Misleading Sales in Salience Markets

Dongwoo Lee*

School of Economics & China Center for Behavioral Economics and Finance
Southwestern University of Finance and Economics, China

November 27, 2020

Abstract

This paper studies misleading sales by the low-quality firm who competes against the high-quality firm. Misleading sales intentionally make consumers (mis-)believe that the current price is a limited-time discount price. It generates a decoy for the low-quality product that inflates the reference price in consumers' minds. We show that the low-quality firm can benefit by offering aggressive discount rates when consumers are salient thinkers who place a higher weight on a standing-out attribute.

Keywords: Misleading Sales, Salience, Salient Attribute, Decoy Good

JEL Classification: L13, L15, D21

1 Introduction

Discount pricing strategy is one of the most common ways in which firms may consider increasing profit across all industries. The strategy is often associated with price discrimination to deal with different willingness-to-pay of consumers. Recently, Bordalo et al. (2013) provide the novel insight that some retailers may inflate regular prices by advertising misleading sales. The intuitive explanation is that retailers can attract consumers' attention by placing a decoy into the mind of the consumer, which makes misleading discount prices seem appealing.[1] Moreover, Bordalo et al. (2013) demonstrate that misleading sales are useful only for high-quality products under moderate discount rates.

However, we often observe price discounts for low-quality products in the real world. Marketing literature observe that firms with low-quality products frequently choose discount pricing as a marketing strategy (Raju et al., 1990; Koçaş and Bohlmann, 2008). In this paper, we show that aggressive misleading sales can be useful for low-quality products at the expense of demand for

*CONTACT: Dongwoo Lee Email: dwlee05@gmail.com Address: Southwestern University of Finance and Economics, Office 302, Hongyuan Building, Chengdu, Sichuan 611130, China

[1]We can also observe field evidence consistent with this insight. The survey from Consumers' Checkbook in 2018 reports that many retailers in the US advertised misleading sales more than 75 percent of the time during they visits over 2017 and 2018. See details at https://www.checkbook.org/washington-area/sale-fail/

high-quality products – even when actual selling prices of both products are fixed. In other words, the demand for low-quality products can be boosted if a firm offers higher discounts, which is consistent with the observation by Rajiv et al. (2002).

The consumers in our model perceive two attributes – quality and price – for every commodity in the markets. By following the seminal work of Bordalo et al. (2013), we further assume that consumers are salient thinkers who place a higher weight on an attribute standing out more compared to reference quality and price. We refer to a market with salient-thinking consumers as the salience market. In salience markets, we have a fringe firm producing a low-quality good competing against a brand manufacturer producing a high-quality good. In our model, the fringe firm does not have any pricing power, i.e., the firm cannot set the price of its own product. However, the fringe firm may advertise misleading discount rates without any costs. This is a common situation since firms start advertisement of their products once specific details of their products including qualities and prices are finalized.

To get a benefit from misleading sales, a fringe firm with a low-quality product may want consumers to turn their attention to prices not qualities. We show that consumers do not switch their attention to prices when difference in qualities between a high-quality product and a low-quality product is large enough. Then, consumers do not care how much discount rate on a low-quality product is. However, when difference in qualities between the products is not that large, we show that aggressive discount rates inflate the reference price to which consumers switch their attention for both high-quality and low-quality products. Given this situation, a fringe firm can boost the profit from misleading sales.

2 Model

We closely follow the model in Herweg et al. (2017, 2018). We study a market in which a fringe firm competes against a brand manufacturer. Both firms sell products represented by two attributes: quality q and price p. The fringe firm produces a good with low quality q_l at constant marginal cost c_l, and the brand manufacturer produces a good with high quality $q_h > q_l$ at constant marginal cost $c_h > c_l$. Finally, the fringe firm sells its own product only at a fixed price $p_l \in (c_l, p_h)$. We study the profit maximization for the fringe firm in the short-run, where both firms take qualities and prices of their products as a given in the specific period. In sum, qualities, prices and costs are exogenously determined in our model as $q_l < q_h$ and $c_l < p_l < p_h$.[2]

There is a mass of consumers normalized to one. Consumers are heterogeneous in the sense that each consumer has a different willingness-to-pay for quality $\theta \in [\underline{\theta}, \overline{\theta}]$ distributed according to the cumulative density function $F : [\underline{\theta}, \overline{\theta}] \to [0, 1]$. We assume that consumers are salient thinkers who focus more on some stand-out attribute $a \in \{q, p\}$ by the reference good. In specific, we denote the reference good in the market by $(\bar{q}, \bar{p})$, where $\bar{q}$ and $\bar{p}$ are the average quality and the average

[2]Admittedly, the setting with fixed qualities and prices could be very strong. However, firms start advertisement after finalizing the details of their products including qualities and prices. In our work, we show that firms may increase profits in the situation.

price in a given choice set. The salience of attribute $a \in \{q, p\}$ for good (q_k, p_k) is then determined by the salience function $\sigma(a_k, \bar{a})$ introduced by Bordalo et al. (2013, 2016).

