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Hindered settling velocity of cohesive/non-cohesive sediment mixtures

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ABSTRACT

New methods are proposed for predicting the hindered settling conditions encountered by concentrated suspensions containing mixtures of sand particles and mud flocs. These methods, based on two-fraction formulations, are developed by consideration of the settling characteristics of monodisperse and polydisperse solid particle suspensions applied to cohesive/non-cohesive mixtures of mud flocs and sand particles. The behaviour of these predictive methods is evaluated over a wide range of mixture conditions and compared with existing formulations, with their parametric dependence on the relative volumetric concentrations and floc/particle sizes for the mud and sand constituents established. The results indicate that consideration of the full return flow effects generated by both fractions provides the best modelling framework for predicting the hindered settling conditions over a wide range of sand–mud mixtures.

1. Introduction

The settling of negatively-buoyant particles (i.e. those that are heavier than the fluid in which they are suspended) is an important process that has been studied extensively in many natural and industrial systems including rivers, estuaries, sewers, treatment plants and sedimentation tanks etc. In the context of coastal and estuarine processes, accurate prediction of the settling characteristics of both cohesive (mud) flocs and non-cohesive (sand) particles is essential in determining their transport, interactions and fate within the coastal/estuarine waters. These processes, in turn, are of major importance for the maintenance and management of navigation channels, ports and harbours, as well as in assessing the effects of increased turbidity on water quality and aquatic habitats within these environments.

The factors influencing the settling velocity w_s include the particle properties (i.e. size, shape, structure); the particle concentration; the turbulence levels within the fluid and the ambient fluid properties (i.e. viscosity and salinity). For monodisperse particles settling at infinite dilution, discrete particles will settle within still, homogeneous fluid conditions at a terminal fall velocity w_s determined by application of Stokes' law, if the particle Reynolds number Re_p<<1 (e.g. Batchelor, 1967), or by one of various empirically-derived formulae (e.g. Van Rijn, 1984; Cheng, 1997a), if Re_p>1. When the concentration of sediment particles increases, particles cease to behave independently. Instead, their motions are correlated through hydrodynamic and particle-particle interactions, often resulting in settling rates that are lower than that for individual, isolated particles (i.e. hindered settling) (Scott, 1984). Within mud suspensions, hindered settling characteristics have been shown to occur normally for concentrations of a few kg/m^3 or above (e.g. Dankers, 2006).

Numerous processes are known to exist that may affect the settling velocity of individual particles within a concentrated suspension. Previous studies considering concentrated suspensions of cohesive sediment flocs (e.g. Winterwerp, 2002) and non-cohesive sediment particles (e.g. Cheng, 1997b) have shown that these hindered settling characteristics arise predominantly from three distinct processes related to the presence of the cohesive floc/non-cohesive particle concentration, namely: (i) return flow generation and wake formation; (ii) increased mixture viscosity; and (iii) buoyancy effects. In this sense, the settling characteristics of monodisperse non-cohesive or flocculated suspensions have been studied extensively, with most hindered settling models employed for coastal sediment transport calculations being either in the exact form of the Richardson and Zaki (1954) formulae or some related variation.

By contrast, the settling of polydisperse suspensions containing two or more particle species with differing size, density and/or structure is not yet completely understood. A number of theoretical and numerical approaches do however exist by which this problem may be tackled. The most well-known theoretical model is that of Batchelor (e.g. Batchelor, 1982; Batchelor and Wen, 1982) who predicted settling rates for polydisperse dilute suspensions of spheres for low Reynolds numbers. A different approach is based on the consideration of two-phase flow equations, with the Richardson and Zaki relation used to determine the fluid-particle interacting forces due to finite concentration effects (Masliyah 1979; Berres et al. 2004). This latter method has been applied to both non-cohesive particles and cohesive flocs, although fractal properties, which are commonly

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used to describe the floc structure, have not been considered previously.

In this paper, the hindered settling characteristics of monodisperse mud floc and sand particle fractions contained within concentrated suspensions of sand–mud mixtures are studied numerically. The methods considered herein are those of Winterwerp (2002) [see also Winterwerp and Van Kesteren (2004)], which were derived for concentrated mud floc suspensions with relatively low sand content, and two new models that have been adapted from previous work considering the hindered settling characteristics of (i) monodisperse concentrations of non-cohesive sediment particles (e.g. Cheng, 1997b) and (ii) polydisperse suspensions (e.g. Davis and Gecol, 1994). The behaviour of these methods is evaluated for a wide range of sand–mud mixture conditions, showing the parametric influence of relative volumetric concentrations and floc/particle size on the hindered settling velocities of the mud and sand constituents within the mixture.

2. Previous work

2.1. Settling of sand particles and mud flocs at infinite dilution

For natural sand grains settling both within the Stokes range ($Re_p << 1$) and at higher particle Reynolds numbers ($Re_p > 1$), many empirical formulae have been derived to predict the single particle settling velocity $w_{ss,0}$. From the definition of particle Reynolds number $Re_p = w_{ss,0}.D_s/\nu$, Cheng (1997a) derived an expression for $w_{ss,0}$ of the form:

$$w_{ss,0} = \frac{\nu}{D_s}\left(\sqrt{25 + 1.2D_*^2} - 5\right)^{1.5} \quad (1)$$

where D_s is the sand grain size, ν is the kinematic viscosity of the fluid and $D_* = (\Delta g/\nu^2)^{1/3} D_s$ is the non-dimensional particle parameter [within which $\Delta = (\rho_s - \rho_w)/\rho_w$, ρ_s and ρ_w are the sand particle and fluid densities, respectively, and g is the gravitational acceleration].

The settling velocity $w_{sf,0}$ of a single mud floc with a fractal structure in a still, homogeneous fluid was derived by Winterwerp (2002) from the assumption that the floc drag coefficient C_D follows a similar relationship with the floc Reynolds number Re_f $(= w_{sf,0}.D_f/\nu)$ as that for non-cohesive particles. This implies that the flocs are impermeable and so there is no mass transfer between the flocs and the fluid. With knowledge of the differential floc density $\Delta\rho_f (= \rho_f - \rho_w)$, an implicit formula for $w_{sf,0}$ for a single floc was derived from the balance of gravitational and drag forces as

$$w_{sf,0} = \left(\frac{\alpha}{18\beta}\frac{\left(\rho_p - \rho_w\right)g}{\mu}D_p^{3-n_f}\right)\frac{D_f^{n_f-1}}{1 + 0.15\mathrm{Re}_f^{0.687}} \quad (2)$$

where α and β are shape coefficients; n_f is the fractal dimension; D_p and ρ_p are the mud primary particle diameter and density, respectively, and D_f and ρ_f is the mud floc diameter and density, respectively.

Assuming that the form of Eq. (1) can also be applied to single mud flocs [under the same assumption applied by Winterwerp (2002) that the relationship between the particle drag coefficient C_D and Reynolds number Re_p also holds for the mud flocs], the settling velocity of a single mud floc $w_{sf,0}$ may be written as follows:

$$w_{sf,0} = \frac{\nu}{D_f}\left(\sqrt{25 + 1.2D_{*f}^2} - 5\right)^{1.5} \quad (3)$$

where the non-dimensional floc parameter $D_{*f} = (\Delta_f g/\nu^2)^{1/3} D_f$ [with $\Delta_f = (\rho_f - \rho_w)/\rho_w$]. The predictive behaviour of Eqs. (1)–(3) is compared in Fig. 1, with both $w_{sf,0}$ and $w_{ss,0}$ values showing a monotonic increase with mud floc/sand particle diameter within the specified size range $D_f = D_s = 10–1000$ μm.

