Applied Machine Learning: Algorithms, Practice and Theory

Online Learning

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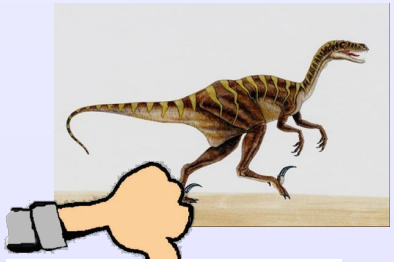
Online Learning

Tyrannosaurus rex

Online Learning

Triceratops

Online Learning



Vel

Tyrannosaurus rex

Formal Setting – Binary Classification

- Instances
  - Images, Sentences
- Labels
  - Parse tree, Names
- Prediction rule
  - Linear predictions rules
- Loss
  - No. of mistakes

$$\mathbf{x} \in \mathcal{X}$$

$$y \in \mathcal{Y} = \{-1 \; ; \; 1\}$$

$$f(\mathbf{x}) = \hat{y}$$

$$\ell(\hat{y}, y) \in \mathbb{R}_+$$

Online Framework

- Initialize Classifier $f_1(\mathbf{x})$
- Algorithm works in rounds $t = 1 \dots T \dots$
- On round t the online algorithm :
  - Receives an input instance $\mathbf{x}_t$
  - Outputs a prediction $f_t(\mathbf{x}_t) = \hat{y}_t$
  - Receives a feedback label y_t
  - Computes loss $\ell(\hat{y}_t, y_t)$
  - Updates the prediction rule $f_t \rightarrow f_{t+1}$

- Goal :
  - Suffer small cumulative loss $\sum_t \ell(\hat{y}_t, y_t)$

Why Online Learning?

- Fast
- Memory efficient - process one example at a time
- Simple to implement
- Formal guarantees – Mistake bounds
- Online to Batch conversions
- No statistical assumptions
- Adaptive

- Not as good as a well designed batch algorithms

Update Rules

- Online algorithms are based on an update rule which defines f_{t+1} from f_t (and possibly other information)
- Linear Classifiers : find $\mathbf{w}_{t+1}$ from $\mathbf{w}_t$ based on the input $(\mathbf{x}_t, y_t)$
- Some Update Rules :
  - Perceptron (Rosenblat)
  - ALMA (Gentile)
  - ROMMA (Li & Long)
  - NORMA (Kivinen et. al)
  - MIRA (Crammer & Singer)
  - EG (Littlestown and Warmuth)
  - Bregman Based (Warmuth)

Three Update Rules

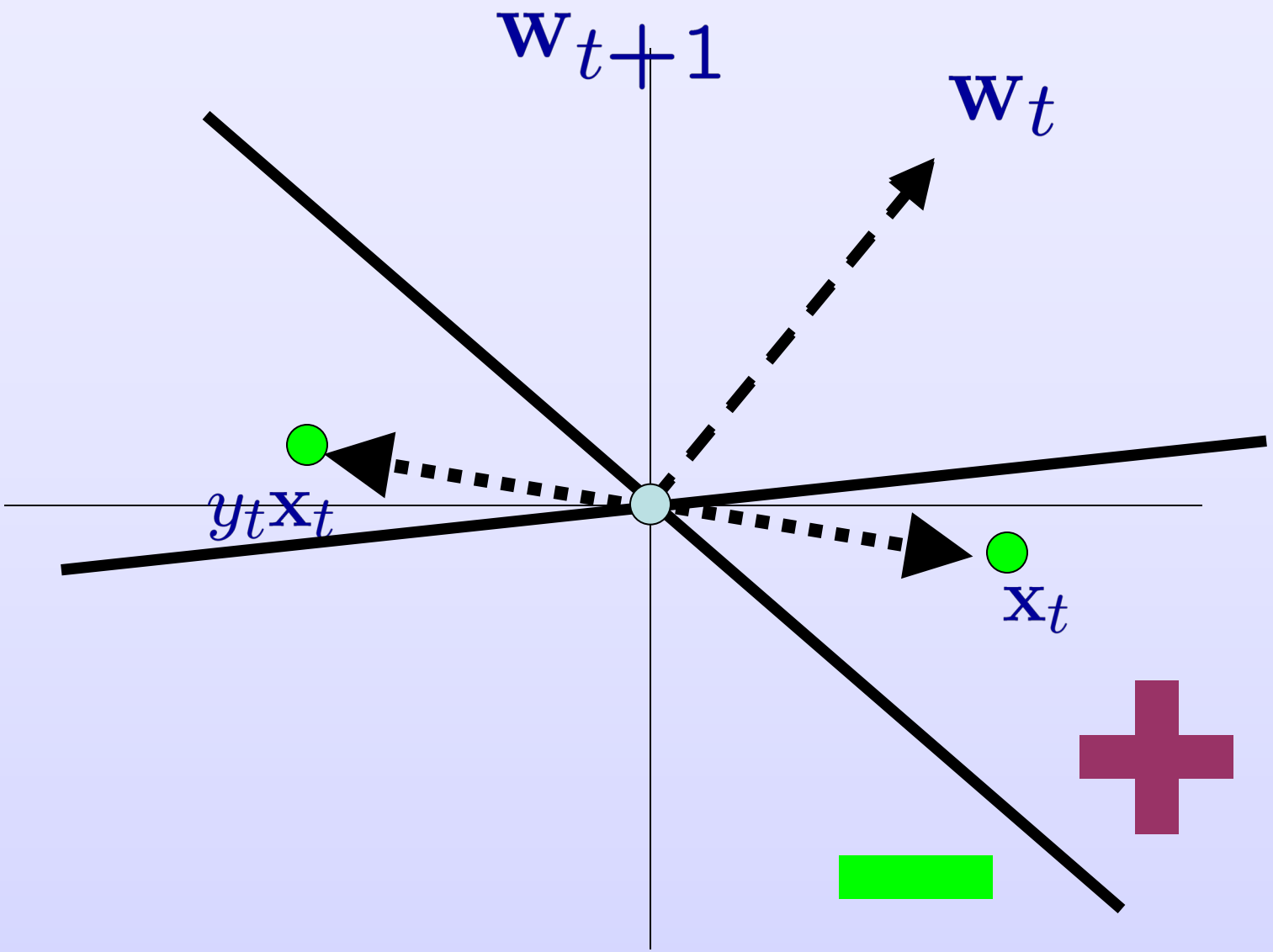
- The Perceptron Algorithm :
  - Agmon 1954; Rosenblatt 1952-1962, Block 1962, Novikoff 1962, Minsky & Papert 1969, Freund & Schapire 1999, Blum & Dunagan 2002
- Hildreth's Algorithm :
  - Hildreth 1957
  - Censor & Zenios 1997
  - Herbster 2002
- Loss Scaled :
  - Crammer & Singer 2001,
  - Crammer & Singer 2002

The Perceptron Algorithm

- If No-Mistake $y_t(\mathbf{w}_t \cdot \mathbf{x}_t) > 0$
  - Do nothing $\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t$
- If Mistake $y_t(\mathbf{w}_t \cdot \mathbf{x}_t) \leq 0$
  - Update $\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t + y_t \mathbf{x}_t$
- Margin after update :

$$y_t(\mathbf{w}_{t+1} \cdot \mathbf{x}_t) \geq y_t(\mathbf{w}_t \cdot \mathbf{x}_t) + \|\mathbf{x}_t\|^2 (\mathbf{x}_t \cdot \mathbf{x}_t)$$

Geometrical Interpretation



Relative Loss Bound

- For any competitor prediction function f^*
- We bound the loss suffered by the algorithm with the loss suffered by f^*

$$\sum_{t=1}^{T} \ell(f_t(\mathbf{x}_t), y_t) \leq \mathcal{R} + C \sum_{t=1}^{T} \ell(f^*(\mathbf{x}_t), y_t)$$

Cumulative Loss Suffered by the Algorithm

Sequence of Prediction Functions

Cumulative Loss of Competitor

Relative Loss Bound

- For any competitor prediction function
- We bound the loss suffered by the algorithm with the loss suffered by f^*

$$\sum_{t=1}^{T} \ell(f_t(\mathbf{x}_t), y_t) \leq \mathcal{R} + \mathcal{C} \sum_{t=1}^{T} \ell(f^*(\mathbf{x}_t), y_t)$$

Inequality
Possibly Large Gap

Regret
Extra Loss

Competitiveness
Ratio

Relative Loss Bound

- For any competitor prediction function
- We bound the loss suffered by the algorithm with the loss suffered by f^*

$$\sum_{t=1}^{T} \ell(f_t(\mathbf{x}_t), y_t) \leq \mathcal{R} + C \sum_{t=1}^{T} \ell(f^*(\mathbf{x}_t), y_t)$$

Grows With T

Constant

Relative Loss Bound

- For any competitor prediction function
- We bound the loss suffered by the algorithm with the loss suffered by f^*

$$\sum_{t=1}^{T} \ell(f_t(\mathbf{x}_t), y_t) \leq \mathcal{R} + \mathcal{C} \min_{f} \left[\sum_{t=1}^{T} \ell(f(\mathbf{x}_t), y_t) \right]$$

Best Prediction Function in hindsight for the data sequence

Remarks

- If the input is inseparable, then the problem of finding a separating hyperplane which attains less then M errors is NP-hard (Open hemisphere)
- Obtaining a zero-one loss bound with a unit competitiveness ratio is as hard as finding a constant approximating error for the Open Hemesphere problem.
- Bound of the *number of mistakes* the perceptron makes with the *hinge loss* of any compitetor

$$\sum_{t=1}^{T} \ell_{01}(f_t(\mathbf{x}_t), y_t) \leq \mathcal{R} + C \sum_{t=1}^{T} \ell_{\text{hinge}}(f(\mathbf{x}_t), y_t)$$

Definitions

- Any Competitor $f^*(\mathbf{x}) = \mathbf{u} \cdot \mathbf{x}$
- The parameters vector $\mathbf{u}$ can be chosen using the input data

$$(\mathbf{x}_1, y_1) \ , \ldots , \ (\mathbf{x}_T, y_T)$$

- The *parameterized* hinge loss of $f^*(\mathbf{x})$ on $(\mathbf{x}_t, y_t)$

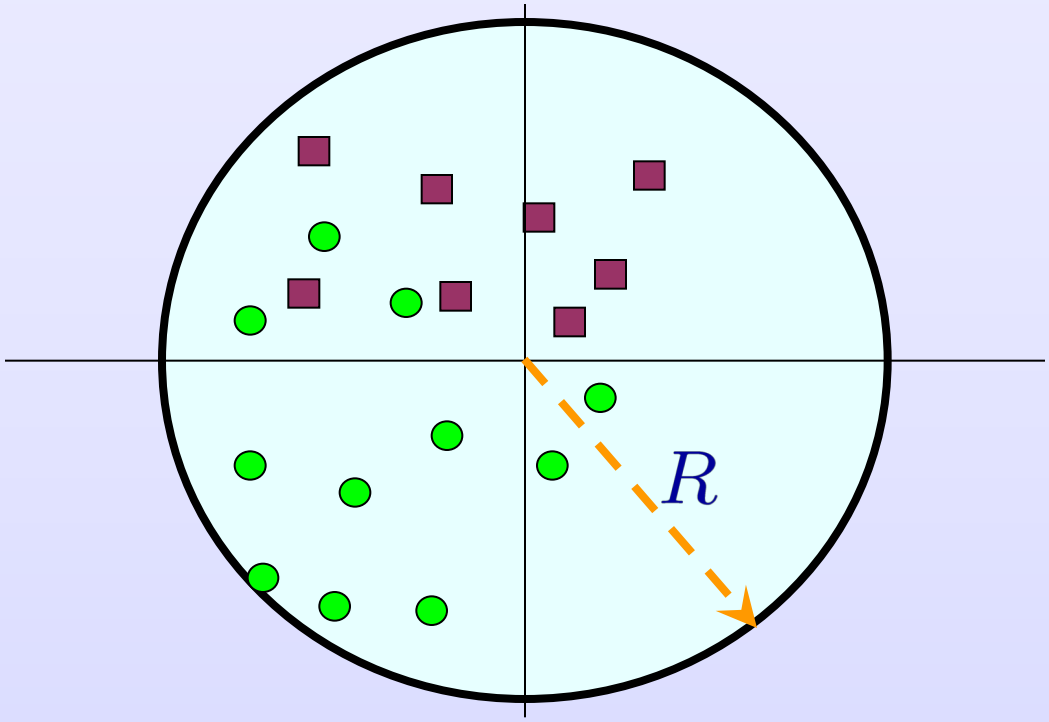
$$\ell^*_\gamma(\mathbf{x}_t, y_t) = \max\{0, \gamma - y_t(\mathbf{u} \cdot \mathbf{x})\}$$

