Control Of A Quadcopter

Daniel Wood

February 20, 2021

1 Generalized Coordinates

When analyzing the dynamics of a quadcopter, two coordinate systems need to be defined, the body & inertial coordinate frames. The inertial frame is earth fixed, this can be thought of as a centralized home position that all motion is relative to, in this case with a center point defined as O. The body frame is a coordinate frame defined from the center of gravity of the quadcopter, this will be defined with a center point of G. Managing two coordinate frames in a dynamics system can be acheived efficiently by introducing a generalized coordinate q, that is comprised of the neccesary variables to fully define the dyanmics of the body frame in the inertial.

A vector can then be related from O to G as $\overrightarrow{r_G}$:

$$\overrightarrow{r_G} = \begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

To relate the motion of the body in the inertial frame, three angles of rotation ψ,θ,ϕ are needed, which are known in literature as Type I Euler angles[cite greenwood]. This leads to the state vector of the system in generalized coordinates q $\in \mathbb{R}^6$:

$$q = \begin{bmatrix} X_G & Y_G & Z_G & \psi & \theta & \phi \end{bmatrix} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \end{bmatrix}$$

In the case of the quadcopter where there are four motors, there are four forces produced by the motors defined in the positive $\overrightarrow{k}$ direction of the body frame, in general the forces are defined as:

$$\overrightarrow{F_i} = \begin{bmatrix} 0 \\ 0 \\ F_i \end{bmatrix} i = 1:4$$

Which will yeild a resultant force in the body frame of:

$$\overrightarrow{F_R} = \sum_{i=1}^{4} \overrightarrow{F_i} - mg$$

These forces will also need to be defined in the inertial frame, this is acheived by the rotation matrix:

$$\overrightarrow{F_R} = R(\psi, \theta, \phi) \cdot \overrightarrow{F_R}$$

Where the rotation matrix is defined as:

$$R(\psi, \theta, \phi) = \begin{bmatrix} \cos\psi\cos\theta & \cos\theta\sin\psi & -\sin\theta \\ \cos\psi\sin\phi\sin\theta - \cos\phi\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\psi\sin\theta & \cos\theta\sin\phi \\ \sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta & \cos\phi\sin\psi\sin\theta - \cos\psi\sin\phi & \cos\phi\cos\theta \end{bmatrix}$$

Which can be expressed in terms of the generalized coordinates q as:

$$R(q) = \begin{bmatrix} \cos(q_4)\cos(q_5) & \cos(q_5)\sin(q_4) & -\sin(q_5) \\ \cos(q_4)\sin(q_6)\sin(q_5) - \cos(q_6)\sin(q_4) & \cos(q_6)\cos(q_4) + \sin(q_6)\sin(q_4)\sin(q_5) & \cos(q_5)\sin(q_6) \\ \sin(q_6)\sin(q_4) + \cos(q_6)\cos(q_4)\sin(q_5) & \cos(q_6)\sin(q_4)\sin(q_5) - \cos(q_4)\sin(q_6) & \cos(q_6)\cos(q_5) \end{bmatrix}$$

which yields the force resultant in the inertial frame as:

$$\overrightarrow{F_R} = \begin{bmatrix} -sin(q_5) \cdot (F_1 + F_2 + F_3 + F_4 - mg) \\ cos(q_5)\sin(q_6) \cdot (F_1 + F_2 + F_3 + F_4 - mg) \\ cos(q_6)\cos(q_5) \cdot (F_1 + F_2 + F_3 + F_4 - mg) \end{bmatrix}$$

The position of the forces in the body frame are defined by the position vectors $\overrightarrow{r_i}$ (i=1:4) as follows:

$$\overrightarrow{r_1} = \frac{a}{2}\overrightarrow{i} + \frac{b}{2}\overrightarrow{j} + 0\overrightarrow{k}$$

$$\overrightarrow{r_2} = -\frac{a}{2}\overrightarrow{i} + \frac{b}{2}\overrightarrow{j} + 0\overrightarrow{k}$$

$$\overrightarrow{r_3} = \frac{a}{2}\overrightarrow{i} - \frac{b}{2}\overrightarrow{j} + 0\overrightarrow{k}$$

$$\overrightarrow{r_4} = -\frac{a}{2}\overrightarrow{i} - \frac{b}{2}\overrightarrow{j} + 0\overrightarrow{k}$$