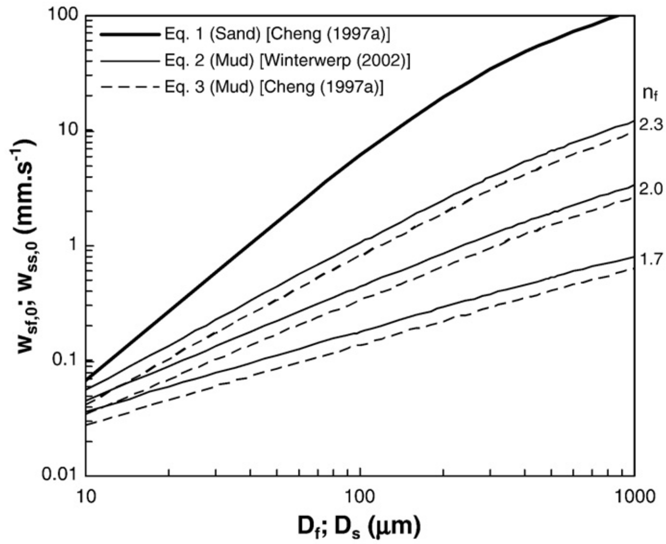

Fig. 1. Comparison of reference settling velocities for single sand particles $w_{ss,0}$ and fractal mud flocs $w_{sf,0}$ at infinite dilution versus particle D_s/floc D_f diameter.

2.2. Hindered settling of monodisperse, single-species suspensions

The standard Richardson and Zaki (1954) relation as applied to a concentrated suspension of sand particles settling in the hindered regime is given by

$$\frac{w_{ss}}{w_{ss,0}} = (1 - \phi_s)^n \quad (4)$$

where w_{ss} is the settling velocity of sand particles dispersed at a volumetric sand concentration $\phi_s = (c_s/\rho_s)$ (with c_s being the sand particle mass concentration and ρ_s being their density). Exponent n is typically determined empirically, with values generally ranging between 2.5 and 5.5 depending on the particle Reynolds number (Richardson and Zaki, 1954). Recently however, Baldock et al. (2004) developed a method for determining n based on grain properties, concentration and fluid viscosity, with the values obtained agreeing satisfactorily with measurements for natural sands. Mehta (1986) suggested the adoption of a Richardson and Zaki-type relation for the prediction of the hindered settling velocity w_{sf} for pure mud suspensions,

$$\frac{w_{sf}}{w_{sf,0}} = \left(1 - k\phi_p\right)^n \quad (5)$$

where ϕ_p $(= c_p/\rho_p)$ is the volumetric concentration of primary mud particles contained within the flocs (c_p being the mass concentration of primary mud particles). It can be shown that the term $k\phi_p$ within Eq. (5) is equivalent to the volumetric mud floc concentration $\phi_f = c_p/c_{gel}$ (c_{gel} being the gelling concentration at which the mud flocs form an interconnected network), and thus Eq. (5) can be rewritten as

$$\frac{w_{sf}}{w_{sf,0}} = \left(1 - \phi_f\right)^n. \quad (6)$$

The hindered settling velocity factor $(1 - \phi_f)^n$ accounts implicitly for the return flow, increased mixture viscosity and buoyancy effects, as noted previously. It remains uncertain, however, whether the range of the exponent n proposed by Richardson and Zaki (1954) or Baldock et al. (2004) can be applied directly to cohesive mud flocs, which differ from non-cohesive (sand) particles in many ways (e.g. irregular shapes; wider distributions of size and density; structural variations).

Recently, Winterwerp (2002) proposed a model that determined separate hindered settling factors for these three mechanisms. In his approach, the "slip" velocity between the fluid and the settling flocs was assumed implicitly to be equal to the terminal fall velocity $w_{sf,0}$ of a discrete floc settling in an infinitely dilute suspension (i.e. $\phi_p=0$), with the resulting hindered settling factor due to the return flow effects being specified by $(1-\phi_f)$. For the hindered settling effects generated by the increased viscosity of the concentrated mud suspension, Winterwerp (2002) adapted the classical formula of Einstein for dilute particle suspensions, replacing the volumetric particle concentration ϕ_s with the mud floc concentration ϕ_f, such that

$$\nu'_f = \nu(1+2.5\phi_f) \tag{7}$$

where ν'_f is the effective viscosity of a suspension of similar particles having the same density as the fluid and a volumetric concentration ϕ_f. With the inclusion of an additional hindered settling factor $(1-\phi_p)$ to account for the increased buoyancy effects, Winterwerp's final hindered settling formula for a concentrated suspension of mud flocs reads

$$\frac{w_{sf}}{w_{sf,0}} = \frac{(1-\phi_f)(1-\phi_p)}{1+2.5\phi_f}. \tag{8}$$

For concentrated suspensions of sand particles, an approach was proposed by Cheng (1997b), based on consideration of the settling particles within the fluid as a two-phase flow problem, with the evaluation of the slip velocity between the particles and the fluid determined from continuity arguments. Adopting this assumption, the return flow w generated by a settling suspension of monodisperse sand particles can be related to the sand settling velocity by the continuity equation,

$$w_{ss}\phi_s + (1-\phi_s)w = 0. \tag{9}$$

Re-arranging Eq. (9) and introducing the sand particle slip velocity w'_s [$=(w_{ss}-w)$], the continuity equation can be recast in the form,

$$w'_s = \frac{w_{ss}}{(1-\phi_s)}. \tag{10}$$

This sand particle slip velocity is then expressed in a form similar to Eq. (1) [i.e. from Cheng (1997b)],

$$w'_s = \frac{\nu'_s}{D_s}\left(\sqrt{25+1.2D'^2_{*s}}-5\right)^{1.5} \tag{11}$$

where ν'_s is the effective sand-fluid mixture viscosity and $D'_{*s}=(\Delta'_s g/\nu'^2_s)^{1/3}D_s$ is the modified non-dimensional sand particle parameter [with $\Delta'_s=(\rho_s-\rho')/\rho'$ and $\rho'=\phi_s\rho_s+(1-\phi_s)\rho_w$].

2.3. Hindered settling of sand–mud suspensions

A full theoretical description of sedimentation for high volumetric concentrations of polydispersed (i.e. multiple-species) particulate suspensions continues to present challenges, in spite of previous contributions by Batchelor (1982) for dilute suspension and by others (e.g. Davis and Gecol, 1994; Ha and Lui, 2002) for concentrated suspensions.