- True hinge loss $\gamma = 1$
- 1-norm and 2-norm of hinge loss

$$\mathcal{L}_1 = \sum_t \ell^*(\mathbf{x}_t, y_t) \qquad \mathcal{L}_2 = \sqrt{\sum_t \left(\ell^*(\mathbf{x}_t, y_t)\right)^2}$$

Geometrical Assumption

- All examples are bounded in a ball of radius R



$$\|\mathbf{x}_t\|^2 \leq R^2 \quad \forall t$$

Perceptron's Mistake Bound

- Bounds : $M \leq (R\|\mathbf{u}\| + \mathcal{L}_2)^2$

$$M \leq \left(R\|\mathbf{u}\| + \sqrt{\mathcal{L}_1}\right)^2$$

$$M \leq R^2\|\mathbf{u}\|^2 + 2\mathcal{L}_1$$

- If the sample is separable then

$$M \leq R^2\|\mathbf{u}\|^2$$

[FS99, SS05]

Proof - Intuition

- Two views :
  - The angle between $\mathbf{u}$ and $\mathbf{w}_t$ decreases with t

$$\frac{\mathbf{u} \cdot \mathbf{w}_t}{\|\mathbf{w}_t\| \, \|\mathbf{u}\|}$$

  - The following sum is fixed

$$M\, R^2 - 2 \sum_t^T \ell^*(\mathbf{x}_t, y_t) + \|\mathbf{w}_{T+1} - \mathbf{u}\|^2$$

as we make more mistakes, our solution is better

Proof

- Define the potential :

$$\Delta_t = \|\mathbf{u} - \mathbf{w}_t\|^2 - \|\mathbf{u} - \mathbf{w}_{t+1}\|^2$$

- Bound it's cumulative sum $\sum_t \Delta_t$ from above and below

Proof

- Bound from above :

$$\begin{aligned}\sum_t \Delta_t &= \sum_t \left[\|\mathbf{u}-\mathbf{w}_t\|^2 - \|\mathbf{u}-\mathbf{w}_{t+1}\|^2\right] \\ &= \|\mathbf{u}-\mathbf{w}_1\|^2 - \|\mathbf{u}-\mathbf{w}_{T+1}\|^2 \\ &\leq \|\mathbf{u}\|^2\end{aligned}$$

Telescopic Sum

Zero Vector

Non-Negative

Proof

- Bound From Below :
  - No error on tth round

$$\begin{aligned}\Delta_t &= \|\mathbf{u}-\mathbf{w}_t\|^2 - \|\mathbf{u}-\mathbf{w}_{t+1}\|^2 \\ &= 0 \geq -2\ell^*_\gamma(\mathbf{x}_t, y_t)\end{aligned}$$

  - Error on tth round

$$\begin{aligned}\Delta_t &= \|\mathbf{u}-\mathbf{w}_t\|^2 - \|\mathbf{u}-\mathbf{w}_t - y_t\mathbf{x}_t\|^2 \\ &= -\|\mathbf{x}_t\|^2 + 2y_t(\mathbf{u}\cdot\mathbf{x}_t) - 2y_t(\mathbf{w}_t\cdot\mathbf{x}_t)\end{aligned}$$

Proof

- We bound each term :

$$\Delta_t = -\|\mathbf{x}_t\|^2 + 2y_t(\mathbf{u}\cdot\mathbf{x}_t) - 2y_t(\mathbf{w}_t\cdot\mathbf{x}_t)$$

$-\|\mathbf{x}_t\|^2 \geq -R^2$

$-2y_t(\mathbf{w}_t\cdot\mathbf{x}_t) \geq 0$

$$\begin{aligned} y_t(\mathbf{u}\cdot\mathbf{x}_t) &\geq \gamma - \max\{0, \gamma - y_t(\mathbf{u}\cdot\mathbf{x}_t)\} \\ &= \gamma - \ell^*_\gamma(\mathbf{x}_t, y_t) \end{aligned}$$

Proof

- Bound From Below :
  - No error on t[th] round

$$\Delta_t \geq -2\ell^*_\gamma(\mathbf{x}_t, y_t)$$

  - Error on t[th] round

$$\Delta_t \geq -2\ell^*_\gamma(\mathbf{x}_t, y_t) - R^2 + 2\gamma$$

- Cumulative bound :

$$\sum_t \Delta_t \geq -2\sum_t \ell^*_\gamma(\mathbf{x}_t, y_t) + M(2\gamma - R^2)$$

Proof

- Putting both bounds together :

$$-2\sum_t \ell^*_\gamma(\mathbf{x}_t, y_t) + M(2\gamma - R^2) \leq \sum_t \Delta_t \leq \|\mathbf{u}\|^2$$

- We use first degree of freedom (and scale) :

$$\mathbf{u} \leftarrow c\mathbf{u} \qquad \gamma \leftarrow c\gamma \qquad \ell^*_\gamma(\mathbf{x}_t, y_t) \leftarrow c\ell^*_\gamma(\mathbf{x}_t, y_t)$$

- Bound :

$$-2c\sum_t \ell^*_\gamma(\mathbf{x}_t, y_t) + M(2c\gamma - R^2) \leq c^2\|\mathbf{u}\|^2$$

Proof

- General Bound :

$$M \leq \frac{c^2\|\mathbf{u}\|^2 + 2c\sum_t \ell_\gamma^*(\mathbf{x}_t, y_t)}{2c\gamma - R^2}$$

- Choose :

$$c = \frac{R^2}{\gamma} \quad , \quad \gamma = 1$$

Objective of SVM

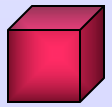
- Simple Bound :

$$M \leq R^2\|\mathbf{u}\|^2 + 2\sum_t \ell^*(\mathbf{x}_t, y_t)$$

Proof

- Better bound : optimize the value of

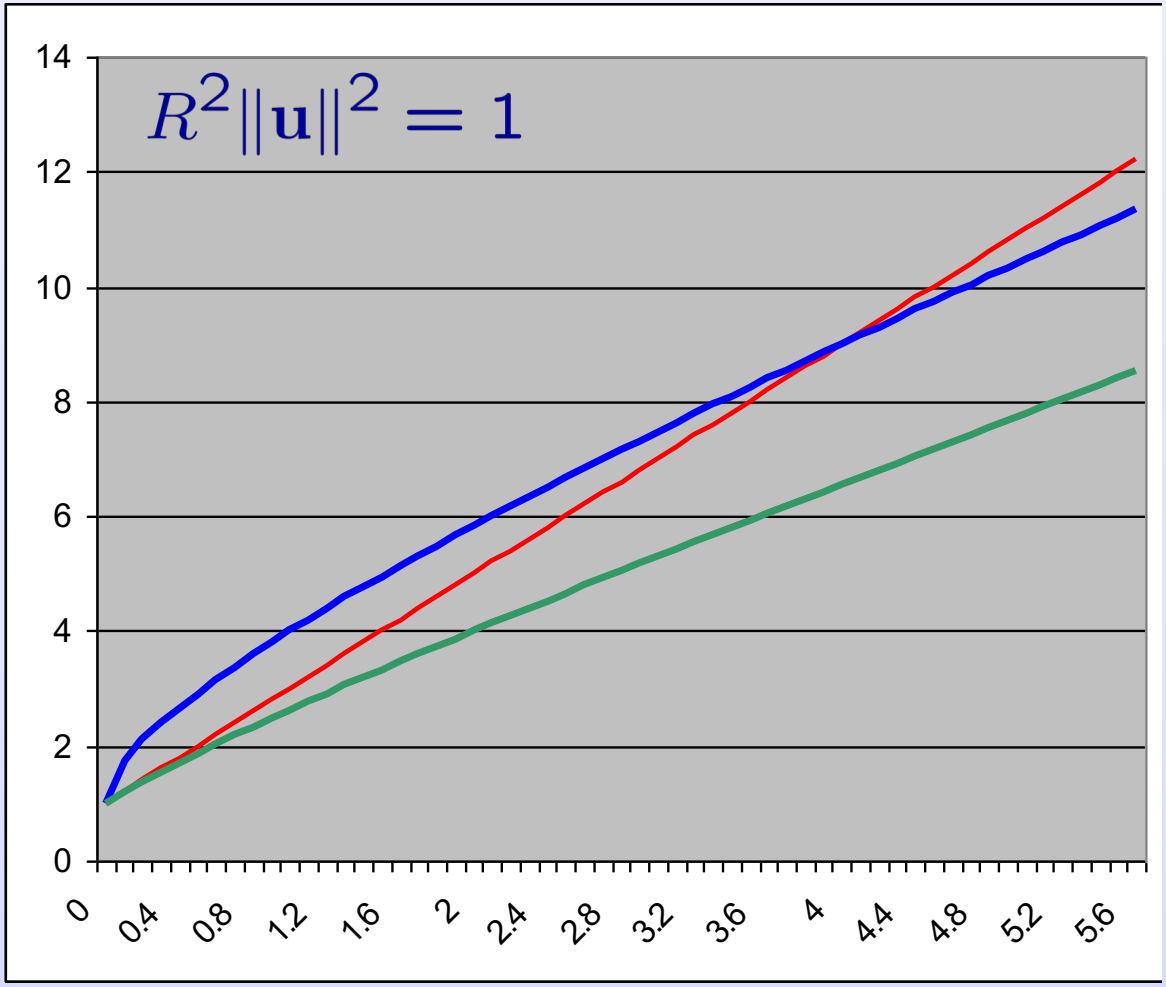
$$M \leq \min_{c} \left\{ \frac{c^2 \|\mathbf{u}\|^2 + 2c \sum_t \ell_\gamma^*(\mathbf{x}_t, y_t)}{2c\gamma - R^2} \right\}$$



Remarks

- Bound does not depend on dimension of the feature vector
- The bound holds for *all* sequences. It is not tight for most real world data
- But, there exists a setting for which it is tight – worst

Three Bounds



$$M \leq R^2\|\mathbf{u}\|^2 + 2\mathcal{L}_1$$

$$M \leq \left(R\|\mathbf{u}\| + \sqrt{\mathcal{L}_1}\right)^2$$

$$M \leq \frac{1}{4}\left(R\|\mathbf{u}\| + \sqrt{\|\mathbf{u}\|^2R^2 + 4\mathcal{L}_1}\right)^2$$

Separable Case

- Assume there exists $\mathbf{u}$ such that $y_t(\mathbf{u} \cdot \mathbf{x}_t) > 0$ for all examples $t = 1 \ldots T$
  Then all bounds are equivalent

$$M \leq R^2 \|\mathbf{u}\|^2$$

- Perceptron makes finite number of mistakes until convergence (not necessarily to $\mathbf{u}$)

