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A study on the numerical model of non-equilibrium sediment transport in unsteady flow

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ABSTRACT: A fully coupled one-dimensional mobile-bed river model for the condition of unsteady flow and non-equilibrium sediment transport with looped network system is created. The governing equations are as follows: continuity, motion, conservation of material in suspension, conservation of bed-material, sediment transport formula and roughness equation. The above equations are solved simultaneously using the Preissmann implicit scheme. Manning roughness coefficient with the bed form (ripple and dune) considered by van Rijn method is calculated at each time step. Applying this fully coupled sediment transport model to Belley reservoir of Rhone river, total trap efficiency (=0.4) gives reasonably good result comparing with the measurement (=0.49).

1. INTRODUCTION

Belley reservoir is located at the upstream part of Rhone river in France. The flushing of Verbois reservoir and Chancy-Pougny reservoir located at the upstream of Belley reservoir is operated every 3 years to evacuate the sediment deposits in these reservoirs. Because of this flushing, 600,000 m^3 of sediments are deposited at Belley reservoir after the measurement of river bed level. (result of flushing in 1990)

The length of Belley reservoir is about 18 km and the upstream reach of 4.5 km is natural river. Artificial canal with the length of 13.5 km is constructed to the downstream of hydraulic power station. (Fig. 1)

The flow is divided into two parts due to island and submerged dike in Cressin reservoir. Because of the velocity difference in these two canals, the sediment transport rates are quite different. The confluence and divergence problem is included in this research because of this reason. Taking into account the interaction of stream flow, sediment transport and bed forms, the effective roughness is used to compute the Chezy or Manning roughness coefficient by the method of van Rijn. (van Rijn, 1984)

As a result of measurement in 1987, 90 % of suspended material is in the range 0-200 μm, D_{50} is in the range 15-25 μm. Bed load, suspended load, settling, pick-up and effective roughness are considered. Van Rijn formula (van Rijn, 1984) is used for the sediment transport of bed load and suspended load.