The inter-related behaviour (including settling characteristics) of the two distinct fractions contained within a suspended mixture of cohesive mud flocs and non-cohesive sand particles has previously been shown to be very complex (e.g. Torfs et al., 1996; Amy et al., 2006; Dankers, 2006). Whilst, for very dilute suspensions, the sand and mud fractions can be assumed to settle independently (Van Ledden, 2003), the fractional settling characteristics for the sand and mud constituents in more concentrated suspensions are not only governed by the total sediment concentration, but also affected critically by their relative fractional content within the mixture. At one extreme, for a suspension containing mainly sand particles with a low mud content (a so-called "sand-rich" suspension), the generated return flow associated with the settling sand particles may be large enough to transport the mud particles/flocs upwards within the fluid (Amy et al., 2006). At the other extreme, for suspensions in which the sand particle content is much lower than the mud particle/floc content (i.e. "mud-rich" suspensions), the settling velocity of the sand particles can be significantly reduced by the increased apparent viscosity and return flow generated by the high mud content within the suspension. The effect of sand particles on the mud fraction settling characteristics in this latter case is however expected to be small, as confirmed experimentally by Dankers (2006) using settling column experiments. Wang et al. (1995) attempted to formulate an expression to describe the settling velocity of non-cohesive sand particles within clay suspensions, proposing a purely empirical equation based on the Richardson and Zaki (1954) and Maude and Whitmore (1958) formulae:

$$\frac{w_{ss}}{w_{ss,0}} = (1-\phi_s)^n(1-\phi_p)^{2.5} \tag{12}$$

where exponent n was a function of particle Reynolds number and had values significantly higher ($n\approx 8$) than Richardson and Zaki (1954) ($n\approx 2.5$–5.5). Wang et al. (1995) found that this model worked well for hindered sand settling within dilute mud suspensions but deviated significantly from experimental data obtained in high concentration mud suspensions. This latter effect is unsurprising given that the influences of the sand and mud fractions are completely decoupled within the expression. In addition, while the second term on the right hand side of Eq. (12) is included to represent the effect of increased viscosity due to the presence of mud, the hindered settling expression does not include any effect from the return flow generated by the mud flocs in the suspension.

3. Model formulations for hindered settling of sand–mud mixtures

3.1. Winterwerp and Van Kesteren "mud-rich" formulation

Recently, Winterwerp and Van Kesteren (2004) proposed a hindered settling model for "mud-rich" suspensions. The basis for this model was the continuity equation for a sand–mud mixture, which has the form

$$w_{ss}\phi_s + w_{sf}\phi_f + (1-\phi_s-\phi_f)w = 0 \tag{13}$$

where w is the return flow generated by the sand–mud suspension. In order to solve Eq. (13) for w_{ss} and w_{sf}, two further equations are required to relate these settling velocities to return flow w. Winterwerp and Van Kesteren (2004) proposed that the slip velocities of the sand particles $(w_{ss}-w)$ and mud flocs $(w_{sf}-w)$ were equal to their terminal settling velocities $w_{ss,0}$ and $w_{sf,0}$ measured in clear fluid, i.e.

$$w_{sf} = w_{sf,0} + w \tag{14a}$$

$$w_{ss} = w_{ss,0} + w. \tag{14b}$$

A further assumption was that the mixture was predominantly mud with only a small sand concentration (i.e. "mud-rich" with $\phi_s << \phi_f$). Solving Eqs. (13) and (14) under these limiting conditions gives the hindered settling formulae for both the fractions of cohesive (i.e. mud) and non-cohesive (i.e. sand) sediments:

$$w_{sf} = \frac{(1-\phi)(1-\phi_p-\phi_s)}{(1+2.5\phi)} w_{sf,0} \tag{15a}$$

$$w_{ss} = \frac{(1-\phi)(1-\phi_p-\phi_s)}{(1-\phi_f)(1+2.5\phi)} (w_{ss,0}-\phi_f w_{sf,0}). \tag{15b}$$

The behaviour of these equations (subsequently referred to as the WVK method) is shown in Fig. 2 for a range of sand–mud mixture conditions. Specifically, Fig. 2(a) and (b) show the variation in the predicted non-dimensional hindered settling ratios $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0}$ [single sand particle and mud floc settling velocities $w_{ss,0}$ and $w_{sf,0}$ are obtained from Eqs. (1) and (2), respectively] versus the total mixture volumetric concentration ϕ (= ϕ_f+ϕ_s). These plots also indicate the parametric effects of (a) mud floc D_f and sand particle D_s sizes (also specified non-dimensionally by D_{*f} and D_{*s} — see below for definitions) and (b) the fractional sand content ϕ_s within the mixture. Fig. 2(a) and (b) show that both $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0}$ decrease monotonically with increasing ϕ, with $w_{sf}/w_{sf,0}$ values tending gradually to zero as the sand–mud mixture gelling concentration is approached (i.e. $\phi \to 1$, $c \to c_{gel}$). However, the corresponding hindered sand ratio $w_{ss}/w_{ss,0}$ is shown to approach a finite value (i.e. $w_{ss}/w_{ss,0} > 0$) at high ϕ values before decreasing sharply to zero at the gelling point. This sharp reduction appears to be an unrealistic effect arising from the ratio of hindrance terms $(1-\phi)/(1-\phi_f)$ in Eq. (15b), which, although generally close to unity, reduce sharply when $\phi \to 1$ (i.e. close to the gelling point). Fig. 2(a) and (b) also indicate that the parametric influence of the relative sand particle/mud floc sizes and fractional sand content ϕ_s within the WVK formulation is largely confined to the hindered sand settling ratio $w_{ss}/w_{ss,0}$, while the hindered mud settling ratio $w_{sf}/w_{sf,0}$ is shown to only vary significantly with the total volumetric concentration ϕ (for the "mud-rich" assumption adopted, i.e. $\phi_s << \phi_f$).

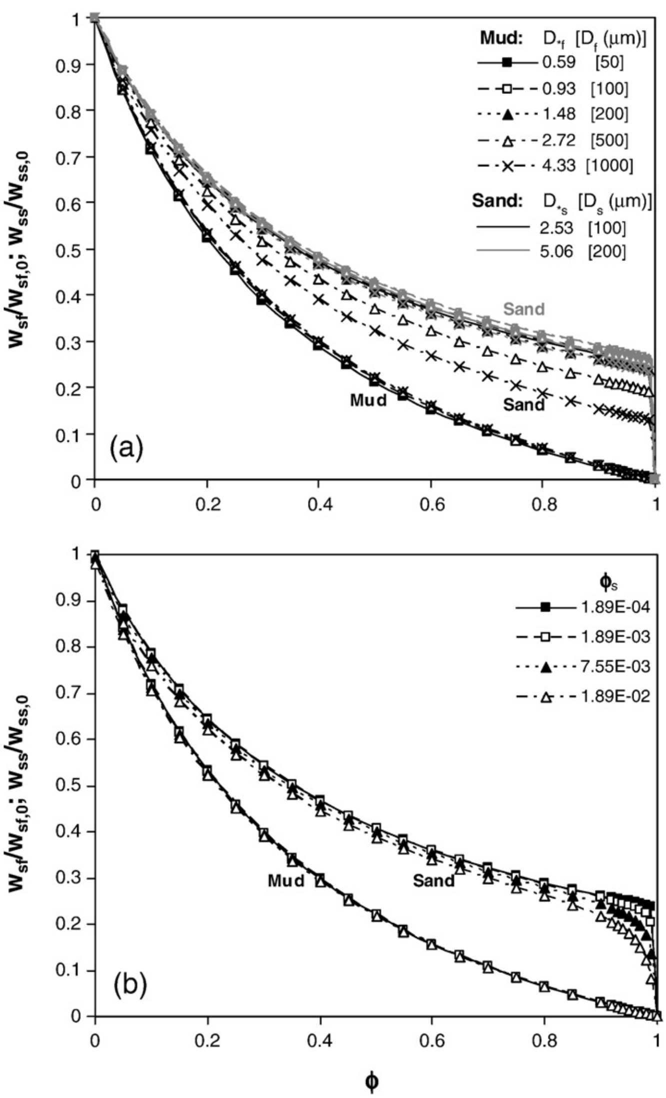

Fig. 2. Hindered settling ratios $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0}$ (obtained by the WVK method) versus mixture concentration ϕ, showing parametric influences of (a) mud floc and sand particle sizes (D_{*f} and D_{*s}) for fixed sand content ϕ_s = 1.89 × 10^{-4} (c_s = 0.5 kg m^{-3}) and (b) variable sand content ϕ_s with fixed mud floc and sand particle sizes [D_{*f} = 1.48 (D_f = 200 μm) and D_{*s} 2.53 (D_s 100 μm)].