Separable Case – Other Quantities

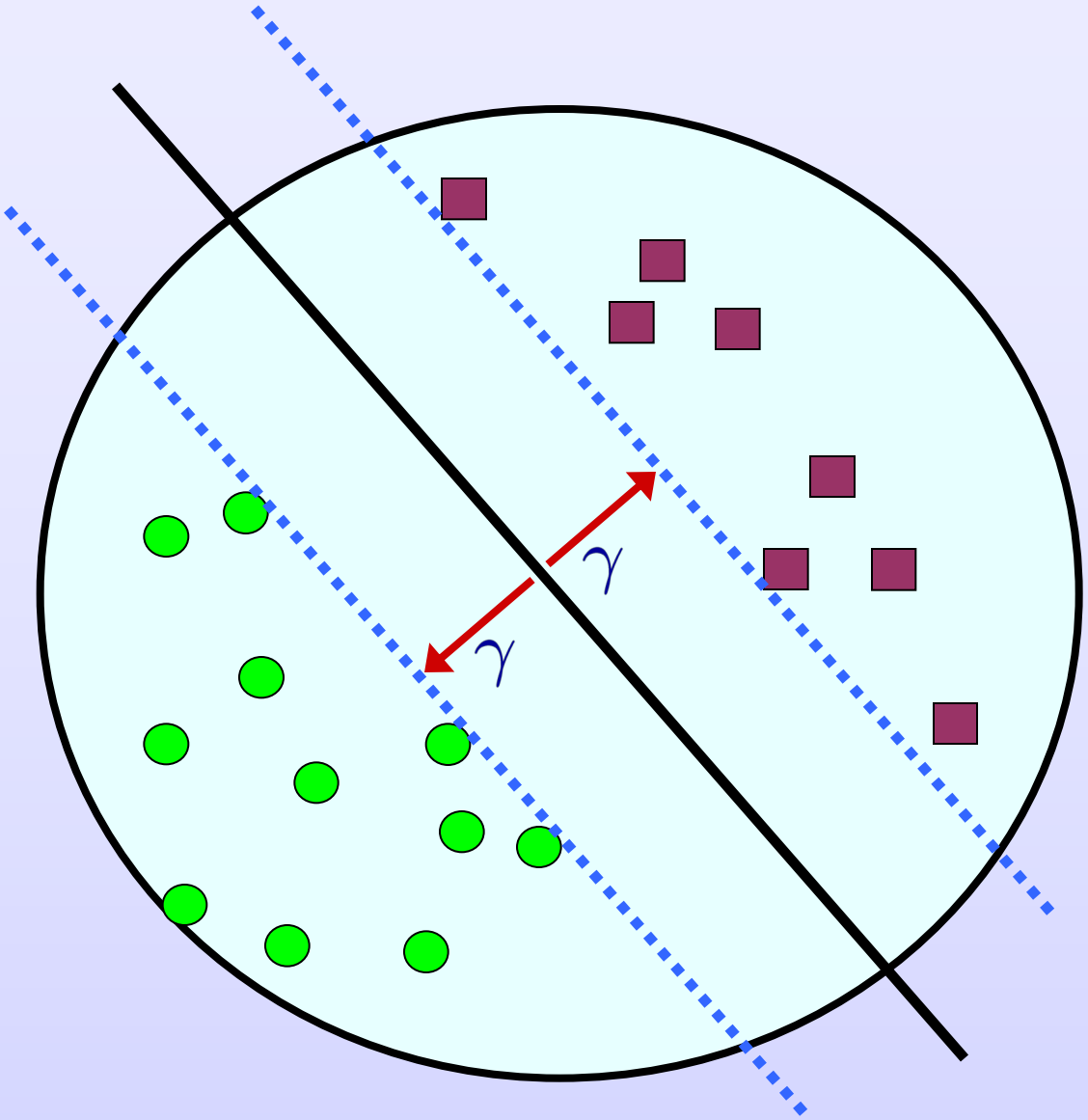
- Use 1st (parameterization) degree of freedom
- Scale the $\mathbf{u}$ such that $\|\mathbf{u}\| = 1$
- Define

$$\gamma = \min_t y_t(\mathbf{u} \cdot \mathbf{x}_t)$$

- The bound becomes

$$M \leq \frac{R^2}{\gamma^2}$$

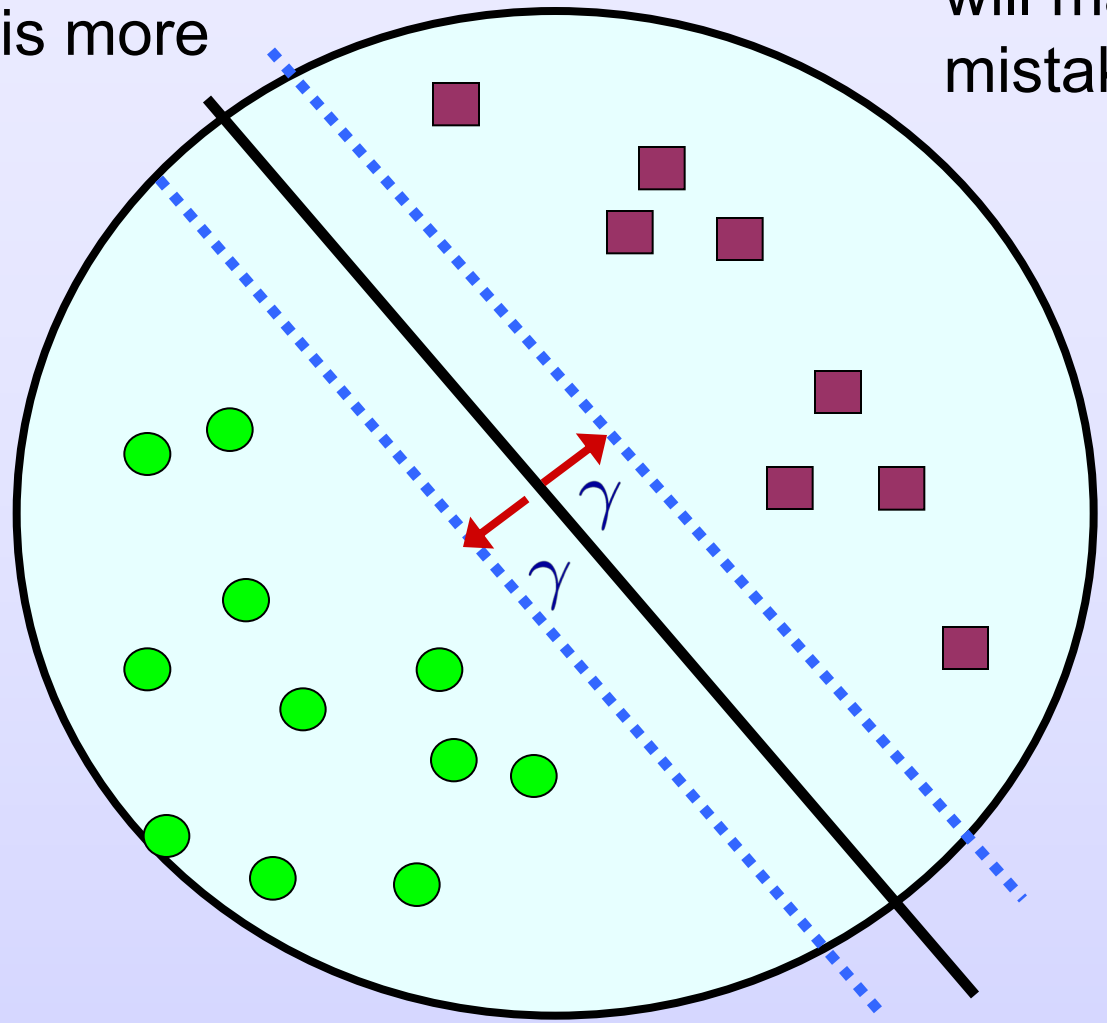
Separable Case - Illustration



separable Case – Illustration

Finding a separating hyperplance is more difficult

The Perceptron will make more mistakes



Inseparable Case

$$
\begin{aligned}
M &\leq \frac{1}{4}\left(R\|\mathbf{u}\| + \sqrt{\|\mathbf{u}\|^2 R^2 + 4\mathcal{L}_1}\right)^2 \\
M &\leq (R\|\mathbf{u}\| + \mathcal{L}_2)^2 \\
M &\leq \left(R\|\mathbf{u}\| + \sqrt{\mathcal{L}_1}\right)^2
\end{aligned}
$$

- Difficult problem implies a large value of $\mathcal{L}_1$, $\mathcal{L}_2$
- In this case the Perceptron will make a large number of mistakes

Perceptron Algorithm

- Extremely easy to implement
- Relative loss bounds for separable and inseparable cases. Minimal assumptions (not iid)
- Easy to convert to a well-performing batch algorithm (under iid assumptions)
- Quantities in bound are not compatible : no. of mistakes vs. hinge-loss.
- Margin of examples is ignored by update
- Same update for separable case and inseparable case.

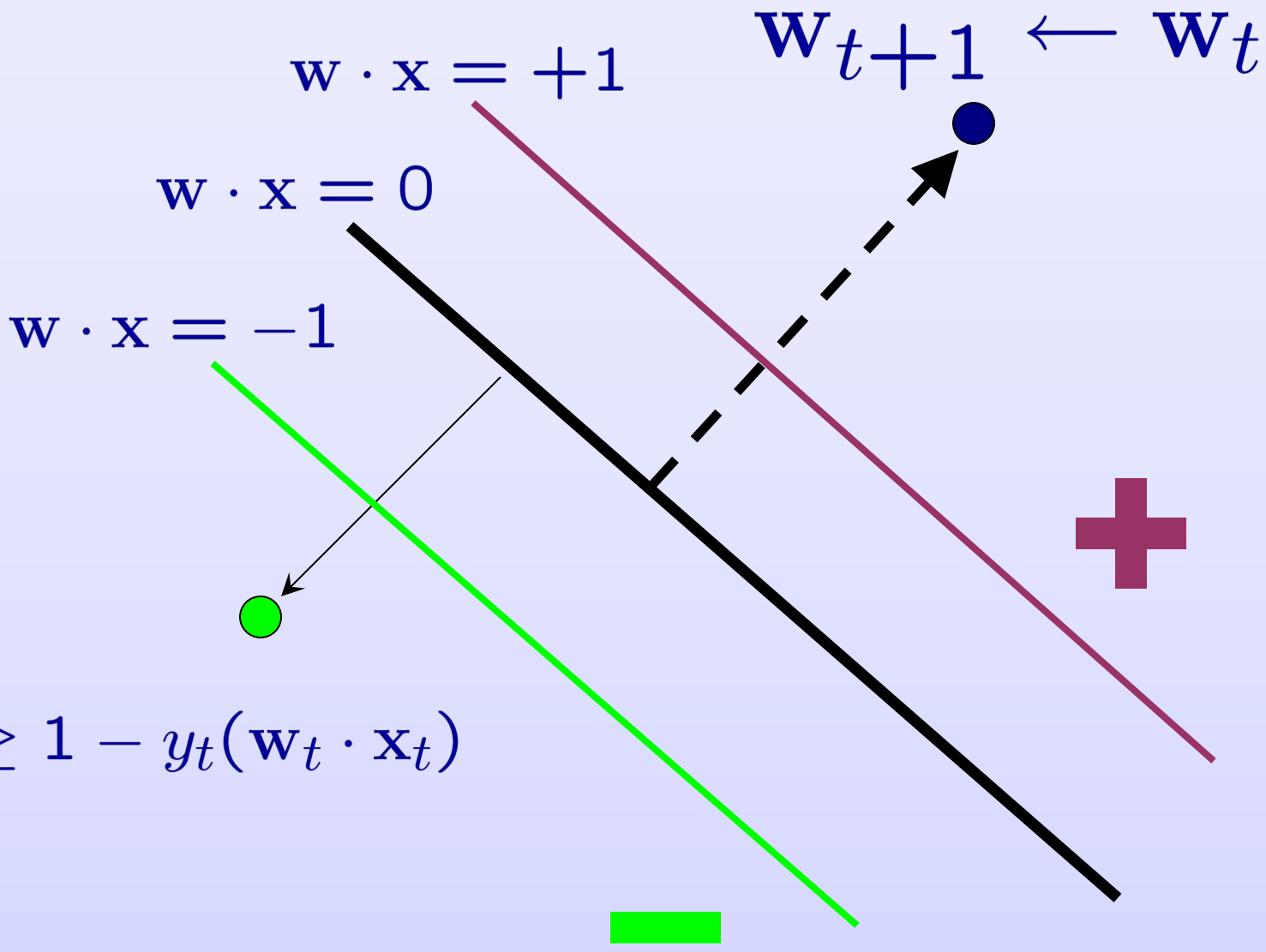
Passive – Aggressive Approach

- The basis for a well-known algorithm in convex optimization due to Hildreth (1957)
  - Asymptotic analysis
  - Does not work in the inseparable case
- Three versions :
  - PA separable case
  - PA-I PA-II inseparable case
- Beyond classification
  - Regression, one class, structured learning
- Relative loss bounds

Motivation

- Perceptron: No guaranties of margin *after* the update
- PA :Enforce a minimal non-zero margin after the update
- In particular :
  - If the margin is large enough (1), then do nothing
  - If the margin is less then unit, update such that the margin *after* the update is *enforced* to be unit

Input Space


$\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t$
$\mathbf{w} \cdot \mathbf{x} = +1$
$\mathbf{w} \cdot \mathbf{x} = 0$
$\mathbf{w} \cdot \mathbf{x} = -1$
$0 \geq 1 - y_t(\mathbf{w}_t \cdot \mathbf{x}_t)$