Using the forces and their position vectors the moment about origin G in the body frame is defined as:

$$\overrightarrow{M_G} = \sum_{i=1}^{4}(\overrightarrow{r_i} \times \overrightarrow{F_i})$$

this will also need to be rotated into the inertial frame with a center point of O yielding:

$$\overrightarrow{M_O} = \overrightarrow{M_G} \cdot R(q) = \begin{bmatrix} cosq_4 cosq_5 M_1 + cosq_5 sinq_4 M_2 \\ -(cosq_6 sinq_4 - cosq_4 sinq_6 sinq_5)M_1 + (cosq_6 cosq_4 + sinq_6 sinq_4 sinq_5)M_2 \\ (sinq_6 sinq_4 + cosq_6 cosq_4 sinq_5)M_1 - (cosq_4 sinq_6 - cosq_6 sinq_4 sinq_5)M_2 \end{bmatrix}$$

$$M_1 = \frac{b}{2}F_1 + \frac{b}{2}F_2 - \frac{b}{2}F_3 - \frac{b}{2}F_4$$

$$M_2 = \frac{a}{2}F_1 - \frac{a}{2}F_2 + \frac{a}{2}F_3 - \frac{a}{2}F_4$$

For a full depiction of the dynamics of the system in the inertial frame first we start with looking at the local angular velocities as expressed in the body frame:

$$\omega_{body} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -sin\theta & 0 & 1 \\ cos\theta cos\phi & cos\phi & 0 \\ cos\phi cos\theta & -sin\phi & 0 \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}$$

Expressing these in the generalized coordinates of q yields:

$$\omega(q) = R(q) \cdot \omega_{body}$$

With the local angular velocities and the moments in the body frame rotated into the inertial, the generalized forces Q_i can be defined by:

$$Q_i = \overrightarrow{F}_R \frac{\partial \overrightarrow{r_G}}{\partial q_i} + \overrightarrow{M}_G \frac{\partial \overrightarrow{\omega}}{\partial \dot{q}_j}, i = 1:6, j = 4:6$$

The Kinetic Energy and the Equations of Motion

The kinetic energy of the quadcopter is defined based on Koenig's Theorem [Cite Greenwood] which states the kinetic energy is the sum of the kinetic energy due to translational velocity of the center of mass as well as the rotation:

$$T(q) = \frac{1}{2}\dot{q}^T M(q)\dot{q}$$

where M(q) is defined as:

$$M(q) = J_v(q)^T m_{rr} J_v(q) + J_w(q)^T m_{\theta\theta} J_w(q)$$

with:

$$J_v(q) = \begin{bmatrix} \frac{\partial q_1}{\partial q_1} & \frac{\partial q_1}{\partial q_2} & \frac{\partial q_1}{\partial q_3} & 0 & 0 & 0 \\ \frac{\partial q_2}{\partial q_1} & \frac{\partial q_2}{\partial q_2} & \frac{\partial q_2}{\partial q_3} & 0 & 0 & 0 \\ \frac{\partial q_3}{\partial q_1} & \frac{\partial q_3}{\partial q_1} & \frac{\partial q_3}{\partial q_1} & 0 & 0 & 0 \end{bmatrix}$$

$$J_w(q) = \begin{bmatrix} 0 & 0 & 0 & -sinq_5 & 0 & 1 \\ 0 & 0 & 0 & cosq_5 sinq_6 & cosq_6 & 0 \\ 0 & 0 & 0 & cosq_5 cosq_6 & -sinq_6 & 0 \end{bmatrix}$$

$$m_{rr} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$$

$$m_{\theta\theta} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

which expands to:

$$T(q) = \frac{1}{2}(m\dot{q_1}^2 + m\dot{q_2}^2 + m\dot{q_3}^2 + \dot{q_4}^2((I_{yy}+I_{zz})cos(q_5)^2 cos(q_6)^2 + I_{xx}sin(q_5)^2) + \dots$$