It can be easily shown from manipulation of Eq. (14) that $(w_{ss}-w_{sf})$ = $(w_{ss,0}-w_{sf,0})$, which implies that the difference between the settling velocities of sand particles and mud flocs is constant at all fractional volumetric concentrations. No theoretical basis can be found for this behaviour in polydisperse suspensions, although either of Eqs. (14a) or (14b) may be adopted in isolation as a satisfactory first approximation for monodisperse suspensions (Hawksley, 1951). In order to treat this problem more rigorously, the relative slip velocities would have to be determined theoretically based either on a momentum equation (e.g. Berres et al., 2004) or on an energy dissipation equation (e.g. Mills and Snabre, 1994).

3.2. Modified Cheng's formulation for "mud-rich" mixtures

As previously indicated, the hindered settling regime for a concentrated sand–mud suspension depends primarily on the fractional contents ϕ_f and ϕ_s of the mud flocs and sand particles, as well as on their respective sizes D_f and D_s. Considering the approach taken by Cheng (1997b), the slip velocities of settling mud flocs w'_f and sand particles w'_s in the sand–mud mixture can be determined from the continuity equation [Eq. (13)] as

$$w'_f = \frac{(1-\phi_s)w_{sf} + \phi_s w_{ss}}{(1-\phi)} \tag{16a}$$

$$w'_s = \frac{(1-\phi_f)w_{ss} + \phi_f w_{sf}}{(1-\phi)}. \tag{16b}$$

As with the WVK method, additional assumptions are required in order to solve Eq. (16) for hindered settling velocities w_{ss} and w_{sf}. Again, considering suspensions with a high mud content (i.e. "mud-rich" mixtures with $\phi_s << \phi_f$), the slip velocity w'_f for a monodisperse concentration of mud flocs can be obtained by applying the same arguments that Cheng (1997b) adopted for monodisperse sand suspensions (see Section 2.2). In this way, the slip velocity of the mud flocs $w'_f = (w_{sf}-w)$ can be expressed [following Eqs. (10) and (11)] as

$$w'_f = \frac{w_{sf}}{(1-\phi_f)} \tag{17}$$

$$w'_f = \frac{\nu'_f}{D_f}\left(\sqrt{25+1.2D'^2_{*f}}-5\right)^{1.5} \tag{18}$$

where $\nu'_f = \nu(1-\phi_f)^{-2.5}$ is the effective mud-fluid mixture viscosity [following Brinkmann (1948)] and $D'_{*f} = (\Delta'_f g/\nu'^2_f)^{1/3} D_f$ is the modified non-dimensional mud floc parameter [with $\Delta'_f = (\rho_f-\rho')/\rho'$ and $\rho' = \phi_f\rho_f + (1-\phi_f)\rho_w$]. Note that Eq. (17) can also be derived by substituting ϕ_s = 0 into Eq. (16a). Using Eq. (18) to define w'_f, the hindered settling velocity w_{sf} of the mud flocs is then predicted from substitution of w'_f into Eq. (17). The corresponding slip velocity of the sand particles w'_s contained within the "mud-rich" suspension can then be estimated by modifying Eq. (11), such that

$$w'_s = \frac{\nu'_f}{D_s}\left(\sqrt{25+1.2D'^{2}_{*s}}-5\right)^{1.5} \tag{19}$$

where $D'_{*s}=(\Delta'_s g/\nu'^2_f)^{1/3}D_s$ is the non-dimensional sand particle parameter defined in the mud-fluid mixture [with $\Delta'_s=(\rho_s-\rho')/\rho'$ and $\rho'=\phi_f\rho_f+(1-\phi_f)\rho_w$, as before]. The hindered settling velocity w_{ss} for the sand fraction is then obtained from re-arranging Eq. (16b),

$$w_{ss} = \frac{(1-\phi)w'_s-\phi_f w_{sf}}{(1-\phi_f)}. \tag{20}$$

The predictive behaviour of this new hindered settling formulation given by Eqs. (17)–(20) is demonstrated in Fig. 3(a) and (b) for the same range of sand–mud mixture conditions as tested previously with the WVK method (see Fig. 2). In contrast with the WVK predictions, both the hindered mud and sand settling ratios $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0}$ [$w_{ss,0}$ and $w_{sf,0}$ being obtained from Eqs. (1) and (3), respectively] decrease to zero as the total volumetric concentration $\phi\to 1$ (i.e. as the sand–mud mixture approaches the gelling concentration). The parametric influence of the mud floc size D_f within the mixture is shown in Fig. 3(a) to result in only small increases in the hindered mud settling ratio $w_{sf}/w_{sf,0}$ (at any given ϕ value), whilst also having a small (but inconsistent) effect on $w_{ss}/w_{ss,0}$ values. By contrast, an increase in the sand particle size D_s results in a significant increase in $w_{ss}/w_{ss,0}$ values (for otherwise identical conditions), whilst having no effect on the hindered mud settling $w_{sf}/w_{sf,0}$ values (as expected for the "mud-rich" definition adopted in the modified Cheng model). Fig. 3(b) indicates that the volumetric sand content ϕ_s within the mixture has only a small parametric influence on both $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0}$, with marginal increases observed in both hindered settling ratios as the sand content is increased. This result arises from the fact that, at any given total mixture concentration ϕ, an increase in sand content ϕ_s corresponds to a reduction in mud floc content ϕ_f, which when substituted into Eqs. (17) and (20), will result in increased w_{sf} and w_{ss} values, respectively.

