Adv Controls HW 2

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1 Problem 1

a .)

$$x = \begin{bmatrix} \alpha \\ \dot{\alpha} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

$$\dot{x} = f(x) + g(x)\tau$$

where,

$$f(x) = \begin{bmatrix} \dot{\alpha} \\ \frac{B_m\dot{\theta}rcos(\alpha)+J_b\dot{\alpha}^2l_psin(\alpha)cos(\alpha)+J_bgsin(\alpha)+\dot{\alpha}^2l_p^3m_psin^2(\alpha)cos(\alpha)+\dot{\alpha}\dot{\theta}l_p^2m_prcos^2(\alpha)+gl_p^2m_psin^2(\alpha)+gm_pr^2sin(\alpha)}{J_bl_p+l_p^3m_psin(\alpha)+l_pm_pr^2sin^2(\alpha)} \\ \dot{\theta} \\ \frac{-B_m\dot{\theta}+\dot{\alpha}^2l_pm_prsin^3(\alpha)-\dot{\alpha}\dot{\theta}l_p^2m_pcos(\alpha)-gm_prsin(\alpha)cos(\alpha)}{J_b+l_p^2m_psin(\alpha)+m_pr^2sin^2(\alpha)} \end{bmatrix}$$

$$g(x,\tau) = \begin{bmatrix} 0 \\ -\frac{rcos(\alpha)}{J_bl_p+l_p^3m_psin(\alpha)+l_pm_pr^2sin^2(\alpha)} \\ 0 \\ \frac{1}{J_b+l_p^2m_psin(\alpha)+m_pr^2sin^2(\alpha)} \end{bmatrix}\tau$$

b .)

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \dot{\alpha} \\ \theta \\ \dot{\theta} \end{bmatrix} = \alpha$$

$$\frac{dy}{dt} = \dot{\alpha}$$

$$\frac{d^2y}{dt^2} = \ddot{\alpha} = f(\tau)$$

Thus relative degree of 2.

c .)

Because of relative degree of 2, a transformation can be defined as:

$$T = \begin{bmatrix} \phi(x) \\ L_f^0h(x) \\ L_f^1h(x) \end{bmatrix} = \begin{bmatrix} \eta \\ \xi_1 \\ \xi_2 \end{bmatrix}$$

$$\xi_1 = L_f^0h(x) = \alpha$$

$$\xi_2 = L_f^1h(x) = \dot{\alpha}$$

$$\frac{\partial \phi(x)}{\partial x} G(x,\tau) = 0$$

substituting $G(x,\tau)$:

$$\frac{\partial \phi(x)}{\partial \alpha}[-\frac{\tau r cos(\alpha)}{J_b l_p + l_p^3 m_p sin(\alpha) + l_p m_p r^2 sin^2(\alpha)}] + \frac{\partial \phi(x)}{\partial \theta}[\frac{\tau}{J_b + l_p^2 m_p sin(\alpha) + m_p r^2 sin^2(\alpha)}] = 0$$

$$\frac{\partial \phi(x)}{\partial \alpha} + \frac{-\tau(J_b l_p + l_p^3 m_p sin(\alpha) + l_p m_p r^2 sin^2(\alpha))}{\tau r cos(\alpha)(J_b + l_p^2 m_p sin(\alpha) + m_p r^2 sin^2(\alpha))}\frac{\partial \phi(x)}{\partial \theta} = 0$$

$$\frac{\partial \phi(x)}{\partial \alpha} + \frac{-l_p}{r cos(\alpha)}\frac{\partial \phi(x)}{\partial \theta} = 0$$

separation of variables and solving yields:

$$\phi(x) = -\theta + \frac{r}{l_p} sin(\alpha) = \eta$$

taking the derivative:

$$\dot{\eta} = -\dot{\theta} + \frac{r}{l_p} cos(\alpha)\dot{\alpha}$$

substituting for $\dot{\alpha}$, $\dot{\theta}$

$$\dot{\eta} = -\dot{\theta} + \frac{r}{l_p} cos(\xi_1)\xi_2$$

then,

$$\dot{\xi_1} = \xi_2$$

$$\dot{\xi_2} =$$

$$y = \xi_1$$

2 Problem 2

a .)

b .)

$$f(x) = \begin{bmatrix} x_1 x_2 - 4x_1 \\ -x_1 x_2 + 2x_2 \end{bmatrix}$$

$$g(x) = \begin{bmatrix} -2x_1 \\ -x_2 \end{bmatrix}$$

$$\dot{S} = 2x_1\dot{x_1} + 2x_2\dot{x_2} - 20\dot{x_1} - 20\dot{x_2}$$

substituting $\dot{x_1}$, $\dot{x_2}$:

$$\dot{S} = 2x_1^2 x_2 - 8x_1^2 - 4x_1^2 u - 2x_1 x_2^2 + 4x_2^2 - 2x_2^2 u + 80x_1 + 40x_1 u - 40x_2 + 20x_2 u < 0$$

$$\dot{S} = 2x_1^2 x_2 - 8x_1^2 - 4x_1^2 u - 2x_1 x_2^2 + 4x_2^2 - 2x_2^2 u + 80x_1 + 40x_1 u - 40x_2 + 20x_2 u > 0$$

and,

$$u(x) = \begin{cases} [\frac{2x_1^2 x_2 - 8x_1^2 - 2x_1 x_2^2 + 4x_2^2 + 80x_1 - 40x_2}{-4x_1^2 - 2x_2^2 + 40x_1 + 20x_2}] & s(x) > 0 \\ -[\frac{2x_1^2 x_2 - 8x_1^2 - 2x_1 x_2^2 + 4x_2^2 + 80x_1 - 40x_2}{-4x_1^2 - 2x_2^2 + 40x_1 + 20x_2}] & s(x) < 0 \end{cases}$$

Figure 1: Phase Portraits, Predator vs. Prey