$$\dots + \dot{q_5}^2(I_{yy}cos(q_6)^2 + I_{zz}sin(q_6)^2) + 2\dot{q_5}\dot{q_4}\sigma + \dot{q_6}^2 I_{xx} - 2\dot{q_6}\dot{q_4}I_{xx}sin(q_5))$$

where:

$$\sigma = \frac{1}{2}(I_{yy} - I_{zz})cos(q_5)sin(2q_6)$$

The potential energy of the drone is defined as:

$$\overrightarrow{V}(q) = mgq_3$$

which allows for the lagrangian to be:

$$\mathcal{L}(q) = T(q) - V(q)$$

yielding,

$$\mathcal{L}(q) = \frac{1}{2}m\dot{q}_1^2 + \frac{1}{2}m\dot{q}_2^2 + \frac{1}{2}m\dot{q}_3^2 +$$

$$\dots(\frac{1}{2}I_{xx}sin^2(q_5) + \frac{1}{2}I_{yy}cos^2(q_6) - (\frac{1}{2}I_{yy} - \frac{1}{2}I_{zz})cos^2(q_5)cos^2(q_6))\dot{q}_4^2 +$$

$$\dots(\frac{1}{2}I_{zz} + \frac{1}{2}(I_{yy} - I_{zz})cos^2(q_6))\dot{q}_5^2 + \frac{1}{2}I_{xx}\dot{q}_6^2 - I_{xx}sin(q_5)\dot{q}_4\dot{q}_6$$

$$\dots + \frac{1}{2}(I_{yy} + I_{zz})cos(q_5)sin(2q_6)\dot{q}_4\dot{q}_5 + gmq_3$$

with $\mathcal{L}(q)$ defined this allows us to find the corresponding Euler-Lagrange equations of motions from the following equation, which is also known as the fundamental holonomic form of Lagrange's equation:

$$\frac{d}{dt}(\frac{\partial \mathcal{L}(q)}{\partial \dot{q}_j}) - \frac{\partial \mathcal{L}(q)}{\partial q_j} = Q_j(q)$$

$$Q_1(q) = m\ddot{q}_1$$

$$Q_2(q) = m\ddot{q}_2$$

$$Q_3(q) = m\ddot{q}_3 - mg$$

$$Q_4(q) = -I_{xx}sin(q_5)\ddot{q}_6 + (-I_{xx} + (I_{yy} - I_{zz})cos^2(q_5))sin(q_6)cos(q_6)\dot{q}_4\dot{q}_6 +$$
$$\dots 2(I_{xx} + (I_{yy} - I_{zz})cos^2(q_6))sin(q_5)cos(q_5)\dot{q}_4\dot{q}_5 + (-0.5I_{yy} + 0.5I_{zz})cos^2(q_5)cos^2(q_6)\ddot{q}_4 +$$
$$\dots(I_{yy} + I_{zz})cos(q_5)sin(2q_6)\ddot{q}_5$$

$$Q_5(q) = (I_{xx} + (I_{yy} - I_{zz})cos^2(q_6))sin(q_5)cos(q_5)\dot{q}_4^2 + (I_{xx} + (I_{yy} + I_{zz})cos(2q_6))cos(q_5)\dot{q}_4\dot{q}_6 +$$
$$\dots(I_{yy} + I_{zz})cos(q_5)sin(2q_6)\ddot{q}_4 - (2I_{yy} - 2I_{zz})sin(q_6)cos(q_6)\dot{q}_5\dot{q}_6 + (I_{zz} + (I_{yy} - I_{zz})cos^2(q_6))\ddot{q}_5$$

$$Q_6(q) = -I_{xx}sin(q_5)\ddot{q}_4 + I_{xx}\ddot{q}_6 - (I_{xx} + (I_{yy} + I_{zz})cos(2q_6))cos(q_5)\dot{q}_4\dot{q}_5 +$$
$$\dots(I_{yy} - I_{zz})sin(q_6)cos(q_6)\dot{q}_5^2 - (I_{yy} + (I_{yy} - I_{zz})cos^2(q_5))sin(q_6)cos(q_6)\dot{q}_4^2$$