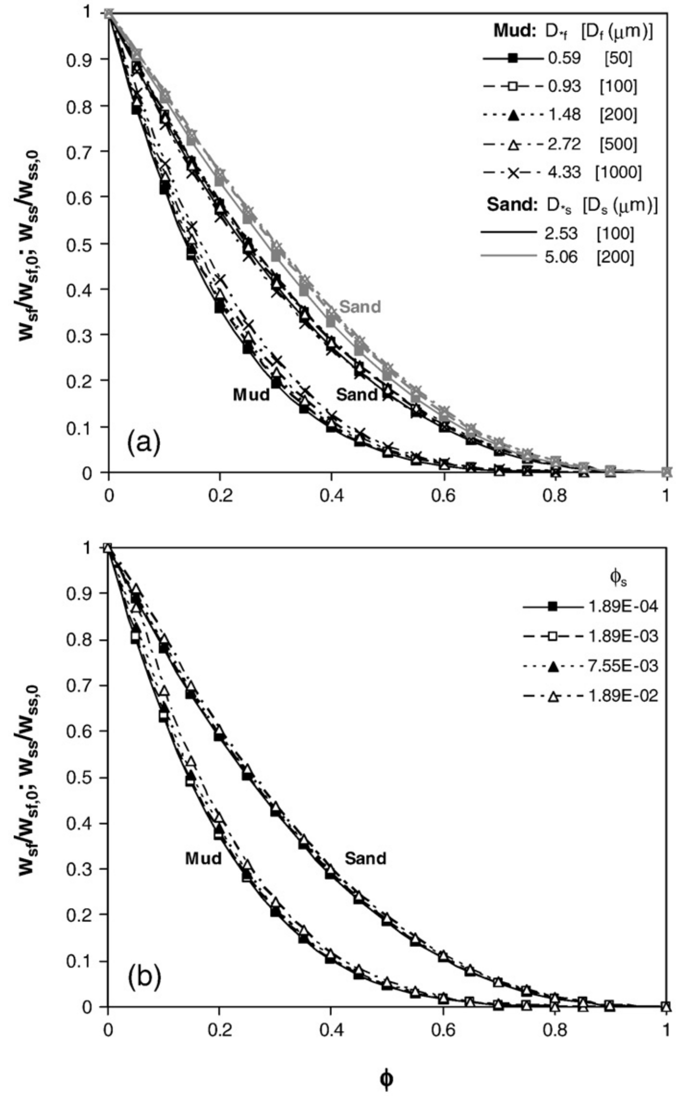

Fig. 3. Hindered settling ratios $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0}$ (obtained by the modified Cheng model) versus mixture concentration ϕ, showing parametric influences of (a) mud floc and sand particle sizes (D_{*f} and D_{*s}) for fixed sand content ϕ_s=1.89×10^{-4} (c_s=0.5 kg m^{-3}) and (b) variable sand content ϕ_s with fixed mud floc and sand particle sizes [D_{*f}=1.48 (D_f=200 μm) and D_{*s}=2.53 (D_s=100 μm)].

3.3. Polydisperse formulation for sand–mud mixtures

In order to remove the restriction associated with the "mud-rich" assumption ($\phi_s<<\phi_f$), as well as to account adequately for the relative size and density effects between the cohesive mud flocs and non-cohesive sand particles, a model formulation that considers the hindered settling velocities of polydisperse concentrated suspensions is proposed. Batchelor (1982) calculated fractional settling velocities w_{si} for a suspension containing m different particle types to the first order in respect to their corresponding fractional volume concentrations ϕ_j within the polydisperse mixture for small particle Péclet numbers Pe (=Ua/D, with U being a typical particle velocity, a the particle radius and D the particle diffusion coefficient), proposing the following:

$$w_{si} = w_{si,0}\left(1+\sum_{j=1}^{m} S_{ij}\phi_j\right) \tag{21}$$

where $w_{si,0}$ is the terminal settling velocity of a solitary particle from fraction i; S_{ij} is an empirical sedimentation parameter generally dependent on the particle diameter ratio λ_{ij} (=D_j/D_i), the reduced density ratio γ_{ij} [=$(\rho_j-\rho_w)/(\rho_i-\rho_w)$] and the particle Péclet number Pe; and ϕ_j is the volumetric concentration of fraction j. The particle Péclet number compares the ratio of particle advection to particle diffusion and thus provides an estimate as to whether particle motions are predominantly advective ($Pe>>1$) or dominated by randomly diffusive (Brownian) motions ($Pe<<1$). Davis and Gecol (1994) extended the work of Batchelor (1982) and Batchelor and Wen (1982) through consideration of the Richardson–Zaki-type relation [e.g. Eq. (4)], proposing the following relationship for w_{si}:

$$w_{si} = w_{si,0}(1-\phi)^{-S_{ii}}\left(1+\sum_{j\neq i}^{m}(S_{ij}-S_{ii})\phi_j\right). \tag{22}$$

In applying this type of polydisperse hindered settling model to concentrated sand–mud suspensions, it is assumed that the mud flocs and sand particles contained within the suspension are individually monodisperse (i.e. represented by single volumetric concentrations ϕ_f and ϕ_s and sizes D_f and D_s, respectively). It is also assumed that $Pe>>1$ for each fraction (i.e. non-colloidal). Under these conditions, the sedimentation parameter S_{ij} can be estimated for the mud floc and sand particle fractions by an expression proposed by Ha and Lui (2002), following simulations by Batchelor and Wen (1982):

$$S_{ij} = -2.5 - \left(\lambda_{ij}^2 + 3\lambda_{ij} + 1 - \frac{1.87\lambda_{ij}}{1+0.0024\lambda_{ij}^2}\right)\gamma_{ij} \tag{23}$$

where λ_{ij} and γ_{ij} are the particle diameter and reduced density ratios (defined previously) and subscripts i, j can be replaced by f, s to denote the mud and sand fractions. Thus, for a mixed suspension of monodisperse sand particles and mud flocs, Eq. (22) can be specified for the individual mud and sand constituents as follows,

$$w_{sf} = w_{sf,0}(1-\phi)^{-S_{ff}}\left[1+(S_{fs}-S_{ff})\phi_s\right] \tag{24a}$$

$$w_{ss} = w_{ss,0}(1-\phi)^{-S_{ss}}\left[1+(S_{sf}-S_{ss})\phi_f\right] \tag{24b}$$

in which S_{fs} and S_{sf} (note: $S_{fs} \neq S_{sf}$) are given according to Eq. (23) as:

$$S_{fs} = -2.5 - \left[\left(\frac{D_s}{D_f}\right)^2 + 3\left(\frac{D_s}{D_f}\right) + 1 - \frac{1.87(D_s/D_f)}{1+0.0024(D_s/D_f)^2}\right]\left(\frac{\rho_s-\rho_w}{\rho_f-\rho_w}\right) \tag{25a}$$

$$S_{sf} = -2.5 - \left[\left(\frac{D_f}{D_s}\right)^2 + 3\left(\frac{D_f}{D_s}\right) + 1 - \frac{1.87(D_f/D_s)}{1+0.0024(D_f/D_s)^2}\right]\left(\frac{\rho_f-\rho_w}{\rho_s-\rho_w}\right). \tag{25b}$$

Assuming the densities of the sand particles and mud primary particles are equal (e.g. $\rho_s = \rho_p$), the density difference terms in Eqs. (25a) and (25b) can be related to the mud primary particle/mud floc size ratio (e.g. Winterwerp, 2002), such that

$$\left(\frac{\rho_p-\rho_w}{\rho_f-\rho_w}\right) = \left(\frac{D_f}{D_p}\right)^{3-n_f};\ \left(\frac{\rho_f-\rho_w}{\rho_p-\rho_w}\right) = \left(\frac{D_p}{D_f}\right)^{3-n_f}. \tag{26}$$

It is clear from these relative density terms that S_{fs} can be orders of magnitude larger than S_{sf} [i.e. through $(D_f/D_p)^{3-nf} >> (D_p/D_f)^{3-nf}$]. It is therefore expected that S_{fs} will have a greater (and more varied) influence on the hindered settling characteristics of the mud fraction than S_{sf} will have on the sand fraction. It can also be shown that both sedimentation parameters S_{fs} and S_{sf} exhibit a maximum value at $D_f/D_s = 0.1$, with the steep reduction in S_{fs} and S_{sf} values for $D_f/D_s < 0.1$ due to the small floc sizes in this region, which influences Eqs. (25a) and (25b) through large size ratios D_s/D_f and D_p/D_f, respectively. Conversely, the monotonic reduction in S_{fs} and S_{sf} values for $D_f/D_s > 0.1$ results from the presence of large flocs with small densities, influencing Eqs. (25a) and (25b) through large size ratios $D_f/D_p >> 1$ and $D_f/D_s > 1$, respectively. The two sedimentation parameters S_{ff} and S_{ss} specified in Eq. (24), can be regarded as being equivalent to the exponent n within monodisperse hindered settling formulations [e.g. Richardson and Zaki (1954), Eqs. (4) and (6)], although these are clearly applied within a polydisperse (sand–mud) suspension. In the absence of specific experimental values for S_{ff} and S_{ss}, initial estimates can be obtained from Eq. (23) (substituting for $\lambda_{ij} = 1$ and $\gamma_{ij} = 1$), yielding constant values of $S_{ff} = S_{ss} = -5.63$. This assumption that the sedimentation parameters are both constant and equal is deemed acceptable as a first approximation. The sensitivity of modified DG model predictions to the values of S_{ff} and S_{ss} specified is tested later (Section 4.2).