Definition 1 *The continuous salience function $\sigma(\cdot, \cdot)$ satisfies the following properties:*

(1) (ordering) For any $x, x', y, y' \in \mathbb{R}_+$ with $[x, y] \subset [x', y']$, it is true that $\sigma(x, y) < \sigma(x', y')$.

(2) (homogeneity of degree zero) For any $x, y \in \mathbb{R}_+$ with $x \neq y$ and $t > 0$, it is true that $\sigma(tx, ty) = \sigma(x, y)$.

(3) (symmetry) For any $x, y, z \in \mathbb{R}_+$ such that $|x - y| = |z - y|$, it is true that $\sigma(x, y) = \sigma(z, y)$.

The property of ordering shows that an attribute becomes more salient if the attribute has a greater difference from an average level. Homogeneity of degree zero guarantees scale independence that scailing an attribute and its average level together does not change its salience. As mentioned by Bordalo et al. (2013, 2016), ordering and homogeneity of degree zero imply diminishing sensitivity, in the sense that salience decreases as an attribute uniformly increases across all goods in a choice set. Lastly, the property of symmetry ensures that salience is determined by proportional difference from the average value of the attribute, which allows us to compare salience between a higher attribute and a lower attribute compared to its average value.[3]

The salience attribute between quality q and price p is determined by which attribute is more distinct from the reference good. We say that q (p) is salient for good (q_k, p_k) if and only if

$$\sigma(q_k, \bar{q}) > (<)\ \sigma(p_k, \bar{p}),$$

where $\bar{q}$ and $\bar{p}$ are the average quality and price in a choice set. When we have $\sigma(q_k, \bar{q}) = \sigma(p_k, \bar{p})$, both attributes are equally salient for good (q_k, p_k).

Depending on the rank between quality and price salience, salient-thinking consumers perceive the utility weighted by salience distortion δ. From good (q_k, p_k) in a choice set $\mathcal{C}$, the salient thinker's payoff is

$$u(q_k, p_k \mid \theta, \mathcal{C}) = \begin{cases} \theta q_k - \delta p_k & \text{if } \sigma(q_k, \bar{q}) > \sigma(p_k, \bar{p}) \\ \delta\theta q_k - p_k & \text{if } \sigma(q_k, \bar{q}) < \sigma(p_k, \bar{p}) \\ \theta q_k - p_k & \text{if } \sigma(q_k, \bar{q}) = \sigma(p_k, \bar{p}) \end{cases},$$

where salience distortion $\delta \in [0, 1]$ measures the degree of salient thinking. As δ becomes smaller, the consumer puts more relative weight on the salient attribute. When $\delta = 0$, the consumer is an extreme salient thinker who only cares about a salient attribute. When $\delta = 1$, the consumer behaves like a rational consumer whose willingness-to-pay for quality price is not distorted.

We assume that consumers' willingness-to-pay for quality is sufficiently dispersed so that both the fringe firm and brand manufacturer have positive market shares.

[3]Symmetry property is assumed in Bordalo et al. (2016) only for the case of two goods only. However, we apply this property to compare salience between attributes for the case of three goods.

Assumption 1 *It holds that*

$$\underline{\theta} < \frac{1}{\delta}\frac{p_h - p_l}{q_h - q_l} < \overline{\theta}. \tag{1}$$

Under this assumption, the market structure (i.e., the fringe firm and the brand manufacturer) is maintained in equilibrium.

Lastly, we further assume throughout the paper that *walking away* option is not optimal for consumers for the sake of simplicity of analysis.

Assumption 2 *The consumers always purchase one of the two goods, that is,*

$$\min\{\underline{\theta}q_l - p_l, \underline{\theta}q_h - p_h\} > 0.$$

This assumption ensures the calculation for the demand of low-quality product comfortable, which is essential to compute the profit for the fringe firm.

3 Analysis

In the market, the fringe firm produces the low-quality good (q_l, p_l), whereas the brand manufacturer produces the high-quality good (q_h, p_h). Hence, consumers face a choice set $\mathcal{C} = \{(q_l, p_l), (q_h, p_h)\}$. It is shown by Proposition 1 in Bordalo et al. (2013) that the same attribute is salient for both goods when there are only two goods in a choice set.[4] Given this situation, Herweg et al. (2018) demonstrate that the brand manufacturer would induce a quality-salient environment if the salience distortion is weak, $\delta \geq p_l/q_l$, and a price-salient environment if the salience distortion is strong, $\delta < p_l/q_l$, in order to maximize the profit.[5]

In our analysis, we consider misleading sales as a marketing strategy for the fringe firm. It generates a decoy for the low-quality product with artificial regular price in consumers' minds, in the sense that consumers believe there is another good with the same quality as the low-quality product but which costs more. With this decoy, a new choice set contains three alternatives under which each product may have a different salient attribute. In other words, it is possible that the low-quality product is price salient while the high-quality product is quality salient.