State Space Representation

To define a controller, the dynamics of the system must be first put into state space form:

$$x = [q, \dot{q}]^T$$

and,

$$\dot{x} = [\frac{dq}{dt}, \frac{d\dot{q}}{dt}]^T$$

where,

$$\frac{d\dot{q}_1}{dt} = \frac{Q_1}{m}$$

$$\frac{d\dot{q}_2}{dt} = \frac{Q_2}{m}$$

$$\frac{d\dot{q}_3}{dt} = g + \frac{Q_3}{m}$$

$$\frac{d\dot{q}_4}{dt} = \frac{A_1}{(I_{zz} + (I_{yy} - I_{zz})cos^2(q_6))(2I_{xx}sin^2(q_5) + (I_{yy} - I_{zz})cos^2(q_5)cos^2(q_6))}$$

where:

$$\begin{gathered} A_1 = -(I_{yy} + I_{zz})((I_{xx} + (I_{yy} - I_{zz})cos^2(q_6))sin(q_5)cos(q_6)\dot{q}_4^2 + \\ \ldots(I_{xx} + (I_{yy} + I_{zz})cos(2q_6))cos(q_5)\dot{q}_4\dot{q}_6 + \\ \ldots(-I_{yy} + I_{zz})sin(2q_6)\dot{q}_5\dot{q}_6 - Q_5)sin(2q_6)cos(q_5) - \\ \ldots(2I_{zz} + 2(I_{yy} - I_{zz})cos^2(q_6))((-I_{xx} + (I_{yy} - I_{zz})cos^2(q_6))sin(2q_5)\dot{q}_4\dot{q}_5 + \\ (I_{xx} + (I_{yy} - I_{zz})cos^2(q_5))sin(q_6)cos(q_6)\dot{q}_4\dot{q}_6 + Q_4)sin(q_5) - \\ \ldots(2I_{zz} + 2(I_{yy} - I_{zz})cos^2(q_6))((I_{xx} + (I_{yy} + I_{zz})cos(2q_6))cos(q_5)\dot{q}_4\dot{q}_5 + (-0.5I_{yy} + 0.5I_{zz})sin(2q_6)\dot{q}_5^2 + \\ (-0.125I_{yy} + 0.125I_{zz})sin(2q_5 - 2q_6) + (0.125I_{yy} - 0.125I_{zz})sin(2q_5 + 2q_6) + (1.5I_{yy} - 0.5I_{zz})sin(q_5)cos(q_5)\dot{q}_4^2) \end{gathered}$$

$$\frac{d\dot{q}_5}{dt} = \frac{A_2}{2I_{xx}sin^2(q_5)(I_{yy}cos^2(q_6) + I_{zz}sin^2(q_6)) + cos^2(q_5)cos^2(q_6)(I_{yy}^2(sin^2(q_6) + 1) + I_{yy}I_{zz}(6sin^2(q_6) - 1)) + I_{zz}^2 sin^2(q_6)}$$

where,

$$\begin{gathered} A_2 = (0.5I_{yy} + 0.5I_{zz})((-0.5I_{yy} + 0.5I_{zz})sin(2q_6)\dot{q}_5^2 + (I_{xx}cos(q_5) + (0.5I_{yy} + 0.5I_{zz})cos(q_5)cos(2q_6))\dot{q}_4\dot{q}_5 + \\ \ldots((-0.125I_{yy} + 0.125I_{zz})sin(2q_5 - 2q_6) + (0.125I_{yy} - 0.125I_{zz})sin(2q_5 + 2q_6) + \\ \ldots(0.75I_{yy} - 0.25I_{zz})sin(2q_6))\dot{q}_4^2 + Q_6)sin(2q_5)sin(2q_6) + (I_{yy} + I_{zz})((-I_{xx} + (-I_{yy} + I_{zz})cos^2(q_6))sin(2q_5)\dot{q}_4\dot{q}_5 + \\ \ldots(I_{xx} + (-0.5I_{yy} + 0.5I_{zz})cos^2(q_5))sin(2q_6)\dot{q}_4\dot{q}_6 + Q_4)sin(2q_6)cos(q_5) - \\ \ldots(2I_{xx}sin^2(q_5) + (I_{yy} - I_{zz})cos^2(q_5)cos^2(q_6))((0.5I_{xx} + (0.5I_{yy} - 0.5I_{zz})cos^2(q_6))sin(2q_5)\dot{q}_4^2 + \\ \ldots(-I_{yy} + I_{zz})sin(2q_6)\dot{q}_5\dot{q}_6 + (I_{xx}cos(q_5) + (I_{yy} + I_{zz})cos(q_5)cos(2q_6))\dot{q}_4\dot{q}_6 - Q_6) \end{gathered}$$