Input Space vs. Version Space

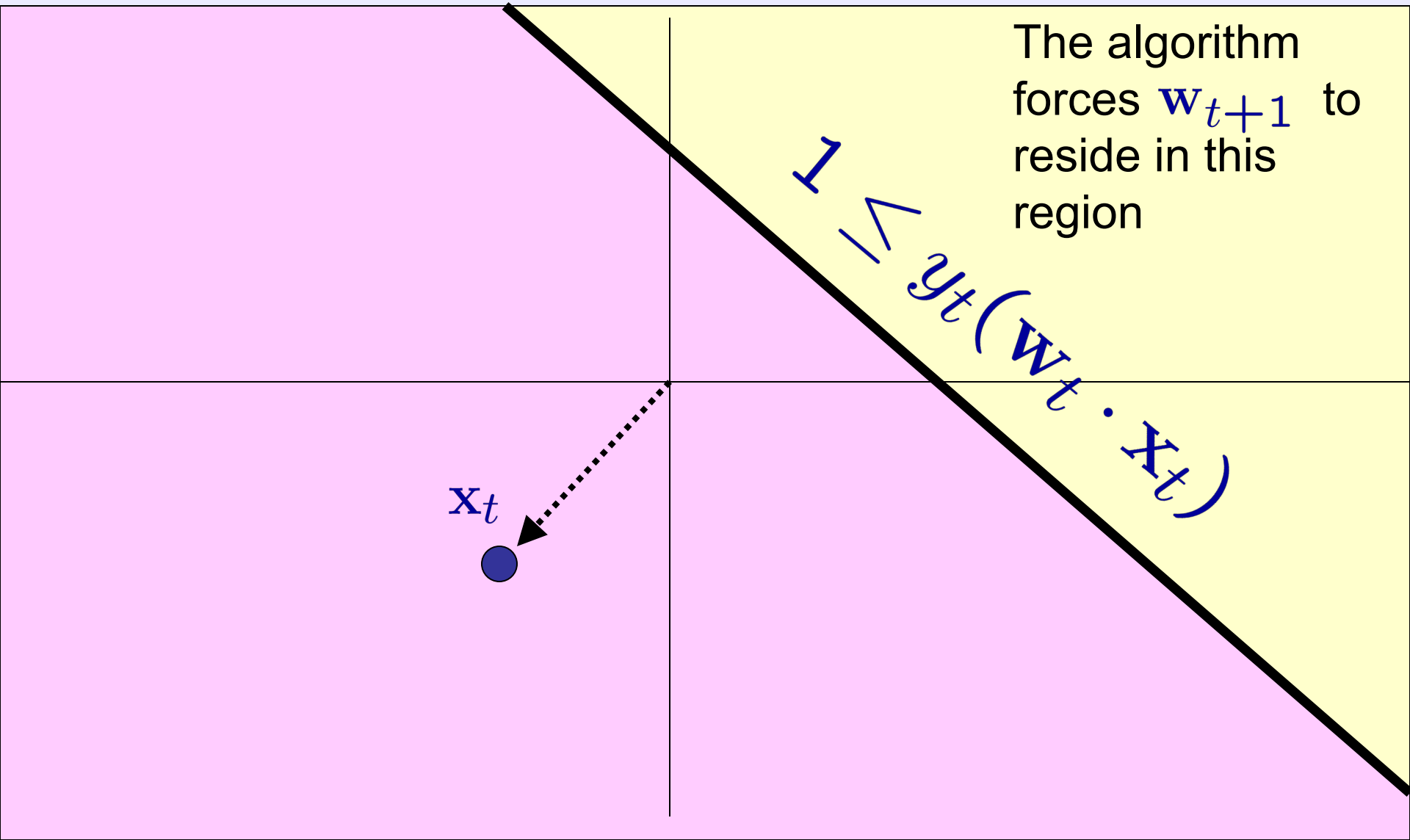
- Input Space :
  - Points are input data $y_t \mathbf{x}_t$
  - One constraint is induced by weight vector $\mathbf{w}$
  - Primal space
  - Half space = all input examples that are classified correctly by a given predictor (weight vector)

$$\{y\mathbf{x} \ : \ \mathbf{w} \cdot (y\mathbf{x}) \geq 0\}$$

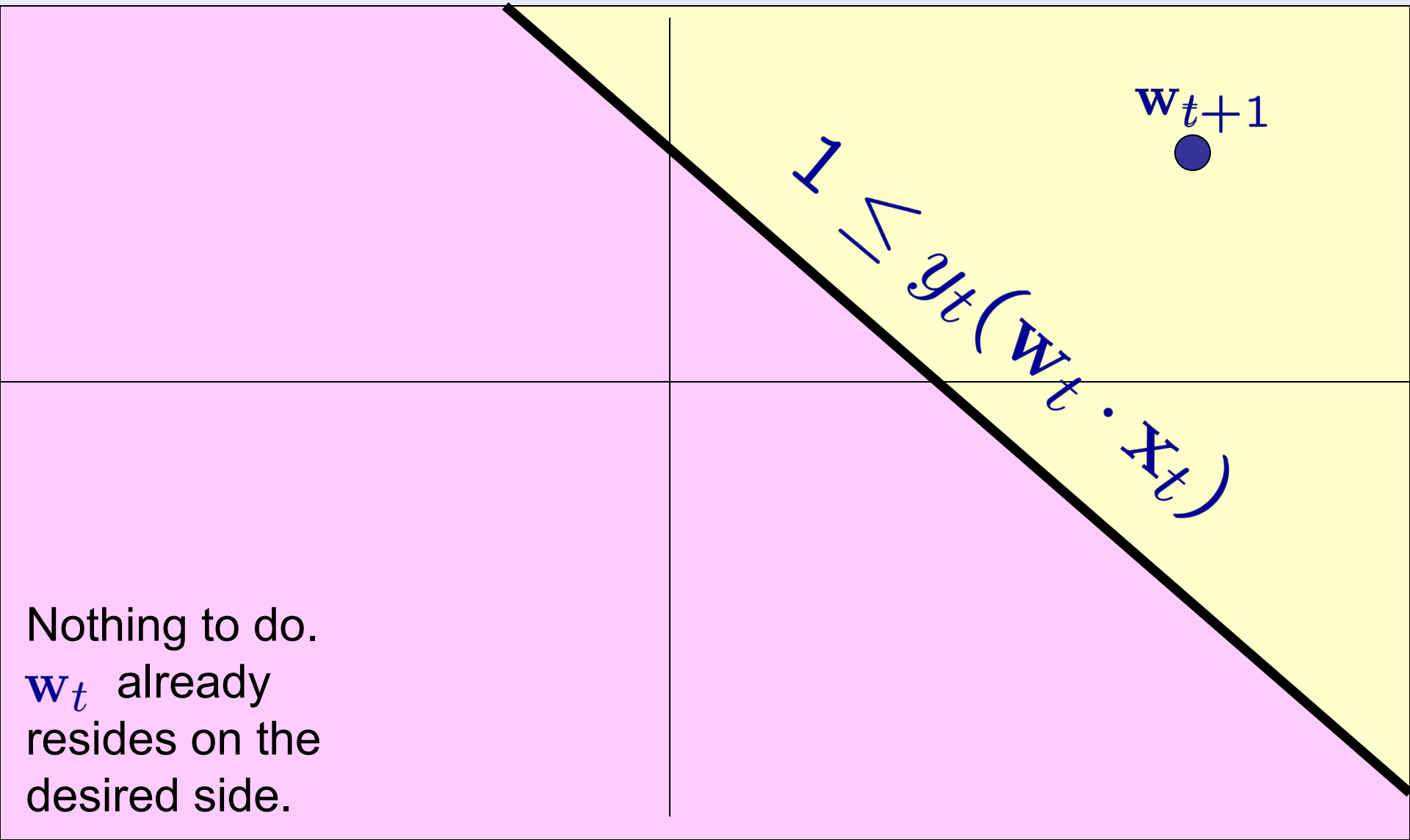
- Version Space :
  - Points are weight vectors $\mathbf{w}$
  - One constraints is induced by input data $y_t \mathbf{x}_t$
  - Dual space
  - Half space = all predictors (weight vectors) that classify correctly a given input example

$$\{\mathbf{w} \ : \ \mathbf{w} \cdot (y\mathbf{x}) \geq 0\}$$

Weight Vector (Version) Space

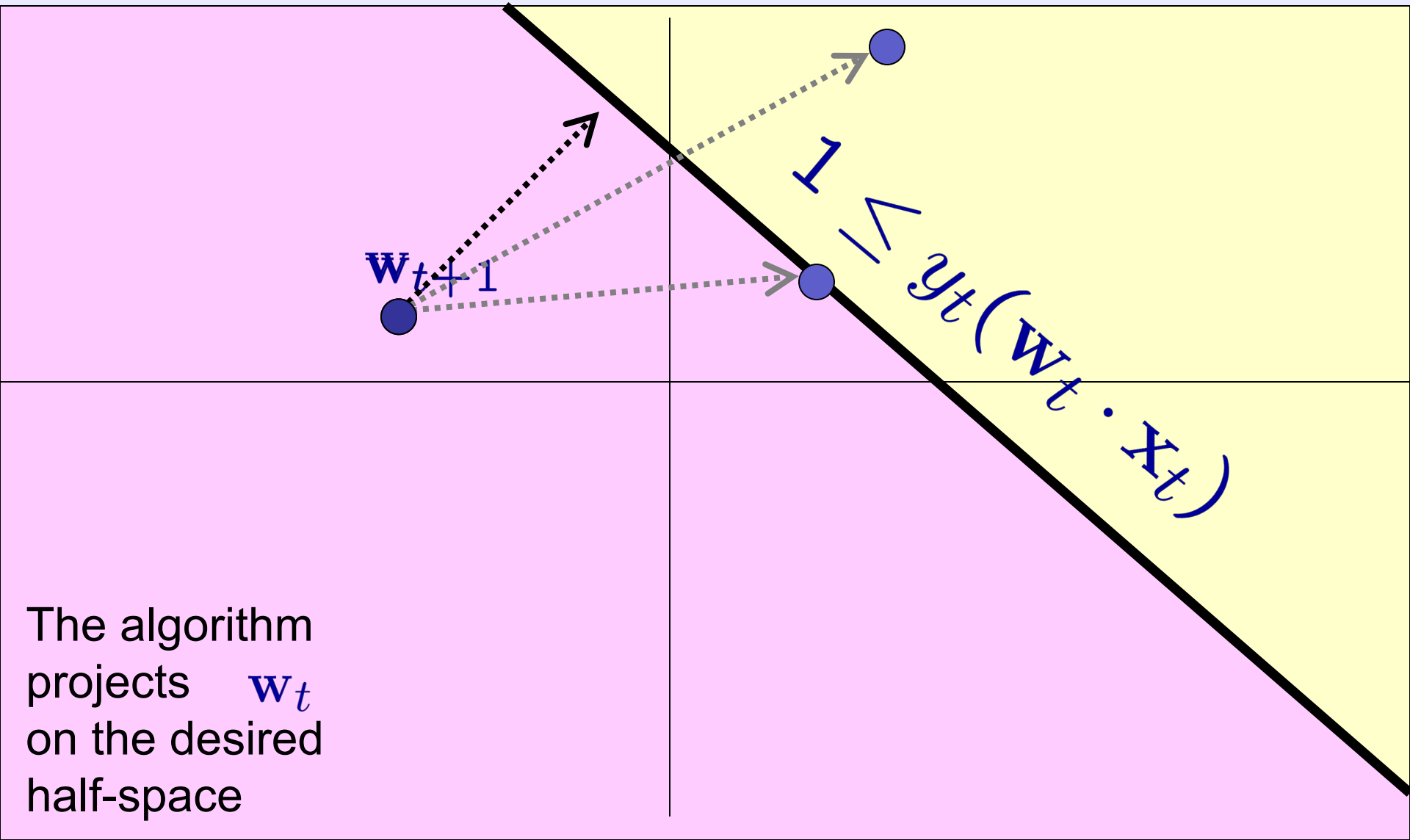


Passive Step



Nothing to do.
$\mathbf{w}_t$ already resides on the desired side.