As Eqs. (24a) and (24b) include implicitly all hindered settling effects including return flow generation, increased viscosity and buoyancy of the sand–mud mixture, a correction is required to account for the increased buoyancy effect resulting from the sum of the volumetric particle concentrations $(\phi_p + \phi_s)$ rather than from the total mixture volumetric concentration $\phi = (\phi_f + \phi_s)$. The proposed correction applied to both fractional equations is therefore given below:

$$w_{sf} = w_{sf,0}(1-\phi)^{-(S_{ff}+1)}(1-\phi_p-\phi_s)\left[1+(S_{fs}-S_{ff})\phi_s\right] \tag{27a}$$

$$w_{ss} = w_{ss,0}(1-\phi)^{-(S_{ss}+1)}(1-\phi_p-\phi_s)\left[1+(S_{sf}-S_{ss})\phi_f\right]. \tag{27b}$$

Thus, exponents $-(S_{ss}+1)$ and $-(S_{ff}+1)$ within Eq. (27) can be regarded as an equivalent n value for hindered effects that arise solely due to the total volumetric concentration of the sand–mud mixture. The predictive behaviour of this new model [subsequently referred to as the modified Davis and Gecol (DG) model] is shown in Fig. 4 to have a similar qualitative variation with ϕ as the modified Cheng model (Fig. 3), with hindered settling ratios $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0} \to 0$ as $\phi \to 1$ (i.e. as mixture gelling conditions are approached). Fig. 4(a) indicates that the parametric influence of the relative mud floc D_f and sand particle D_s sizes [included through sedimentation parameters S_{fs} and S_{sf} — Eq. (25)] are relatively minor and shows no clear trend in the majority of cases (except for sand–mud mixtures containing large

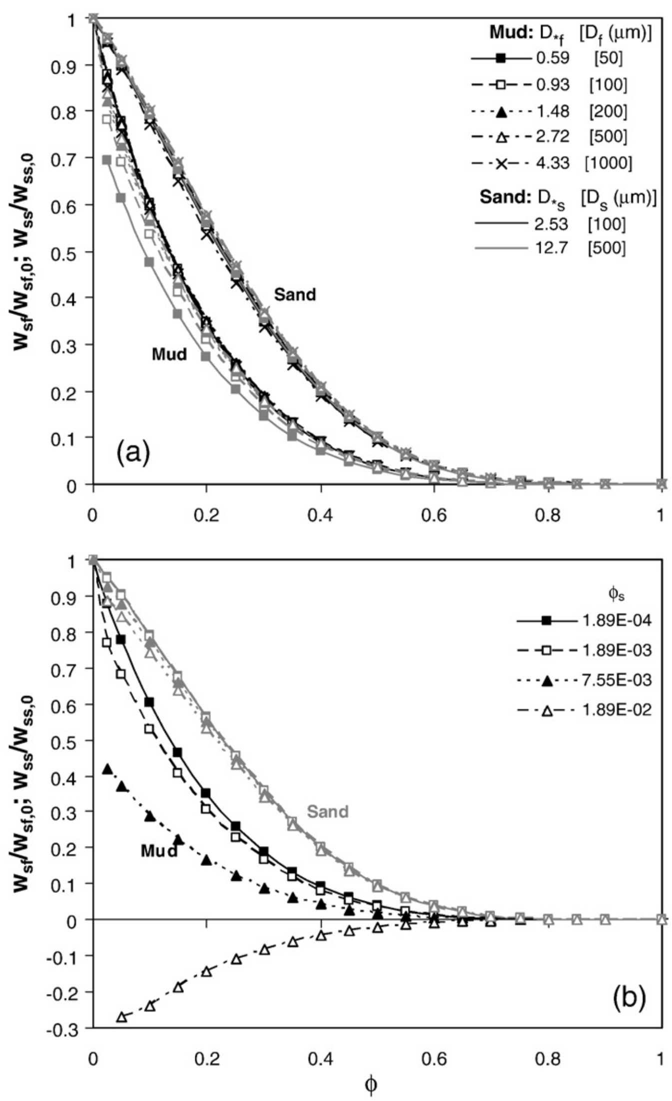

Fig. 4. Hindered settling ratios $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0}$ (obtained by the modified DG model) versus mixture concentration ϕ, showing parametric influences of (a) mud floc and sand particle sizes (D_{*f} and D_{*s}) for fixed sand content $\phi_s = 1.89 \times 10^{-4}$ ($c_s = 0.5$ kg m^{-3}) and (b) variable sand content ϕ_s with fixed mud floc and sand particle sizes [$D_{*f} = 1.48$ ($D_f = 200$ μm) and $D_{*s} = 2.53$ ($D_s = 100$ μm)].

Table 1
Sand–mud mixture characteristics and parameters for model simulations

D_p (μm)	D_f (μm)	D_s (μm)	c_p (kg m^{-3})	c_s (kg m^{-3})	ρ_s, ρ_p (kg m^{-3})	ρ_f (kg m^{-3})	ρ_w (kg m^{-3})	c_{gel} (kg m^{-3})	ν (m^2 s^{-1})	n_f
5	10–1000	100, 200	0.5–50	0.5–50	2650	1008–1825	1000	13.3–1325	1.0x 10^{-6}	2.0

sand particles and small mud flocs, where the hindered mud settling ratio $w_{sf}/w_{sf,0}$ values are clearly reduced). By contrast, the parametric influence of the sand content ϕ_s within the mixture is shown in Fig. 4(b) to result in a considerable variation in the behaviour of the hindered mud settling ratio $w_{sf}/w_{sf,0}$ from that predicted by the previous formulations. Specifically, while the modified DG model behaves in a similar qualitative manner to these "mud-rich" simulations at low sand contents ϕ_s, the hindered settling ratio $w_{sf}/w_{sf,0}$ values are shown in Fig. 4(b) to decrease significantly with increasing ϕ_s, becoming negative as sand content ϕ_s increases further. This latter prediction indicates that, when the sand concentration ϕ_s becomes sufficiently high, the mud flocs within the suspension are being transported upwards rather than settling downwards. The mechanism responsible for this upward transport of mud flocs is likely to be related to the strength of return flow generated by the presence of a high sand content in the mixture, which may exceed the settling velocity of the mud flocs. In either case (upward or downward transport of mud flocs), the transport velocity diminishes as the total mixture concentration ϕ approaches the gelling concentration (i.e. $\phi \to 1$), where $w_{sf}/w_{sf,0} \to 0$, as before.

