Quadcopter Physical Parameter Identification and Altitude System Analysis

Zaki Mustapa[1], Shakir Saat[2], S. H. Husin[3], Thoriq Zaid [4]
Faculty of Electronic and Computer Engineering
Universiti Teknikal Malaysia Melaka, Malaysia
m.zaki5377@gmail.com

***Abstract*— This paper presents a general quadcopter model. To achieve this, some experimental works have been done. In particular, the reverse engineering approach is applied to find the parameters of the quadcopter system. The first experiment is to obtain the amount of force generated on the propeller via a lift force test. Then the moment of inertia is calculated through a simple test bifilar pendulum concept. Then the suitability of speed is tested to overcome the force of gravity. The motor speed at hovering level is also checked to know the exact motor speed for hovering purpose. The simulation results which are conducted in Matlab/Simulink show the effectiveness of the proposed approach.**

Keywords—quadcopter modelling, quadcopter altitude, bifilar test, lift test, altitude analysis.

I. Introduction

The Unman Aerial Vehicle (UAV) robot has gained demand on the worldwide markets. There are two types of UAV which is fixed wings and rotary wings. Nowadays, one of the most popular rotary wings UAV is quad copter. It has more advantages in term of their capability to take off and landing in vertical condition [1]. In addition, it has good mobility, simple mechanics, and ability on load capacity.

The quadcopter system is nonlinear system, which is the speed of the motor are varies the torque and force onto the rigid body of quadcopter. The force and torque that produced by the motor propeller will determine the behavior of the quadcopter [2]. The objective of this paper is to understand model of quadcopter system and to focus on altitude quadcopter system analysis by conducting several experiment. The Newton Eular Formalism will be use for the physical derivation. The kinematics and dynamics of quadcopter will be considered in modeling the system especially in movement activities [3].

The study of quadcopter modeling will assist to understand the behavior of the quadcopter system. The value of minimum speed of hover will be determined by simulate the altitude quadcopter system in Simulink. To build an altitude and attitude quadcopter controller, the system modeling must be known perfectly [3]. All the procedure to getting the unknown parameter that require in physical model are explained.

II. Dynamics and Modeling of a Quadcopter

A. Basic concept

Quadcopter consist of 4 set motor which each one has the same specification of the propeller. The quadcopter frame designs in symmetric to make the model of quadcopter simple. In a quadcopter system, only the speed of the motor is controlled. The attitude and altitude that produced by the system only based on the speed of motor propeller [4].

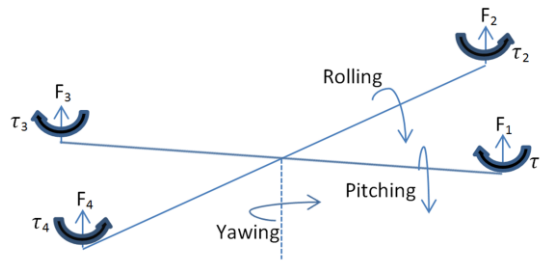

Fig. 1. Quadcopter system

Figure 1 shows the simple quadcopter system. Each motor produces a force (F) and torque (τ). Each motor behavior will be determined the movement rolling, pitching or yawing [5].

TABLE I: Movement Characteristic.

Moment	Direction	M_1	M_2	M_3	M_4
Thrusting	Up	$\Omega_H+\Delta_a$	$\Omega_H+\Delta_a$	$\Omega_H+\Delta_a$	$\Omega_H+\Delta_a$
	Down	$\Omega_H-\Delta_b$	$\Omega_H-\Delta_b$	$\Omega_H-\Delta_b$	$\Omega_H-\Delta_b$
Rolling	Right	Ω_H	$\Omega_H-\Delta_c$	Ω_H	$\Omega_H+\Delta_c$
	Left	Ω_H	$\Omega_H+\Delta_d$	Ω_H	$\Omega_H-\Delta_d$
Pitching	Forward	$\Omega_H-\Delta_e$	Ω_H	$\Omega_H+\Delta_e$	Ω_H
	Backward	$\Omega_H+\Delta_f$	Ω_H	$\Omega_H-\Delta_f$	Ω_H
Yawing	Clockwise	$\Omega_H-\Delta_g$	$\Omega_H+\Delta_g$	$\Omega_H-\Delta_g$	$\Omega_H+\Delta_g$
	C.clockwise	$\Omega_H+\Delta_h$	$\Omega_H-\Delta_h$	$\Omega_H+\Delta_h$	$\Omega_H-\Delta_h$

Table 1 shows the movement characteristic that produces by change of motor speed. The minimum speed of hover (Ω_H) is used as the speed reference. The increament or decreament of speed (Δ) are same to produce the balance thrust in a moment.