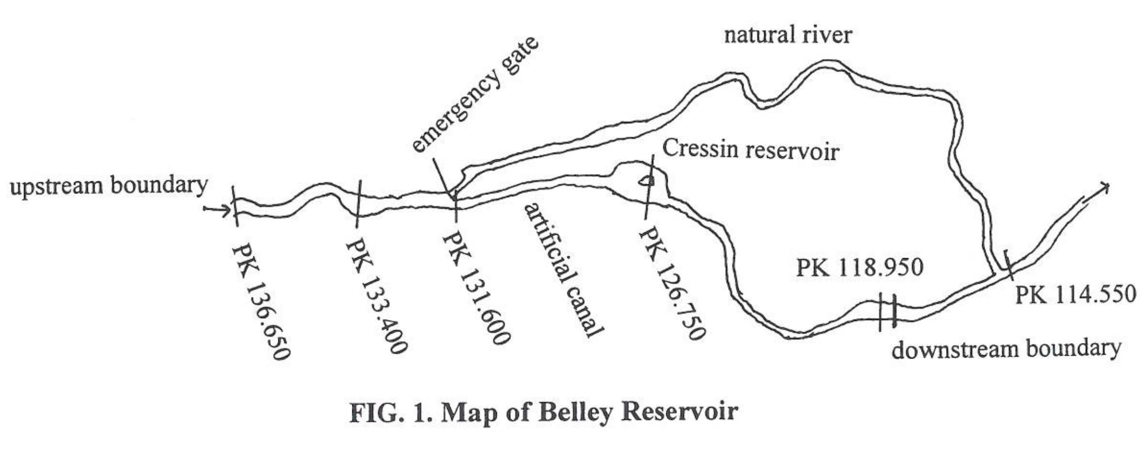


FIG. 1. Map of Belley Reservoir

Fig. 1.5 **AMÉNAGEMENT