4. Results and discussions

Within this section, the hindered fractional settling velocities w_{sf} and w_{ss} for sand–mud suspensions with a specified range of mixture characteristics (see Table 1) are predicted and compared using each of the three methods, namely: (i) the WVK method [Eqs. (15a) and

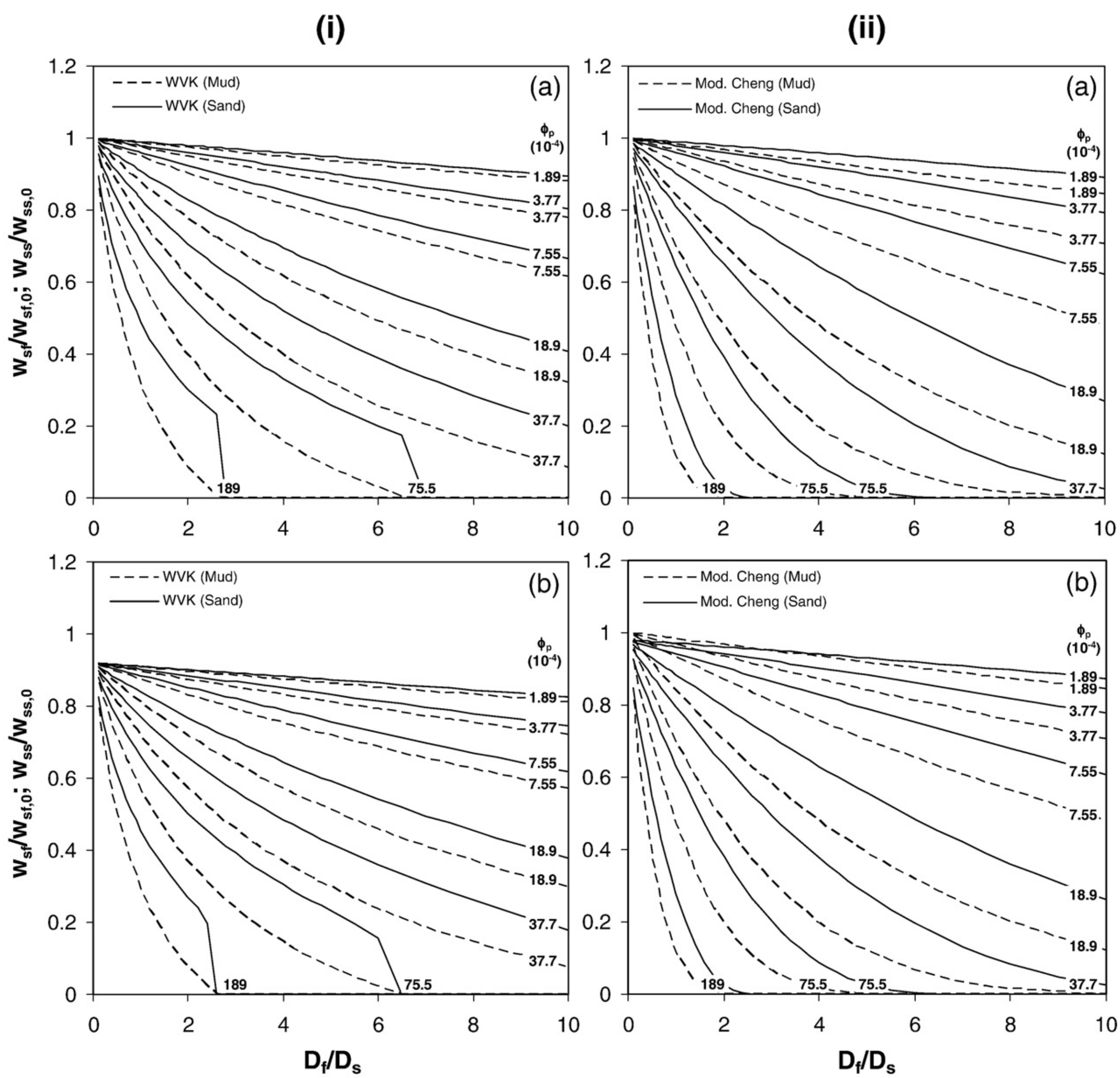

Fig. 5. Hindered settling ratios $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0}$ versus size ratio D_f/D_s (D_s = 100 μm) showing predictions from (i) the WVK method and (ii) the modified Cheng model, for sand content ϕ_s values of (a) 1.84×10^{-4} and (b) 1.89×10^{-2}. (Mud content ϕ_p values as shown).

(15b)]; (ii) the modified Cheng method [Eqs. (17)–(20)]; and (iii) the modified Davis and Gecol (DG) method [Eqs. (25)–(27)].

4.1. Parametric simulations

Within the model computations of the hindered settling for sand–mud suspensions, the mixture characteristics can be defined adequately through (i) the relative mud floc to sand particle size ratio D_f/D_s and (ii) the relative mud and sand volumetric concentrations ϕ_p, ϕ_f and ϕ_s [i.e. ϕ_p and $\phi_s = 1.89\times10^{-4} - 1.89\times10^{-2}$; $\phi_f = 3.77\times10^{-4} - 3.76$ for parameter ranges given in Table 1 (note that within the hindered settling regime, total volumetric mixture concentration $\phi = \phi_f + \phi_s \le 1$)].

Fig. 5(i) and (ii) shows the predicted variations in $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0}$, obtained from the "mud-rich" formulations of the WVK and modified Cheng methods, respectively, over a relative mud floc to sand particle size range $D_f/D_s = 0.1$ - 10 (with $D_f = 10$–1000 μm and $D_s =$ 100 μm, Table 1). Within these figures, predictions are provided for a range of mud concentrations ϕ_p (see Table 1), while the sand content ϕ_s within the mixtures is increased by two orders of magnitude [i.e. $c_s = 0.5$ kg m^{-3} in Fig. 5(i)(a) and (ii)(a); $c_s = 50$ kg m^{-3} in Fig. 5(i)(b) and (ii)(b)]. Both methods provide similar predictions, with consistent trends showing monotonic reductions in $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0}$ values as the relative size ratio D_f/D_s and fractional mud concentration ϕ_p increase. The relative insensitivity of both these methods to the sand content ϕ_s within the mixture is again highlighted by the marked similarity between Fig. 5(i)(a) and (i)(b) and between 5(ii)(a) and (ii)(b), indicating the significant limitation imposed by the "mud-rich" assumption adopted in both formulations, whereby the hindering effects of the sand fraction are not accommodated adequately in either the WVK or modified Cheng models. Fig. 5(i)(a) and (i)(b) also illustrate the peculiarity arising within the WVK predictions, where a sharp reduction in the hindered sand settling ratio $w_{ss}/w_{ss,0}$ is observed for mixtures with high mud content ϕ_p when a specific size ratio D_f/D_s is attained (i.e. specific floc size D_f for a fixed sand particle size D_s). As noted previously, this marked reduction occurs at specific mixture conditions that are close to the gelling concentration ($\phi \to 1$), where the hindrance ratio $(1-\phi)/(1-\phi_f)$ in Eq. (15b) decreases sharply from values close unity at $\phi < 1$ to zero at $\phi = 1$.