3.1 Misleading Sales

Recall that the fringe firm cannot change the quality or price of its own product. Our model allows for the fringe firm to advertise the rate of misleading sales to maximize the profit by increasing the demand.

[4] Quality is salient for both goods if and only if $q_h/p_h > \bar{q}/\bar{p}$.

[5] For the analysis in Herweg et al. (2018), the brand manufacturer can set the price of its own product, the firm should take into account both the price and the demand for own product. For a detailed explanation, see Proposition 1 in Herweg et al. (2018).

Suppose the fringe firm advertises p_l as the discounted price under the discount rate $\alpha \in (0,1)$. This advertisement may give rise to a decoy for the fringe good with the artificial regular price:

$$decoy = (q_l, p_l/\alpha) \text{ with } \alpha \in (0,1).$$

Price p_l is regarded as the discounted price from $(p_l/\alpha) \times \alpha = p_l$. For instance, consider a retailer who advertises \$10 as a 50% discount from the regular price for a red wine. Consumers may (mis-)believe that the regular price would be \$20. In this way, the fringe firm frames an additional option $(q_l, p_l/\alpha)$ in a choice set for consumers. It is immediate that $p_l/\alpha > p_l$ for any $\alpha \in (0,1)$. Hence, misleading sales may inflate the reference price for salient-thinking consumers.

To study the impact of misleading sales, we assume that both low-quality and high-quality products are quality salient when consumers have a choice set $\{(q_l, p_l), (q_h, p_h)\}$ without misleading sales, which sets the upper bound for the proportional difference in price p_h/p_l. At the same time, we restrict that p_h/p_l is not too small.

Assumption 3 *It holds that*

$$\frac{\bar{q}_d}{q_l} < \frac{p_h}{p_l} < \frac{q_h}{q_l}, \tag{2}$$

where $\bar{q}_d = (q_h + 2q_l)/3$.

The first inequality in Assumption 3 restricts that p_h/p_l is high enough to switch consumers' attention on low-quality good to the price under aggressive misleading sales. Meanwhile, the second inequality in Assumption 3 ensures that salient-thinking consumers focus on quality when having only (q_l, p_l) and (q_h, p_h) in a choice set.

We introduce the profit maximization problem for the low-quality firm. We denote by $D^{(a_l,a_h)}(\mathcal{C})$ the demand for the low-quality product under salience pair $(a_l, a_h) \in \{q,p\} \times \{q,p\}$ in a choice set $\mathcal{C}$, where a_l and a_h indicate the salience attribute for the low-quality good and for the high-quality good, respectively. Then, the fringe firm's problem is:

$$\max_{\alpha \in (0,1)} (p_l - c_l) \times D^{(a_l,a_h)}(\mathcal{C}(\alpha)), \tag{3}$$

subject to:

$$\mathcal{C}(\alpha) = \{(q_l, p_l), (q_h, p_h), (q_l, p_l/\alpha)\}, \tag{CSC}$$

$$\max\{u(q_l, p_l \mid \theta, \mathcal{C}(\alpha)), u(q_h, p_h \mid \theta, \mathcal{C}(\alpha))\} \geq u(q_l, p_l/\alpha \mid \theta, \mathcal{C}(\alpha)),\ \forall \theta \in [\underline{\theta}, \overline{\theta}]. \tag{DC}$$

The objective function in (3) describes how the fringe firm's profit is determined. Since the selling price p_l and the production cost c_l are fixed, the only way to increase the profit is to control the salience pair (a_l, a_h) optimally by choosing the discount rate $\alpha \in (0,1)$. The choice-set constraint (CSC) describes which choice set is given to salient-thinking consumers. The choice set contains a decoy $(q_l, p_l/\alpha)$, which varies depending on $\alpha \in (0,1)$. Finally, the decision constraint (DC) guarantees that no consumers choose the decoy good.

We first assert that misleading sales cannot alter salient thinkers' attention regarding the high-quality product to price if the proportional difference in qualities is too large.

Proposition 1 *Price is never salient for the high-quality good if* $q_h/q_l \geq 4$*.*

All proofs are collected in the Appendix. Intuitively, consumers only care about quality if products exhibit enough difference in qualities.