DU HAUT-RHÔNE**

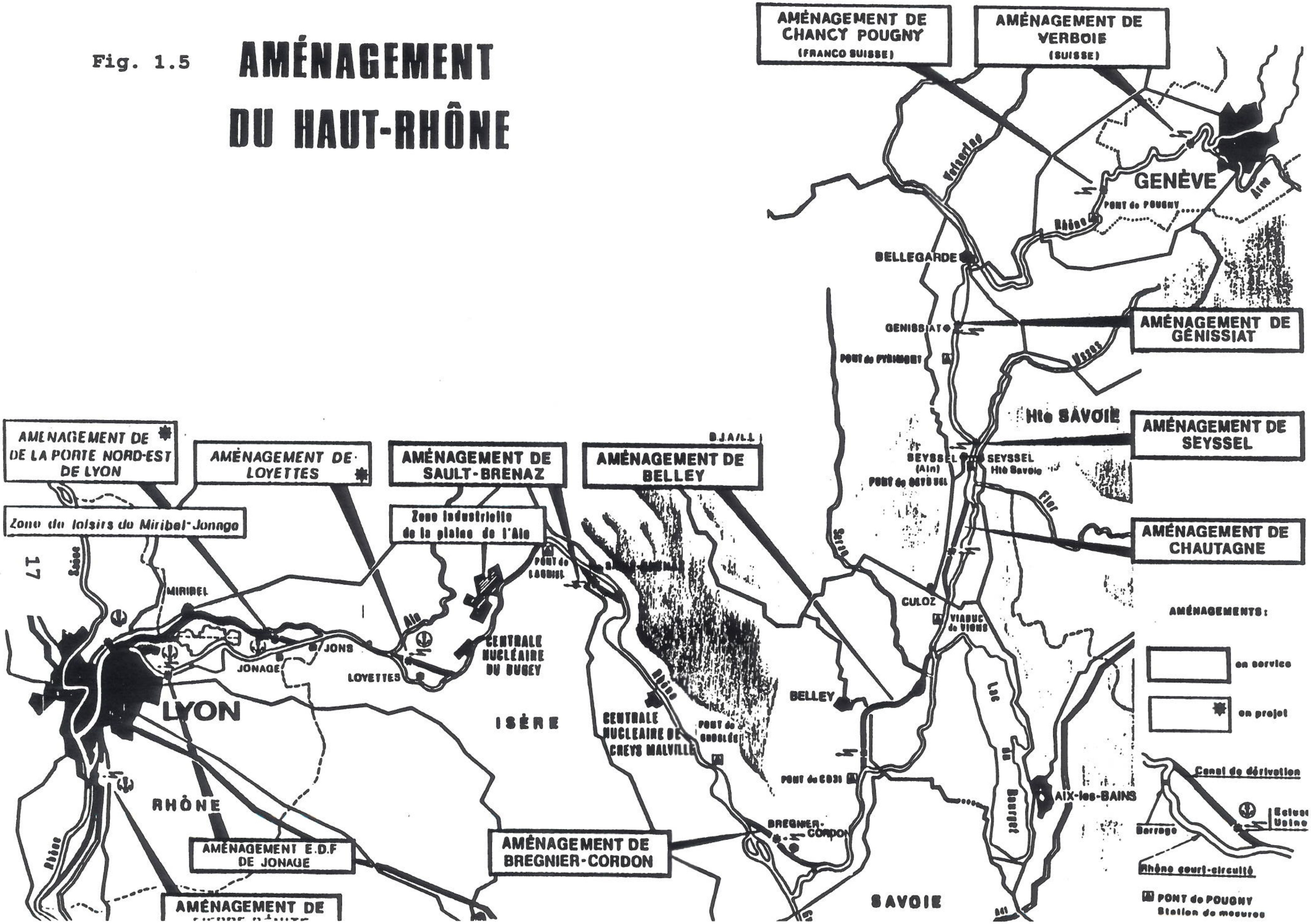
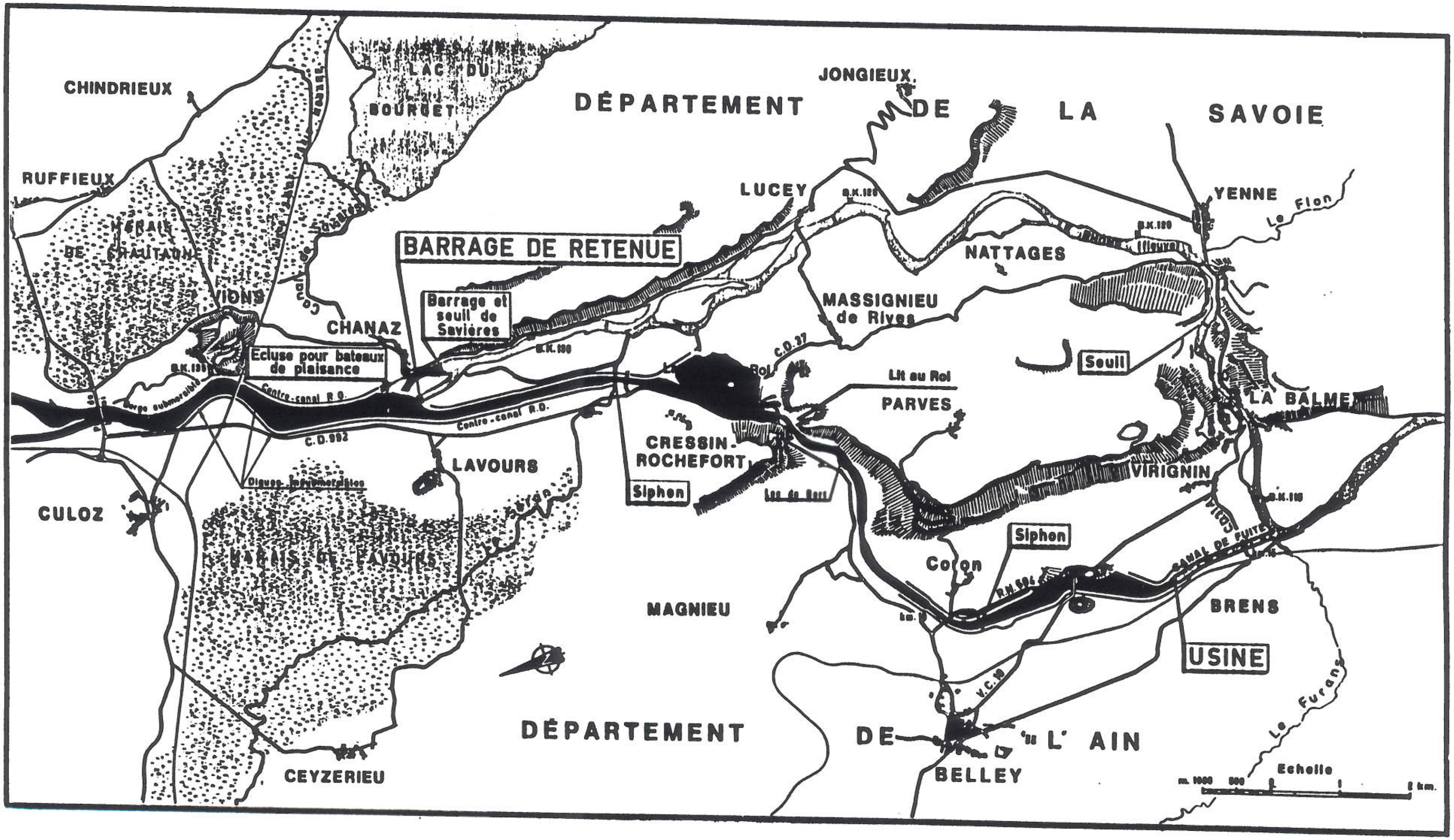


