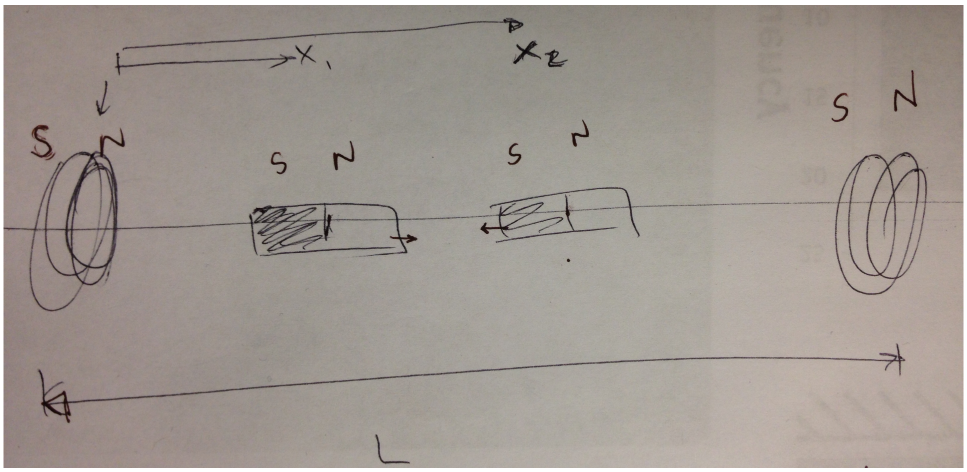
1D electromagnets for control of 2 permanent magnetics analysis

Denise Wong

October 13, 2014



μ_0 : permeability constant $4\pi \times 10^{-7} Tm/A$
$\vec{\mu_j}$: magnetic dipole moment for magnet j, Am^2
I_i : current through coil i, A
R_i : radius of coil i, m
$C_{perm} = \frac{\mu_0 I_p R_p^2}{2}$: constant representing permanent magnet strength

Electromagnet

The magnetic field on-axis of the coil is given by:

$$B_{loop} = \frac{\mu_0 I R^2}{2\left(z^2 + R^2\right)^{3/2}}$$

z : distance from the center of the fixed electromagnet to the center of the free permanent magnet.

Permanent Magnet

Permanent Magnet modeled using classical mechanics

Let force between 2 magnets be expressed as:

$$\vec{F} = \frac{\mu_0 m_1 m_2}{4\pi r^2}\hat{r}$$

where m_1, m_2 are the magnitude of the magnetic poles of permanent magnet 1 and 2 measured in Ampere-meters (Am). Written in the above coordinate frame for the 1D problem, the force in the x-direction acting on magnet 1:

$$F_{x,1} = \frac{\mu_0 m_1 m_2}{4\pi\left(x_2 - x_1\right)^2}$$

Force exerted on each magnet

Relating the magnetic field, $\vec{B}$ to the force, F:

$$F = \vec{\mu} \cdot \nabla \vec{B}$$

For a 1D system as shown above, this simplifies to:

$$F_x = \mu \frac{d\vec{B}}{dx}$$

The force exerted on magnet 1 is given by:

$$F_{x,1} = \mu_1 \left(-\frac{3x_1 \mu_0 I_1 R_1^2}{\left(x_1^2 + R_1^2\right)^{\frac{5}{2}}} - \frac{3\left(x_1 - L\right)\mu_0 I_2 R_2^2}{\left(\left(x_1 - L\right)^2 + R_2^2\right)^{\frac{5}{2}}} \right) + \frac{\mu_0 m_1 m_2}{4\pi \left(x_2 - x_1\right)^2}$$

The force exerted on magnet 2 is given by:

$$F_{x,2} = \mu_2 \left(-\frac{3x_2 \mu_0 I_1 R_1^2}{\left(x_2^2 + R_1^2\right)^{\frac{5}{2}}} - \frac{3\left(x_2 - L\right)\mu_0 I_2 R_2^2}{\left(\left(x_2 - L\right)^2 + R_2^2\right)^{\frac{5}{2}}} \right) - \frac{\mu_0 m_1 m_2}{4\pi \left(x_1 - x_2\right)^2}$$

Setting the above equations so that force is 0 to find the equilibrium point and writing these 2 equations in matrix form to solve for the input current:

$$\begin{bmatrix} -\frac{\mu_0 m_1 m_2}{4\pi(x_2-x_1)^2} \\ \frac{\mu_0 m_1 m_2}{4\pi(x_2-x_1)^2} \end{bmatrix} = \begin{bmatrix} -\mu_1 \frac{3x_1\mu_0 R_1^2}{\left(x_1^2+R_1^2\right)^{\frac{5}{2}}} & -\mu_1 \frac{3(x_1-L)\mu_0 R_2^2}{\left((x_1-L)^2+R_2^2\right)^{\frac{5}{2}}} \\ -\mu_2 \frac{3x_2\mu_0 R_1^2}{\left(x_2^2+R_1^2\right)^{\frac{5}{2}}} & -\mu_2 \frac{3(x_2-L)\mu_0 R_2^2}{\left((x_2-L)^2+R_2^2\right)^{\frac{5}{2}}} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

By inspecting the determinant of the 2x2 matrix, it can be shown that a solution for the 2 currents through the coil exists as long as $x_1 \neq x_2$, a configuration that is not physically possible.