To see how misleading sales affect the demand for the low-quality product, we proceed our analysis in a situation where the proportional difference in qualities is not too large, i.e., $q_h/q_l < 4$. In choice set $\mathcal{C}(\alpha) = \{(q_l, p_l), (q_h, p_h), (q_l, p_l/\alpha)\}$, the average quality and the average price are

$$\bar{q}_d = (2q_l + q_h)/3 \ \text{ and } \ \bar{p}_d(\alpha) = (p_l + p_h + p_l/\alpha)/3.$$

While the average quality is constant, the average price increases in $\alpha \in (0, 1)$. In other words, the average price in consumers' minds increases as the fringe firm advertises a higher discount rate (lower α).

We next explain how salience pair (a_l, a_h) for the low-quality and high-quality products changes as the discount rate increases. When the discount rate is very small, namely $\alpha \lesssim 1$, quality is salient for both the low-quality and the high-quality products. This is because the proportional difference in quality is greater than the proportional difference in price by Assumption 3. The low-quality product becomes price salient first, as α decreases. Then, the high-quality product also changes to be price salient at a lower level of α.

Proposition 2 *Suppose that the proportional difference in qualities is not that large, namely* $q_h/q_l < 4$*. From misleading sales, the most preferred salience pair for the fringe firm is* (p, p) *which can be attained by aggressive discount rate* $\alpha < \alpha^*$*, where*

$$\alpha^* = \frac{4q_l p_l - q_h p_l}{4q_h p_h + q_h p_l + 2q_l p_h - 4q_l p_l} \in (0, 1). \tag{4}$$

This result provides an intuitive explanation why the low-quality firm advertises its low price in the market. In the proof of Proposition 2 in Appendix, we show that the salience pair changes in the order of (q, q), (p, q), and (p, p) as α decreases. Moreover, the salience pair (p, p) maximizes the fringe firm's profit in (3) in the sense that $D^{(p,p)}(\mathcal{C}(\alpha)) > D^{(q,q)}(\mathcal{C}(\alpha)) > D^{(p,q)}(\mathcal{C}(\alpha))$. Hence, the fringe firm advertises higher discount rates to further draw consumers' attention to price, which boosts the demand for the low-quality product.[6]

The final task is to constrain consumers not to buy a decoy. This requires that each consumer prefers either the low-quality good or the high-quality good to $(q_l, p_l/\alpha)$. Interestingly, the decision constraint (DC) is trivially satisfied under aggressive discount rates $\alpha < \alpha^*$ that make the decoy

[6] Bordalo et al. (2013) assume the discount rate of misleading sales in $[p_l/p_h, 1]$, which results in a different prediction from our model.

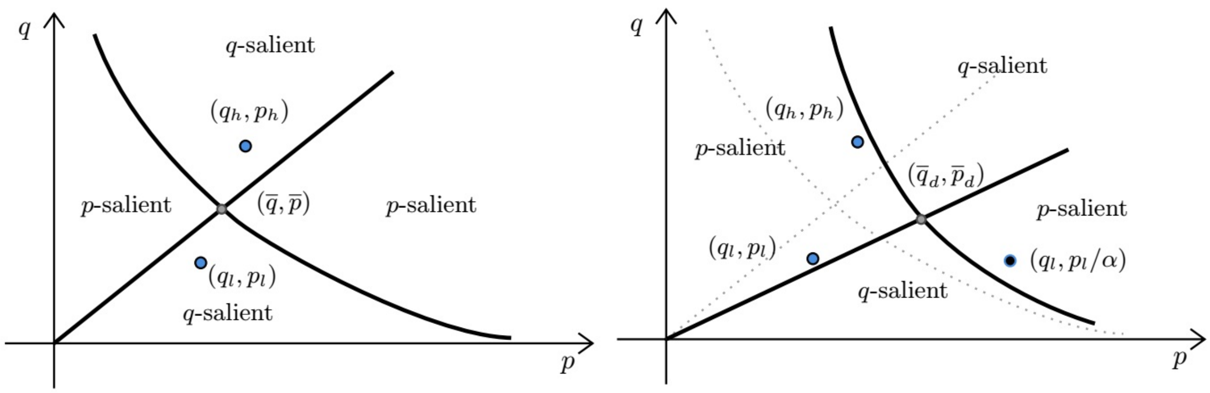

Figure 1: Misleading Sales

price salient.[7] For every θ, it holds

$$u(q_l, p_l|\theta, \mathcal{C}(\alpha)) = \delta\theta q_l - p_l > \delta\theta q_l - p_l/\alpha = u(q_l, p_l/\alpha|\theta, \mathcal{C}(\alpha)). \tag{5}$$

Hence, misleading sales always makes the decoy good dominated by the low-quality good.