Aggressive Step



The algorithm projects $\mathbf{w}_t$ on the desired half-space

Aggressive Update Step

- Set $\mathbf{w}_{t+1}$ to be the solution of the following optimization problem :

$$\mathbf{w}_{t+1} = \min_{\mathbf{w}} \frac{1}{2}\|\mathbf{w} - \mathbf{w}_t\|^2$$
$$\text{s.t.} \quad y_t(\mathbf{w} \cdot \mathbf{x}_t) \geq 1$$

- The Lagrangian :

$$\mathcal{L}(\mathbf{w}, \tau) = \frac{1}{2}\|\mathbf{w} - \mathbf{w}_t\|^2 + \tau(1 - y_t(\mathbf{w} \cdot \mathbf{x}_t))$$

- Solve for the dual :

$$\max_{\tau \geq 0} \min_{\mathbf{w}} \mathcal{L}(\mathbf{w}, \tau)$$

Aggressive Update Step

- Optimize for $\mathbf{w}$:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \mathbf{w} - \mathbf{w}_t + \tau y_t \mathbf{x}_t$$

- Set the derivative to zero
- Substitute back into the Lagrangian :

$$\mathcal{L}(\max_{\tau \geq 0}) = -\frac{1}{2}\|\mathbf{x}_t\|^2 \tau^2 + \tau(1 - y_t(\mathbf{w}_t \cdot \mathbf{x}_t))$$

- Dual optimization problem

Aggressive Update Step

- Dual Problem :

$$\max_{\tau \geq 0} \quad -\frac{1}{2}\|\mathbf{x}_t\|^2\tau^2 + \tau(1 - y_t(\mathbf{w}_t \cdot \mathbf{x}_t))$$

- Solve it :

$$\tau = \max\left\{0, \frac{1 - y_t(\mathbf{w}_t \cdot \mathbf{x}_t)}{\|\mathbf{x}_t\|^2}\right\}$$

- What about the constraint?

Alternative Derivation

- Additional Constraint (linear update) :

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \tau y_t \mathbf{x}_t$$

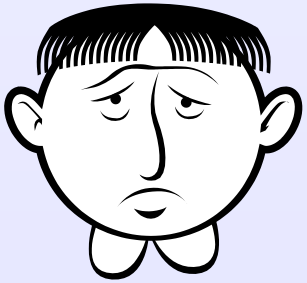
- Force the constraint to hold as equality

$$1 = y_t((\mathbf{w}_t + \tau y_t \mathbf{x}_t) \cdot \mathbf{x}_t) = y_t(\mathbf{w}_t \cdot \mathbf{x}_t) + \tau \|\mathbf{x}_t\|^2$$

- Solve :

$$\tau = \frac{1 - y_t(\mathbf{w}_t \cdot \mathbf{x}_t)}{\|\mathbf{x}_t\|^2}$$

Passive-Aggressive Update



$$\mathbf{w}_{t+1} = \mathbf{w}_t + \tau y_t \mathbf{x}_t$$

$$y_t(\mathbf{w}_t \cdot \mathbf{x}_t) \geq 1$$

$$y_t(\mathbf{w}_t \cdot \mathbf{x}_t) < 1$$

$$\tau = 0$$

$$\tau = \frac{1 - y_t(\mathbf{w}_t \cdot \mathbf{x}_t)}{\|\mathbf{x}_t\|^2}$$

Perceptron vs. PA

- Common Update :

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \tau y_t \mathbf{x}_t$$

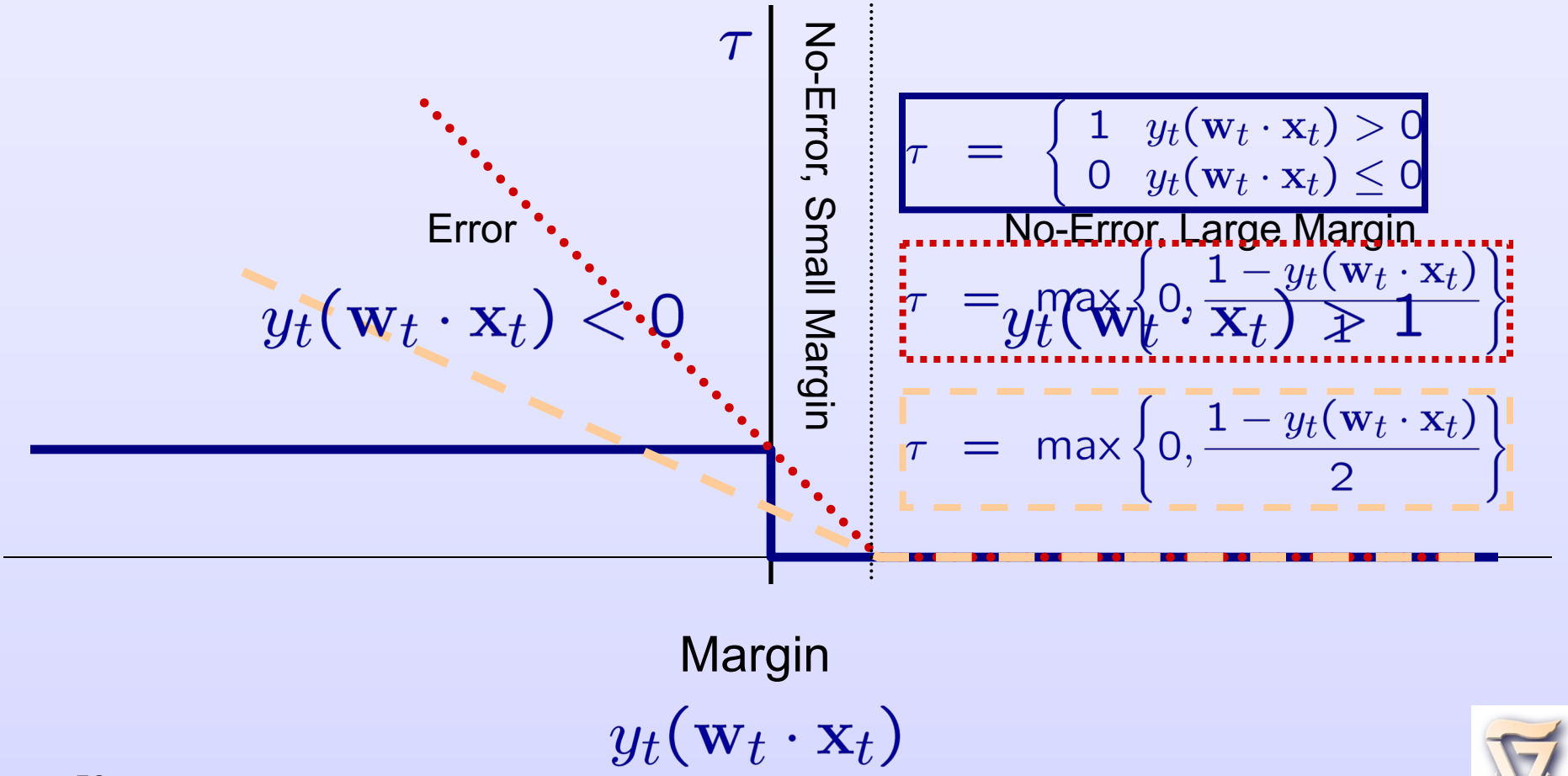
- Perceptron

$$\tau = \begin{cases} 1 & y_t(\mathbf{w}_t \cdot \mathbf{x}_t) > 0 \\ 0 & y_t(\mathbf{w}_t \cdot \mathbf{x}_t) \leq 0 \end{cases}$$

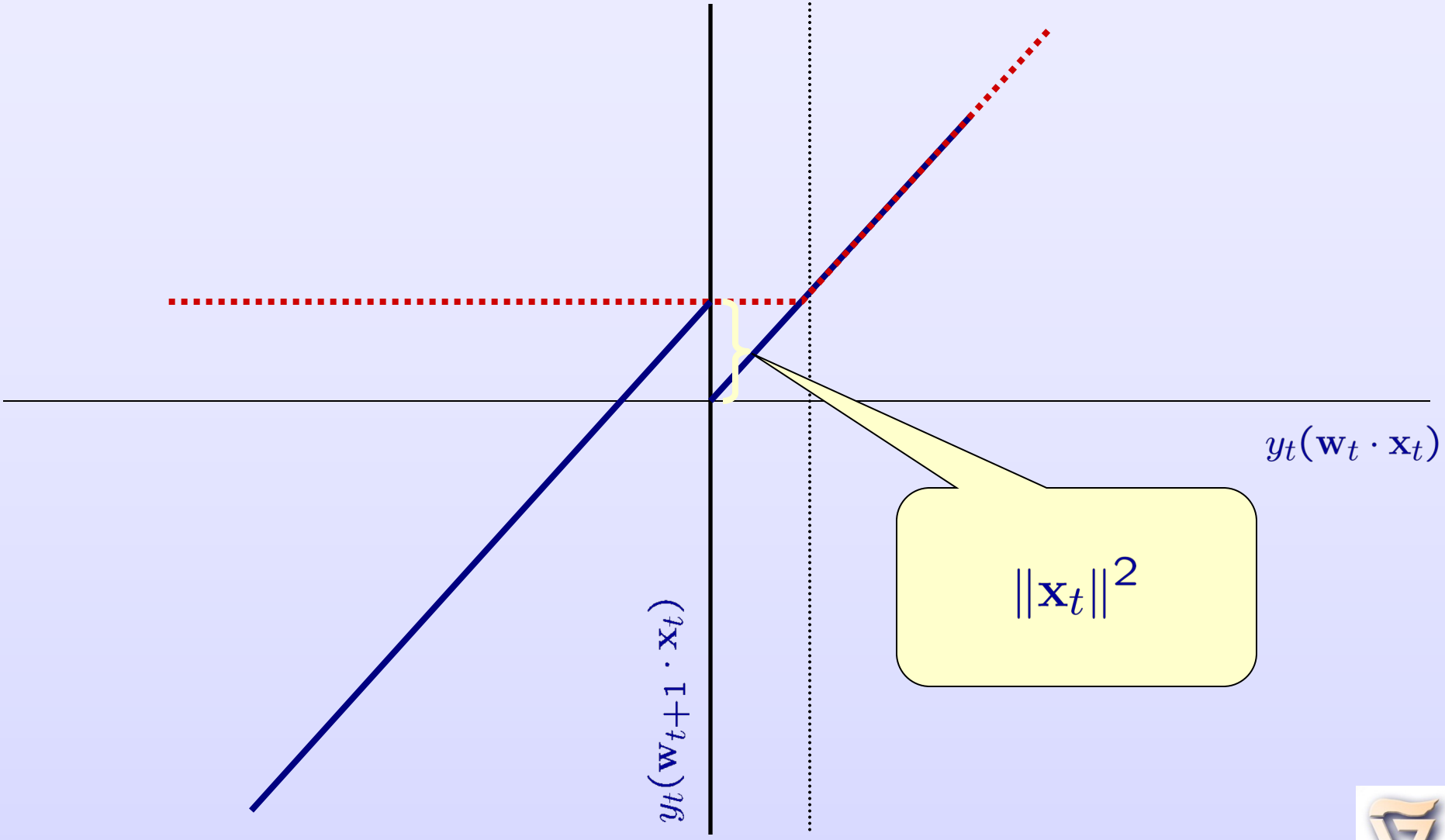
- Passive-Aggressive

$$\tau = \max\left\{0, \frac{1 - y_t(\mathbf{w}_t \cdot \mathbf{x}_t)}{\|\mathbf{x}_t\|^2}\right\}$$