$$\frac{d\dot{q}_6}{dt} = \frac{A_3}{\begin{gathered} I_{xx}(2I_{xx}((I_{yy} - I_{zz})cos^2(q_6) + I_{zz})sin^2(q_5) + (I_{yy} - I_{zz})((I_{yy} - I_{zz})cos^2(q_6) \\ + I_{zz})cos^2(q_5)cos^2(q_6) + 2(I_{yy} + I_{zz})^2 sin^2(q_6)cos^2(q_5)cos^2(q_6)) \end{gathered}}$$

where,

$$\begin{gathered} A_3 = -0.25I_{xx}(I_{yy} + I_{zz})((-I_{yy} + I_{zz})sin(2q_6)\dot{q}_5\dot{q}_6 + (0.5I_{xx}sin(2q_5) + (I_{yy} - I_{zz})sin(q_5)cos(q_5)cos^2(q_6))\dot{q}_4^2 + \\ \ldots(I_{xx}cos(q_5) + (I_{yy} + I_{zz})cos(q_5)cos(2q_6))\dot{q}_4\dot{q}_6 - Q_5)sin(2q_5)sin(2q_6) - \\ \ldots 2I_{xx}(I_{zz} + (I_{yy} - I_{zz})cos^2(q_6))((-0.5I_{xx} + (-I_{yy} + I_{zz})cos^2(q_6))sin(2q_5)\dot{q}_4\dot{q}_5 + \\ \ldots(I_{xx} + (-0.5I_{yy} + 0.5I_{zz})cos^2(q_5))sin(2q_6)\dot{q}_4\dot{q}_6 + Q_4)sin(q_5) + \end{gathered}$$

$$\ldots((I_{yy}-I_{zz})(I_{zz}+(I_{yy}-I_{zz})cos^2(q_6))+2(I_{yy}+I_{zz})^2 sin(q_6))((I_{xx}+(I_{yy}+I_{zz})cos(2q_6))cos(q_5)\dot{q}_4\dot{q}_5+$$
$$\ldots(-0.5I_{yy}+0.5I_{zz})sin(2q_6)\dot{q}_5^2+((-0.25I_{yy}+0.25I_{zz})sin(2q_5-2q_6)+(0.25I_{yy}+0.25I_{zz})sin(2q_5+2q_6)+$$
$$\ldots(0.75I_{yy}-0.25I_{zz})sin(2q_6))\dot{q}_4^2+Q_6)cos^2(q_5)cos^2(q_6)$$

To control the dynamics of the system the state space needs to be written in the following form:

$$\dot{x}=F(q,\dot{q})+G(q,\dot{q})Q$$

where $G(q,\dot{q})$ is a function of the states, and the generalized forces Q_{1x12} are the system inputs.

$$G(q,\dot{q})=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&\frac{1}{m}&0&0&0&0&0\\0&0&0&0&0&0&0&\frac{1}{m}&0&0&0&0\\0&0&0&0&0&0&0&0&\frac{1}{m}&0&0&0\\0&0&0&0&0&0&0&0&0&g_{1010}&g_{1011}&g_{1012}\\0&0&0&0&0&0&0&0&0&g_{1110}&g_{1111}&g_{1112}\\0&0&0&0&0&0&0&0&0&g_{1210}&g_{1211}&g_{1212}\end{bmatrix},Q=\begin{bmatrix}0\\0\\0\\0\\0\\0\\Q_1\\Q_2\\Q_3\\Q_4\\Q_5\\Q_6\end{bmatrix}$$