Finally, we summarize the solution for the fringe firm's profit maximization problem as follows.[8]

Remark 1 *The low-quality firm can benefit from misleading sales. The discount rate should be huge enough to switch consumers attention on the high-quality good to price. This aggressive discount inflates the reference price in the market, which makes the low-quality good more attractive to salient thinkers.*

To illustrate the impact of misleading sales in Proposition 2, consider the salience function proposed by Bordalo et al. (2013):

$$\sigma(a_k, \bar{a}) = \frac{|a_k - \bar{a}|}{\bar{a}}. \tag{6}$$

The entire product space $(q, p) \in \mathbb{R}^2_+$ is divided into four areas in Figure 1. Based on the average quality and price, the north and south areas are where products are quality salient, while the west and east areas are where products are price salient. The left panel describes the two products (q_h, p_h) and (q_l, p_l) are quality salient as Assumption 3 suggests. Now, when a decoy $(q_l, p_l/\alpha)$ is introduced in a choice set, the reference point is changed to $(\bar{q}_d, \bar{p}_d)$. Accordingly, the four distinct areas of the product space are adjusted. The right panel describes the existing two products (q_h, p_h) and (q_l, p_l) become price salient under the new reference point $(\bar{q}_d, \bar{p}_d)$. Furthermore, the decoy $(q_l, p_l/\alpha)$ is also price salient, hence, it is dominated by the fringe good (q_l, p_l).

[7] This comes from immediate calculation.

[8] One may wonder what happens if the brand manufacturer also uses misleading sales. The brand manufacturer would like to induce quality salient for its own product. However, as long as their selling prices are fixed, the fringe firm can induce price salient for both products. This is because quality variations are fixed while (misleading) price variations can be increased across products in the choice set.

4 Discussion

We have shown that – under the assumption that advertising misleading sales is the only decision – a fringe firm can inflate the reference price to salience-thinking consumers via aggressive discount rates. As a consequence, this switches consumers' attention to prices for both fringe and brand goods, which benefits the fringe firm in salience markets. However, it is of great interest to consider a situation where the low-quality firm can determine the product price, as well as misleading sales, which affects the reference price to salience-thinking consumers.[9] The salience pair (p, p) is maximizing the profit of the low-quality firm for any p_l such that $\frac{q_l p_h}{q_h} = \underline{p_l} < p_l < \overline{p_l} = \frac{q_l p_h}{\bar{q}_d}$ by Assumption 3 and Proposition 2. Thus, endogenizing the choice of the product price and the discount rate, (p_l, α), in our model is to find the optimal p_l under the salience pair (p, p), and then to choose α accordingly so as to keep the reference price high enough. Notice that $\alpha^*(p_l)$ is a function of p_l, and $D^{(a_l,a_h)}(p_l, \mathcal{C}(\alpha))$ is a function (p_l, α) now.

The optimal price p_l^* can be obtained from the firm's maximization problem:

$$\max_{\underline{p_l}<p_l<\overline{p_l}} (p_l - c_l) \times D^{(p,p)}(p_l, \mathcal{C}(\alpha)). \tag{7}$$

Once the optimal p_l^* is determined, the low-quality firm can then choose any discount rate $\alpha < \alpha^*(p_l^*)$ for misleading sales to maintain the salience pair (p, p). Moreover, we can see how the price changes the threshold of the discount rate $\alpha^*(p_l)$, that is,

$$\frac{\partial \alpha^*(p_l)}{\partial p_l} > 0. \tag{8}$$

Hence, the increase in the price permits the less aggressive discount rate to maintain the salience pair (p, p). This comes from the fact that the increase in p_l inflates the reference price, $\bar{p}_d(p_l, \alpha) = (p_l + p_l/\alpha + p_h)/3$, high enough to keep consumers' attention on prices at less aggressive discount rate. In this way, we can find the low-quality firm's optimal choice of (p_l, α).

Acknowledgment

I would like to thank Hans Haller for his valuable comments and fruitful discussions.

References

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer (2013), "Salience and consumer choice." *Journal of Political Economy*, 121, 803–843.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer (2016), "Competition for attention." *Review of Economic Studies*, 83, 481–513.

[9] The author thanks the referee for pointing out this issue.

Herweg, Fabian, Daniel Müller, and Philipp Weinschenk (2017), "Salience, competition, and decoy goods." *Economics Letters*, 153, 28–31.

Herweg, Fabian, Daniel Müller, and Philipp Weinschenk (2018), "Salience in markets." In *Handbook of Behavioral Industrial Organization*, Edward Elgar Publishing, Massachusetts, USA.

Koçaş, Cenk and Jonathan D Bohlmann (2008), "Segmented switchers and retailer pricing strategies." *Journal of Marketing*, 72, 124–142.

Rajiv, Surendra, Shantanu Dutta, and Sanjay K Dhar (2002), "Asymmetric store positioning and promotional advertising strategies: Theory and evidence." *Marketing Science*, 21, 74–96.

Raju, Jagmohan S, Venkatesh Srinivasan, and Rajiv Lal (1990), "The effects of brand loyalty on competitive price promotional strategies." *Management Science*, 36, 276–304.