Perceptron vs. PA

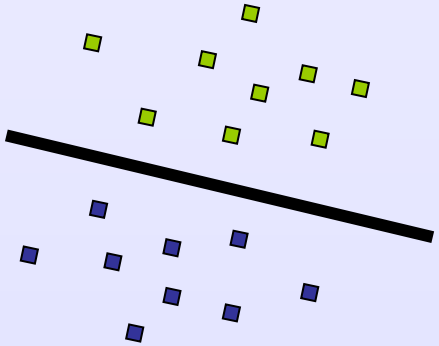


Perceptron vs. PA



Three Decision Problems

Classification

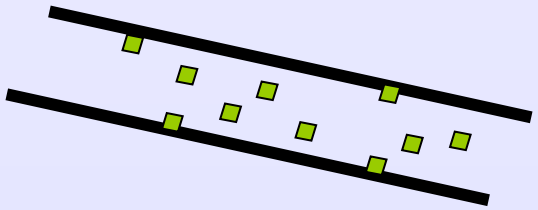


$$y_t \mathbf{w} \cdot \mathbf{x}_t \geq \widetilde{\epsilon}$$

$$\mathbf{z}_t = (\mathbf{x}_t, y_t)$$

$$(\mathbf{x}_t \in \mathcal{R}^n, y_t \in \{-1, 1\})$$

Regression

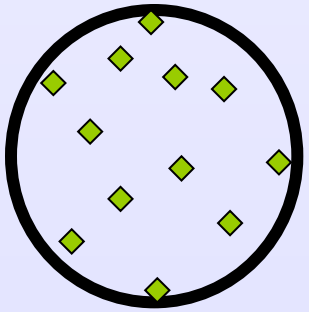


$$|\mathbf{w} \cdot \mathbf{x}_t - y_t| \leq \widetilde{\epsilon}$$

$$\mathbf{z}_t = (\mathbf{x}_t, y_t)$$

$$(\mathbf{x}_t \in \mathcal{R}^n, y_t \in \mathcal{R})$$

Uniclass



$$\|\mathbf{y}_t - \mathbf{w}\| \leq \widetilde{\epsilon}$$

$$\mathbf{z}_t = \mathbf{y}_t$$

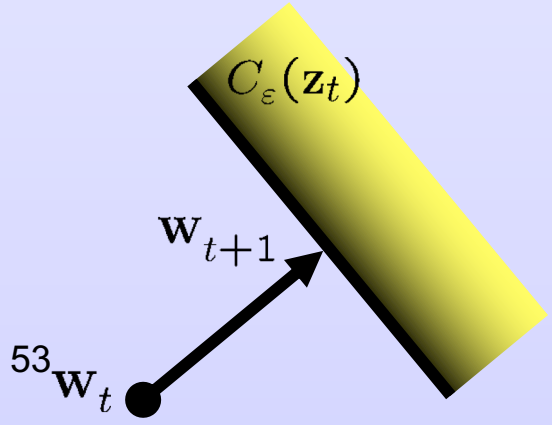
$$\mathbf{y}_t \in \mathcal{R}^n$$

The Passive-Aggressive Algorithm

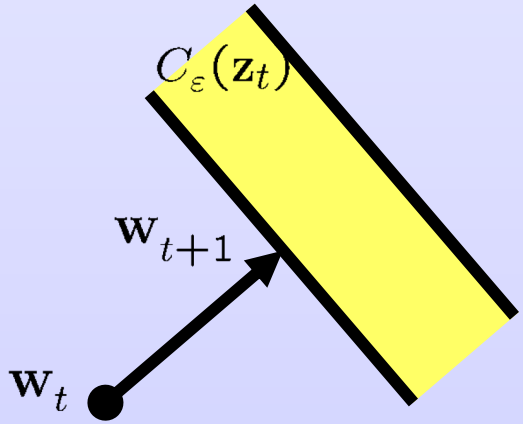
- Each example defines a set of consistent hypotheses: $C_\varepsilon(\mathbf{z}_t) = \{\mathbf{w} \mid \delta(\mathbf{w}; \mathbf{z}_t) \leq \varepsilon\}$
- The new vector $\mathbf{w}_{t+1}$ is set to be the projection of $\mathbf{w}_t$ onto $C_\varepsilon(\mathbf{z}_t)$

$$\mathbf{w}_{t+1} = \underset{\mathbf{w}}{\operatorname{argmin}} \|\mathbf{w} - \mathbf{w}_t\| \quad \text{s.t.} \quad \mathbf{w} \in C_\varepsilon(\mathbf{z}_t)$$

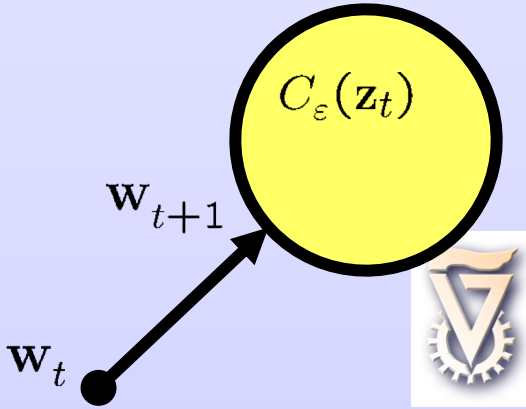
Classification



Regression



Uniclass



Loss Bound

- Assume there exists $\mathbf{u}$ such that

$$y_t(\mathbf{u} \cdot \mathbf{x}_t) \geq 1 \quad \forall t = 1 \ldots T$$

- Assume :

$$\|\mathbf{x}_t\| \leq R \quad \forall t = 1 \ldots T$$

- Then :

$$\sum_t \left(\ell(\mathbf{w}_t \cdot \mathbf{x}_t, y_t)^2 \right) \leq R^2 \|\mathbf{u}\|^2$$

- Note :

$$\ell_{01}(\mathbf{w}_t \cdot \mathbf{x}_t, y_t) \leq \ell(\mathbf{w}_t \cdot \mathbf{x}_t, y_t) \leq R^2 \|\mathbf{u}\|^2$$

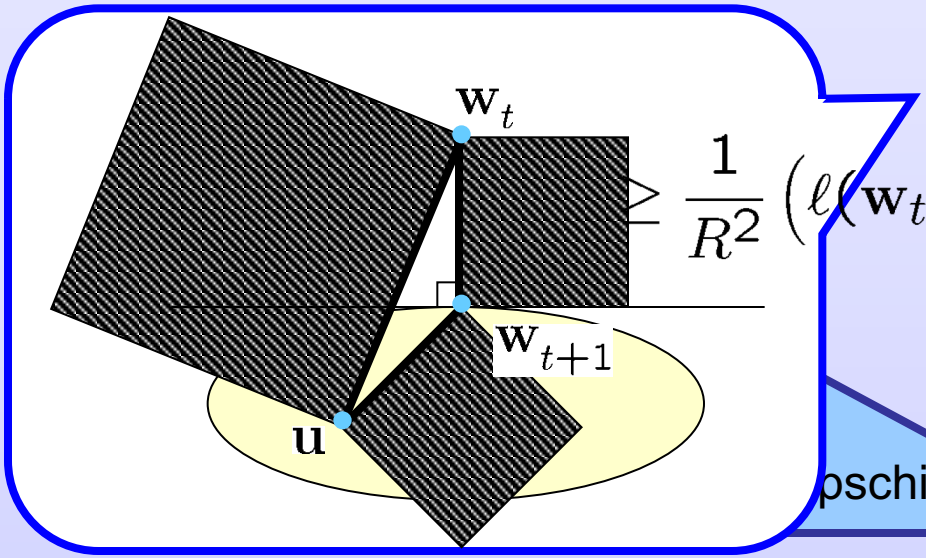
Proof Sketch

- Define:
- Upper bound:
- Lower bound:

$$\Delta_t = \|\mathbf{w}_t - \mathbf{u}\|^2 - \|\mathbf{w}_{t+1} - \mathbf{u}\|^2$$

$$\sum_{t=1}^{T} \Delta_t \leq \|\mathbf{w}_1 - \mathbf{u}\|^2$$

$$\Delta_t = \|\mathbf{w}_t - \mathbf{u}\|^2 - \|\mathbf{w}_{t+1} - \mathbf{u}\|^2$$



$$\geq \|\mathbf{w}_t - \mathbf{w}_{t+1}\|^2$$

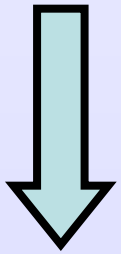
$$\geq \frac{1}{R^2}\left(\ell(\mathbf{w}_t \cdot \mathbf{x}_t\ ,\ y_t) - \underbrace{\ell(\mathbf{w}_{t+1} \cdot \mathbf{x}_t\ ,\ y_t)}_{=0}\right)^2$$

pschitz Condition

Proof Sketch (Cont.)

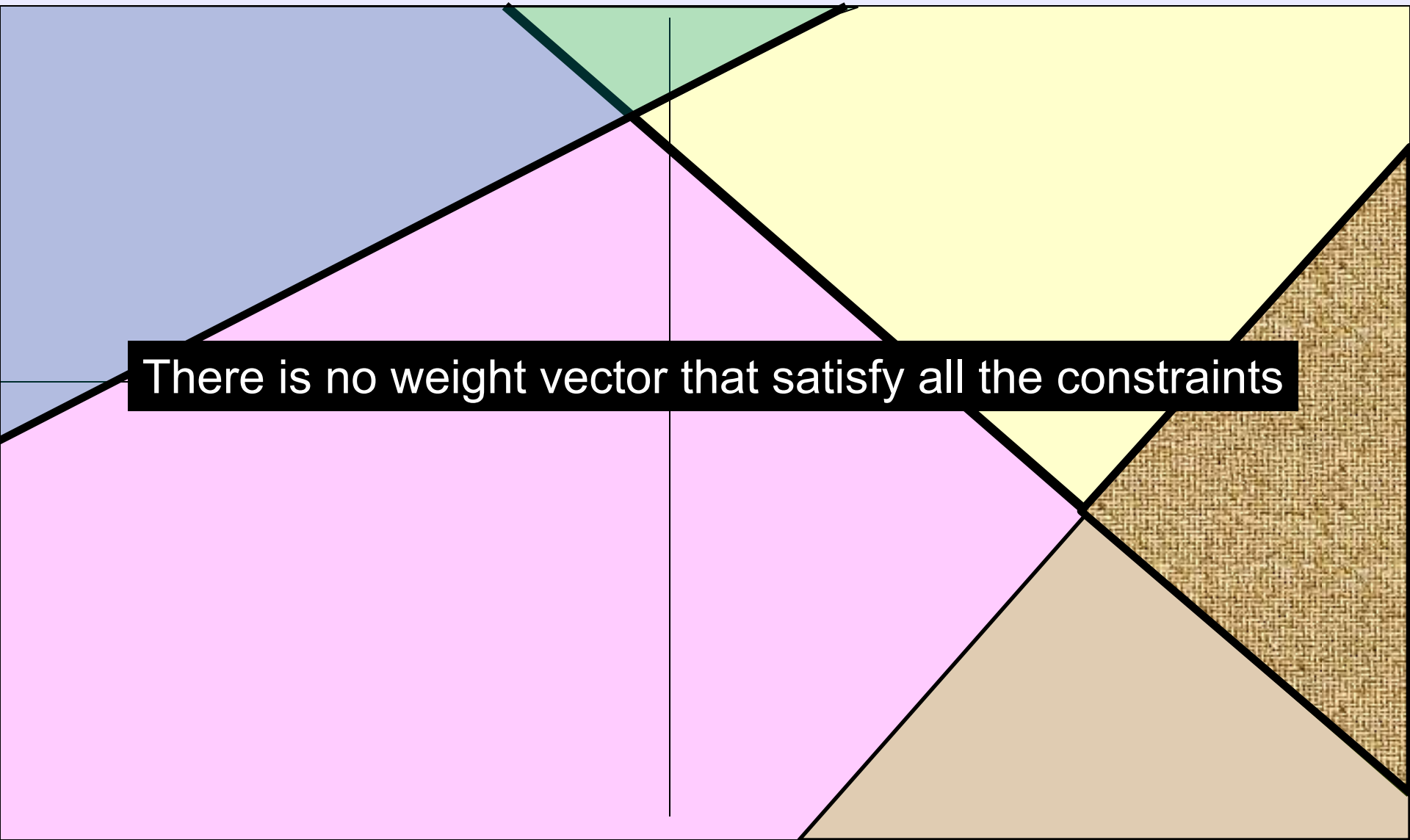
- Combining upper and lower bounds

$$\frac{1}{R^2}\sum_{t=1}^{T}\left(\ell(\mathbf{w}_t\cdot\mathbf{x}_t\ ,\ y_t)\right)^2\ \leq\ \sum_{t=1}^{T}\Delta_t\ \leq\ \left\|\mathbf{w}_1-\mathbf{u}\right\|^2$$