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B. Frame

- The inertial frame (E- frame)
  A frame with coordinating base on fix earth coordinates, which refers to the center of gravity (linear quadcopter).
- The body fixed frame (B- frame)
  A frame based on the body frame of quadcopter. These frames are pointing at the center of mass of the quadcopter body (angular velocity quadcopter).

These two frames are used as references in 6 degree of freedom (DOF) rigid body equations. The equations need to express the dynamic respect to a composed of a linear quadcopter base on E-frame and angular quadcopter base on B-frame [6].

C. Three basic attitudes of quadcopter

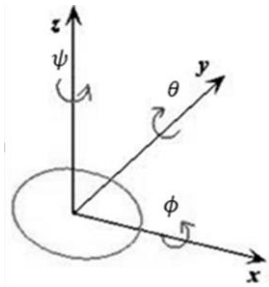

Fig. 2. Rotation.

Figure 2 shows the rotation each axis to produce roll, pitch, yaw movement.

- Roll (ϕ): - Base on the right hand rotation rule, roll movement is about the rotation of the x axis:

$$R_0^{p0}(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \tag{1}$$

- Pitch (θ): - Base on the right hand rotation rule, pitch movement is about the rotation of the y axis.

$$R_0^{p1}(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \tag{2}$$

- Yaw (ψ): - Based on the right hand rotational rules, yaw movement is about the rotation of the z axis.

$$R_0^{p2}(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

To find the Euler angles from translation of $\phi, \theta, and\ \psi$. The transformation is defined from the inertial frame to the body frame.

$$R_b^e(\phi,\theta,\psi) = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \tag{4}$$

$$= \begin{bmatrix} c\phi c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta s\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\theta & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi s\psi & c\phi c\theta \end{bmatrix}$$

$$R_e^b = (R_b^e)^T = \begin{bmatrix} c\phi c\psi & s\phi s\theta s\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\theta & c\phi s\theta s\psi - s\phi s\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \tag{5}$$

D. 6 DOF equations with Newton-Eular formalism.

6 degrees of freedom rigid body required to represent the quadcopter movement system. Newton-Euler formalism is used to derive the quadcopter rigid body system.

$$\begin{bmatrix} mI_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} \omega^B \times (mV^B) \\ \omega^B \times (I\omega^B) \end{bmatrix} = \begin{bmatrix} F^B \\ \tau^B \end{bmatrix} \tag{6}$$

The $mI_{3\times3}$ Mass identity matrix. I is 3 dimension inertia matrixes (x, y and z). Since the quadcopter designed symmetric in 3 axis, inertia matrix can be written by assume $I_{xy} = I_{xz} = I_{yz} = 0$, therefore:

$$I = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad ; \quad mI_{3\times3} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \tag{7}$$

The second group of matrix is about Coriolis-Centripetal matrix. The Coriolis only affect from the angular velocity vector for the body fix frame only. To derive 3D angular velocity vector, skew-symmetric technique is applied.

$$[V \quad \omega]^T = [u \quad v \quad w \quad p \quad q \quad r]^T \tag{8}$$

The value of linear velocity quadcopter (V) in body fix frame is equal to zero.

$$\begin{bmatrix} \omega^B \times (mV^B) \\ \omega^B \times (I\omega^B) \end{bmatrix} = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & -S(I\omega^B) \end{bmatrix} \tag{9}$$

To derive 3D angular velocity vector, skew-symmetric technique is applied.

$$\begin{bmatrix} \omega^B \times (mV^B) \\ \omega^B \times (I\omega^B) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_z r & -I_y q \\ 0 & 0 & 0 & -I_z r & 0 & I_x p \\ 0 & 0 & 0 & I_y q & -I_x p & 0 \end{bmatrix} \tag{10}$$

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On a body of quadcopter, force, F^B and torque vector affected by gravitational, gyroscopic and main input vector as referred in the (11)[7].

$$\begin{bmatrix} F^B \\ \tau^B \end{bmatrix} = G + (O \times \Omega) + \begin{bmatrix} R_e^b & 0 \\ 0 & I_{3\times3} \end{bmatrix} [U] \tag{11}$$

- First contribution is the gravity force where it is only changing to the quadcopter linear only. The gravity force is effected of the vertical (z) vector. The gravity force being opposite to the thrust force. So the gravitational vector can write as shown below:-

$$G = \begin{bmatrix} 0 \\ 0 \\ -ma_g \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