Appendix - Proofs

Proof of Proposition 1: Suppose the high-quality good is price salient in $\mathcal{C}(\alpha) = \{(q_l, p_l), (q_h, p_h), (q_l, p_l/\alpha)\}$. By symmetry of σ, $\sigma(q_h, \bar{q}_d) < \sigma(p_h, \bar{p}_d(\alpha))$ implies $(q_h - \bar{q}_d)/\bar{q}_d < (\bar{p}_d(\alpha) - p_h)/\bar{p}_d(\alpha)$, where $q_h > \bar{q}_d = (2q_l + q_h)/3$ and $p_h < \bar{p}_d(\alpha) = (p_l + p_h + p_l/\alpha)/3$.[10] Then, it must be true that

$$\bar{q}_d p_h < (2\bar{q}_d - q_h)\bar{p}_d(\alpha). \tag{9}$$

However, $q_h/q_l \geq 4$ implies that $2\bar{q}_d \leq q_h$. Contradiction. $\square$

Proof of Proposition 2: The proof is completed throughout *Claim 1* to *Claim 3* below.

Claim 1: Salience pair $(a_l, a_h) = (q, p)$ cannot be obtained by adding a decoy $(q_l, p_l/\alpha)$ for any $\alpha \in (0, 1)$.

Proof: (i) Suppose that $\bar{p}_d(\alpha) = (p_l + p_h + p_l/\alpha)/3 \leq p_h$, which is equivalent to $\alpha \geq p_l/(2p_h - p_l)$. We want to show that the high-quality product cannot be price salient.

Assume that (q_h, p_h) is price salient, $\sigma(q_h, \bar{q}_d) < \sigma(p_h, \bar{p}_d(\alpha))$, where $\bar{q}_d = (2q_l + q_h)/3$. By homogeneity of degree zero of σ, we have $\sigma(q_h/\bar{q}_d, 1) < \sigma(p_h/\bar{p}_d(\alpha), 1)$, which implies $q_h/\bar{q}_d < p_h/\bar{p}_d(\alpha)$. Then, it follows that $2q_l p_h > \left(1 + \frac{1}{\alpha}\right) q_h p_l$ for some $\alpha \in \left(\frac{p_l}{2p_h - p_l}, 1\right)$. However, since $q_l p_h < q_h p_l$ by Assumption 3 and $1 + \frac{1}{\alpha} > 2$ for any $\alpha < 1$, the inequality cannot be satisfied. Therefore, (q, p) cannot be achieved for $\bar{p}_d(\alpha) \leq p_h$.

(ii) Suppose that $\bar{p}_d(\alpha) > p_h$, equivalently, $\alpha < p_l/(2p_h - p_l)$. We want to show that the low-quality product cannot be quality salient.

Assume that (q_l, p_l) is quality salient, $\sigma(q_l, \bar{p}_d) > \sigma(p_l, \bar{p}_d(\alpha))$, which is equivalent to $\sigma(1, \bar{q}_d/q_l) > \sigma(1, \bar{p}_d(\alpha)/p_l)$. Then, it follows that $\bar{q}_d/q_l > \bar{p}_d(\alpha)/p_l$ for some $\alpha < \frac{p_l}{2p_h - p_l}$, which requires the range of α as $\frac{q_l p_l}{q_l p_l + q_h p_l - q_l p_h} < \alpha < \frac{p_l}{2p_h - p_l}$. However, Assumption 3 implies $\frac{q_l p_l}{q_l p_l + q_h p_l - q_l p_h} > \frac{p_l}{2p_h - p_l}$. Therefore, (q, p) cannot be achieved for $\bar{p}_d(\alpha) > p_h$. $\square$

Claim 2: Salience pair $(a_l, a_h) \in \{(q, q), (p, q), (p, p)\}$ is achieved by adding a decoy $(q_l, p_l/\alpha)$ for any $\alpha \in (0, 1)$. Specifically,

$$(a_l, a_h) = \begin{cases} (q, q) & \text{if } \alpha \in \left(\frac{q_l p_l}{q_l p_l + q_h p_l - q_l p_h}, 1\right), \\ (p, q) & \text{if } \alpha \in \left(\frac{4q_l p_l - q_h p_l}{4q_h p_h + q_h p_l + 2q_l p_h - 4q_l p_l}, \frac{q_l p_l}{q_l p_l + q_h p_l - q_l p_h}\right), \\ (p, p) & \text{if } \alpha \in \left(0, \frac{4q_l p_l - q_h p_l}{4q_h p_h + q_h p_l + 2q_l p_h - 4q_l p_l}\right). \end{cases}$$

Proof: (i) Suppose $\alpha \in \left(\frac{q_l p_l}{q_l p_l + q_h p_l - q_l p_h}, 1\right)$, that is, $\bar{q}_d/q_l > \bar{p}_d(\alpha)/p_l$ under which $p_h > \bar{p}_d(\alpha)$.

