
1	3408
2	4959
3	X
4	2023
5	1953
6	7634
7	6003
8	1028
9	2308

3c

first insertions

0	→ 8958
1	→ 3408
2	→ 2023
3	→ 6033 → 2308 → 1028
4	
5	→ 5763
6	
7	
8	→ 6003
9	→ 1953

2 deletions

2 probes

0	→ 8958
1	→ 3408
2	→ 2023
3	→ 2308 → 1028
4	
5	
6	
7	
8	→ 6003
9	→ 1953

final insertions

2 probes

0	→ 8958
1	→ 3408
2	→ 2023
3	→ 2308 → 1028
4	
5	
6	→ 4959
7	
8	7634 → 6003
9	→ 1953

3d

first insertions

0	8958
1	3408
2	2023
3	1028
4	2308
5	5763
6	6033
7	
8	6003
9	1953

2 deletion

0	8958
1	3408
2	2023
3	1028
4	2308
5	X
6	X
7	
8	6003
9	1953

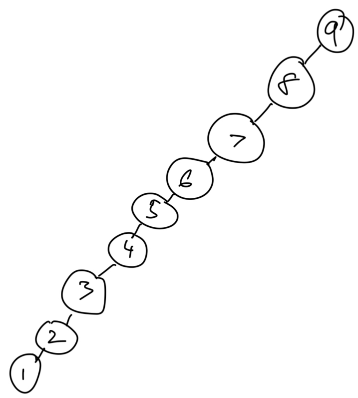
5 probes

2 insertion

0	8958
1	3408
2	2023
3	1028
4	2308
5	7634
6	4959
7	
8	6003
9	1953

9 probes

Problem 4.



(a)

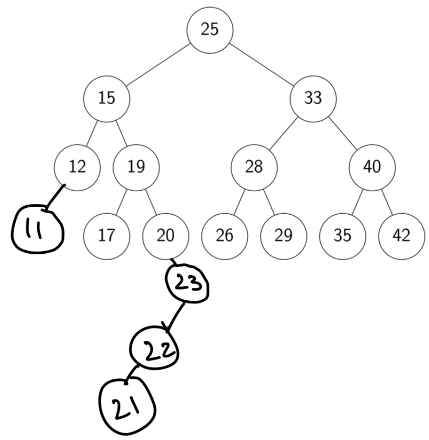
Assuming n, n-1, n-2, ... 1 are inserted into a BST, the runtime of this would be O($\sum_{i=1}^{n} 1$). This is because the first element inserted would be an O(1) operation, then O(2), and so on until O(n). The resulting runtime is O($\sum_{i=1}^{n} 1$) = O($\frac{n(n+1)}{2}$) = O(n^2).

To get big theta we need to prove that the runtime is also $\Omega(n^2)$. Ω for this would be the same, because there is no better way to insert the elements in descending order; we have to traverse the left child every time.

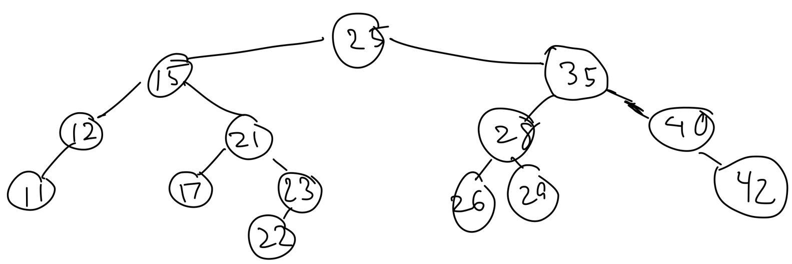
f(n) = O(n^2) and g(n) = n^2

When C = 1, and k = 1, $n^2 \leq C \cdot n^2$

Therefore, the runtime is $\Theta(n^2)$.



(b)

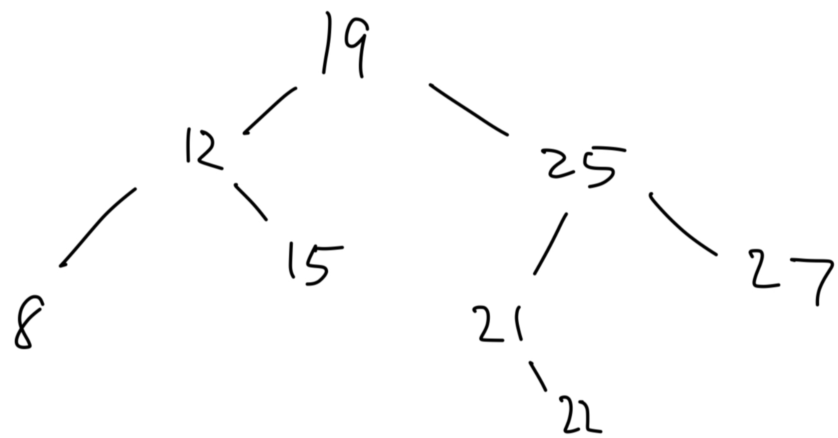


(c)

del 19: 19 had 2 children, so we replace 19 with min(right), which is 20. 20 has one child, so we replace 20 with 21. 21 has no children, so we delete it.

del 33: 33 had 2 children, so we replace 33 with min(right), which is 35. 35 has no children, so we delete it.

del 20: 20 has one child, so we take the min of the right subtree, which is 21, then delete 21.



(d)