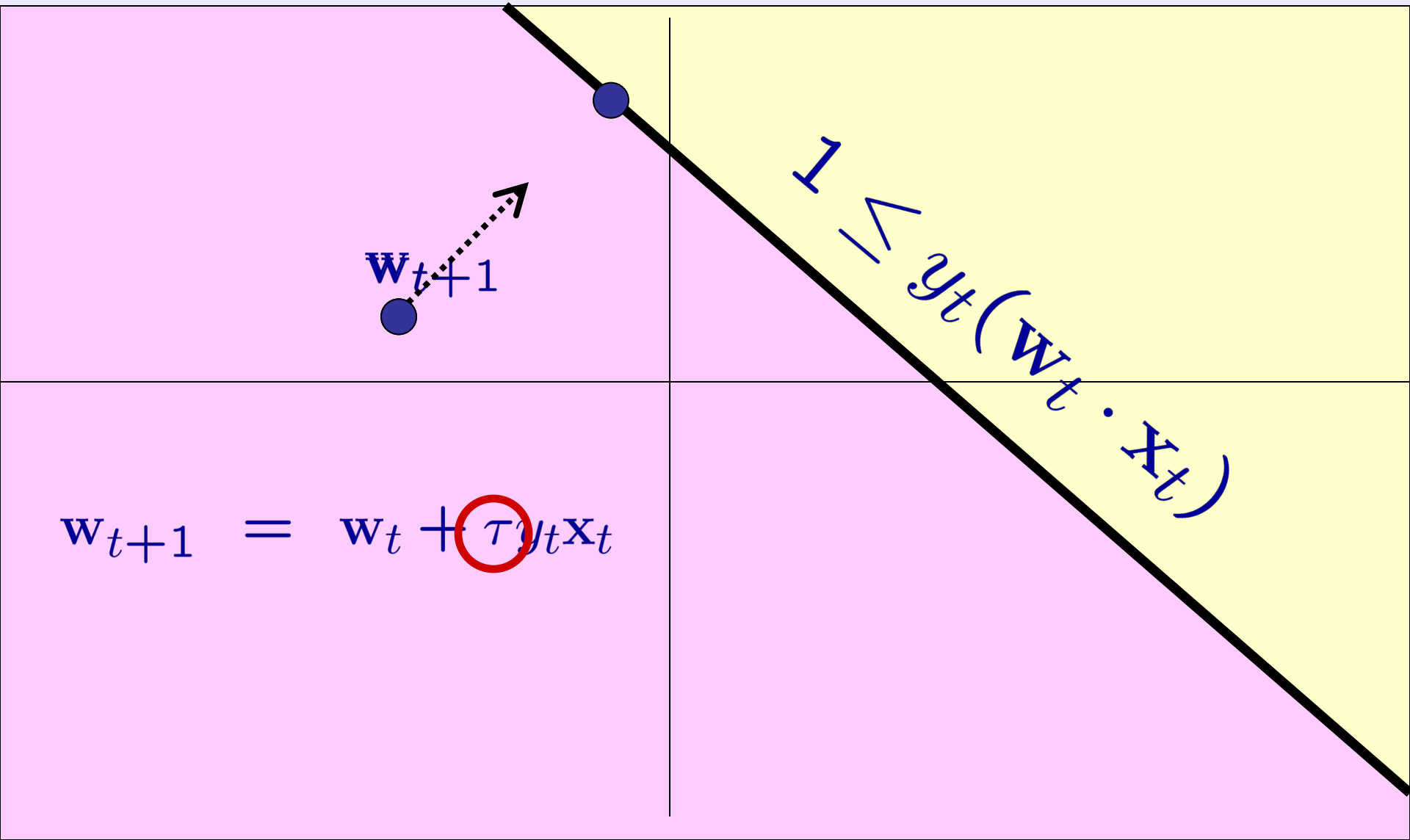


$$\sum_{t=1}^{T}\left(\ell(\mathbf{w}_t\cdot\mathbf{x}_t\ ,\ y_t)\right)^2\ \leq\ R^2\ \left\|\mathbf{w}_1-\mathbf{u}\right\|^2$$

Unrealizable case



Unrealizable Case



Unrealizable Case

$$\mathbf{w}_{t+1} = \min_{\mathbf{w}} \quad \frac{1}{2}\|\mathbf{w} - \mathbf{w}_t\|^2 + C\xi^2$$
$$\text{s.t.} \quad y_t(\mathbf{w} \cdot \mathbf{x}_t) \geq 1 - \xi \qquad \xi \geq 0$$

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \tau y_t \mathbf{x}_t$$

ξ

$$\min\left\{C, \max\left\{0, \frac{1 - y_t(\mathbf{w}_t \cdot \mathbf{x}_t)}{\|\mathbf{x}_t\|^2}\right\}\right\}$$

ξ^2

$$\max\left\{0, \frac{1 - y_t(\mathbf{w}_t \cdot \mathbf{x}_t)}{\|\mathbf{x}_t\|^2 + C}\right\}$$

Loss Bound for PA-I

- Mistake bound :

$$M \leq \max\left\{R^2, \frac{1}{C}\right\}\left(2\|\mathbf{u}\|^2 + 2C\sum_t \ell(\mathbf{x}_t \cdot \mathbf{u}\ ,\ y_t)\right)$$

- Optimal value, set $C = \frac{1}{R^2}$

$$M \leq \left(2R^2\|\mathbf{u}\|^2 + 2\sum_t \ell(\mathbf{x}_t \cdot \mathbf{u}\ ,\ y_t)\right)$$

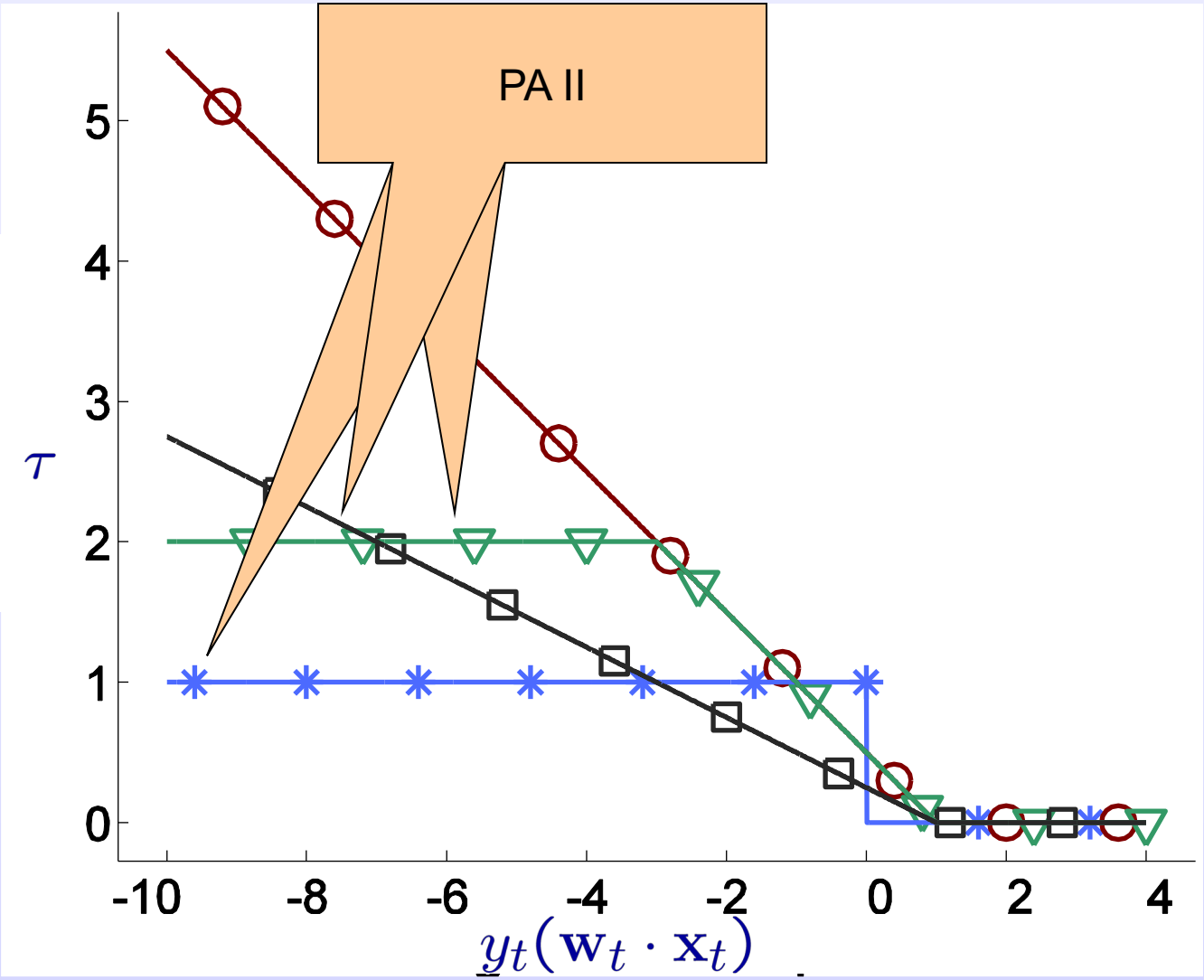
Loss Bound for PA-II

- Loss bound :

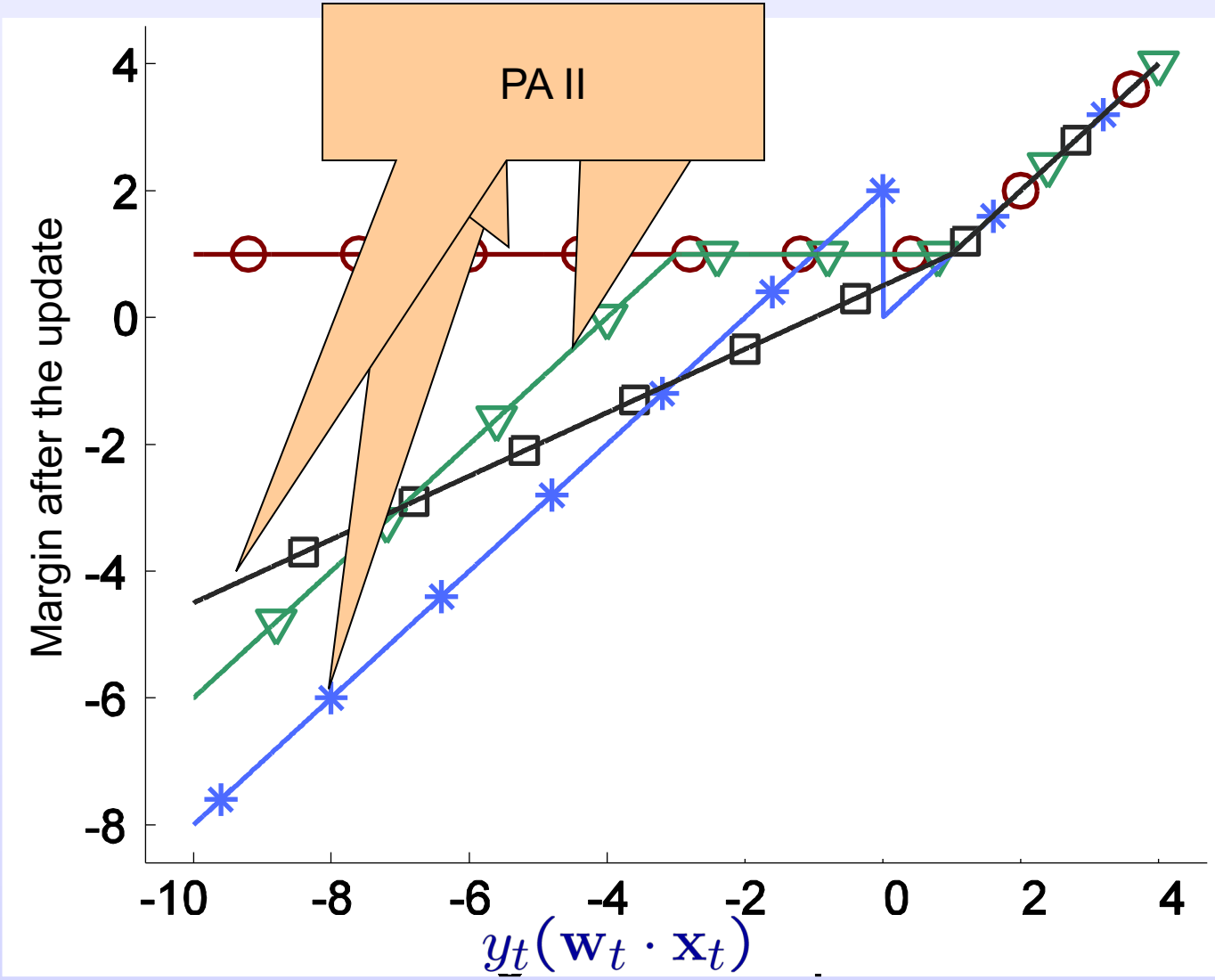
$$\sum_t \left(\ell(\mathbf{w}_t \cdot \mathbf{x}_t \ , \ y_t)^2\right)$$
$$\leq \ (C + R^2)\|\mathbf{u}\|^2 + \left(1 + \frac{R^2}{C}\right) \sum_t \left(\ell(\mathbf{u} \cdot \mathbf{x}_t \ , \ y_t)^2\right)$$