Consider $\sigma(q_l, \bar{q}_d)$ and $\sigma(p_l, \bar{p}_d(\alpha))$, which are equivalent to $\sigma(1, \bar{q}_d/q_l)$ and $\sigma(1, \bar{p}_d(\alpha)/p_l)$, respectively, by homogeneity of degree zero of σ. From $\bar{q}_d/q_l > \bar{p}_d(\alpha)/p_l$, it is immediate that

[10] It is shown that the high-quality is not price salient when $p_h > \bar{p}_d(\alpha)$ in the proof of Proposition 2.

$\sigma(q_l, \bar{q}_d) > \sigma(p_l, \bar{p}_d(\alpha))$. Hence, the low-quality product is quality salient.

Consider now $\sigma(q_h, \bar{q}_d)$ and $\sigma(p_h, \bar{p}_d(\alpha))$, which are equivalent to $\sigma(q_h/\bar{q}_d, 1)$ and $\sigma(p_h/\bar{p}_d(\alpha), 1)$, respectively. Assumption 3 implies $2q_lp_h < \left(1 + \frac{1}{\alpha}\right) q_hp_l$, which results in $q_h/\bar{q}_d > p_h/\bar{p}_d(\alpha)$. Hence, the high-quality product is quality salient.

(ii) Suppose $\alpha \in \left(\frac{4q_lp_l - q_hp_l}{4q_hp_h + q_hp_l + 2q_lp_h - 4q_lp_l}, \frac{q_lp_l}{q_lp_l + q_hp_l - q_lp_h}\right)$. Notice that

$$\frac{p_l}{2p_h - p_l} \in \left(\frac{4q_lp_l - q_hp_l}{4q_hp_h + q_hp_l + 2q_lp_h - 4q_lp_l}, \frac{q_lp_l}{q_lp_l + q_hp_l - q_lp_h}\right),$$

and $\bar{p}_d(\alpha) = p_h$ when $\alpha = \frac{p_l}{2p_h - p_l}$. We then have $p_h > \bar{p}_d(\alpha)$ when $\alpha \in \left(\frac{p_l}{2p_h - p_l}, \frac{q_lp_l}{q_lp_l + q_hp_l - q_lp_h}\right)$, and $p_h \leq \bar{p}_d(\alpha)$ when $\alpha \in \left(\frac{4q_lp_l - q_hp_l}{4q_hp_h + q_hp_l + 2q_lp_h - 4q_lp_l}, \frac{p_l}{2p_h - p_l}\right]$.

First, assume that $\alpha \in \left(\frac{p_l}{2p_h - p_l}, \frac{q_lp_l}{q_lp_l + q_hp_l - q_lp_h}\right)$ under which $\bar{q}_d/q_l < \bar{p}_d(\alpha)/p_l$ and $p_h > \bar{p}_d(\alpha)$. Consider $\sigma(q_l, \bar{q}_d)$ and $\sigma(p_l, \bar{p}_d(\alpha))$, which are equivalent to $\sigma(1, \bar{q}_d/q_l)$ and $\sigma(1, \bar{p}_d(\alpha)/p_l)$, respectively. From $\bar{q}_d/q_l < \bar{p}_d(\alpha)/p_l$, it holds $\sigma(q_l, \bar{q}_d) < \sigma(p_l, \bar{p}_d(\alpha))$. Hence, the low-quality product is price salient. Consider now $\sigma(q_h, \bar{q}_d)$ and $\sigma(p_h, \bar{p}_d(\alpha))$, which are equivalent to $\sigma(q_h/\bar{q}_d, 1)$ and $\sigma(p_h/\bar{p}_d(\alpha), 1)$. Assumption 3 implies $2q_lp_h < \left(1 + \frac{1}{\alpha}\right) q_hp_l$, which results in $q_h/\bar{q}_d > p_h/\bar{p}_d(\alpha)$. Hence, the high-quality product is quality salient.

