Modeling and Control of a Tilting Quadcopter

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This article addresses modeling and tracking control for a tilting quadcopter in the presence of parametric uncertainties and external disturbances. We propose the novel concept of a tilting quadcopter, which suggests that the rotational and translational movements can be controlled independently. A complete dynamic model is developed, where parametric uncertainties and external disturbances are taken into consideration. Then, an adaptive fast finite-time control is proposed to provide robust, chattering-free, and fast convergence tracking performances. All the tracking errors can fast converge into arbitrary small neighborhoods around the origin in finite-time proved by a modified Lyapunov finite-time stability theory, and an adaptive scheme is synthesized to compensate for the effect of uncertainties. Finally, comparative simulations are carried out to illustrate the effectiveness and the robustness of the proposed controller.

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I. INTRODUCTION

Quadcopters have seen a boost in popularity and are widely applied both in military and civil applications due to their maneuverability and versatility [1]–[3]. However, quadcopters possess fewer control inputs than available degrees of freedom, where only the Cartesian position and the yaw angle can be controlled independently [4]. Current trends require them interacting with hostile and cluttered environments, such as object grasping, searching in ruins, and narrow space operation. Such underactuated vehicles not only limit their flying ability in free or cluttered space, but also degrade their performance within hostile environments [5]. This indicates that exploring fully actuated vehicles is essential to improve the performance and efficiency in tasks.

All rotors of conventional multirotors are aligned in a single plane, which constrains the force generated by propellers to a single direction and, thereby, the translational and rotational movements cannot be controlled independently. Motivated by the above requirements, several actuation strategies have been proposed over the past few years. One straightforward concept is adding additional rotors in other directions. A nonplanar vehicle with six or eight-rotor configurations in different planes is proposed in [6]–[8], where the vehicles are able to provide full force and torque in all three dimensions. In [9] and [10], a vehicle with two central counter-rotating coaxial propellers and three variable ducted fans is proposed, where the altitude and yaw movements, and lateral, roll, and pitch movements can be controlled by two sets of rotors, respectively. The tilt-wing has the ability to vertical take-off and landing like a helicopter and high-speed cruise flight like an airplane as in [11] and [12]. Inspired by these concepts, a variety of multirotors equipped with tilting mechanisms (tilting quadcopter) are proposed to improve flight performances. A novel actuation concept is first proposed in [13], where the propellers are able to tilt along the axes with respect to (w.r.t.) the main body. It introduces four additional servo motors to achieve the desired force and torque in any direction. Flight tests, although preliminary, clearly show the capabilities of this novel concept in tracking [5]. A wide attitude transition flight for pitch angle from 0° to 90° is then discussed. Two switching control systems are proposed w.r.t. the attitude because the maximum control torque decreases as the pitch increases [14], [15]. To further improve the control bandwidth, a dual-axis tilting propeller, where gyroscopic torque, thrust vectoring, and differential thrusting are integrated to drive the vehicle, is introduced in [16]. This modified tilting quadcopter can still complete the task even with half of its actuators damaged [17]. The fault-tolerance is dramatically improved over the conventional quadcopter by using tilting mechanisms.

The tilting mechanisms make the vehicle fully actuated, namely, decoupling the translational and rotational movements. Reliability and feasibility are dramatically improved while interacting with the environment in tough conditions. However, these tilt-design vehicles make the dynamics more complex, and it is a challenging task to design a proper controller. A simple dynamic model including lift force and counteracting torque are developed in [18] to show its superiority in tracking over the underactuated quadcopter. The high-speed spinning rotor generates gyroscopic effects, which is able to increase the maximum available moment [19], [20]. Furthermore, the additional aerodynamic effects including the hub force and rolling moment are analyzed [21]. In addition, several controllers have been developed to improve their tracking performance in the past literature. In [22], two cases, including roll angle and horizontal movement in y-direction, are proposed. It succeeds in moving along the two cases by using proportional-integral derivative (PID) control. The input–output linearization is applied to address the nonlinearities in [23]. A mixed sensitivity H_∞ optimal controller is synthesized to solve the effect of uncertainties in model and external disturbances [24]. Based on [25], a type of robust integral of the sign of the error control is proposed to overcome the chattering problem while preserving the asymptotic output tracking in [26].

Although these existing controllers are asymptotically stable, the tracking errors converge to the equilibrium as time approaches infinity. In recent years, the finite-time stability problems have received significant attention for achieving high precision and fast transient performance [27]. Up to now, terminal sliding mode control [28]–[30] and fast terminal sliding mode control [31]–[33] are widely developed, which enables the tracking errors to converge to the equilibrium in finite time. Inspired by these results, the Lyapunov finite-time stability theory is first proposed in [34] and [35]. Due to the decreasing convergence rate of fractional powers at a distance of the origin, a modified Lyapunov finite-time stability theory combining the linear term and the fractional term is proposed in [36]. Since then, on the base of the Lyapunov finite-time theory, considerable research has been performed in many applications, such as nonlinear servo systems [37], manipulators [38]–[40], and quadcopters [41]–[43]. Considering parametric uncertainties and external disturbances, many scholars focus on developing continuous adaptive finite-time controllers for a class of nonlinear systems [44]–[48], where the chattering problem is eliminated in these references.

Motivated by the above discussion, note that the performance of the aforementioned tilting quadcopter is limited because of the push structure as designed in [16] and [17], which is used to change the direction of the propeller. Large tilting angles are not possible for these mechanisms due to the existence of the rigid arm and the propeller. Thus, a novel concept of tilting quadcopter is first proposed in this article to obtain a more agile version. In addition, a modified Lyapunov finite-time stability theory is proposed in this article, where additional power terms are integrated within the conventional theory. The proposed controller can guarantee the fast convergence both at a distance and at a close range of the equilibrium. Compared with the existing literature, the main contributions of this article include the following.

1) We first propose a novel concept of a tilting quadcopter, where not only translational and rotational movements can be controlled independently, but also Push structures are able to tilt along corresponding axes with large angles compared with [16] and [17], which further improves the reliability and the agility of the fully actuated vehicle.
2) A complete dynamic model of the tilting quadcopter with parametric uncertainties and external disturbances is developed. Few references above consider the side effect of the varied skeleton while tilting their mechanisms, where the uncertain moment of inertia exists. In addition, the aerodynamic damping with uncertain coefficients is also considered.
3) To compensate for the effects of uncertainties and external disturbances, an adaptive fast finite-time control (AFFTC) is developed for the tilting quadcopter to track arbitrary trajectories in space. We further propose a modified Lyapunov finite-time stability theory, where high convergence rate could be obtained in the whole tracking process. Compared with [37]–[48], the tracking errors can very quickly converge into small neighborhoods around the origin in finite-time both at a distance and close of the equilibrium. And a robust, chattering-free, and fast convergence tracking performance is achieved.

The rest of this article is organized as follows. Section II presents the design of the novel tilting quadcopter and develops its complete dynamics with parametric uncertainties and external disturbances. The main results are given in Section III, where an AFFTC is proposed for the tilting quadcopter to achieve fast convergence tracking performance in finite time. In Section IV, comparative simulation results are presented. Finally, we draw a conclusion in Section V.

Notations: Throughout this article, let $\mathbb{R}^{n\times n}$ denote n-dimensional matrix. Let $z^k = [|z_1|^k\mathrm{sgn}(z_1),\ |z_2|^k\mathrm{sgn}(z_2),\ \ldots,\ |z_n|^k\mathrm{sgn}(z_n)]^{\mathrm{T}}$, where $z = [z_1, z_2, \ldots, z_n]^{\mathrm{T}}$ and $\mathrm{sgn}(\cdot)$ is the sign function. Let $K_m = \mathrm{diag}[k_{m1}, k_{m2}, k_{m3}]$ and $\bar{K}_m = \mathrm{diag}[\bar{k}_{m1}, \bar{k}_{m2}, \bar{k}_{m3}]$ denote positive-definite diagonal matrices with $m = 1, \ldots, 6$.

II. DYNAMIC MODELING

A tilting quadcopter consisting of four main rigid bodies in relative motion is proposed: the body, propeller, servo, and push components as shown in Fig. 1. Such a novel mechanical structure could provide propulsion in any direction by tilting servo and push parts. This suggests a fully actuated vehicle, namely, decoupling the translational and rotational movements.

A. Preliminary Definitions

In this article, $R_W : \{o_w,\ x_w,\ y_w,\ z_w\}$ is the world reference frame; $R_B : \{o_b,\ x_b,\ y_b,\ z_b\}$ is the body frame; $R_{S_i} : \{o_{s_i},\ x_{s_i},\ y_{s_i},\ z_{s_i}\}$ and $R_{P_i} : \{o_{p_i},\ x_{p_i},\ y_{p_i},\ z_{p_i}\}$, $(i = 1\ldots4)$ are the servo frame and the push frame associated to the

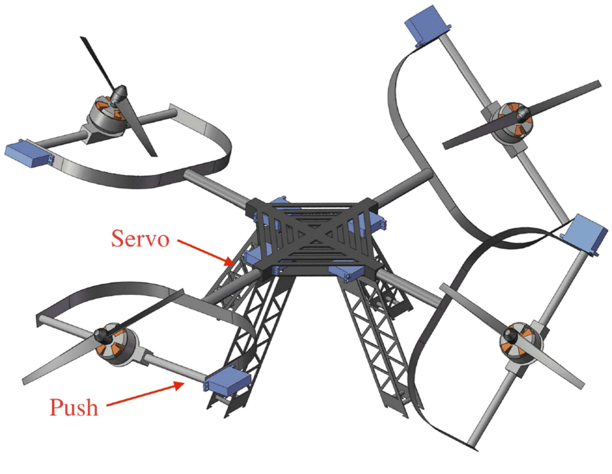

Fig. 1. Tilting quadcopter vehicle.

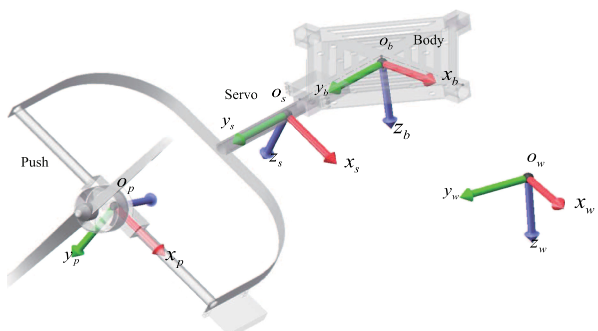

Fig. 2. Reference frame of third propeller group.

ith propeller group, with y_{s_i} and x_{p_i} representing the tilting actuation axes, respectively, as shown in Fig. 2.

To represent the attitude, we select Euler formulation, and the rotation order is Z–Y–X, which means first yaw ψ, then pitch θ, and, finally, roll ϕ angle. The orientation of the inertia frame w.r.t. the body frame is formulated as

$$R_{E2B} = \begin{bmatrix} \mathrm{c}\theta\mathrm{c}\psi & \mathrm{c}\theta\mathrm{s}\psi & -\mathrm{s}\theta \\ -\mathrm{c}\phi\mathrm{s}\psi + \mathrm{s}\phi\mathrm{s}\theta\mathrm{c}\psi & \mathrm{c}\phi\mathrm{c}\psi + \mathrm{s}\phi\mathrm{s}\theta\mathrm{s}\psi & \mathrm{s}\phi\mathrm{c}\theta \\ \mathrm{s}\phi\mathrm{s}\psi + \mathrm{c}\phi\mathrm{s}\theta\mathrm{c}\psi & -\mathrm{s}\phi\mathrm{c}\psi + \mathrm{c}\phi\mathrm{s}\theta\mathrm{s}\psi & \mathrm{c}\phi\mathrm{c}\theta \end{bmatrix} \tag{1}$$

with the denotations $\mathrm{c}\cdot = \cos(\cdot)$ and $\mathrm{s}\cdot = \sin(\cdot)$.

The rotational kinematics can be obtained from the following equation:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \mathrm{s}\phi\mathrm{t}\theta & \mathrm{c}\phi\mathrm{t}\theta \\ 0 & \mathrm{c}\phi & -\mathrm{s}\phi \\ 0 & \dfrac{\mathrm{s}\phi}{\mathrm{c}\theta} & \dfrac{\mathrm{c}\phi}{\mathrm{c}\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = {R_1}^{-1}\omega_b \tag{2}$$

where $\eta = [\phi, \theta, \psi]^{\mathrm{T}}$ denotes the attitude vector and $\omega_b = [p, q, r]^{\mathrm{T}}$ denotes the angle velocity projected in R_B. From (2), the relationship between η and ω_b is $\dot{\eta} = R_1^{-1}\omega_b$.

Assumption 1 The inverse matrix of R_1 exists if condition $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ holds. Moreover, in this article, we also assume ϕ and ψ within $(-\frac{\pi}{2}, \frac{\pi}{2})$.

B. Dynamic Modeling

According to Euler–Lagrange equations, the dynamic model of the tilting quadcopter can be separated into rotational and translational subsystems as follows:

$$\begin{aligned} J(\eta)\ddot{\eta} + C(\eta, \dot{\eta})\dot{\eta} &= \tau - \tau_f + R_1^{\mathrm{T}}\Delta_1 \\ m\ddot{\xi} &= R_{E2B}^T u_T + F_g - F_f + R_{E2B}^T\Delta_2 \end{aligned} \tag{3}$$

where τ and u_T are the total torque and force control inputs; $F_g = [0, 0, mg]^{\mathrm{T}}$ and $\xi = [x, y, z]^{\mathrm{T}}$ is the position vector in R_W; $J(\eta) \in \mathbb{R}^{3\times 3}$ is the effective inertial matrix, and $C(\eta, \dot{\eta}) \in \mathbb{R}^{3\times 3}$ is the Coriolis and Centrifugal force matrix; m is the mass of this omnidirectional flying vehicle (ODFV); Δ_1 and Δ_2 are external disturbances in R_B; $\tau_f = R_1^{\mathrm{T}}N_1(\omega_b)$ and $F_f = R_{E2B}^{\mathrm{T}}N_2(v_b)$ with $v_b = R_{E2B}\dot{\xi}$, denoting the linear velocity in R_B, are the aerodynamic damping torque and force, respectively. According to [49], the $N_1(\omega_b)$ and $N_2(v_b)$ are given by

$$\begin{aligned} N_1(\omega_b) &= \begin{bmatrix} g_1 + g_2|p| & 0 & 0 \\ 0 & g_3 + g_4|q| & 0 \\ 0 & 0 & g_5 + g_6|r| \end{bmatrix} \omega_b \\ N_2(v_b) &= \begin{bmatrix} d_1 + d_2|v_{bx}| & 0 & 0 \\ 0 & d_3 + d_4|v_{by}| & 0 \\ 0 & 0 & d_5 + d_6|v_{bz}| \end{bmatrix} v_b \end{aligned} \tag{4}$$

where d_i and g_i, $i = 1, \ldots, 6$ are positive constants, and $v_b = [v_{bx}, v_{by}, v_{bz}]^{\mathrm{T}}$. Then, using the property of angular momentum being independent of frame, we can derive $J = R_1^T I_v R_1$, where $I_v = \mathrm{diag}(I_{xx}, I_{yy}, I_{zz})$ is the moment of inertia of the whole vehicle. If the readers are interested in $J(\eta)$ and $C(\eta, \dot{\eta})$, more details can be found in Appendix A.

When in an outdoor flight, the aerodynamic damping could significantly affect the performance of the vehicle and could not be ignored as flying indoors. From the expressions of τ_f and F_f, the damping is related to the angle and linear velocities. However, the complex environment makes it unable to develop their precise models, which means that the coefficients g_i and d_i in damping models would deviate from their true values, but within a known bounded range. Although the tilting system has brought some advantages over the conventional quadcopter, some side effects appear. From Fig. 1, the vehicle skeleton is varied while the push and servo parts are tilted along x_{pi} and y_{si}. The inherent property like the moment of inertia I_v would be varied within a known bounded range. Through the above analysis of ODFV, parametric uncertainties exist in the dynamic model of the tilting quadcopter.

For our design later, the following properties of the dynamics (3) are proposed.

Property 1 There exist positive constants k_a, $j_{\min}$, and $j_{\max}$ such that for all $\eta, \zeta \in \mathbb{R}^3$

1) The inertial matrix is symmetric, positive definite, and is bounded from below and above, i.e.,

$j_{\min}I \leq J(\eta) \leq j_{\max}I$, where $j_{\min}$ and $j_{\max} \in \mathbb{R}$ denote the minimum and maximum eigenvalues of $J(\eta)$.
2) The matrix $\dot{J}(\eta) - 2C(\eta, \dot{\eta})$ is skew-symmetric, i.e., $\zeta^{\mathrm{T}}(\dot{J}(\eta) - 2C(\eta, \dot{\eta}))\zeta = 0$.
3) The orientation subsystem in (3) can be parameterized as

$$J(\eta)\ddot{\eta} + C(\eta, \dot{\eta})\dot{\eta} = Y(\eta, \dot{\eta}, \ddot{\eta})\vartheta \tag{5}$$

where $Y(\bullet) \in \mathbb{R}^{3\times\ell}(\ell \geq 1)$ represents the regressor matrix including the known functions of $\eta, \dot{\eta}, \ddot{\eta}$; and $\vartheta \in R^{\ell}$ is the parametric uncertainties in ℓ-dimension.

For the Properties 1), 2), and 3), the readers can refer to [50] for more details. The proof of Property 3) can be found in Appendix A.

Assumption 2 It is assumed that inequalities $|\Delta_1| \leq \bar{\Delta}_1$ and $|\Delta_2| \leq \bar{\Delta}_2$ hold, where $\bar{\Delta}_1$ and $\bar{\Delta}_2$ are vectors with positive elements.

III. ADAPTIVE INTEGRAL FINITE-TIME CONTROLLER DESIGN

In this section, an AFFTC is proposed for the tilting quadcopter with parametric uncertainties and external disturbances. Thus, this novel vehicle can track arbitrary desired trajectories, and achieve high-precision as well as fast convergence, robust, and chattering-free performance. The controller design for rotational and translational movements are, then, developed respectively.

A. Preliminaries

Definition 1 Consider a nonlinear system as

$$\dot{\xi} = f(\xi, u),\ f(0,0) = 0,\ \xi \in \mathbb{R}^n,\ u \in \mathbb{R}^m \tag{6}$$

where u is the control input and $f : D \to \mathbb{R}^n$ is continuous on an open neighborhood D of the origin. The equilibrium $\xi = 0$ is semiglobal practical finite-time stable (SGPFS), if there exists $\varrho > 0$ and a setting time $T < \infty$ for every initial state $\xi(t_0) = \xi_0$ such that $\|\xi(t)\| \leq \varrho$ when $t \geq t_0 + T$ [51].

Lemma 1 Inspired by Bhat *et al.* [35] and Yu *et al.* [36], we further propose a modified Lyapunov finite-time stability theory. Consider the system (6), for a continuous positive definite function $V(\xi)$, which satisfies the following differential inequality:

$$\dot{V}(\xi) \leq -aV(\xi) - bV^{\beta}(\xi) - dV^{\gamma}(\xi) + \varepsilon \tag{7}$$

where a, b, d, and ε are positive constants, $0 < \beta < 1$, and $1 < \gamma < 2$. Then, the nonlinear system (6) is SGPFS (for proof, please see Appendix B).

Remark 1 The Lyapunov finite-time stability theories could be divided into two categories. One is $\dot{V}(\xi) \leq -bV(\xi)^{\beta}$ proposed in [35]. The other is $\dot{V}(\xi) \leq -aV(\xi) - bV^{\beta}(\xi)$ in [36], which makes the system converge faster than before when the system states are at a distance of the equilibrium. Motivated by these theories, we propose a modified Lyapunov finite-time stability theory as (7). It is noted that the modified theory is a combined result: when the system states stay at a distance from the origin, $-dV^{\gamma}(\xi)$ dominates over $-aV(\xi) - bV^{\beta}(\xi)$ and, thus, guarantees fast convergence rate and when the state is closed to the equilibrium, the term $-aV(\xi) - bV^{\beta}(\xi)$ could guarantee fast convergence. Thus, the modified Lemma 1 could guarantee fast convergence rate and improve the tracking performance in the whole process compared with the theories before. In order to illustrate the performance of the modified theory, we compare four cases: $\dot{V} = -bV^{\beta}$; $\dot{V} = -aV - bV^{\beta}$; $\dot{V}(\xi) = -bV^{\beta}(\xi) - dV^{\gamma}(\xi)$; and $\dot{V}(\xi) = -aV(\xi) - bV^{\beta}(\xi) - dV^{\gamma}(\xi)$. The convergence performances are shown in Fig. 3, where the result of the modified theory could fast converge during the whole process.

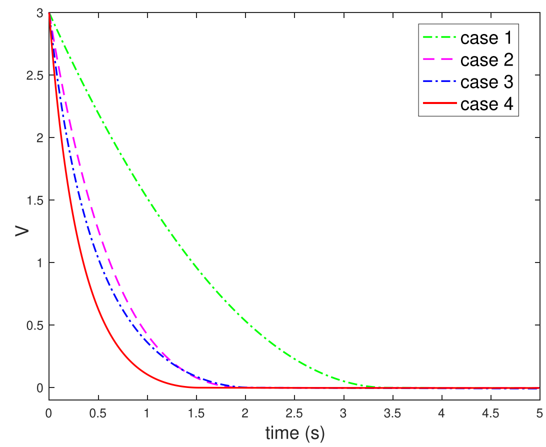

Fig. 3. Convergence performances of four cases.

Lemma 2 For $r_i \in R$, $i = 1, 2, \ldots, n$, and $0 < p_1 < 1$, the following inequality holds [52]:

$$(|r_1| + \cdots + |r_n|)^{p_1} \leq |r_1|^{p_1} + \cdots + |r_n|^{p_1}. \tag{8}$$

Lemma 3 Let r_i, $i = 1 \ldots n$ are all positive numbers, and $0 < p_2 < 2$, then the following inequality holds:

$$\left(\sum_{i=1}^{n} r_i^2\right)^{p_2} \leq \left(\sum_{i=1}^{n} r_i^{p_2}\right)^2 \leq n\left(\sum_{i=1}^{n} r_i^{2p_2}\right). \tag{9}$$

Lemma 4 Define the discontinuous projection as [53]

$$\dot{\hat{\theta}}_i = \mathrm{Proj}(\Gamma\upsilon) = \begin{cases} 0 & \text{if } \theta_i = \theta_{i\max} \text{ and } \upsilon > 0 \\ 0 & \text{if } \theta_i = \theta_{i\min} \text{ and } \upsilon < 0 \\ \Gamma\upsilon & \text{otherwise} \end{cases} \tag{10}$$

where Γ is any positive-definite diagonal matrices. The following properties should be guaranteed no matter what function of υ is developed.

1) $\Omega_{\theta_i} := \{\hat{\theta}_i \in \mathbb{R} \,|\, \theta_{i\min} \leq \hat{\theta}_i \leq \theta_{i\max}\ \ \forall t\}$, and
2) $\tilde{\theta}_i^T[\upsilon - \Gamma^{-1}\mathrm{Proj}(\Gamma\upsilon)] \leq 0\ \ \forall\upsilon$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ is the error between the true value θ_i and the estimated value $\hat{\theta}_i$ updated by adaptive law.

B. Rotational Subsystem Control

According to the rotational subsystem equation in (3), let $z_1 = \eta - \eta_d$ be the attitude tracking error in which $\eta_d = [\phi_d, \theta_d, \psi_d]^{\mathrm{T}}$ is the desired attitude. Then, we define a virtual control $\alpha_1(\eta)$ and we have

$$z_2 = \dot{\eta} - \alpha_1(\eta) \tag{11}$$

where the expression of $\alpha_1(\eta)$ is given as

$$\alpha_1(t) = \dot{\eta}_d - K_1 z_1 - K_2 z_1^{\beta_1} - K_3 z_1^{\gamma_1} \tag{12}$$

where $\eta_d = [\phi_d, \theta_d, \psi_d]^{\mathrm{T}}$ is the desired orientation angles, $0 < \beta_1 < 1$, and $1 < \gamma_1 < 3$. Then, submitting (11) and (12) into (3), we have

$$\begin{aligned}\dot{z}_1 &= z_2 + \alpha_1(t) - \dot{\eta}_d\\ J(\eta)\dot{z}_2 &= \tau - \tau_f - C(\eta, \dot{\eta})\dot{\eta} - J(\eta)\dot{\alpha}_1(t) + R_1^{\mathrm{T}}\Delta_1.\end{aligned} \tag{13}$$

For the dynamics described above, an AFFTC is given as

$$\begin{aligned}\tau &= \tau_1 + \tau_2 + \tau_3\\ \tau_1 &= -z_1 - H_1(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d)\hat{\theta}_1\\ \tau_2 &= -K_4 z_2 - K_5 z_2^{\beta_1} - K_6 z_2^{\gamma_1}\\ \tau_3 &= -S(\mathrm{diag}[\mathrm{sgn}(z_2)]G_1(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d))\\ \dot{\hat{\theta}}_1 &= \mathrm{Proj}(\Gamma_1 H_1^{\mathrm{T}}(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d) z_2)\end{aligned} \tag{14}$$

where $\Gamma_1 \in R^{9\times 9}$ is positive-definite diagonal matrix; $0 < \beta_1 < 1$; $1 < \gamma_1 < 3$; $\theta_1 = [I_{xx}, I_{yy}, I_{zz}, g_1, g_2, g_3, g_4, g_5, g_6]^{\mathrm{T}}$; $\hat{\theta}_1$ represents the estimated value updated by the adaptive law; The ideal control law of τ_3 is $\tau_3 = \mathrm{diag}[\mathrm{sgn}(z_2)]G_1$, which leads to severe chattering problem and may lead to instability in applications since the discontinuous function $\mathrm{sgn}(z_2)$. To overcome this problem, a continuous approximation function $S(\cdot)$ is proposed, and satisfies the following two conditions [54]:

$$\begin{aligned}&z_2^{\mathrm{T}} S(\mathrm{diag}[\mathrm{sgn}(z_2)]G_1) \geq 0\\ &z_2^{\mathrm{T}}[\mathrm{diag}[\mathrm{sgn}(z_2)]G_1 - S(\mathrm{diag}[\mathrm{sgn}(z_2)]G_1)] \leq \varepsilon_1\end{aligned} \tag{15}$$

where ε_1 is positive scale. The function $H_1(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d) \in \mathbb{R}^{3\times 9}$ is given as

$$H_1(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d)\theta_1 = -\tau_f - J(\eta)\dot{\alpha}_1(t) - C(\eta, \dot{\eta})\alpha_1(t) \tag{16}$$

and $G_1(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d) \in \mathbb{R}^3$ satisfies

$$\begin{aligned}G_1(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d) = |H_1(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d)\\ \times (\theta_{1\max} - \theta_{1\min})| + |R_1^{\mathrm{T}}|\bar{\Delta}_1\end{aligned} \tag{17}$$

where $|\cdot|$ are the absolute value of each component of vector.

THEOREM 1 Considering the rotational subsystem (13), all tracking signals of the closed-loop system are SGPFS, and the tracking errors z_1 and z_2 can fast converge to a small neighborhood around the origin and remain within the compact set $\Omega_{z_1 z_2}$ with AFFTC (14)

$$\Omega_{z_1 z_2} := \left\{ \left(\frac{1}{2} z_1^{\mathrm{T}} z_1 + \frac{1}{2} z_2^{\mathrm{T}} J(\eta) z_2 \right)^{\mu_1} \leq \frac{\varepsilon_1}{\bar{l}_2(1 - c_1)} \right\} \tag{18}$$

where $0 < \mu_1 < 1$; $0 < c_1 < 1$; $\bar{l}_2 = \min(2^{\mu_1} k_{2\min}, \frac{2^{\mu_1} k_{5\min}}{j_{\max}^{\mu_1}})$; $k_{m\min} = \min(k_{m1}, k_{m2}, k_{m3})$ with $i = 2, 5$.

PROOF Considering a Lyapunov function candidate as $V_1(t) = \frac{1}{2} z_1^{\mathrm{T}} z_1$. Taking its time derivative, we have

$$\begin{aligned}\dot{V}_1(t) &= z_1^{\mathrm{T}}\dot{z}_1 = z_1^{\mathrm{T}}(z_2 + \alpha_1(\eta) - \dot{\eta}_d)\\ &= z_1^{\mathrm{T}} z_2 - z_1^{\mathrm{T}} K_1 z_1 - z_1^{\mathrm{T}} K_2 z_1^{\beta_1} - z_1^{\mathrm{T}} K_3 z_1^{\gamma_1}\\ &\leq z_1^{\mathrm{T}} z_2 - k_{1\min}\sum_{i=1}^{3} z_{1i}^2 - k_{2\min}\sum_{i=1}^{3} |z_{1i}|^{\beta_1+1}\\ &\quad - k_{3\min}\sum_{i=1}^{3} |z_{1i}|^{\gamma_1+1}\\ &\leq z_1^{\mathrm{T}} z_2 - k_{1\min}\sum_{i=1}^{3} z_{1i}^2 - k_{2\min}\sum_{i=1}^{3} \left(z_{1i}^2\right)^{\mu_1}\\ &\quad - k_{3\min}\sum_{i=1}^{3} \left(z_{1i}^2\right)^{\mu_2}\end{aligned} \tag{19}$$

where $k_{m\min} = \min(k_{m1}, k_{m2}, k_{m3})$ with $m = 1, 2, 3$; $\mu_1 = \frac{\beta_1+1}{2}$ and $\mu_2 = \frac{\gamma_1+1}{2}$. Then, we consider the Lyapunov function candidate as

$$V_2(t) = V_1(t) + \frac{1}{2} z_2^{\mathrm{T}} J(\eta) z_2. \tag{20}$$

Differentiating (20) w.r.t. time leads to

$$\begin{aligned}\dot{V}_2(t) &= \dot{V}_1(t) + \frac{1}{2} z_2^{\mathrm{T}}\dot{J}(\eta) z_2 + z_2^{\mathrm{T}} J(\eta)\dot{z}_2\\ &= \dot{V}_1(t) + \frac{1}{2} z_2^{\mathrm{T}}\dot{J}(\eta) z_2 + z_2^{\mathrm{T}}(\tau - \tau_f - C(\eta, \dot{\eta})\dot{\eta}\\ &\quad - J(\eta)\dot{\alpha}_1(t) + R_1^{\mathrm{T}}\Delta_1)\\ &= \dot{V}_1(t) + \frac{1}{2} z_2^{\mathrm{T}}(\dot{J}(\eta) - 2C(\eta, \dot{\eta})) z_2\\ &\quad + z_2^{\mathrm{T}}(\tau - \tau_f - J(\eta)\dot{\alpha}_1(t)\\ &\quad - C(\eta, \dot{\eta})\alpha_1(t) + R_1^{\mathrm{T}}\Delta_1)\\ &= \dot{V}_1(t) + \frac{1}{2} z_2^{\mathrm{T}}(\dot{J}(\eta) - 2C(\eta, \dot{\eta})) z_2\\ &\quad + z_2^{\mathrm{T}}\left(\tau + H_1(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d)\theta_1 + R_1^{\mathrm{T}}\Delta_1\right).\end{aligned} \tag{21}$$

Submitting the AFFTC (14) and referring to Property 2) into (21) yields

$$\begin{aligned}\dot{V}_2(t) &= -z_1^{\mathrm{T}} K_1 z_1 - z_1^{\mathrm{T}} K_2 z_1^{\beta_1} - z_1^{\mathrm{T}} K_3 z_1^{\gamma_1} - z_2^{\mathrm{T}} K_4 z_2\\ &\quad - z_2^{\mathrm{T}} K_5 z_2^{\beta_1} - z_2^{\mathrm{T}} K_6 z_2^{\gamma_1}\\ &\quad + z_2^{\mathrm{T}}\left(\tau_3 + H_1(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d)\tilde{\theta}_1 + R_1^{\mathrm{T}}\Delta_1\right).\end{aligned} \tag{22}$$

From (17), we have

$$z_2^{\mathrm{T}}(\tau_3 + H_1\tilde{\theta}_1 + R_1^{\mathrm{T}}\Delta_1) \le z_2^{\mathrm{T}} \left[-S(\mathrm{diag}[\mathrm{sgn}(z_2)]G_1) + H_1\tilde{\theta}_1 + R_1^{\mathrm{T}}\Delta_1\right] \le z_2^{\mathrm{T}}[-S(\mathrm{diag}[\mathrm{sgn}(z_2)]G_1) + \mathrm{diag}[\mathrm{sgn}(z_2)]G_1] \le \varepsilon_1. \quad (23)$$

Substituting (23) to (22) leads to

$$\begin{aligned}\dot{V}_2(t) \le& -z_1^{\mathrm{T}}K_1z_1 - z_1^{\mathrm{T}}K_2z_1^{\beta_1} - z_1^{\mathrm{T}}K_3z_1^{\gamma_1} - z_2^{\mathrm{T}}K_4z_2 - z_2^{\mathrm{T}}K_5z_2^{\beta_1} - z_2^{\mathrm{T}}K_6z_2^{\gamma_1} + \varepsilon_1\\ \le& -k_{1\,\min}\sum_{i=1}^{3} z_{1i}^2 - k_{4\,\min}\sum_{i=1}^{3} z_{2i}^2 + \varepsilon_1 - k_{2\,\min}\sum_{i=1}^{3}|z_{1i}|^{\beta_1+1} - k_{5\,\min}\sum_{i=1}^{3}|z_{2i}|^{\beta_1+1}\\ &- k_{3\,\min}\sum_{i=1}^{3}|z_{1i}|^{\gamma_1+1} - k_{6\,\min}\sum_{i=1}^{3}|z_{2i}|^{\gamma_1+1}\\ \le& -k_{1\,\min}\sum_{i=1}^{3} z_{1i}^2 - \frac{k_{4\,\min}}{j_{\max}}\sum_{i=1}^{3} j_{\max}z_{2i}^2 + \varepsilon_1\\ &- k_{2\,\min}\sum_{i=1}^{3}(z_{1i}^2)^{\mu_1} - \frac{k_{5\,\min}}{j_{\max}^{\mu_1}}\sum_{i=1}^{3}\left(j_{\max}z_{2i}^2\right)^{\mu_1}\\ &- k_{3\,\min}\sum_{i=1}^{3}(z_{1i}^2)^{\mu_2} - \frac{k_{6\,\min}}{j_{\max}^{\mu_2}}\sum_{i=1}^{3}\left(j_{\max}z_{2i}^2\right)^{\mu_2}. \end{aligned} \quad (24)$$

Applying Lemma 2 and Lemma 3, we have

$$\begin{aligned}\dot{V}_2(t) \le& -\bar{l}_1\left(\frac{1}{2}\sum_{i=1}^{3} z_{1i}^2 + \frac{1}{2}\sum_{i=1}^{3} j_{\max}z_{2i}^2\right) - \bar{l}_2\left(\frac{1}{2}\sum_{i=1}^{3} z_{1i}^2 + \frac{1}{2}\sum_{i=1}^{3} j_{\max}z_{2i}^2\right)^{\mu_1}\\ &- \bar{l}_3\left(\frac{1}{2}\sum_{i=1}^{3} z_{1i}^2 + \frac{1}{2}\sum_{i=1}^{3} j_{\max}z_{2i}^2\right)^{\mu_2} + \varepsilon_1 \end{aligned} \quad (25)$$

where $\bar{l}_1 = \min(2k_{1\,\min}, \frac{2k_{4\,\min}}{j_{\max}})$; $\bar{l}_2 = \min(2^{\mu_1}k_{2\,\min}, \frac{2^{\mu_1}k_{5\,\min}}{j_{\max}^{\mu_1}})$ and $\bar{l}_3 = \min(\frac{2^{\mu_2}k_{3\,\min}}{3}, \frac{2^{\mu_2}k_{6\,\min}}{3j_{\max}^{\mu_2}})$. Property 1) indicates that $j_{\min}||z_2||^2 \le z_2^{\mathrm{T}}J(\eta)z_2 \le j_{\max}||z_2||^2$, and then we have

$$\begin{aligned}\dot{V}_2(t) \le& -\bar{l}_1\left(\frac{1}{2}\sum_{i=1}^{n} z_{1i}^2 + \frac{1}{2}z_2^TJ(\eta)z_2\right) - \bar{l}_2\left(\frac{1}{2}\sum_{i=1}^{n} z_{1i}^2 + \frac{1}{2}z_2^TJ(\eta)z_2\right)^{\mu_1}\\ &- \bar{l}_3\left(\frac{1}{2}\sum_{i=1}^{n} z_{1i}^2 + \frac{1}{2}z_2^TJ(\eta)z_2\right)^{\mu_2} + \varepsilon_1\\ \le& -\bar{l}_1V_2(t) - \bar{l}_2V_2^{\mu_1}(t) - \bar{l}_3V_2^{\mu_2}(t) + \varepsilon_1. \end{aligned} \quad (26)$$

According to Lemma 1, the tracking errors z_1 and z_2 could fast converge into a small neighborhood around the origin in finite time. This completes the proof. ■

C. Translational Subsystem Control

From the translational subsystem in (3), let $\bar{z}_1 = \xi - \xi_d$ as the position tracking errors where $\xi_d = [x_d, y_d, z_d]^{\mathrm{T}}$ is the desired position. We introduce a virtual control $\alpha_2(t)$ and define a second error variable as $\bar{z}_2 = \dot{\xi} - \alpha_2(\xi, \xi_d)$. Give the virtual control α_2 as

$$\alpha_2(t) = \dot{\xi}_d - \bar{K}_1\bar{z}_1 - \bar{K}_2\bar{z}_1^{\beta_2} - \bar{K}_3\bar{z}_1^{\gamma_2} \quad (27)$$

where $\bar{K}_1, \bar{K}_2$, and $\bar{K}_3$ are positive-definite diagonal matrices, $0 < \beta_2 < 1$ and $1 < \gamma_2 < 3$. The translational dynamics of ODFV can be described as

$$\begin{aligned}\dot{\bar{z}}_1 &= \bar{z}_2 + \alpha_2(t) - \dot{\xi}_d\\ \dot{\bar{z}}_2 &= \frac{1}{m}\left(R_{E2B}^Tu_T + F_g + H_2(\upsilon_b)\theta_2 + R_{E2B}^T\Delta_2\right) - \dot{\alpha}_2(t)\end{aligned} \quad (28)$$

where $\theta_2 = [d_1, d_2, d_3, d_4, d_5, d_6]^{\mathrm{T}}$ and $H_2(\upsilon_b)\theta_2 = -F_f$ with $H_2 \in \mathbb{R}^{3\times 6}$. We give the AIFTC and adaptive law as follows:

$$\begin{aligned}u_T &= u_{T1} + u_{T2} + u_{T3}\\ u_{T1} &= -R_{E2B}(F_g + H_2(\dot{\xi})\hat{\theta}_2 + m\bar{z}_1 - m\dot{\alpha}_2(t))\\ u_{T2} &= -mR_{E2B}(\bar{K}_4\bar{z}_2 + \bar{K}_5\bar{z}_2^{\beta_2} + \bar{K}_6\bar{z}_2^{\gamma_2})\\ u_{T3} &= -R_{E2B}S(\mathrm{diag}[\mathrm{sgn}(\bar{z}_2)]G_2(\dot{\xi}))\\ \dot{\hat{\theta}}_2 &= \mathrm{Proj}\left(\frac{1}{m}\Gamma_2H_2^{\mathrm{T}}(\dot{\xi})\bar{z}_2\right)\end{aligned} \quad (29)$$

where $\bar{K}_j$, $j = 3\ldots 6$ are positive-definite diagonal matrices; and $\hat{\theta}_2$ is the estimated parameter by adaptive law. $G_2(\dot{\xi})$ is given as

$$G_2(\dot{\xi}) = |H_2(\dot{\xi})(\theta_{2\max} - \theta_{2\,\min})| + |R_{E2B}^T|\bar{\Delta}_2 \quad (30)$$

where $|\cdot|$ are the absolute value of each component of vector.

THEOREM 2 We consider the translational subsystem (28). All tracking signals of the closed-loop system are SGPFS and the tracking errors $\bar{z}_1$ and $\bar{z}_2$ can fast converge into an arbitrary small neighborhood $\bar{\Omega}_{\bar{z}_1\bar{z}_2}$ around the region with AFFTC (29)

$$\bar{\Omega}_{\bar{z}_1\bar{z}_2} := \left\{\left(\frac{1}{2}\bar{z}_1^{\mathrm{T}}\bar{z}_1 + \frac{1}{2}\bar{z}_2^{\mathrm{T}}\bar{z}_2\right)^{\mu_3} \le \frac{\varepsilon_2}{m\bar{b}_2(1-c_2)}\right\} \quad (31)$$

where $0 < \mu_3 < 1$; $0 < c_2 < 1$; $\bar{b}_2 = \min(2^{\mu_3}\bar{k}_{2\,\min}, 2^{\mu_3}\bar{k}_{5_{\min}})$; and $\bar{k}_{m\min} = \min(\bar{k}_{m1}, \bar{k}_{m2}, \bar{k}_{m3})$ with $m = 2, 5$ and ε_2 is positive constant.

The proof of this theorem is omitted for clarity and conciseness. If readers are interested in its proof, please refer to the stability analysis in the rotational subsystem.

TABLE I
Parameters for Simulations

Parameters	Values	Parameters	Values
m	2.878 kg	g_1, g_3, g_5	0.2
I_{xx}	0.04463 kgm^2	g_2, g_4, g_6	0.3
I_{yy}	0.04463 kgm^2	d_1, d_3, d_5	1.0
I_{zz}	0.04463 kgm^2	d_2, d_4, d_6	1.0

TABLE II
Attitude Controller Parameters for Simulations

Parameters	Values	Parameters	Values
K_1	diag(0.3, 0.3, 0.3)	$I_{xx\min}$	0.03 kgm^2
K_2	diag(0.4, 0.4, 0.4)	$I_{yy\min}$	0.03 kgm^2
K_3	diag(0.7, 0.7, 0.7)	$I_{zz\min}$	0.06 kgm^2
K_4	diag(0.3, 0.3, 0.3)	$\hat{I}_{xx}(0)$	0.06 kgm^2
K_5	diag(0.4, 0.4, 0.4)	$\hat{I}_{yy}(0)$	0.06 kgm^2
K_6	diag(0.7, 0.7, 0.7)	$\hat{I}_{zz}(0)$	0.10 kgm^2
$I_{xx\max}$	0.08 kgm^2	$g_{\max}$	0.50
$I_{yy\max}$	0.08 kgm^2	$g_{\min}$	0.10
$I_{zz\max}$	0.12 kgm^2	$\bar{g}_{\max}$	0.70
β_1	0.5	$\bar{g}_{\min}$	0.10
γ_1	1.5	$\hat{g}_i(0)$	0.30

IV. Simulation Results

In this section, comparative simulations are conducted to demonstrate the superiority of the proposed AFFTC over the conventional adaptive finite-time control (AFTC). To test the full controllability over six degrees of freedom (6DOF), the vehicle is commanded to track the desired rotational and translational trajectories, simultaneously. Simulation results are shown below. To establish the basic dynamics, some parameters for simulation are given in Table I.

A. Rotational Subsystem Simulation Results

The initial attitude of the vehicle is $\eta(0) = [0, 0, 0]^{\mathrm{T}}$. The desired attitude is commanded as $\eta_d = [\cos(0.6t - 1), 0.8\cos(0.5t - 0.5), \cos(0.55t)]^{\mathrm{T}}$. According to the dynamics of the rotational subsystem (3) and AFFTC (14), the controller parameters are given in Table II, where $g_{i\min} \leq g_i \leq g_{i\max}$ $(i = 1, 3, 5)$; $\bar{g}_{i\min} \leq g_i \leq \bar{g}_{i\max}$ $(i = 2, 4, 6)$ hold. The adaptive gain matrix is designed as $\Gamma_1 = \mathrm{diag}(0.01, 0.01, 0.01, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1)$. The external disturbance Δ_1 is $\Delta_1 = [0.03\sin(1.2t), 0.05\sin(1.3t), 0.02\sin(t)]^{\mathrm{T}}$. We first give the attitude tracking performance with the proposed AFFTC in Figs. 5–7, where the AFTC is developed based on the conventional Lyapunov finite-time stability theory $\dot{V}(\xi) \leq -aV(\xi) - bV^{\beta}(\xi)$, and this comparative controller is given as

$$\begin{aligned}
\tau' &= \tau_1' + \tau_2' + \tau_3' \\
\tau_1' &= -z_1' - H_1(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d)\hat{\theta}_1 \\
\tau_2' &= -K_4 z_2' - K_5 (z_2')^{\beta_1} \\
\tau_3' &= -S(\mathrm{diag}[\mathrm{sgn}(z_2')]G_1(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d)) \\
\dot{\hat{\theta}}_1 &= \mathrm{Proj}(\Gamma_1 H_1^{\mathrm{T}}(\eta, \dot{\eta}, \eta_d, \dot{\eta}_d) z_2')
\end{aligned} \tag{32}$$

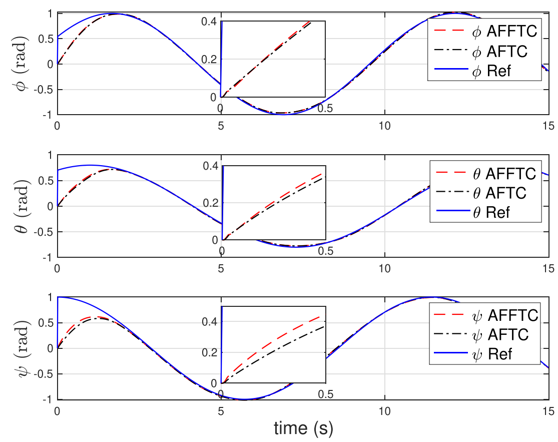

Fig. 4. Tracking performance of attitude.

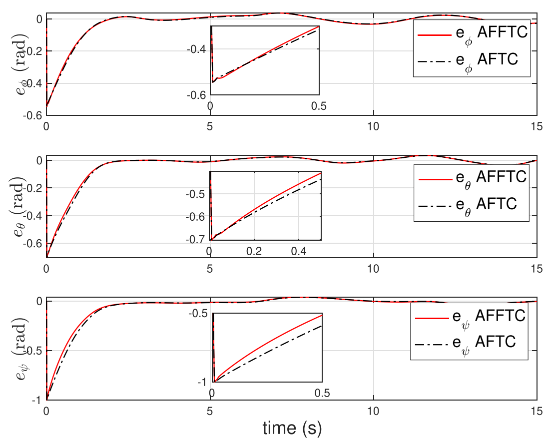

Fig. 5. Tracking errors of attitude.

where $z_2' = \dot{\eta} - \alpha_1'$ with $\alpha_1' = \dot{\eta}_d - K_1 z_1' - K_2 (z_1')^{\beta_1}$. Note that the AFTC removes all γ_1 powers, and to ensure rationality, the controller parameters in AFTC should be consistent with the AFFTC as in Table II. For the continuous function $S(\cdot)$, in this article, we define its function as

$$S(\mathrm{diag}[\mathrm{sgn}(z_2)]G_1) = \mathrm{diag}[G_1]\mathrm{sat}\left(\mathrm{diag}[G_1]\frac{z_2}{4\varepsilon_1}\right) \tag{33}$$

where $\mathrm{sat}(\cdot)$ is a continuous saturation function of each component to obtain a continuous approximation of the ideal discontinuous action $\mathrm{diag}[\mathrm{sgn}(z_2)]G_1$.

Comparative simulation results are shown in Figs. 4–6. Under the conditions of parametric uncertainties and external disturbances, both controllers could well track the desired attitude and the tracking errors z_1 and z_2 converge into a small neighborhood around the origin in finite time without chattering problem. From Figs. 4 and 5, although the improvements of tracking performances are not obvious, the tracking errors of AFFTC could faster converge when the system states at a distance of the origin. Due to the Assumption 1 to avoid the singularity problem, the attitude commands are bounded within $(-\frac{\pi}{2}, \frac{\pi}{2})$. Thus, the terms of γ_1 powers are unable to dominate over linear terms and β_1 powers terms when the system states are at a close range of

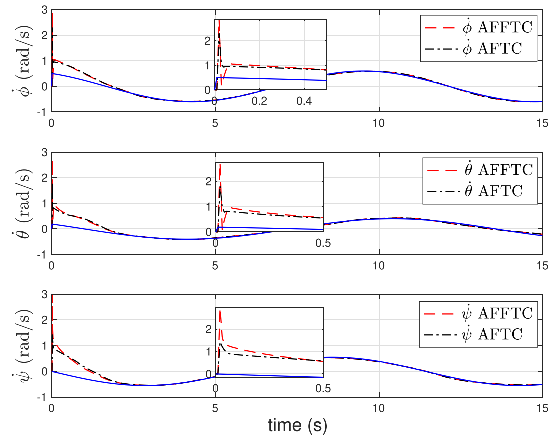

Fig. 6. Tracking performance of angular velocity.

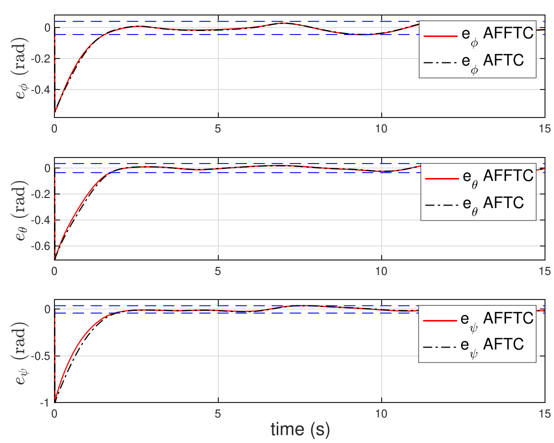

Fig. 7. Tracking errors under variations of ideal parameters.

the equilibrium. In the translational simulation results, we can better show the improvements of the proposed AFFTC.

To further verify the effectiveness and robustness of the AFFTC, we consider about 30% variation to the ideal parameters I_v and g_i as $I_{xx} = 0.04463 + 0.013\sin(1.3t)$, $I_{yy} = 0.04463 + 0.013\sin(t)$, $I_{zz} = 0.08844 + 0.013\sin(1.7t)$, $g_1 = 0.2 + 0.07\sin(t)$, $g_2 = 0.5$, $g_3 = 0.2 + 0.06\sin(1.5t)$, $g_4 = 0.5 + 0.15\sin(0.8t)$, $g_5 = 0.2$, and $g_6 = 0.5 + 0.1\sin(2t)$. As shown in Fig. 7, the tracking errors are all enveloped in a small region around the origin, which illustrates the robustness and effectiveness of the proposed AFFTC.

B. Translational Subsystem Simulation Results

The tilting quadcopter is commanded to track the desired position and attitude trajectories, simultaneously. This section would show the simulation results of the position tracking performance. The initial position is set as $\xi(0) = [0, 0, 0]^{\mathrm{T}}$, and the desired position trajectories are set as $\xi_d = [5\sin(0.6t), 6\sin(0.4t), 5\sin(0.7t)]^{\mathrm{T}}$. The parameters needed for the AFFTC are given in Table III. The coefficients in the aerodynamic damping force satisfies $d_{\min} \le d_i \le d_{\max}$ $(i = 1, \dots, 6)$, and the adaptive gain matrix is set

TABLE III
Position Controller Parameters for Simulations

Parameters	Values	Parameters	Values
$\bar K_1$	$\mathrm{diag}(0.8, 0.8, 0.8)$	$d_{\max}$	1.5
$\bar K_2$	$\mathrm{diag}(0.5, 0.5, 0.5)$	$d_{\min}$	0.5
$\bar K_3$	$\mathrm{diag}(0.5, 0.5, 0.5)$	$\hat d_i(0)$	0.6
$\bar K_4$	$\mathrm{diag}(0.8, 0.8, 0.8)$	β_2	0.5
$\bar K_5$	$\mathrm{diag}(0.5, 0.5, 0.5)$	γ_2	1.5
$\bar K_6$	$\mathrm{diag}(0.5, 0.5, 0.5)$		

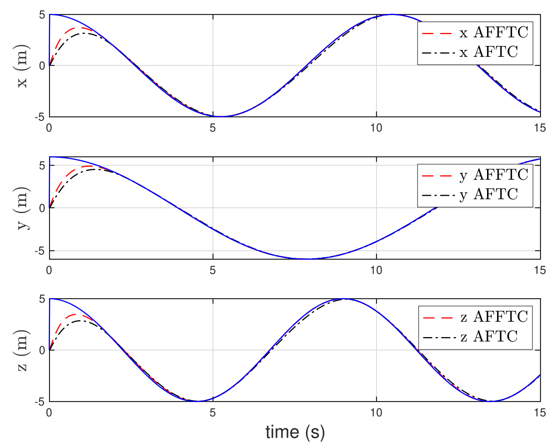

Fig. 8. Tracking performance of position.

as $\Gamma_2 = \mathrm{diag}(1, 1, 1, 1, 1, 1)$. The controller of the conventional AFTC is given as follows:

$$
\begin{aligned}
u'_T &= u'_{T1} + u'_{T2} + u'_{T3} \\
u'_{T1} &= -R_{E2B}(F_g + H_2(\dot\xi)\hat\theta_2 + m\dot{\bar z}'_1 - m\dot\alpha'_2 \\
u'_{T2} &= -mR_{E2B}(\bar K_4 \bar z'_2 + \bar K_5(\bar z'_2)^{\beta_2}) \\
u'_{T3} &= -R_{E2B}S(\mathrm{diag}[\mathrm{sgn}(\bar z'_2)]G_2(\dot\xi)) \\
\dot{\hat\theta}_2 &= \mathrm{Proj}\left(\frac{1}{m}\Gamma_2 H_2^{\mathrm{T}}(\dot\xi)\bar z'_2\right),
\end{aligned} \tag{34}
$$

where $\bar z'_2 = \dot\xi - \alpha_2$ with $\alpha'_2 = \dot\xi_d - \bar K_1 \bar z'_1 - \bar K_2(\bar z'_1)^{\beta_2}$. The tracking performance of ODFV are given as follows.

The simulation results illustrate the superiority of the tilting quadcopter over the conventional quadcopter. The tilting quadcoper has the ability to control translational and rotational movements independently. From Fig. 8, it can be obvious that the tracking performance of the AFFTC is improved dramatically compared with the conventional controller. The AFFTC can obtain fast convergence rate both at a distance or at a close range of the equilibrium as shown in Fig. 9 without chattering problem. From Fig. 10, the tilting quadcopter can fast converge to the desired velocities in the whole process compared with the tracking performance of the AFTC. Thus, the AFFTC indeed improves the tracking performance and obtains a fast convergence rate in the whole tracking process.

We further add 50% variations to the ideal parameters d_i as $d_1 = 1 + 0.5\sin(t)$, $d_2 = 1 + 0.5\sin(1.5t)$, $d_3 = 1$, $d_4 = 1 + 0.5\sin(0.6t)$, $d_5 = 1 + 0.5\sin(2t)$, $d_6 = 1$. From Fig. 11, the tracking errors are all enveloped in a small region

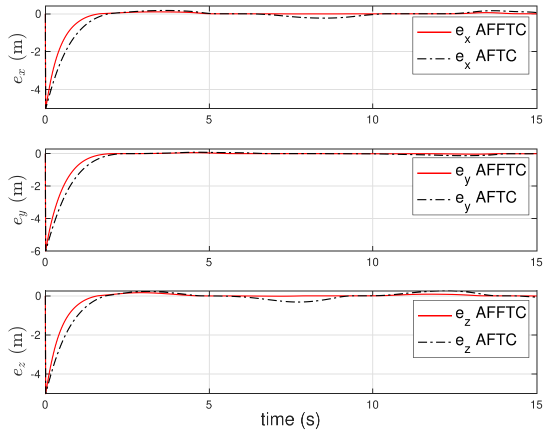

Fig. 9. Tracking error of position.

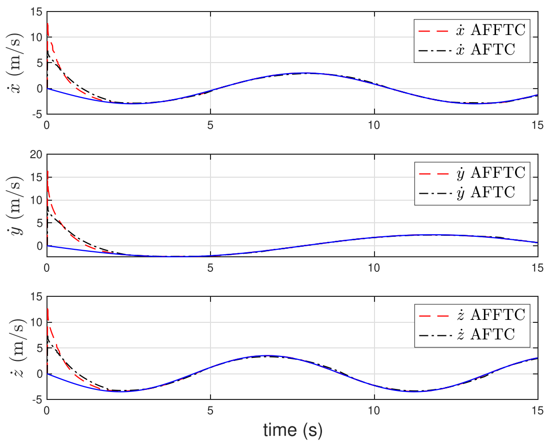

Fig. 10. Tracking performance of velocity.

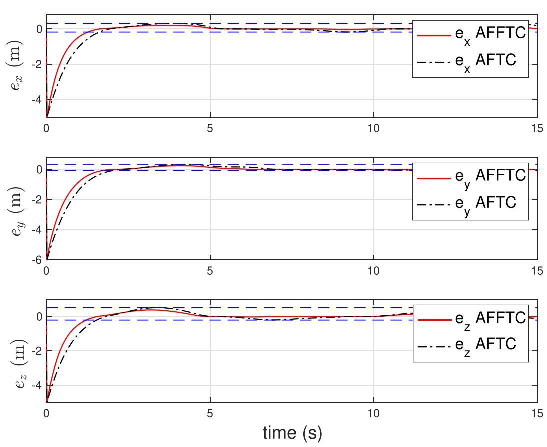

Fig. 11. Tracking errors under variations of ideal parameters d_i.

around the origin, which demonstrates the effectiveness and robustness of the proposed controller.

The above simulation results have demonstrated that the control approach proposed in this article is effective and robust, even with parametric uncertainties and external disturbances. AFFTC can achieve fast convergence both at a distance and at a close range of the equilibrium compared with the conventional controller. The tilting quadcopter can track the translational and rotational movements independently, which indicates that this novel vehicle can be applied in many applications in the future.

V. CONCLUSION

In this article, we first propose a novel concept of the tilting quadcopter with full controllability over 6 DOF. The propellers can tilt along corresponding axes with large angles, and, thus, improve the reliability and agility of the vehicle. A complete dynamic model is developed with parametric uncertainties and external disturbances. Then, an AFFTC based on the modified Lyapunov finite-time stability theory is proposed, which can obtain fast convergence rate both at a distance and at a close range of the equilibrium without chattering problem. Finally, comparative simulation results have demonstrated the robustness and effectiveness of the proposed AFFTC. In future work, we will extend our approach to practical situations, such as control constraints and fault-tolerant problem.