- Second contribution is the gyroscopic effects which are come from torque each of motor propeller rotation. To avoid the gyroscopic effects, speed of 4 motors must be produce balance algebraic sum which is the two of motor rotate clockwise and another two motor rotate counter clockwise. However the gyroscopic effects are used to produce yawing movement. In another case, when the motor speed is varied for others movement, such as rolling and pitching, an undesirable gyroscopic effect also produced. In that case, only the angular velocity part is involved as the expression below.:-

$$O\Omega = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ q & -q & q & -q \\ -p & p & -p & p \\ 0 & 0 & 0 & 0 \end{bmatrix} \Omega \tag{13}$$

Where;

$$\Omega = [\Omega_1 \quad \Omega_2 \quad \Omega_3 \quad \Omega_4]^T \tag{14}$$

- The third contribution of the force and torque vector is a main input vector of the system. The aerodynamic theories determine that both force and torques change linearly to the squared propeller velocity. Hence, the mathematic model of the motor propeller required. There are two aerodynamic parameters included in this system which is the thrust vector and drag factor that produced by each propeller. For quadcopter movement system, only has 4 basic movement vector, where is U_1 is the total force produced by 4 motor propellers. U_2 is represent the rolling torque produced by the velocity of motor 4 and motor 2. U_3 is refer to the pitching torque produced by the velocity of motor 3 and motor 1. U_4 is about the yawing torque.

$$U = \begin{bmatrix} U_1 \\ U_1 \\ U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ bl(\Omega_4^2 - \Omega_2^2) \\ bl(\Omega_3^2 - \Omega_1^2) \\ d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{bmatrix} \tag{15}$$

The input U_1 affects will be involved into three linear vectors through the transformation matrixR_e^b. The product of main input can be derived in matrix form. The input U_2 is used to generate rolling movement, while U_3 and U_4 is used to generate pitching and yawing movement. U_1 In this system, in term of thrust force, while U_2, U_3, U_4 represent of torque to produce the movement.

$$\begin{bmatrix} R_e^b & 0 \\ 0 & I_{3\times3} \end{bmatrix} [U] = \begin{bmatrix} (\sin\psi \sin\phi + \cos\psi \sin\theta \cos\phi)U_1 \\ (-\cos\psi \sin\phi + \sin\psi \sin\theta \cos\phi)U_1 \\ (\cos\phi \cos\theta)U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \tag{16}$$

Thus the generalized acceleration vector can be summarized as below.

$$\begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} mI_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I \end{bmatrix}^{-1}$$

$$\left(G + (O\Omega) + \begin{bmatrix} R_e^b & 0 \\ 0 & I_{3\times3} \end{bmatrix} [U] - \begin{bmatrix} \omega^B \times (mV^B) \\ \omega^B \times (I\omega^B) \end{bmatrix} \right) \tag{17}$$

The quadcopter model can be simplified to make the system easy to analyze. Since the change of roll and pitch angle is too small compared to the hovering angle, the change of angular velocity value can be assumed to zero. Hence, quadcopter angular velocity of E-frame equal to the angular velocity for B-frame. Another term to be simplified is Coriolis effect, because it is too small to obtain. Thus the model will be simplified as below:-

$$\begin{bmatrix} \ddot{X} = (\sin\psi \sin\phi + \cos\psi \sin\theta \cos\phi)\frac{U_1}{m} \\ \ddot{Y} = (-\cos\psi \sin\phi + \sin\psi \sin\theta \cos\phi)\frac{U_1}{m} \\ \ddot{Z} = -g + (\cos\phi \cos\theta)\frac{U_1}{m} \\ \dot{p} = \frac{U_2}{I_x} \\ \dot{q} = \frac{U_3}{I_y} \\ \dot{r} = \frac{U_3}{I_z} \end{bmatrix} \tag{18}$$

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Where;

$$\begin{bmatrix} U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = bl(-\Omega_2^2 + \Omega_4^2) \\ U_3 = bl(-\Omega_1^2 + \Omega_3^2) \\ U_4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \end{bmatrix} \quad (19)$$

III. PHYSICAL PARAMETER DETERMINATION

After getting quadcopter model equations, the values of the parameters have to be identified. The parameters to be identified are the constant parameters in the model equations. The constant parameter is the acceleration of gravity (g), mass (m) of body quadcopter, and inertial (I) parameters. Aerodynamic parameters of the constant thrust factor (b), drag factor (d), and the distance between the motor and the quadcopter centers[8]. There are three values of inertia parameters of movement, which is rolling due to x-axis, pitching due to y-axis and yawing due to z-axis.