Fig. 1.6 **PLAN D'ENSEMBLE DE L'AMENAGEMENT**



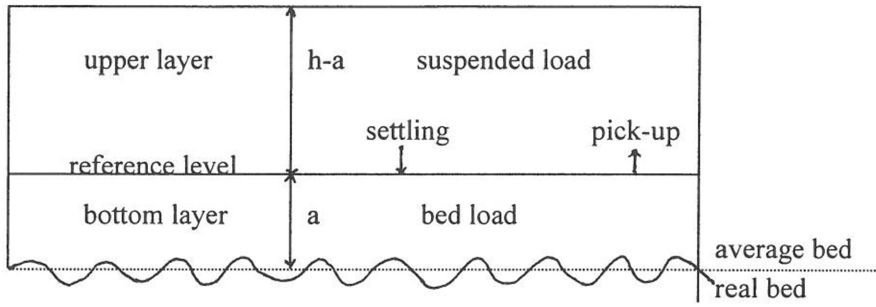


FIG. 2. Definition Sketch in the Sediment Transport Model

2. THEORY

2.1 Description of Sediment Transport Model

A phenomenon considered in this model is given in the following figure. (FIG. 2)

2.2 Governing Equations

The one-dimensional conservation equations for the sediment transport can be described by the following primary four equations:

Fluid continuity equation:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = B\frac{\partial z}{\partial t} + \frac{\partial Q}{\partial x} = q \tag{1}$$