- Similar proof technique as of PA
- Bound can be improved similarly to the Perceptron

Four Algorithms



Four Algorithms



Concluding Remarks

- Batch vs. Online
  - Two phases: Training and then Test
  - Single continues process
- Statistical Assumption
  - Distribution over examples
  - All sequences
- Conversions
  - Online -> Batch
  - Batch -> Online

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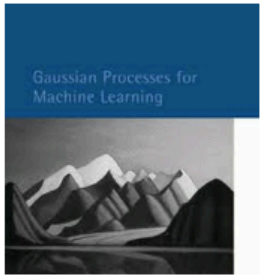

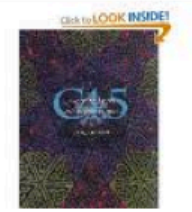

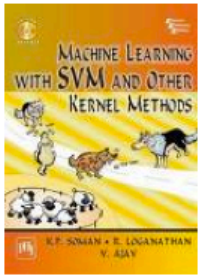

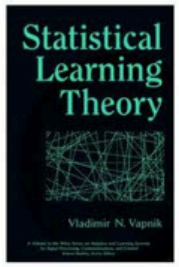

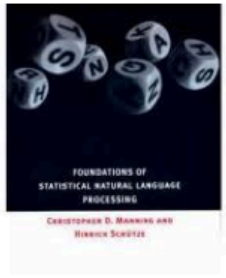


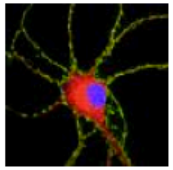
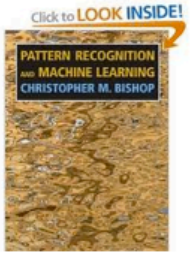

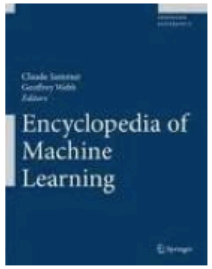

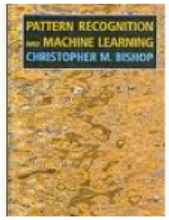

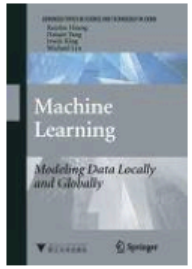

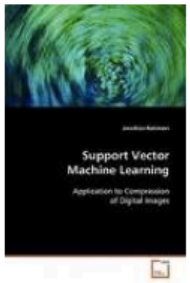



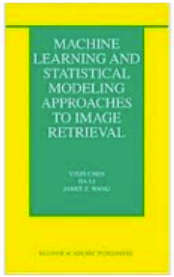


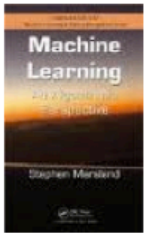

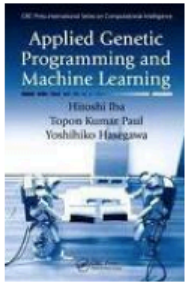

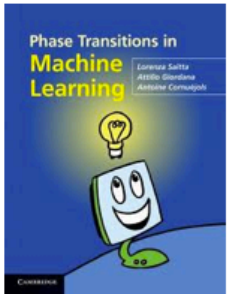

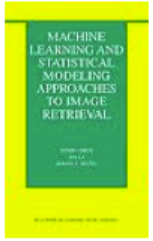

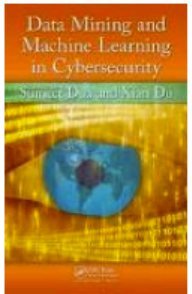




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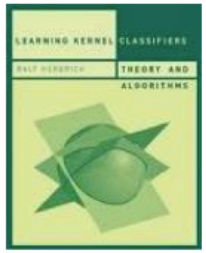

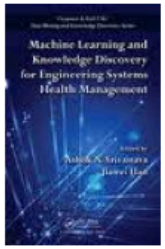

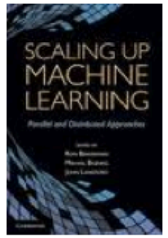

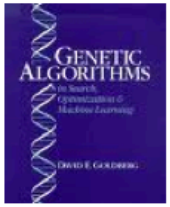

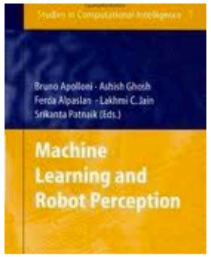

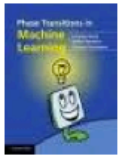

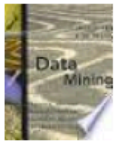

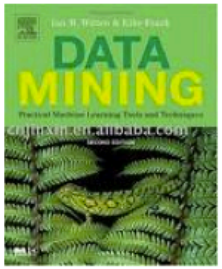

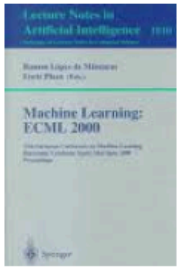

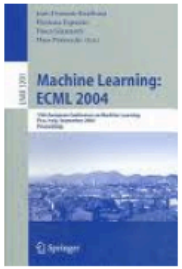

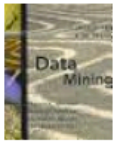

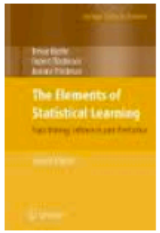

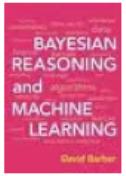

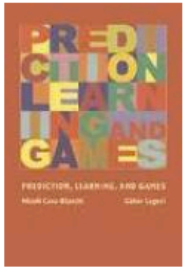

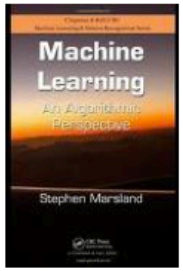

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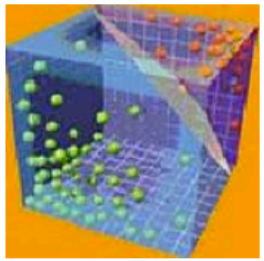

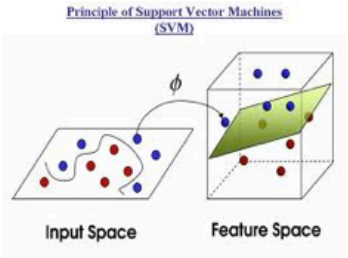

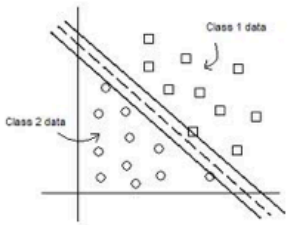

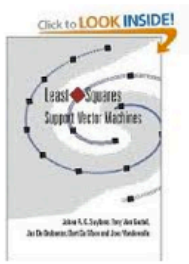

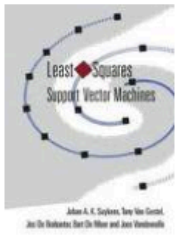

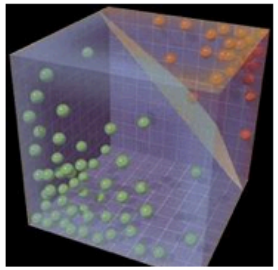

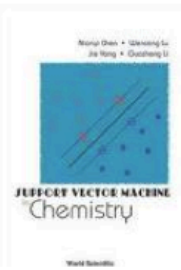

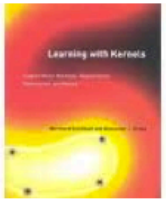

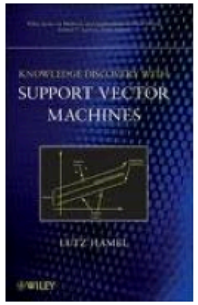

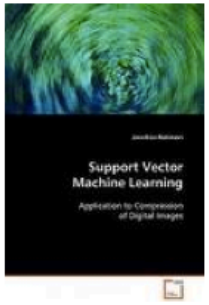

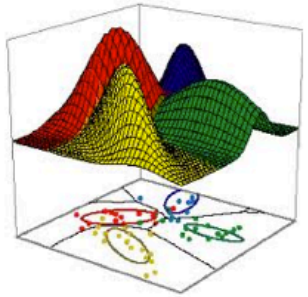

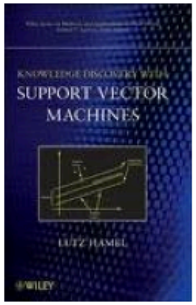

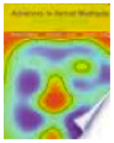

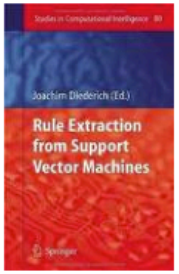

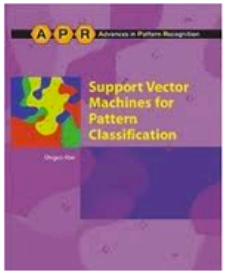

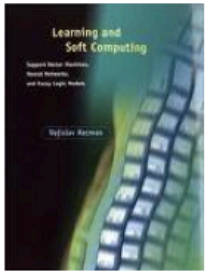

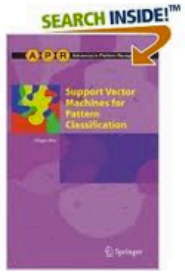

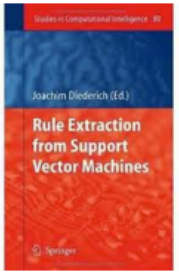

Page 2


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Support Vector Machines


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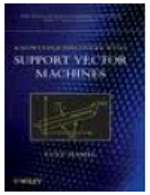
Pattern Recognition with Support Vector Machines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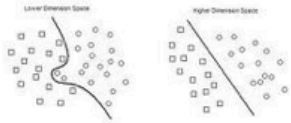
Support Vector Machines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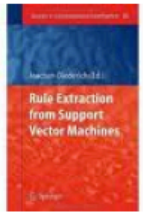
Automatic Text Categorization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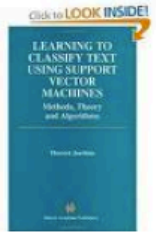
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LEARNING TO CLASSIFY TEXT USING SUPPORT VECTOR MACHINES
Methods, Theory and Algorithms


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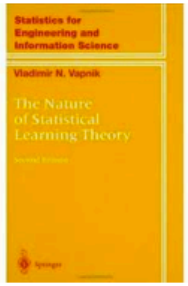
Support Vector Machines for Pattern Classification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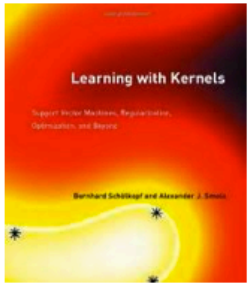
The Nature of Statistical Learning Theory


Pattern Recognition with Support Vector Machines


Support Vector Machines for Pattern Classification


Learning with Kernels


Support Vector Machines and Their Applications in
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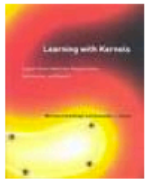
Bio-Inspired Credit Risk Analysis


Learning with Kernels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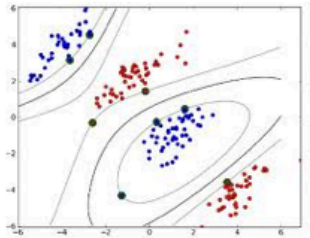

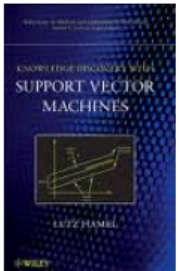
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