A. Moment of inertia

There are 2 methods to determine value of inertia parameter which is by using moment of inertia calculation and by bifilar pendulum experiment. This paper is focus on the bifilar pendulum experimental method.

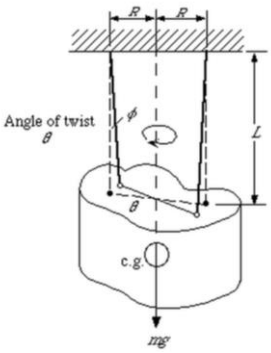

Fig. 3. Bifilar pendulum test[9]

The Figure 3 shows the bifilar pendulum system to measure the value of inertia. Based on the bifilar experiment, two ropes used to string up the quadcopter body. Then the quadcopter body is held at a certain degree by hand, and then releases the quadcopter body. Due to the tension of the rope, the system will be swung about the vertical axis of the system. The inertia can be calculated by the bifilar equation [9].

$$I = \frac{mgT^2b^2}{4\pi^2 L} \quad (20)$$

Where:-
m=mass of quadcopter body, (kg)
g=acceleration due to gravity, (m/s^2)
T=period of swing in the 1 cycle (s) determine from IMU data.
b=radius between the rope point (m)
L=the rope length (m)

The quadcopter design is in symmetrical. Value of the inertia parameter for rolling are same for the pitching movement.

TABLE II: CONSTANT PARAMETER

Parameter	Value	Unit
m	1.00	kg
g	9.81	m/s^2
b	0.20	m
L	1.50	m

Table 2 shows the value of each constant parameter in bifilar test.

TABLE III: MOMENT OF INERTIA VALUE

Moment of inertia	Period per cycle (s)	Value (kgm^2)
I_x	1.40	0.0398
I_y	1.40	0.0398
I_z	1.85	0.0421

Table 3 shows the result for the moment of inertia for rolling, pitching and yawing rotation. The value will be used in control system design.

B. Thrust force factor

Thrust factor can be determined by 2 methods, one is by blade element theory calculation, and another is by force lift test experiment. For this paper, force lift test method is chosen, because the blade element analysis required a lot of environments and blade specification variable, and the experiment is about the real situation environment, so the parameter will be much more reliable [10].

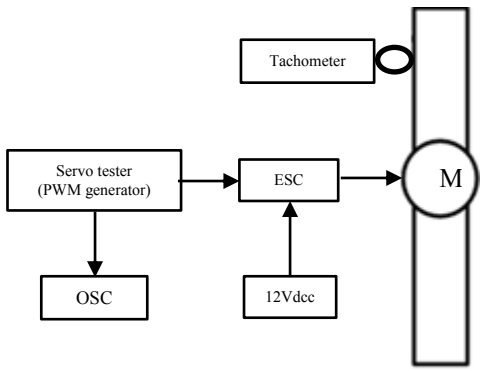

Fig. 4. Lift test method for Brushless motor

The Figure 4 shows the experimental method to measure the relationship of force produce by the speed of the propeller. The electronic speed controller used while triggering by PWM generator. And the speed of the propeller will be determined by micro tachometer. The value of thrust force is recorded by changing the speed of the propeller.

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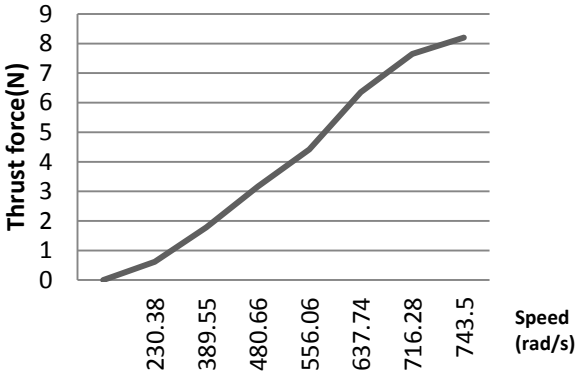

Fig. 5. Force VS speed of propeller

The Figure 5 shows the relationship between thrust force produced by the speed of the propeller. The theory of aerodynamic already proves that the thrust force are varied proportionally by squaring velocity. To define the constant value of the relationship, the equation below is applied.

$$b = \frac{f}{\Omega^2} \tag{21}$$

Thus;

$$b = 1.50 \times 10^{-5}$$

Based on the aerodynamic theory, drag factor value are depending to the lift factor by a ratio $\frac{d}{b} = \tan\alpha$, where α is the attack angle of the propeller [11]. The value of α equal to 15°, and the value of drag factor is equal to $d = 4.02 \times 10^{-6}$.