Fluid motion equation:

$$\frac{\partial z}{\partial t} + \frac{1}{gA}\frac{\partial}{\partial x}\left(\beta\frac{Q^2}{A}\right) + \frac{Q^2}{K^2} + \frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{Ke}{2g}\frac{\partial (Q/A)^2}{\partial x} = 0 \tag{2}$$

Suspended material conservation equation:

$$\frac{\partial CA}{\partial t} + \frac{\partial Qs}{\partial x} = S - qC \tag{3}$$

Bed material conservation equation:

$$(1-p)\frac{\partial Ab}{\partial t} + \frac{\partial Qb}{\partial x} + S = 0 \tag{4}$$

in which x = streamwise coordinate; t = time; A = wetted cross-sectional area; Q = discharge; Z = water-surface elevation above a datum; B = flow width; q = lateral flow; g = gravitational acceleration; β = momentum correction coefficient; K = conveyance; Ke = coefficient of expansion-contraction; C = average sediment concentration; Qs = volumetric suspended load; S = sediment flux between bottom layer and waterstream; Ab = bottom layer cross-section; Qb = bed load.

In addition to the above primary four equations, the van Rijn formula (part III, 1984) is used as sediment transport formula for suspended load and bed load as follows:

$$Qs = 0.012\ [(V-Vcr)/\{(s-1)g\ D_{50}\}^{0.5}]^{2.4} D_{50} D_*^{-0.6}\ VBh^{-0.2} \tag{5}$$

$$Qb = 0.005\ [(V-Vcr)/\{(s-1)g\ D_{50}\}^{0.5}]^{2.4} D_{50}^{\ 1.2} VB \tag{6}$$

in which s = specific density; V = mean flow velocity; Vcr = critical mean flow velocity based on Shield's criterion; D_{50} = median grain size of bed material; D_* = particle diameter; h = flow depth.

Roughness coefficient due to friction resistance uses Manning roughness coefficient derived from overall Chezy coefficient considering grain roughness and bed form roughness:

$$C' = 18 \log (12R / ks) \quad \text{or} \quad n = R^{1/6} / C' \tag{7}$$

in which C' = Chezy roughness coefficient; R = hydraulic radius; ks = effective roughness; n = Manning roughness coefficient.

The sediment flux between bottom layer and waterstream are given by Armanini and Di Silvio (1988) as follows:

$$S = (Qse - Qs) / L_* \tag{8}$$

in which Qse = suspended load in equilibrium conditions; Qs = actual suspended load; L_* = characteristic length which depends on the flow characteristic and sediment size.

In summary, a one-dimensional morphological system is described by the following seven equations:

A. Fluid continuity equation B. Fluid motion equation C. Suspended material conservation equation
D. Bed material conservation equation E. Sediment transport equation (Bed load and Suspended load)
F. Alluvial roughness equation G. Sediment flux equation between bottom layer and water stream

2.3 Discretization

The above seven equations for fully coupled non-equilibrium sediment transport model are discretized using the Preissmann scheme of implicit finite difference. This scheme use the following approximations to the derivatives:

$$\frac{\partial f}{\partial t} = \frac{1}{\Delta t} [\psi \Delta f_{i+1} + (1-\psi) \Delta f_i] \tag{9}$$

$$\frac{\partial f}{\partial x} = \frac{1}{\Delta x} [(f_{i+1} + \theta\Delta f_{i+1}) - (f_i + \theta\Delta f_i)] \tag{10}$$

$$f(x,t) = \psi (f_{i+1} + \theta\Delta f_{i+1}) + (1-\psi)(f_i + \theta\Delta f_i) \tag{11}$$

in which i and n = gird point, θ = the weighting factors for time, ψ = the weighting factors for space.

In applications of the Preissmann scheme, it is supposed that all the function f(Z, Q, Zb, C) in the discretized algebraic equations are known at time level $n\Delta t$ and are differentiable with respect to Z, Zb, Q and C. Using a Taylor series expansion, the finite difference approximation leads to a system of four algebraic equations for every pair of points (i, i+1). One can obtain the linearized system for a pair of adjacent points (i, i+1).

2.4 Solution for Algebraic System and Boundary Condition

The system of four algebraic equations is solved for all computational points by the double sweep method which is often used to solve the St. Venant equations in fixed bed modeling.