Next, assume that $\alpha \in \left(\frac{4q_lp_l - q_hp_l}{4q_hp_h + q_hp_l + 2q_lp_h - 4q_lp_l}, \frac{p_l}{2p_h - p_l}\right]$ under which $p_h \leq \bar{p}_d(\alpha)$. Consider $\sigma(q_l, \bar{q}_d)$ and $\sigma(p_l, \bar{p}_d(\alpha))$, which are equivalent to $\sigma(1, \bar{q}_d/q_l)$ and $\sigma(1, \bar{p}_d(\alpha)/p_l)$, respectively. From Assumption 3 and $p_h \leq \bar{p}_d(\alpha)$, we have $\frac{\bar{q}_d}{q_l} < \frac{p_h}{p_l} \leq \frac{\bar{p}_d(\alpha)}{p_l}$. Hence, the low-quality product is price-salient. Consider now $\sigma(q_h, \bar{q}_d)$ and $\sigma(p_h, \bar{p}_d(\alpha))$. As $q_h > \bar{q}_d$ and $p_h \leq \bar{p}_d(\alpha)$, we compare $\frac{q_h - \bar{q}_d}{\bar{q}_d}$ and $\frac{\bar{p}_d(\alpha) - p_h}{\bar{p}_d(\alpha)}$ by symmetry of σ. Notice $\frac{q_h - \bar{q}_d}{\bar{q}_d}$ and $\frac{\bar{p}_d(\alpha) - p_h}{\bar{p}_d(\alpha)}$ are identical when $\alpha = \frac{4q_lp_l - q_hp_l}{4q_hp_h + q_hp_l + 2q_lp_h - 4q_lp_l}$. Since $\frac{\bar{p}_d(\alpha) - p_h}{\bar{p}_d(\alpha)}$ strictly decreases as α increases, it holds $\sigma(q_h, \bar{q}_d) > \sigma(p_h, \bar{p}_d(\alpha))$ for $\alpha \in \left(\frac{4q_lp_l - q_hp_l}{4q_hp_h + q_hp_l + 2q_lp_h - 4q_lp_l}, \frac{p_l}{2p_h - p_l}\right]$. Hence, the high-quality product is quality salient.

(iii) Suppose $\alpha \in \left(0, \frac{4q_lp_l - q_hp_l}{4q_hp_h + q_hp_l + 2q_lp_h - 4q_lp_l}\right)$ under which $p_h < \bar{p}_d(\alpha)$. Consider $\sigma(q_l, \bar{q}_d)$ and $\sigma(p_l, \bar{p}_d(\alpha))$, which are equivalent to $\sigma(1, \bar{q}_d/q_l)$ and $\sigma(1, \bar{p}_d(\alpha)/p_l)$, respectively. From Assumption 3 and $p_h < \bar{p}_d(\alpha)$, it follows $\frac{\bar{q}_d}{q_l} < \frac{p_h}{p_l} < \frac{\bar{p}_d(\alpha)}{p_l}$. Hence, the low-quality product is price salient. Consider now $\sigma(q_h, \bar{q}_d)$ and $\sigma(p_h, \bar{p}_d(\alpha))$. We have shown above that $\sigma(q_h, \bar{q}_d) = \sigma(p_h, \bar{p}_d(\alpha))$ when $\alpha = \frac{4q_lp_l - q_hp_l}{4q_hp_h + q_hp_l + 2q_lp_h - 4q_lp_l}$. Since $\bar{p}(\alpha)$ strictly increases as α decreases, it should be true that $\sigma(q_h, \bar{q}_d) < \sigma(p_h, \bar{p}_d(\alpha))$ for $\alpha \in \left(0, \frac{2q_lp_l + q_hp_l}{8q_hp_h - q_hp_l - 2q_lp_h - 2q_lp_l}\right)$ by ordering of σ. Hence, the high-quality product is price salient. $\square$

Claim 3: Among salience pair $(a_l, a_h) \in \{(q,q),(p,q),(p,p)\}$, the fringe prefers (p,p) the most.

Proof: We list the following cases to compare the demand for the fringe good.

(i) Under (q,q), a consumer purchases the low-quality product if

$$\theta q_h - \delta p_h < \theta q_l - \delta p_l, \tag{10}$$

which is equivalent to $\theta < \delta \frac{p_h - p_l}{q_h - q_l} \equiv \hat{\theta}^{(q,q)}$. This yields the demand for the low-quality product $D^{(q,q)} = F(\hat{\theta}^{(q,q)})$ by Assumption 2.

(ii) Under (p,q), a consumer purchases the low-quality product if

$$\theta q_h - \delta p_h < \delta \theta q_l - p_l, \tag{11}$$

which is equivalent to $\theta < \frac{\delta p_h - p_l}{q_h - \delta q_l} \equiv \hat{\theta}^{(q,p)}$. This yields $D^{(q,p)} = F(\hat{\theta}^{(q,p)})$ by Assumption 2.

(iii) Under (p,p), a consumer purchases the low-quality product if

$$\delta \theta q_h - p_h < \delta \theta q_l - p_l, \tag{12}$$

which is equivalent to $\theta < \frac{1}{\delta} \frac{p_h - p_l}{q_h - q_l} \equiv \hat{\theta}^{(p,p)}$. This yields $D^{(p,p)} = F(\hat{\theta}^{(p,p)})$ by Assumption 2.

Since $\hat{\theta}^{(p,p)} > \hat{\theta}^{(q,q)} > \hat{\theta}^{(p,q)}$, it is immediate that $D^{(p,p)} > D^{(q,q)} > D^{(p,q)}$. Therefore, the fringe prefers (p,p) the most. $\square$