Two-Dimensional Total Sediment Load Model Equations

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Abstract: An unsteady total load equation is derived for use in depth-averaged sediment transport models. The equation does not require the load to be segregated a priori into bed and suspended but rather automatically switches to suspended load, bed load, or mixed load depending on a transport mode parameter consisting of local flow hydraulics. Further, the sediment transport velocity, developed from available data, is explicitly tracked, and makes the equation suitable for unsteady events of sediment movement. The equation can be applied to multiple size fractions and ensures smooth transition of sediment variables between bed load and suspended load for each size fraction. The new contributions of the current work are the consistent treatment of sediment concentration in the model equation and the empirical definition of parameters that ensure smooth transitions of sediment variables between suspended load and bed load.

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Introduction

There are two general categories of sediment transport model equations used to simulate the movement of sediment in natural rivers. One set of transport model equations separates the total sediment load into suspended and bed load, whereas the other combines the two modes of transport and tracks only the total load. In this note, "transport model equation" or "transport equation" refers to the partial differential mass-balance equations that are used to track the movement of sediment. Transport equations do not refer to the empirical sediment capacity formulas that are used to predict equilibrium sediment transport rates.

The first category of sediment transport equations separates sediment into bed load and suspended load based upon empirical criteria. Separate transport equations are used for suspended and bed load. For example, the Exner equation is used to predict the movement of bed load and an advection-dispersion equation is used for suspended sediment. The Exner equation ignores the time rate of change of the sediment concentration. The advection-dispersion equation usually assumes that the sediment moves at the same velocity as the fluid. One such example of the separate treatment of bed load and suspended load is Spasojevic and Holly (1990).

The second category simulates the movement of suspended load and bed load with a single model equation. Some examples are Wu (2004) and Armanini and Di Silvio (1988). It should be noted that both the separate transport equation method and the single model equation approaches can be applied to multiple size fractions.

Here, we propose a single transport equation that will simulate bed load, suspended load, or mixed load. The equation can be applied to multiple size fractions and ensures smooth transition of sediment variables between bed load and suspended load for each size fraction. The new contributions of the current work are the consistent treatment of sediment concentration in the model equation and the empirical definition of parameters that ensure smooth transitions of sediment variables between suspended load and bed load.

Derivation of the Total Load Model Equation

A general three-dimensional advection-diffusion equation may be used to describe the ensemble averaged sediment concentration transport as

$$\frac{\partial c}{\partial t}+\frac{\partial u_s c}{\partial x}+\frac{\partial v_s c}{\partial y}+\frac{\partial w_s c}{\partial z}=\frac{\partial}{\partial x}\left(d_x\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(d_y\frac{\partial c}{\partial y}\right)+\frac{\partial}{\partial z}\left(d_z\frac{\partial c}{\partial z}\right) \quad (1)$$

where u_s, v_s, and w_s=ensemble averaged sediment velocities in the x-, y-, and z-directions, respectively; c denotes the ensemble averaged volumetric sediment concentration; and d_i=turbulent diffusion coefficient of particles $[i \in (x,y,z)]$. The ensemble average is an average over a number of realizations of a flow. The ensemble average is used instead of a time average because it does not require the definition of a specific time period or control volume [for example, see Drew (1983)]. In addition, the ensemble average results in fewer correlations then time averaging. Time averaging is equivalent to ensemble averaging if the flow is statistically stationary and volume averaging is equivalent if the flow is spatially uniform.

In this note, we use the concentration-weighted depth average to develop the equations. That is, given a generic variable χ, it can be split into its concentration-weighted average and a deviation: $\chi=X+\chi''$, where X concentration-weighted average and

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χ''=deviation of χ from X. The concentration-weighted average X is defined as

$$X=\frac{1}{\bar{c}h}\int_0^h c\chi dz \tag{2}$$

Application of the conventional averaging to Eq. (1) leads to

$$\frac{\partial hC}{\partial t}+\frac{\partial U_s hC}{\partial x}+\frac{\partial V_s hC}{\partial y}=\frac{\partial}{\partial x}\left(hD_x\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(hD_y\frac{\partial C}{\partial y}\right)+S_e \tag{3}$$

where U_s, V_s=concentration-weighted depth-averaged sediment velocities and $C=\bar{c}$=conventional depth-averaged sediment concentration, $\bar{c}=(\int_0^h c dz)/h$. The parameters D_x and D_y=mixing coefficients of suspended sediment in the x- and y-directions, respectively, that consist of both turbulent diffusion and dispersion due to depth averaging. A common alternative averaging method is to define the velocity-weighted concentration as in Wu et al. (2006)