Suppose that the discharge and concentration variations for upstream section i become the linear function of Z_i, Zb_i and C_i. It has the following relationship:

$$\Delta Q_i = F_i \Delta Z_i + G_i \Delta Zb_i + H_i \Delta C_i + K_i \tag{12}$$

$$\Delta C_i = F_j \Delta Z_i + G_j \Delta Zb_i + H_j \Delta Q_i + K_j \tag{13}$$

in which F_i, F_j, G_i, G_j, H_i, H_j, K_i and K_j are known coefficients for the given time.

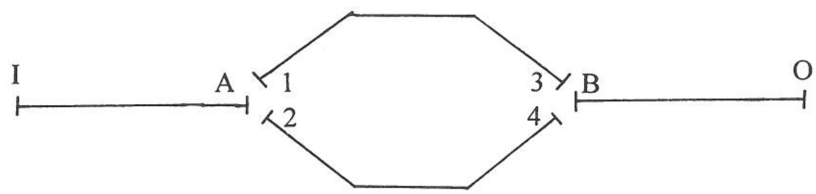


FIG. 3. Looped Network

Using the above relationship, ΔQ_i, ΔZ_i, ΔC_i and ΔZb_i can be eliminated from the four algebraic equations. The following relationship is obtained:

$$\Delta Q_{i+1} = F_{i+1}\, \Delta Z_{i+1} + G_{i+1}\, \Delta Zb_{i+1} + H_{i+1}\, \Delta C_{i+1} + K_{i+1} \tag{14}$$

$$\Delta C_{i+1} = F_{j+1}\, \Delta Z_{i+1} + G_{j+1}\, \Delta Zb_{i+1} + H_{j+1}\, \Delta Q_{i+1} + K_{j+1} \tag{15}$$

The procedure can be applied from upstream to downstream for N computational points. A system of 4 (N-1) equations with 4N unknowns is obtained. If 4 boundary conditions are given, 2 for i = 1 and 2 for i = N, all coefficients $(F, G, H, K)_{i+1,\, j+1}$ can be computed by forward sweep. Unknown value (ΔZ_{i+1}, ΔZb_{i+1}, ΔC_{i+1}, ΔQ_{i+1}) can be computed for all sections by backward sweep.

2.5 Looped Network with Internal Boundary Condition

The looped network has the points of confluence and divergence of tributaries or canals. Double sweep method is applied for the problem of confluence and divergence because of Cressin reservoir.
For divergence (A12), the following compatibility condition is used: (FIG. 3)

$$Z_A = Z_1 = Z_2,\ Zb_A = Zb_1 = Zb_2,\ C_A = C_1 = C_2,\ Q_A = Q_1 + Q_2 \tag{16}$$

For confluence (B34), the following compatibility condition is used:

$$Z_B = Z_3 = Z_4,\ Zb_B = Zb_3 = Zb_4,\ C_3Q_3 + C_4Q_4 = C_BQ_B,\ Q_B = Q_3 + Q_4 \tag{17}$$

3. APPLICATION

The simulated results of mobile bed unsteady flow model are compared with the measurement using the Manning roughness coefficient, n. To estimate n value for mobile bed, the soil conservation service (SCS) method is used as a basic n (=0.02) for initial value. After the model calculate n value with van Rijn formula as a basic n and additional n is added to basic n considering channel irregularity. (French, 1986) The variation of water level calculated by the model gives good result comparing with the measurement. (FIG.4)
The size distribution for bed material and suspended load are:

Bed material: D_{16} = 0.034 mm, D_{50} = 0.150 mm, D_{84} = 0.288 mm, D_{90} = 0.375 mm
Suspended load: D_{16} = 3.4 μm, D_{50} = 11.5 μm, D_{84} = 28.0 μm, D_{90} = 36.0 μm

Using Δt = 5 minute and D = 11.5 μm for the condition of Q = 700 m^3/s, Z = 233.93 m, Zb = 255 m and varying C, the concentration variation along the Belley reservoir shows as follows: (FIG. 5)

The flushing operation in 1990 is lasted about 4 days. The discharge during 4 days changes with minimum Q = 512 m^3/s and maximum Q = 990 m^3/s. During flushing the concentration variation is observed with minimum C = 0.7 g/l and maximum C = 9 g/l. The concentration variation for 5 μm, 11.5 μm, 20 μm, and 30 μm are also given in the following figure. (FIG.6)