Potential Function

The potential function can be used to explore the stability of a system. It is related to the force on a system by:

$$F = -\frac{dU}{dx}$$

Given that for magnets, $\vec{F_j} = \vec{\mu_j} \bullet \frac{d\vec{B}}{dx}$, the potential function, U, is a function of the magnetic field, B. Assuming that $\vec{\mu}_j$ is a constant, the potential function in the x direction for the j-th permanent magnet:

$$U_{x,j} = -\int_{x_i}^{x} F_x dx + U(x_i) = -\mu_{x,j} B_x$$

Permanent magnet modeled using classical mechanics

To compute the potential function component from this interaction:

$$U_{perm} = -\int_{x_i}^{x} F dx + U\left(x_i\right) = -\int_{x_i}^{x_1} \frac{\mu_0 m_1 m_2}{4\pi \left(x_2 - x_1\right)^2} dx_1 + U\left(x_i\right) = -\left[-\frac{\mu_0 m_1 m_2}{4\pi \left(x_1 - x_2\right)} \right]_{x_i}^{x_1} + U\left(x_i\right)$$

Letting $-\frac{\mu_0 m_1 m_2}{4\pi(x_i - x_2)} + U\left(x_i\right) = 0$.

The expression for the potential function for magnet 1 in the above system is therefore:

$$U_1 = \mu_1 \left(\frac{\mu_0 I_1 R_1^2}{2\left(x_1^2 + R_1^2\right)^{\frac{3}{2}}} + \frac{\mu_0 I_2 R_2^2}{2\left(\left(x_1 - L\right)^2 + R_2^2\right)^{\frac{3}{2}}} \right) + \frac{\mu_0 m_1 m_2}{4\pi\left(x_2 - x_1\right)}$$

The expression for the potential function for magnet 2 in the above system is therefore:

$$U_2 = \mu_1 \left(\frac{\mu_0 I_1 R_1^2}{2\left(x_2^2 + R_1^2\right)^{\frac{3}{2}}} + \frac{\mu_0 I_2 R_2^2}{2\left(\left(x_2 - L\right)^2 + R_2^2\right)^{\frac{3}{2}}} \right) + \frac{\mu_0 m_1 m_2}{4\pi\left(x_1 - x_2\right)}$$

Stability analysis

The Hessian matrix is computed to determine the stability of the equilibrium point, where $\frac{dU}{dx_1} = 0$:

$$H = \begin{bmatrix} \frac{\partial^2 U}{\partial x_1^2} & \frac{\partial^2 U}{\partial x_1 \partial x_2} \\ \frac{\partial^2 U}{\partial x_2 \partial x_1} & \frac{\partial^2 U}{\partial x_2^2} \end{bmatrix}$$

For magnet 1:

$$H_1 = \begin{bmatrix} \frac{\partial^2 U_1}{\partial x_1^2} & \frac{\partial^2 U_1}{\partial x_1 \partial x_2} \\ \frac{\partial^2 U_1}{\partial x_2 \partial x_1} & \frac{\partial^2 U_1}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} \left(\frac{-3}{\left((x_1-L)^2+R_2^2\right)^{\frac{5}{2}}} + \frac{15(x_1-L)^2}{\left((x_1-L)^2+R_2^2\right)^{\frac{7}{2}}} + \frac{15x_1^2}{\left(R_1^2+x_1^2\right)^{\frac{7}{2}}} - \frac{3}{\left(R_1^2+x_1^2\right)^{\frac{5}{2}}} + \frac{2}{(x_2-x_1)^3} \right) & -\frac{2}{(x_2-x_1)^3} \\ -\frac{2}{(x_2-x_1)^3} & \frac{2}{(x_2-x_1)^3} \end{bmatrix}$$

For magnet 2:

$$H_2 = \begin{bmatrix} \frac{\partial^2 U_2}{\partial x_1^2} & \frac{\partial^2 U_2}{\partial x_1 \partial x_2} \\ \frac{\partial^2 U_2}{\partial x_2 \partial x_1} & \frac{\partial^2 U_2}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} \frac{2}{(x_1-x_2)^3} & -\frac{2}{(x_1-x_2)^3} \\ -\frac{2}{(x_1-x_2)^3} & \left(\frac{-3}{\left((x_2-L)^2+R_2^2\right)^{\frac{5}{2}}} + \frac{15(x_2-L)^2}{\left((x_2-L)^2+R_2^2\right)^{\frac{7}{2}}} + \frac{15x_2^2}{\left(R_1^2+x_2^2\right)^{\frac{7}{2}}} - \frac{3}{\left(R_1^2+x_2^2\right)^{\frac{5}{2}}} + \frac{2}{(x_1-x_2)^3} \right) \end{bmatrix}$$

Solving for the eigenvalues: