Applied Mechanics and Materials
ISSN: 1662-7482, Vol. 619, pp 267-272
doi:10.4028/www.scientific.net/AMM.619.267

Submitted: 2014-06-23
Accepted: 2014-06-23
Online: 2014-08-18

Experiment of the State Variable Feedback for a Quadrotor

Veeravat Faramee[a] and Viboon Sangveraphunsiri[b]

Department of Mechanical Engineering, Chulalongkorn University
254 Phayathai Road, Patumwan, Bangkok 10330, Thailand

[a]veeravat.F@Student.chula.ac.th, [b]viboon.s@eng.chula.ac.th

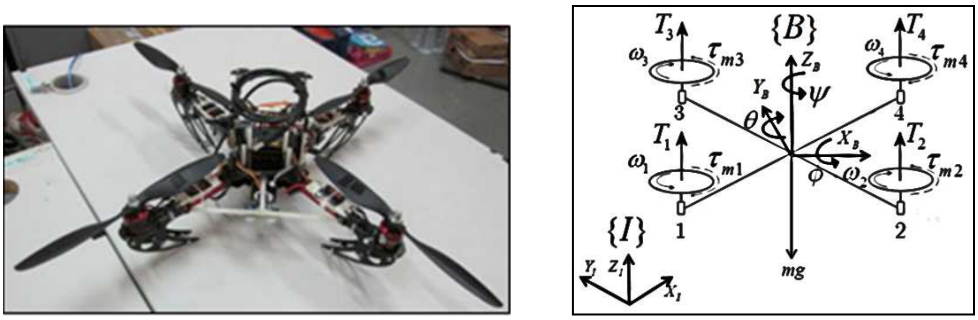
Keywords: Quadrotor; Multiple Input Multiple Output; State Space Control.

Abstract. This work is the design of the robust attitude stabilization of a quadrotor. The dynamics model of the quadrotor is normally non-linear and it is difficult to obtain an accurate dynamic model. The linearized model is obtained based on data from experiments. The Multiple Input Multiple Output (MIMO) state variable feedback control is implemented to stabilize the system. The dynamic model based on approximated torques (roll, pitch, and yaw) as inputs and the dynamic model based on rotational speeds of the four rotors are used for design the controller. The experimental results from both dynamic model has very similar characteristic and perform much better than the conventional controller.

Introduction

Unmanned Aerial Vehicles (UAVs) [1] are remotely piloted or self-piloted aircraft. They are used by the military and civilians for many tasks such as providing assistance, exploring resources, taking photograph. The quadrotor usually has two pairs of propellers, which works in contra-rotation in order to create torque refuting at hovering. Each pair of propellers are arranged as shown in Fig. 1a.

The motions of quadrotor are defined in the cumulative rotation of angles ϕ (Roll) in x-axis, θ (Pitch) in y-axis, and φ (Yaw) in z-axis. Fig. 1b shows the quadrotor and its rotors motions. Roll moment is accomplished by changing the 1, 2 and 3, 4 propeller's speed. Pitch moment is accomplished by changing the 1, 3 and 2, 4 propeller's speed. Yaw moment is more subtle, as it results from the difference in the counter-torque between each pair of propellers. Thus, increasing or decreasing the four propeller's speeds together generates vertical motion.



(a) (b)

Figure 1 Quadrotor description (a) experiment platform and (b) motion description.

This paper presents the controller that capable for the attitude stabilization of the quadrotor. The Multiple Input Multiple Output (MIMO) state variable feedback control [2], [6] is studied in detail to meet the specification of the stabilization. Using Lagrange-Euler formalism [3], [4], the non-linear dynamic model can be written as shown in Eq. (1). With results of simulations and experiments, the linearized dynamic model based on approximated torques (roll, pitch, and yaw) as inputs is shown in Eq. (2), and the linearized dynamic model with the rotational speeds of the four rotors as inputs can be obtained from Eq. (1) by neglect the non-linear terms. And because we can obtain experimentally the relationship between each of the motor speed and its input voltage so that the linearized dynamic model can be written in form of Eq. (3) where the inputs of the system are input voltage of each motors. Eq. (2) and (3) are used for design the controller. By assuming that the system torques and speed of rotors have linear relationship. Both dynamic models should performed with similar performance.

System Modeling

The first step, before control development, is try to obtain an adequate dynamic modeling, which is derived using Lagrange-Euler formalism [3]. Under the following assumptions:

- The structure is supposed to be rigid.
- The structure is supposed symmetrical.
- The center of mass and the body fixed frame origin are assumed to coincide.
- The propellers are supposed rigid.
- The thrust and drag are proportional to the square of the propeller speed.

The equation of motion can be written as

$$\begin{aligned}
\ddot{\phi} &= \frac{\boxed{\dot{\psi}\dot{\theta}\left(I_{yy}-I_{zz}\right)}}{I_{xx}} + \frac{\boxed{J_r\dot{\theta}\left(\omega_1+\omega_3-\omega_2-\omega_4\right)}}{I_{xx}} + \frac{bl\left(\left(\omega_3^2+\omega_4^2\right)-\left(\omega_1^2+\omega_2^2\right)\right)}{I_{xx}} \\
\ddot{\theta} &= \frac{\boxed{\dot{\psi}\dot{\phi}\left(I_{zz}-I_{xx}\right)}}{I_{yy}} + \frac{\boxed{J_r\dot{\phi}\left(\omega_2+\omega_4-\omega_1-\omega_3\right)}}{I_{yy}} + \frac{bl\left(\left(\omega_1^2+\omega_3^2\right)-\left(\omega_2^2+\omega_4^2\right)\right)}{I_{yy}} \\
\ddot{\psi} &= \frac{\boxed{\dot{\theta}\dot{\phi}\left(I_{xx}-I_{yy}\right)}}{I_{zz}} + \frac{d\left(\left(\omega_2^2+\omega_3^2\right)-\left(\omega_1^2+\omega_4^2\right)\right)}{I_{zz}}
\end{aligned} \tag{1}$$

where ϕ is roll angle, θ is pitch angle, ψ is yaw angle, I_{xx}, I_{yy}, I_{zz} are inertia about x, y, z axis, b is thrust coefficient, d is drag coefficient, ω_i is angular velocity each motor, J_r is rotor inertia, m is mass, l is arm length.

With some simulations based on Eq. (1), it can be shown that the non-linear terms, the terms in shown in the blocks of Eq. (1), are very small (less than 1 to 10,000) compare to the linear terms. So, the linearized dynamic model can be obtained by neglect the non-linear terms as results shown in Eq. (2) and Eq. (3). Eq. (3) the inputs of the system are converted to input voltage or duty cycle of the motors by using the relation as shown in Eq. (4) which are obtained experimentally.

$$\begin{aligned}
\ddot{\phi} &= \frac{\tau_x}{I_{xx}} \\
\ddot{\theta} &= \frac{\tau_y}{I_{yy}} \\
\ddot{\psi} &= \frac{\tau_z}{I_{zz}}
\end{aligned} \tag{2}$$

$$\begin{aligned}
\ddot{\phi} &= \frac{bl}{I_{xx}} K_{DCtow^2}\left(DC_3+DC_4-DC_1-DC_2\right) \\
\ddot{\theta} &= \frac{bl}{I_{yy}} K_{DCtow^2}\left(DC_1+DC_3-DC_2-DC_4\right) \\
\ddot{\psi} &= \frac{d}{I_{zz}} K_{DCtow^2}\left(DC_2+DC_3-DC_1-DC_4\right)
\end{aligned} \tag{3}$$

where $\tau_{x,y,z}$ are torques acting on the quadrotor, K_{DCtow^2} is constants obtained from Eq. (4) or can be called as the duty cycle to speed coefficient. The command DC_i is duty cycle instructing to each rotor.

From the experimental data, relation between the input voltage or duty circle and the rotor speed can be obtained as following:

$$\begin{aligned}
\omega_1^2 &= 1.3\times10^4 DC_1 - 7.5\times10^5 \\
\omega_2^2 &= 1.3\times10^4 DC_2 - 7.3\times10^5 \\
\omega_3^2 &= 1.3\times10^4 DC_3 - 7.4\times10^5 \\
\omega_4^2 &= 1.3\times10^4 DC_1 - 7.4\times10^5
\end{aligned} \tag{4}$$

Due to the rotors arrangement, the bias are canceled and will not appear in Eq. (3)

The dynamics model in Eq. (2), (3), can be rewritten in a state space description [5] as following:

The state space description for Eq. (2) and Eq. (3)

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix} \begin{bmatrix} \phi \\ \theta \\ \psi \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ 125&0&0 \\ 0&125&0 \\ 0&0&72 \end{bmatrix} \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix} \begin{bmatrix} \phi \\ \theta \\ \psi \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ -3.7&-3.7&3.7&3.7 \\ 3.7&-3.7&3.7&-3.7 \\ -1&1&1&-1 \end{bmatrix} \begin{bmatrix} DC_1 \\ DC_2 \\ DC_3 \\ DC_4 \end{bmatrix} \tag{6}$$

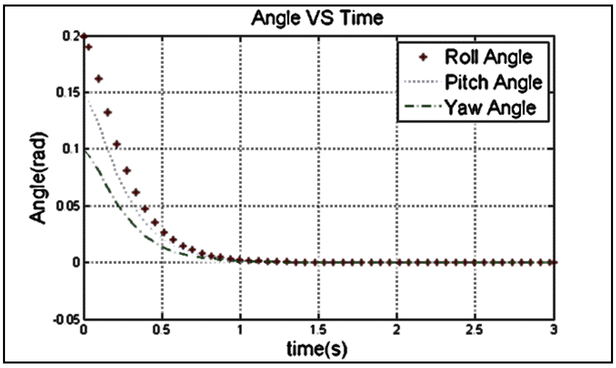
Controller

The model of the quadrotor is a MIMO system (Multiple Input Multiple Output system) and therefore a traditional controller, such as PID controller (Proportional Integral Derivative), would require quite a lot of time for manual tuning. The MIMO state variable [5], [7] is used instead based of the linearized dynamic model using system parameters, the duty cycle to speed coefficients, obtained experimentally. The tuning of the controller gains are far more convenient compare to the PID tuning.

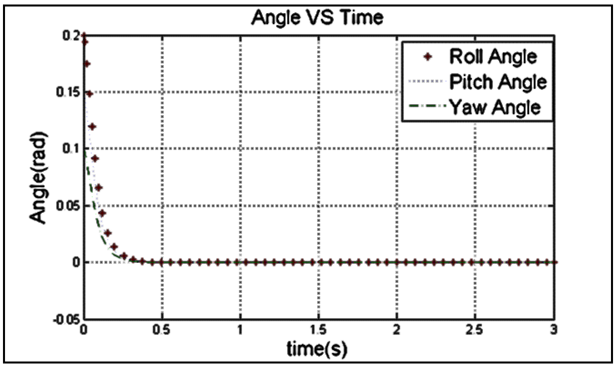
Experiment and Result

The result from simulation is shown in Fig.2a, 2b. The quadrotor can reach the desired stability level within about 1 second. According to simulation of the 2 controllers, reaction of control system, without the duty cycle to speed coefficient, reached the desired level within about 1 second, which is longer than control system with the duty cycle to speed coefficient.

The entire closed loop system of the 2 controllers shown in Fig. 3a, 3b for the dynamic model of Eq. (2) and Eq. (3) respectively. In order to stabilize the quadrotor motion, the output of the controller are torques about 3-axis as shown in Fig. 3a and the duty cycle of the four motors as shown in Fig. 3b.

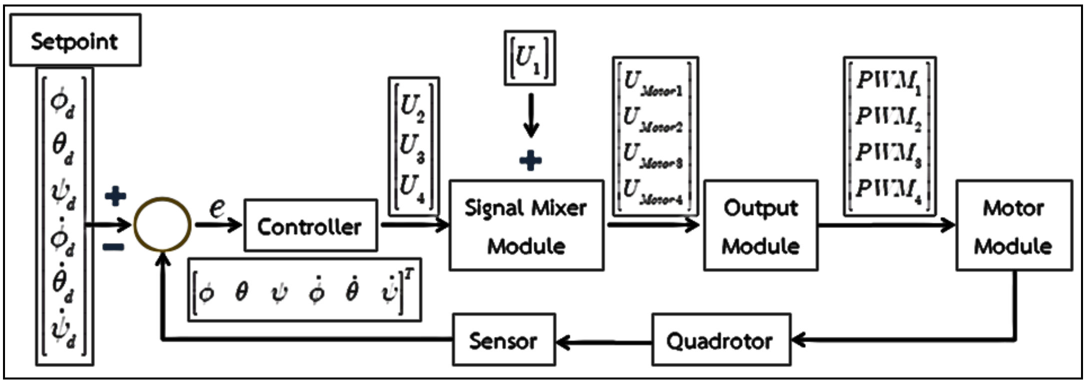


(a)

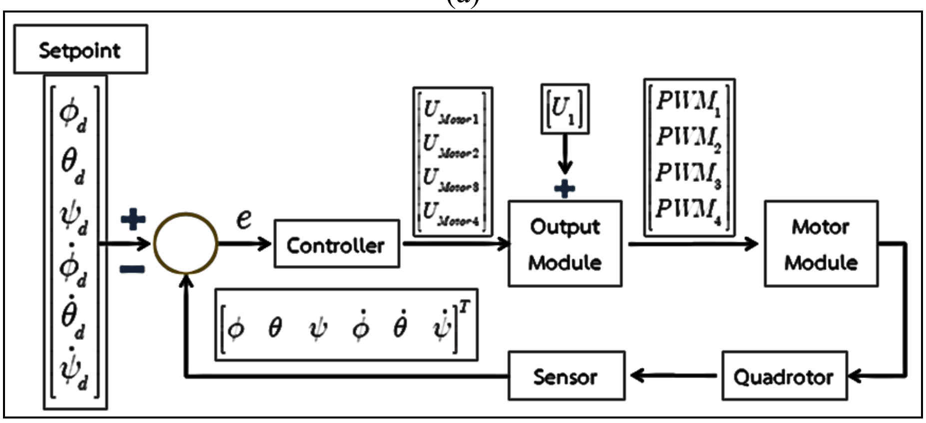


(b)

Figure 2 Simulation results with initial conditions (a) state space control which not utilizes duty cycle to speed coefficient (b) state space control which utilizes duty cycle to speed coefficient.



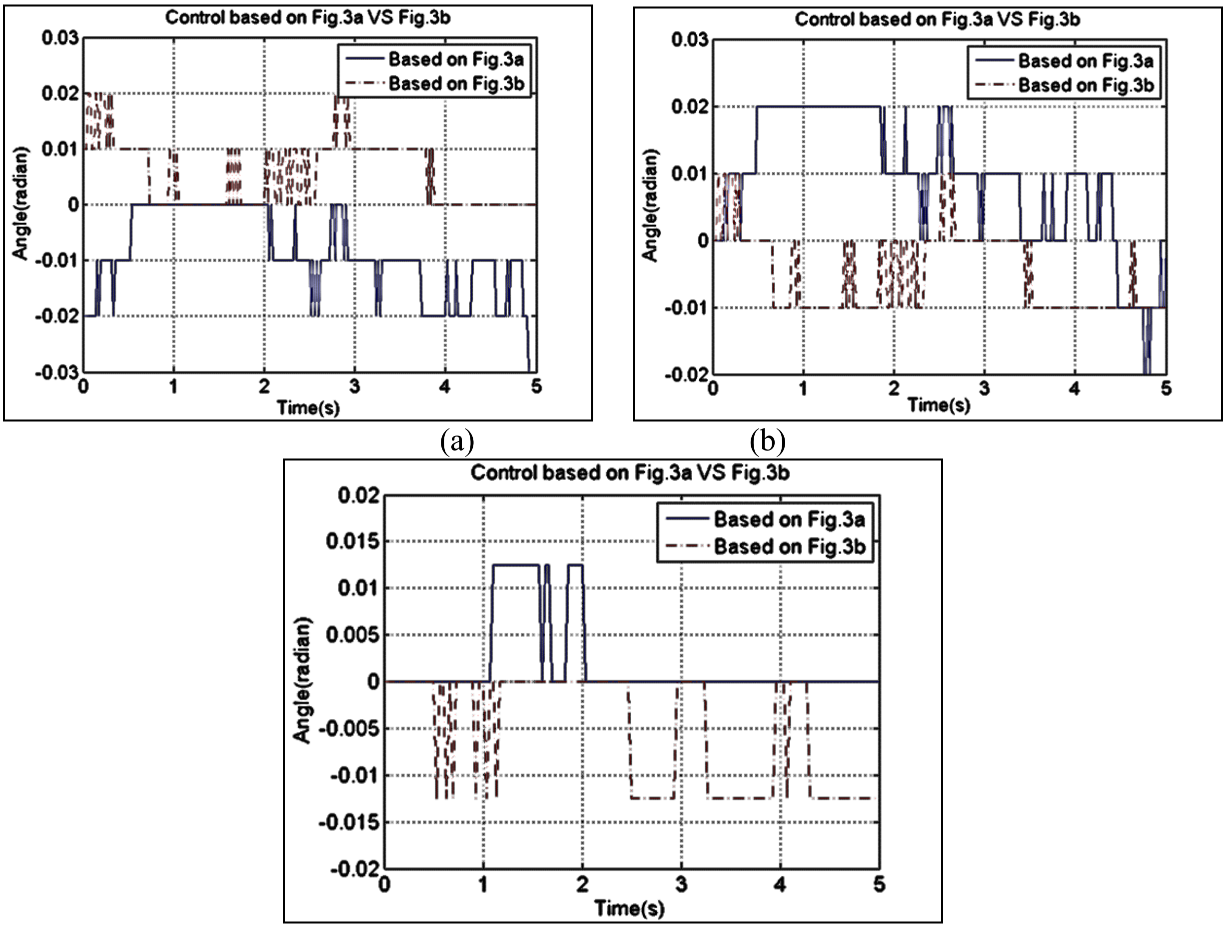
(a)



(b)

Figure 3 The block diagram of (a) state space control which not utilizes duty cycle to speed coefficient. (b) state space control which utilizes duty cycle to speed coefficient.

Fig 5. shows how to generate disturbances. The quadrotor can be maintained stability in all axes as shown in Figure 4a, 4b, 4c, which show the response of the system due to different disturbances with different durations. According to the experiments, it can be shown that the controller tuning for the MIMO state variable feedback control based on both dynamic models are much easier for fine tuning compared to the convention PID controller.



(a) (b)

(c)

Figure 4 Experiment: Attitude controller to maintain stabilization of the quadrotor with adding disturbance (a) roll angle, (b) pitch angle, and (c) yaw angle.

(a) (b) (c)

Figure 5 Quadrotor Platform test with disturbance (a) 1s. (b) 2s. (c) 3s. Respectively.

Summary

In this paper, we presented the controller design for the robust attitude stabilization of a quadrotor. The state variable feedback control is used for design the controllers for stabilizing the system based on two types of linearized models. The linearized models are obtained from experimental data so that they are accurate enough to be used for design the controllers. Experiments are conducted by simulation and real-time testing for a quadrotor platform at hovering. As it can be seen from the experimental results, the ability to control the orientation angles of system using these controllers are very promising. Our next goal is to enhance the control with position controller and to develop a fully autonomous vehicle.

Acknowledgement

This research is supported by National Research University Project, Office of Higher Education Commission (WCU-013-HR-57) and Regional Center of Robotics Technology, Chulalongkorn University.

References

[1] UAVs. New world vista: Air and space for 21st century. Human systems and biotechnology systems, 7.0:17-18, 1997.

[2] Pero Castillo, Rogelio Lozano and Alejandro E. Dzul (2005). Modelling and Control of Mini-Flying, ISBN: 185233957, Springer-Verlag London.

[3] S.Bouabdallah, “Design and control of quadrotors with application to autonomous flying,” Ph.D. dissertation, EPFL, 2006.

[4] Thorhallur Tomas Buchholz and Dagur Gretarsson (2009). Construction of a Four Rotor Helicopter Control System, Master’s Thesis, Technical University of Denmark.

[5] Viboon Sangveraphunsiri. Control of Dynamic System, 3rd Edition, Chulapress, Bangkok, 2013.

[6] Roman Czyba, “Attitude Stabilization of an Indoor Quadrotor”, EMAV, 2009.

[7] Sang-hyun Lee, Seung Hoon Kang and Youdan Kim, “Trajectory Tracking Control of Quadrotor UAV,” in *International Conference on Control, Autonomous and Systems,* (KINTEX,Gyeonggi-do,Korea), Oct.26-29, 2011.