APPENDIX A

The matrices $J(\eta)$ and $C(\eta, \dot{\eta})$ are given as follows:

$$J(\eta) = [j_{ij}], \quad C(\eta, \dot{\eta}) = [c_{ij}], \quad (i, j = 1, 2, 3) \tag{35}$$

with

$$\begin{aligned}
&j_{11} = I_{xx};\ j_{12} = 0;\ j_{13} = -I_{xx}\mathrm{s}\theta \\
&j_{21} = 0;\ j_{22} = I_{yy}\mathrm{c}^2\phi + I_{zz}\mathrm{s}^2\phi \\
&j_{23} = \left(I_{yy} - I_{zz}\right)\mathrm{c}\phi\mathrm{s}\phi\mathrm{c}\theta \\
&j_{31} = -I_{xx}\mathrm{s}\theta;\ \mathrm{j}_{32} = \left(\mathrm{I}_{yy} - \mathrm{I}_{zz}\right)\mathrm{c}\phi\mathrm{s}\phi\mathrm{c}\theta \\
&j_{33} = I_{xx}\mathrm{s}^2\theta + \mathrm{I}_{yy}\mathrm{s}^2\phi\mathrm{c}^2\theta + \mathrm{I}_{zz}\mathrm{c}^2\phi\mathrm{c}^2\theta \\
&c_{11} = 0;\ c_{12} = \frac{1}{2}\left(I_{yy} - I_{zz}\right)\left(\dot{\theta}\mathrm{s}2\phi - \dot{\psi}\mathrm{c}2\phi\mathrm{c}\theta\right) \\
&c_{13} = -I_{xx}\dot{\theta}\mathrm{c}\theta - \frac{1}{2}\left(I_{yy} - I_{zz}\right)\left(\dot{\theta}\mathrm{c}2\phi\mathrm{c}\theta + \dot{\psi}\mathrm{s}2\phi\mathrm{c}^2\theta\right) \\
&c_{21} = \frac{1}{2}I_{xx}\dot{\psi}\mathrm{c}\theta \\
&c_{22} = -\left(I_{yy} - I_{zz}\right)\dot{\phi}\mathrm{s}2\phi + \frac{1}{2}\left(I_{yy} - I_{zz}\right)\dot{\psi}\mathrm{c}\phi\mathrm{s}\phi\mathrm{s}\theta \\
&c_{23} = \left(I_{yy} - I_{zz}\right)\left(\dot{\phi}\mathrm{c}2\phi\mathrm{c}\theta - \frac{1}{2}\dot{\theta}\mathrm{s}\phi\mathrm{c}\phi\mathrm{s}\theta\right) + \frac{1}{2}I_{xx}\dot{\phi}\mathrm{c}\theta \\
&\qquad - \left(I_{xx} - I_{yy}\mathrm{s}^2\phi - I_{zz}\mathrm{c}^2\phi\right)\dot{\psi}\mathrm{c}\theta\mathrm{s}\theta \\
&c_{31} = -I_{xx}\dot{\theta}\mathrm{c}\theta \\
&c_{32} = \left(I_{yy} - I_{zz}\right)\left(\dot{\phi}\mathrm{c}2\phi\mathrm{c}\theta - \dot{\theta}\mathrm{s}\phi\mathrm{c}\phi\mathrm{s}\theta\right) \\
&c_{33} = I_{xx}\dot{\theta}\mathrm{s}2\theta + \left(I_{yy} - I_{zz}\right)\dot{\phi}\mathrm{s}2\phi\mathrm{c}^2\theta \\
&\qquad - \left(I_{yy}\mathrm{s}^2\phi + I_{zz}\mathrm{c}^2\phi\right)\dot{\theta}\mathrm{s}2\theta.
\end{aligned} \tag{36}$$

All elements in (35) are all linear in I_{xx}, I_{yy}, and I_{zz}. We can abstract the matrix $Y(\eta, \dot{\eta}, \ddot{\eta})$ as follows, if we select all the parametric uncertainties as $\vartheta = [I_{xx}, I_{yy}, I_{zz}]^{\mathrm{T}}$

$$Y(\eta, \dot{\eta}, \ddot{\eta}) = [y_{i,j}]\ (i, j = 1, 2, 3) \tag{37}$$

with

$$
\begin{aligned}
y_{11} &= \ddot{\phi} - \mathrm{s}\theta\ddot{\psi} - \dot{\psi}\mathrm{c}\theta\dot{\theta}\\
y_{12} &= \dot{\theta}\mathrm{c}\phi\mathrm{s}\phi\dot{\theta} - \dot{\psi}\mathrm{c}2\phi\mathrm{c}\theta\dot{\theta} - \dot{\psi}\mathrm{c}\phi\mathrm{s}\phi\mathrm{c}^2\theta\dot{\psi}\\
y_{13} &= -\dot{\theta}\mathrm{c}\phi\mathrm{s}\phi\dot{\theta} + \dot{\psi}\mathrm{c}2\phi\mathrm{c}\theta\dot{\theta} + \dot{\psi}\mathrm{c}\phi\mathrm{s}\phi\mathrm{c}^2\theta\dot{\psi}\\
y_{21} &= \dot{\psi}\mathrm{c}\theta\dot{\phi} - \dot{\psi}\mathrm{s}\theta\mathrm{c}\theta\dot{\psi}\\
y_{22} &= \mathrm{c}^2\phi\ddot{\theta} + \mathrm{c}\phi\mathrm{s}\phi\mathrm{c}\theta\ddot{\psi} + \dot{\psi}\mathrm{c}2\phi\mathrm{c}\theta\dot{\phi} - \dot{\phi}\mathrm{s}2\phi\dot{\theta}\\
&\quad + \dot{\psi}\mathrm{s}^2\phi\mathrm{c}\theta\mathrm{s}\theta\dot{\psi}\\
y_{23} &= \mathrm{s}^2\phi\ddot{\theta} - \mathrm{c}\phi\mathrm{s}\phi\mathrm{c}\theta\ddot{\psi} - \dot{\psi}\mathrm{c}2\phi\mathrm{c}\theta\dot{\phi} + \dot{\phi}\mathrm{s}2\phi\dot{\theta}\\
&\quad + \dot{\psi}\mathrm{c}^2\phi\mathrm{s}\theta\mathrm{c}\theta\dot{\psi}\\
y_{31} &= -\mathrm{s}\theta\ddot{\phi} + \mathrm{s}^2\theta\ddot{\psi} - \dot{\theta}\mathrm{c}\theta\dot{\phi} + \dot{\theta}\mathrm{s}2\theta\dot{\psi}\\
y_{32} &= \mathrm{c}\phi\mathrm{s}\phi\mathrm{c}\theta\ddot{\theta} + \mathrm{s}^2\phi\mathrm{c}^2\theta\ddot{\psi} + \dot{\psi}\mathrm{c}^2\theta\mathrm{s}2\phi\dot{\phi} - \dot{\theta}\mathrm{c}\phi\mathrm{s}\phi\mathrm{s}\theta\dot{\theta}\\
&\quad + \dot{\phi}\mathrm{c}2\phi\mathrm{c}\theta\dot{\theta} - \dot{\theta}\mathrm{s}^2\phi\mathrm{s}2\theta\dot{\psi}\\
y_{33} &= -\mathrm{c}\phi\mathrm{s}\phi\mathrm{c}\theta\ddot{\theta} + \mathrm{c}^2\phi\mathrm{c}^2\theta\ddot{\psi} - \dot{\psi}\mathrm{c}^2\theta\mathrm{s}2\phi\dot{\phi}\\
&\quad + \dot{\theta}\mathrm{c}\phi\mathrm{s}\phi\mathrm{s}\theta\dot{\theta} - \dot{\phi}\mathrm{c}2\phi\mathrm{c}\theta\dot{\theta} - \dot{\theta}\mathrm{c}^2\phi\mathrm{s}2\theta\dot{\psi}. \qquad (38)
\end{aligned}
$$

APPENDIX B

From (7), for $\forall\, 0 < c < 1$, we have

$$
\begin{aligned}
\dot{V}(\xi) &\le -aV(\xi) - bcV^{\mu} - dV^{\gamma}(\xi)\\
&\quad - (b(1-c)V^{\mu}(\xi) - \varepsilon). \qquad (39)
\end{aligned}
$$

We consider the following two cases: $\Omega = \{\xi | V^{\mu}(\xi) \le \frac{\varepsilon}{b(1-c)}\}$ and $\bar{\Omega} = \{\xi | V^{\mu}(\xi) > \frac{\varepsilon}{b(1-c)}\}$.

Case 1: If $\xi \in \bar{\Omega}$, we have

$$
\begin{aligned}
\dot{V}(\xi) &\le -aV(\xi) - bcV^{\mu}(\xi) - dV^{\gamma}(\xi)\\
&\le -aV(\xi) - bcV^{\mu}(\xi). \qquad (40)
\end{aligned}
$$

By dividing both sides of the (40) to term $V^{\mu}(\xi)$ yields

$$
\int_{t_0}^{t} \frac{V^{-\mu}(\xi)}{bc + aV^{1-\mu}(\xi)} \mathrm{dV} \le -\int_{t_0}^{t} \mathrm{dt}. \qquad (41)
$$

Then, we have

$$
T = t_0 + \frac{1}{a(1-\mu)} \ln\left(\frac{bc + aV^{1-\mu}(t_0)}{bc}\right). \qquad (42)
$$

This means that $\xi \in \Omega$ for all $t \ge T$.

Case 2: If $\xi \in \Omega$, according to case 1, the system state ξ does not exceed the set Ω.

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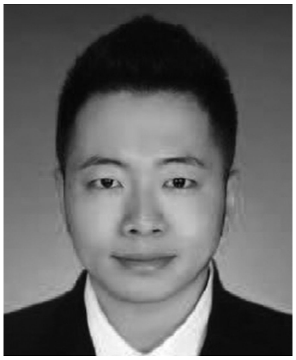

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