$$\hat{C}=\frac{1}{V_t h}\int_0^h c v_t dz \tag{4}$$

where V_t=resultant depth-averaged flow velocity and v_t=resultant local flow velocity. The transport equation then becomes

$$\frac{\partial h\hat{C}V_t/V_{st}}{\partial t}+\frac{\partial \cos(\alpha)V_t h\hat{C}}{\partial x}+\frac{\partial \sin(\alpha)V_t h\hat{C}}{\partial y}=\frac{\partial}{\partial x}\left(hD_x\frac{\partial \hat{C}}{\partial x}\right)+\frac{\partial}{\partial y}\left(hD_y\frac{\partial \hat{C}}{\partial y}\right)+S_e \tag{5}$$

where V_{st}=resultant sediment velocity and α=direction angle of sediment transport (relative to x axis). The benefit of using the depth-averaged concentration to develop the equations instead of the velocity-weighted concentration is that one does not need to divide by the sediment velocity in the time derivative term. Armanini and Di Silvio (1988) recommend eliminating the V_t/V_{st} term in the time derivative. Eliminating this term, however, from Eq. (5) leads to an inconsistent treatment of the sediment concentration and the time derivative term may be incorrect. In addition, the diffusion term in the depth-averaged concentration formulation is a function of the depth-average concentration rather than a velocity-weighted concentration. Using the velocity-weighted concentration adds an extra velocity term to the diffusion term that is difficult to calculate. Therefore, the concentration-weighted method, Eq. (3), is used throughout this note.

Eq. (3) is valid for both suspended and bed load transport as depth averaging includes both loads. Two parameters are introduced to reformulate the equation amenable to numerical solution: the transport mode parameter f and the sediment-to-flow velocity ratio β. The f parameter represents the ratio of suspended portion to the total sediment concentration for a single size class; it ranges from 0 for pure bed load to 1 for pure suspended load. The β parameter is defined as

$$\beta\equiv\frac{\sqrt{U_s^2+V_s^2}}{\sqrt{U^2+V^2}}\equiv\frac{V_{st}}{V_t} \tag{6}$$

where U and V=depth-averaged flow velocities in the x- and y-directions, respectively, and V_{st} and V_t=total depth-averaged sediment and flow velocities, respectively. The final total load model equation may be written as

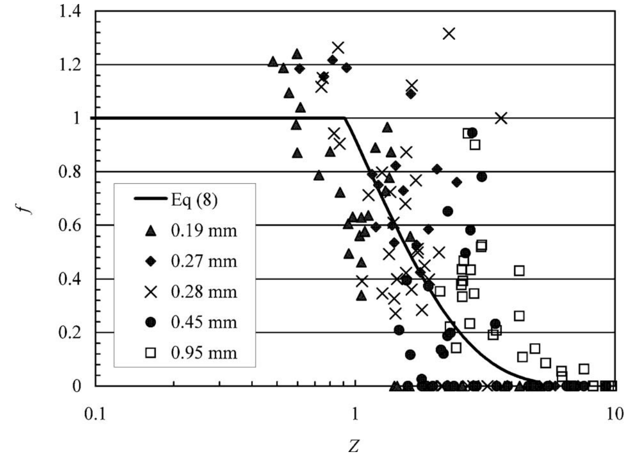

Fig. 1. Comparison between the experimental data collected by Guy et al. (1966) and Eq. (8)

$$\frac{\partial hC}{\partial t}+\frac{\partial \cos(\alpha)\beta V_t hC}{\partial x}+\frac{\partial \sin(\alpha)\beta V_t hC}{\partial y}=\frac{\partial}{\partial x}\left(hfD_x\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(hfD_y\frac{\partial C}{\partial y}\right)+S_e \tag{7}$$

This equation can be applied to each grain size within a sediment mixture. There is no need to a priori specify whether a particular size class behaves as a bed load or suspended load. To provide model closure, four variables still need to be determined: the direction angle of sediment transport α (relative to the x axis), the transport mode parameter f, the velocity ratio β, and the source term S_e. The next section will describe the empirical equations necessary to determine these variables and close the model equation.

Model Closure

The sediment transport mode parameter $f(0\leq f\leq 1)$ specifies how much of a sediment size class is transported as bed load, suspended load, or mixed load. The parameter f is similar to the "allocation coefficient" introduced by Holly and Rahuel (1990). To estimate the value of f, we used the experimental results of Guy et al. (1966), who performed laboratory experiments where both total bed material load and suspended load were collected in an 8 ft wide flume. We found f to be primarily a function of the suspension parameter, $Z=\omega_s/(\kappa u_\tau)$, where ω_s=particle fall velocity; κ=von Kármán constant (assumed here to be a constant =0.41); and u_τ=frictional shear velocity. The function was derived by a best fit to the data

$$f=\min(1,2.5e^{-Z}) \tag{8}$$

A comparison between Eq. (8) and the experimental data is shown as Fig. 1. Laursen (1958) and Van Rijn (1984b) also found f to be primarily dependent upon Z. The considerable scatter in the data is due partially to the data collection technique itself. For example, it is theoretically impossible that f is larger than unity, but the data have many values of f larger than this constraint, some values as large as 1.3. Eq. (8) assumes that the fraction of sediment in suspension immediately responds to changing flow conditions. In reality, there are spatial and temporal lags as the bed load transitions to suspended load and vice versa.

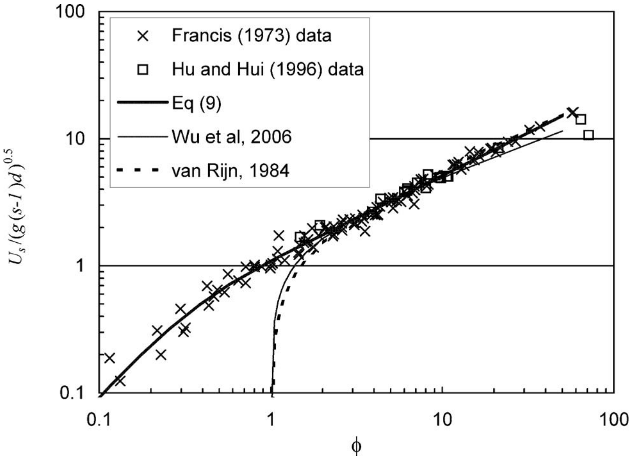

Fig. 2. Comparison between the experimental data collected by Francis (1973) and Hu and Hui (1996) and Eq. (9). The functions proposed by Wu et al. (2006) and Van Rijn (1984a) are also shown.

The velocity of sediment relative to the fluid is denoted by β. For bed load transport, some flume data were available from Francis (1973) and Hu and Hui (1996), who performed experiments measuring the velocity of rolling and saltating particles in a laboratory flume. The data spanned the range of $Z=2$ to $Z=50$. These data were used to find a formula to represent the sediment-to-flow velocity ratio and the following relationship was fit to the experimental data:

$$\frac{U_{s,\text{bed}}}{\sqrt{g(s-1)d}} = 1.1\phi^{0.67}[1-\exp(-5\phi)] \quad \text{for } \phi < 20 \tag{9}$$

where $\phi=\theta/\theta_r$; $\theta=\tau_b/(\gamma(s-1)d)$=Shields parameter ($\tau_b$=bed-shear stress; γ=specific weight of water; d=particle diameter; and s=specific gravity of sediment); and θ_r=reference nondimensional shear stress. The reference nondimensional shear stress is the Shields parameter at which there is a low but measurable reference transport rate, such as defined in Parker (1990). The ratio of bed load velocity to streamwise velocity can be expressed by rearranging Eq. (9)

$$\beta_{\text{bed}} = \frac{u_\tau}{V_t}\frac{1.1\phi^{0.17}[1-\exp(-5\phi)]}{\sqrt{\theta_r}} \quad \text{for } \phi < 20 \tag{10}$$

A comparison of Eq. (9) with the available data is shown in Fig. 2. The formulations of Wu et al. (2006) and Van Rijn (1984a) are also shown. For $\phi>1$, all formulations are similar; however, for $\phi<1$, the formulations of Wu et al. and van Rijn predict zero velocities, whereas Eq. (9) predicts positive velocities. The prediction of nonnegative velocity for $\phi<1$ is consistent with the transport equations of Parker (1990) and Wilcock and Crowe (2003) that predict low, but nonzero, transport rates for $\phi<1$. It should be noted that the reference nondimensional shear stress was estimated for each particle type and bed roughness. The values used are listed in Table 1. Further work is necessary to extend the applicability of Eqs. (9) and (10) to sediment mixtures.

In field applications of this formula, the reference nondimensional shear stress may need to be calibrated to individual field sites. This function also assumes that the sediment velocity responds instantaneously to the flow velocity. To calculate the time required for sediment particles to respond to changes in flow velocity, it would be necessary to solve the depth-averaged momentum equation for bed load sediment. Such a method was developed by Di Cristo et al. (2006), who derived a momentum equation for bed load that accounts for the inertia of bed load particles. Here, we assumed the inertia of bed load particles is not significant.

Table 1. Shields Parameters Used in Fig. 2

Particle type	Reference shear stress (θ_r)
Hu and Hui (1996) smooth bed	0.01
Hu and Hui (1996) rough bed	0.025
Smooth particles and smooth bed [Francis (1973), Series M and N]	0.017
Natural gravel [Francis (1973), Series K and C]	0.035
Angular [Francis (1973), Series A]	0.07
Angular [Francis (1973), Series L, LL]	0.058

Sumer (1974) performed an analytical analysis for the case when the particle stays in suspension in the main body of the flow. By integrating the sediment concentration over the depth assuming a Rouse profile, the velocity ratio for the suspended load was found to be a function of the suspension parameter Z and was derived analytically as

$$\beta_{\text{sus}} = 1 + \frac{u_\tau}{\kappa V_t}[\Psi(2) - 1 - \Psi(1-Z)] \quad \text{for } Z < 1 \tag{11}$$

where Ψ=psi function [e.g., Tables of Abramowitz and Stegun (1968), p. 267 may be used to calculate the function]. An exponential function was fit to Eq. (11) to simplify the computation of β_{sus}

$$\beta_{\text{sus}} = 1 + \frac{u_\tau}{2\kappa V_t}[1-\exp(2.7Z)] \quad \text{for } Z < 1 \tag{12}$$

Eq. (12) was found to reproduce β_{sus} in Eq. (11) to within 2% for $Z<0.7$ and $u_\tau/(\kappa V_t)<0.5$. Greimann et al. (1999) showed that the Rouse profile is not valid near the bed due to particle inertia effects and the Rouse profile will overpredict concentrations near the bed for large particles. Therefore, Eq. (12) would underpredict the sediment velocity for larger particles. In addition, there is a discontinuity in sediment velocity between bed load and suspended load using Eqs. (10) and (12). As a simple remedy to this situation, the relative velocity for all particles is computed as

$$\beta = \max(\beta_{\text{sus}}, \beta_{\text{bed}}) \tag{13}$$

where Z must be less than 1 when computing β_{sus}, and ϕ must be less than 20 when computing β_{bed}. This ensures that the sediment velocity is a continuous function as it transitions from bed load to suspended load. It also ensures the sediment velocity increases with increasing shear stress or decreasing particle size. The writers believe that additional experimental data and analysis is necessary to develop formulations for the sediment velocity between bed load to suspended load.

The sediment transport direction angle, α, may be assumed to be coincident with the depth-averaged velocity direction for pure suspended load. It can deviate, however, from the flow velocity for bed load due to transverse bed slope and secondary currents at the bottom of the channel. Several approaches may be used to include the effect of gravity and secondary flow. One approach was introduced by Struiksma et al. (1985):

$$\tan \alpha = \frac{\sin \delta - \dfrac{1-f}{0.85\sigma_s\sqrt{\theta}}\dfrac{\partial z_b}{\partial y}}{\cos \delta - \dfrac{1-f}{0.85\sigma_s\sqrt{\theta}}\dfrac{\partial z_b}{\partial x}} \quad (14)$$

where σ_s=shape factor of the particles (from 1 to 2). The angle of the bed-shear stress, δ, is calculated as

$$\delta = \tan^{-1}\left(\frac{V}{U}\right) - (1-f)\tan^{-1}\left[\frac{2}{\kappa^2}\left(1 - \frac{n\sqrt{g}}{\kappa h^{1/6}}\right)\frac{h}{R_c}\right] \quad (15)$$

where the second term on the right-hand side=deviation of the bed-shear stress due to the secondary flow; n=Manning's roughness coefficient; and R_c=local radius of curvature of flow streamlines. It is assumed that for $f=1$, the sediment load velocity direction is exactly the flow direction. The dispersion of suspended sediment caused by helical flow may be lumped into the mixing coefficients, D_x and D_y, which are not discussed here.

Similar to Armanini and Di Silvio (1988) and Wu (2004), the following source term is used:

$$S_e = \frac{1}{L_{\text{tot}}}(q_{\text{tot}}^* - \beta V_t hC) \quad (16)$$

where L_{tot}=total effective adaptation length and q_{tot}^*=equilibrium capacity for total load transport rate. To compute the total effective adaptation length, Armanini and Di Silvio used $L_{\text{tot}} = \zeta V_t h/\omega_s$ and Wu (2004) proposed $L_{\text{tot}}=\max(L_b, \zeta V_t h/\omega_s)$, where L_b=bed load adaptation length (Holly and Rahuel 1990) and ζ=parameter that controls the rate of suspended load exchange. The writers have used various functions to compute the value of the adaptation length for bed load, L_b (e.g., Wu 2004; Phillips and Sutherland 1989). There is also more work necessary to define L_b for a general flow in a natural river. While some have used constants for ζ (Han and He 1990), Galappatti and Vreugdenhil (1985) and Armanini and Di Silvio (1988) suggest that ζ is dependent upon the ratio of fall velocity to shear velocity and the relative roughness height. The relationship proposed by Armanini and Di Silvio (1988) is

$$\zeta = \eta + (1-\eta)\exp(-1.5\eta^{-\frac{1}{6}}\varpi_s/u_\tau) \quad (17)$$

where η=relative roughness height, computed as: $\eta = 33\exp(-1-\kappa V_t/u_\tau)$. Additional numerical and laboratory experiments are necessary to determine the validity of Eq. (17) and its influence on the solution in field situations. The function in Eq. (17) was derived by assuming that the sediment behaves as pure suspended load and that interactions with the bed are negligible. Armanini and Di Silvio (1986) also showed that an equation similar to Eq. (17) was less accurate for $Z>1$. In this note, we recommend the following formula for the adaptation length:

$$L_{\text{tot}} = (1-f)L_b + f\zeta V_t h/\omega_s \quad (18)$$

The adaptation length function in Eq. (18) has the advantage that the bed load adaptation length is exactly recovered for large Z and the suspended load adaptation length is exactly recovered for small Z.

Conclusions

A single total load transport equation, Eq. (7), is developed that can simultaneously model suspended load, bed load, or mixed load. To implement the transport equation, four variables were defined (f, α, S_E, and β) and empirical formulas have been developed to determine their values. The fraction of suspended load to total load, f, is related to the suspension parameter, Z in Eq. (8). The ratio of sediment velocity, β, to fluid velocity is computed by Eq. (13). The relationship for sediment velocity is unique in that it does not force the sediment velocity to zero below the reference shear stress. In addition, the equation ensures that the sediment velocity is a continuous function. The sediment transport direction angle, α, is determined by Eq. (14). Finally, the sediment source term, S_E, is computed as in Eq. (16). The source term ensures a smooth transition in the adaptation length between suspended load and bed load.

There are several advantages to using a single transport equation to model sediment load. The transport mode parameter (f) makes it possible to use a single equation to represent suspended load, bed load, and mixed load simultaneously. The velocity of sediment movement is explicitly tracked and there is a continuous description of the velocity from suspended to bed load. In addition, a single total load sediment transport capacity is used instead of separate suspended and bed load capacities. The primary benefit, however, may be in that it simplifies the accounting of sediment and may speed the development and testing of computer models.

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Notation

The following symbols are used in this technical note:

- c = sediment concentration;
- d_x = diffusion coefficient in x-direction;
- f = transport mode parameter, fraction of suspended load to total load;
- g = acceleration of gravity;
- h = flow depth;
- L_b = bed load adaptation length;
- L_{tot} = adaptation length of total load;
- n = Manning's roughness coefficient;
- q_{sx} = total sediment load transport rate in x-direction;
- q_b^* = equilibrium value for the bed load transport rate;
- R_c = local radius of curvature;
- S_e = erosional source term;
- t = time;
- u, v, w = local time averaged velocity of water in x-, y-, and z-directions;
- u_s, v_s, w_s = local time averaged velocity of sediment in x-, y-, and z-directions;
- u_τ = shear velocity;
- V_{st} = total sediment velocity;
- V_t = total flow velocity;
- X = concentration-weighted depth-averaged value of χ;
- x, y, z = distance in Cartesian coordinate system;
- Z = suspension parameter;
- α = angle of sediment transport;
- β = ratio of sediment velocity to flow velocity;
- δ = angle of the bed-shear stress;

ζ = parameter in nonequilibrium suspended sediment source term;
θ = Sheilds parameter;
θ_r = critical Sheilds parameter;
σ_f = shape factor;
κ = von Kármán constant (=0.41);
ϕ = ratio of Shields parameter to the critical Shields parameter;
Ψ = psi function; and
ω_s = sediment fall velocity.

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