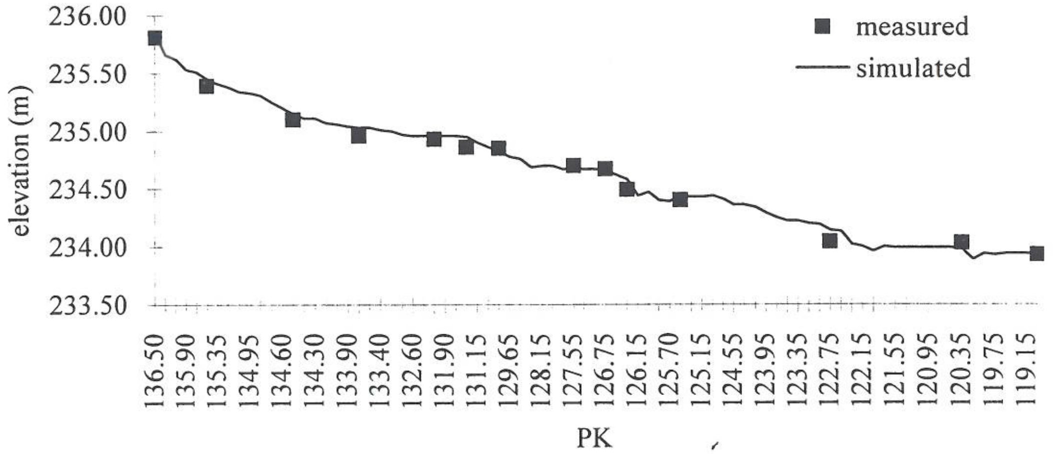


FIG. 4. Comparison of Water Surface Level

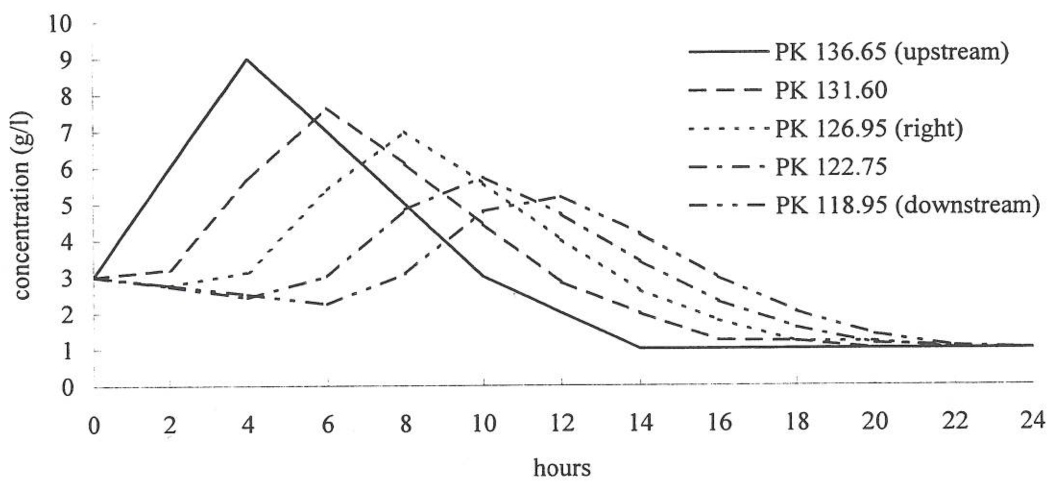


FIG. 5. Concentration Variation

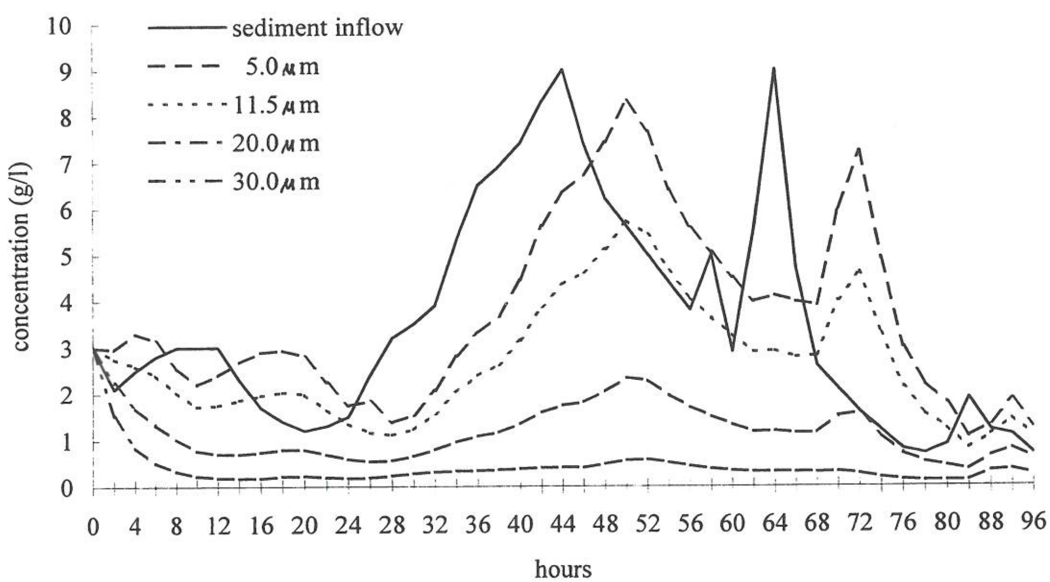


FIG. 6. Concentration Variation at Downstream Depending on Sediment Diameter

TABLE 1. Trap Efficiency of Sediment

Diameter(μm)	Inflow(m^3)	Deposit(m^3)	Outflow(m^3)	Trap(%)
5.0	295,000	3,000	292,000	1
11.5	295,000	84,000	211,000	28
20.0	295,000	201,000	94,000	68
30.0	295,000	284,000	31,000	89

The simulation result shows that D = 5 μm is almost transported to downstream, but D = 30 μm is almost deposited in reservoir. Suppose that sediment inflow is 295,000 m^3 and the size distribution is classified four classes such as D = 5 μm (30 %), D = 11.5 μm (30 %), D= 20 μm (20%) and D = 30 μm (20 %), the total trap efficiency is obtained as follows: (TABLE 1)

Trap efficiency = deposit/inflow = 119,000 / 295,000 = 40 % = 0.4

4. RESULT

A fully coupled one-dimensional sediment transport model for unsteady flow and non-equilibrium condition with looped network is created. It is applied to the Belley reservoir of Rhone river in France.
The results of simulation of this model show that D = 5 μm is almost pass through the reservoir and D = 30 μm is deposited about 89 % in the reservoir. The trap efficiency of simulation (=0.4) gives reasonably good result comparing with the measurement (=0.49) in 1990.
This model can be applicable for the non-equilibrium condition such as flushing where D_{50} of bed material is different from D_{50} of inflow sediment.

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6. REFERENCES

Armanini, A., and Di Silvio, G. (1988). "A One-dimensional model for the transport of a sediment mixture in non-equilibrium condition.", *J. of Hydraulic Research*, Vol. 26, No. 3, pp. 276-286.
Cunge, J.A., Holly, F.M. Jr,Verwey, A. (1980). *Practical aspects of computational river hydraulics*. Pitman.
French, R.H. (1986). *Open-channel hydraulics.* McGraw-Hill.
Holly, F.M. Jr, and Rahuel, J.L. (1990). "New numerical/physical framework for mobile-bed modelling.", *J. of Hydraulic Research*, Vol. 28, No. 4, pp. 404-411.
Lee, S.H. (1993). *Modelisation numerique du transport solide en ecoulement non-permanent.* Thesis presented to Institut National Polytechnique de Grenoble (INPG) in partial fulfillment of the requirements for the Doctor degree (in French).
Van der Berg, J.A., and van Gelder, A. (1993). "Prediction of suspended bed material transport in flows over silt and very fine sand.", *Water Resources Research*, Vol. 29, No. 5, pp. 1401-1403.
Vanoni, V.A. (1977). *Sedimentation Engineering*. ASCE Task Committee.
Van Rijn, L.C. (1984). "Sediment transport: Part I. Bed load transport." *J. of Hyd. Div.*, ASCE, Vol. 110, No. 10, pp. 1431-1452.
Van Rijn, L.C. (1984). "Sediment Transport: Part II. Suspended load transport." *J. of Hyd. Div.*, ASCE, Vol. 110, No. 11, pp. 1613-1638.
Van Rijn, L.C. (1984). "Sediment transport: Part III. Bed forms and alluvial roughness" *J. of Hyd. Div.*, ASCE, Vol. 110, No. 12, pp. 1749-1754.
Van Rijn, L.C. (1988). "Application of sediment pick-up function." *J. of Hyd. Div.*, ASCE, Vol. 112, No. 9, pp. 1749-1754.