IV. ALTITUDE ANALYSIS

A. Simulation

After each physical parameter that gets from the experiment, the value will be inserted into the equation models. The system can analyze by a simulation using Mat-lab software.

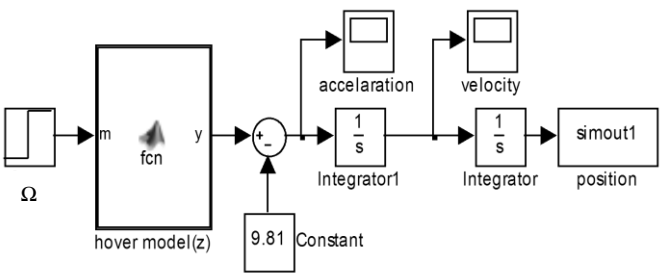

Fig. 6. The altitude system simulates in Simulink.

The Figure 6 is about the altitude system of quad copter which is used to analyze the behavior of quad copter in certain speed of the propellers. The simulation setup based on the below.

$$\ddot{Z} = -g + (\cos\phi\cos\theta)\frac{U_1}{m} \tag{18}$$

$$U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \tag{19}$$

The constant value of the quad copter system which is a mass of the quad copter (m) is 1kg, the thrust factor (b) is 1.50^{-5} and the gravity acceleration(g) is 9.81. In this ideal case, the angle of roll and pitch consider to zero.

B. Result

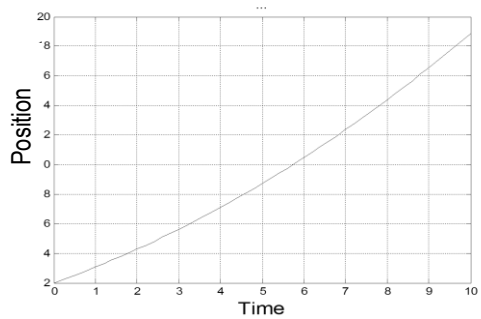

Fig. 7. The position change when $\Omega = 405 rad/s$

Figure 7 shows the positive changing of position for the speed of propeller at $405 rad/s$.

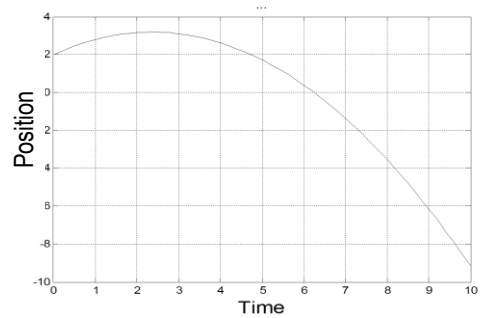

Fig. 7. The position change when $\Omega = 395\ rad/s$

Figure 7 shows the negative changing of position for the speed of propeller at $395 rad/s$.

From the analysis, the minimum hovers speed of the motor (Ω_H) of the quad copter body could be determined. From the basic movement concept, the hovering motor speed are used to be the reference speed of each altitude and attitude control. The result shows the minimum speed of each motor to thrust the load of quad copter body is $\Omega \geq 405 rad/s$, for the payload 1kg, and 1.50^{-5} of the thrust factor.

V. CONCLUSION / FUTURE WORK

A. Conclusion

The analysis of the altitude quadcopter system can be realized by discussing relevant model description. The result of the analysis helps others to start designing a controller that is compatible with the quadcopter system. Furthermore, each model needs to be analyzed by undergoing simulation to understand the behavior of a system. This paper is greatly simplifying the simulation to determine the parameters of a control system for their coincident with the system.

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TABLE III: PARAMETER RESULT

Parameter		Value	Unit
Mass		1	kg
Gravity		9.81	$m\ s^2$
Lift factor		1.50×10^{-5}	$N\ s^2$
Drag factor		4.02×10^{-6}	$N\ m\ s^2$
Inertia	I_x	0.0398	kgm^2
	I_x	0.0398	kgm^2
	I_z	0.0421	kgm^2

Table 3 shows the result of physical parameter.

B. Future work

In the future, a control system will be designed with upgrades to control the altitude of a quadcopter. A sensor to be used for determining the altitude of the control system. Short period of time to reach the required height is a priority for the control system.

VI. ACKNOWLEDGMENT

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