where,

$$g_{1010}=\frac{(I_{yy}-I_{zz})cosq_6{}^2+I_{zz}}{(I_{zz}+(I_{yy}-I_{zz})cos^2(q_6))(2I_{xx}sin^2(q_5)+(I_{yy}-I_{zz})cos^2(q_5)cos^2(q_6))}$$

$$g_{1011}=\frac{(I_{yy}+I_{zz})cosq_5 sin2q_6}{(I_{zz}+(I_{yy}-I_{zz})cos^2(q_6))(2I_{xx}sin^2(q_5)+(I_{yy}-I_{zz})cos^2(q_5)cos^2(q_6))}$$

$$g_{1012}=\frac{2sinq_5((I_{yy}-I_{zz})cosq_6{}^2+I_{zz})}{(I_{zz}+(I_{yy}-I_{zz})cos^2(q_6))(2I_{xx}sin^2(q_5)+(I_{yy}-I_{zz})cos^2(q_5)cos^2(q_6))}$$

$$g_{1110}=\frac{(I_{yy}+I_{zz})cos(q_5)sin(2q_6)}{2I_{xx}sin^2(q_5)(I_{yy}cos^2(q_6)+I_{zz}sin^2(q_6))+cos^2(q_5)cos^2(q_6)(I_{yy}^2(sin^2(q_6)+1)+I_{yy}I_{zz}(6sin^2(q_6)-1))+I_{zz}^2 sin^2(q_6)}$$

$$g_{1111}=\frac{2I_{xx}sin^2(q_5)+(I_{yy}-I_{zz})cos^2(q_5)cos^2(q_6)}{2I_{xx}sin^2(q_5)(I_{yy}cos^2(q_6)+I_{zz}sin^2(q_6))+cos^2(q_5)cos^2(q_6)(I_{yy}^2(sin^2(q_6)+1)+I_{yy}I_{zz}(6sin^2(q_6)-1))+I_{zz}^2 sin^2(q_6)}$$

$$g_{1112}=\frac{\frac{1}{2}(I_{yy}+I_{zz})sin(2q_5)sin(2q_6)}{2I_{xx}sin^2(q_5)(I_{yy}cos^2(q_6)+I_{zz}sin^2(q_6))+cos^2(q_5)cos^2(q_6)(I_{yy}^2(sin^2(q_6)+1)+I_{yy}I_{zz}(6sin^2(q_6)-1))+I_{zz}^2 sin^2(q_6)}$$

$$g_{1210}=\frac{2sin(q_5)((I_{yy}-I_{zz})cos^2(q_6)+I_{zz})}{I_{xx}(2I_{xx}((I_{yy}-I_{zz})cos^2(q_6)+I_{zz})sin^2(q_5)+(I_{yy}-I_{zz})((I_{yy}-I_{zz})cos^2(q_6)+I_{zz})cos^2(q_5)cos^2(q_6)+2(I_{yy}+I_{zz})^2 sin^2(q_6)cos^2(q_5)cos^2(q_6))}$$

$$g_{1211}=\frac{\frac{1}{2}(I_{yy}+I_{zz})sin(2q_5)sin(2q_6)}{I_{xx}(2I_{xx}((I_{yy}-I_{zz})cos^2(q_6)+I_{zz})sin^2(q_5)+(I_{yy}-I_{zz})((I_{yy}-I_{zz})cos^2(q_6)+I_{zz})cos^2(q_5)cos^2(q_6)+2(I_{yy}+I_{zz})^2 sin^2(q_6)cos^2(q_5)cos^2(q_6))}$$

$$g_{1212}=\frac{(I_{yy}-I_{zz})^2cos^2(q_5)cos^4(q_6)+2(I_{yy}+I_{zz})^2sin^2(q_6)cos^2(q_5)cos^2(q_6)+I_{zz}cos^2(q_5)cos^2(q_6)(I_{yy}-I_{zz})}{I_{xx}(2I_{xx}((I_{yy}-I_{zz})cos^2(q_6)+I_{zz})sin^2(q_5)+(I_{yy}-I_{zz})((I_{yy}-I_{zz})cos^2(q_6)+I_{zz})cos^2(q_5)cos^2(q_6)+2(I_{yy}+I_{zz})^2 sin^2(q_6)cos^2(q_5)cos^2(q_6))}$$