Fig. 6(a)–(d) presents predictions of $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0}$ obtained from the modified DG model for a range of volumetric sand concentrations ϕ_s. For mixtures with small sand contents [e.g. Fig. 6(a)], the predicted variations in $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0}$ are shown to be

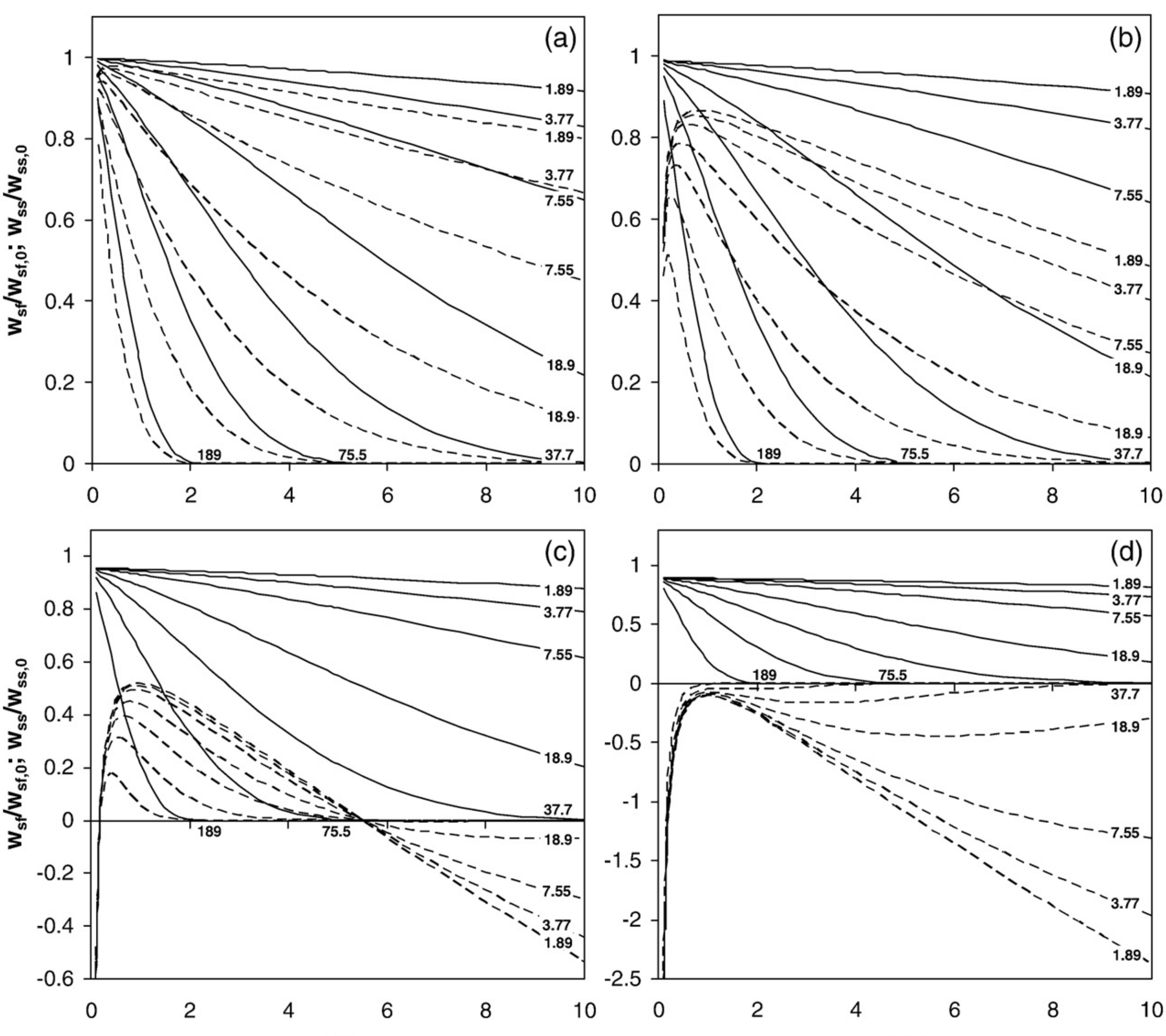

Fig. 6. Hindered settling ratios $w_{sf}/w_{sf,0}$ (− −)and $w_{ss}/w_{ss,0}$ (−) versus size ratio D_f/D_s ($D_s = 100$ μm) showing predictions from the modified Davis and Gecol method for sand content ϕ_s values of (a) 1.89×10^{-4}; (b) 1.89×10^{-3}; (c) 7.55×10^{-3} and (d) 1.89×10^{-2}. Mud content ϕ_p ($\times10^{-4}$) values are as shown.

qualitatively similar to the "mud-rich" predictions of the WVK and modified Cheng methods. However, Fig. 6(b)–(d) indicate the significant effect that the increasing sand content ϕ_s has on the modified DG predictions of the mud floc settling ratio $w_{sf}/w_{sf,0}$. Specifically, Fig. 6(a) and (b) show that increasing ϕ_s by a factor of 10 results in $w_{sf}/w_{sf,0}$ values that are appreciably lower (for specific D_f/D_s and ϕ_p values). With a further 4-fold increase in ϕ_s [Fig. 6(c)], the hindered mud floc settling only remains positive (i.e. downward motion) within the relative size range ~$0.2<D_f/D_s<$~5.5 (i.e. when $D_f \approx 20$–550 μm, for fixed $D_s = 100$ μm), with upward movement of mud flocs (i.e. $w_{sf}/w_{sf,0}<0$) predicted outside this size range. The physical justification for this predicted behaviour is that both smaller, denser mud flocs (i.e. small D_f; high ρ_f) and larger, but significantly less dense flocs (i.e. large D_f; low ρ_f) could be transported upwards in a strong return flow, generated by the increased sand content ϕ_s within the settling suspension. These floc size and density effects are accentuated in suspensions containing very high volumetric sand contents [Fig. 6(d)], where the upward flow of mud flocs (i.e. $w_{sf}/w_{sf,0} \le 0$) is predicted to occur at all relative size ratios D_f/D_s and volumetric mud contents ϕ_p. It is also noted from Fig. 6(c) and (d) that the direction of mud floc transport (i.e. upwards or downwards) does not affect the tendency for settling ratios $w_{sf}/w_{sf,0}$ and $w_{ss}/w_{ss,0} \to 0$ as sand–mud mixture gelling conditions are approached (i.e. $\phi \to 1$).

Fig. 7(a)–(d) presents similar hindered settling predictions from the modified DG model, but with a larger sand particle size D_s (=200 μm; D_f/D_s=0.05–5) specified. Comparing these plots directly with Fig. 6(a)–(d) indicates that, while the overall hindered settling trends are qualitatively similar, a universal reduction is observed in the hindered mud settling $w_{sf}/w_{sf,0}$ values obtained for mixtures containing the larger sand particle size (Fig. 7). This effect is believed to result from an overall increase in return flow strength generated by the larger (and heavier) sand particles, which can induce upward mud floc motions (i.e. $w_{sf}/w_{sf,0}<0$) at lower volumetric sand contents ϕ_s within the mixtures.

It is also informative to consider the relative sensitivity of the hindered sand settling characteristics to the sand content ϕ_s within the sand–mud suspension, as predicted by the modified Cheng and DG models. Comparing equivalent plots within Figs. 5 and 6, the reduction in $w_{ss}/w_{ss,0}$ values associated with the two orders of magnitude increase in ϕ_s [i.e. $\phi_s = 1.89\times10^{-4}$ in Figs. 5(ii)(a) and 6(a) and $\phi_s = 1.89\times10^{-2}$ in Figs. 5 (ii)(b) and 6 (d)] is typically about 2–3%, when predicted by the modified Cheng model, whilst being generally

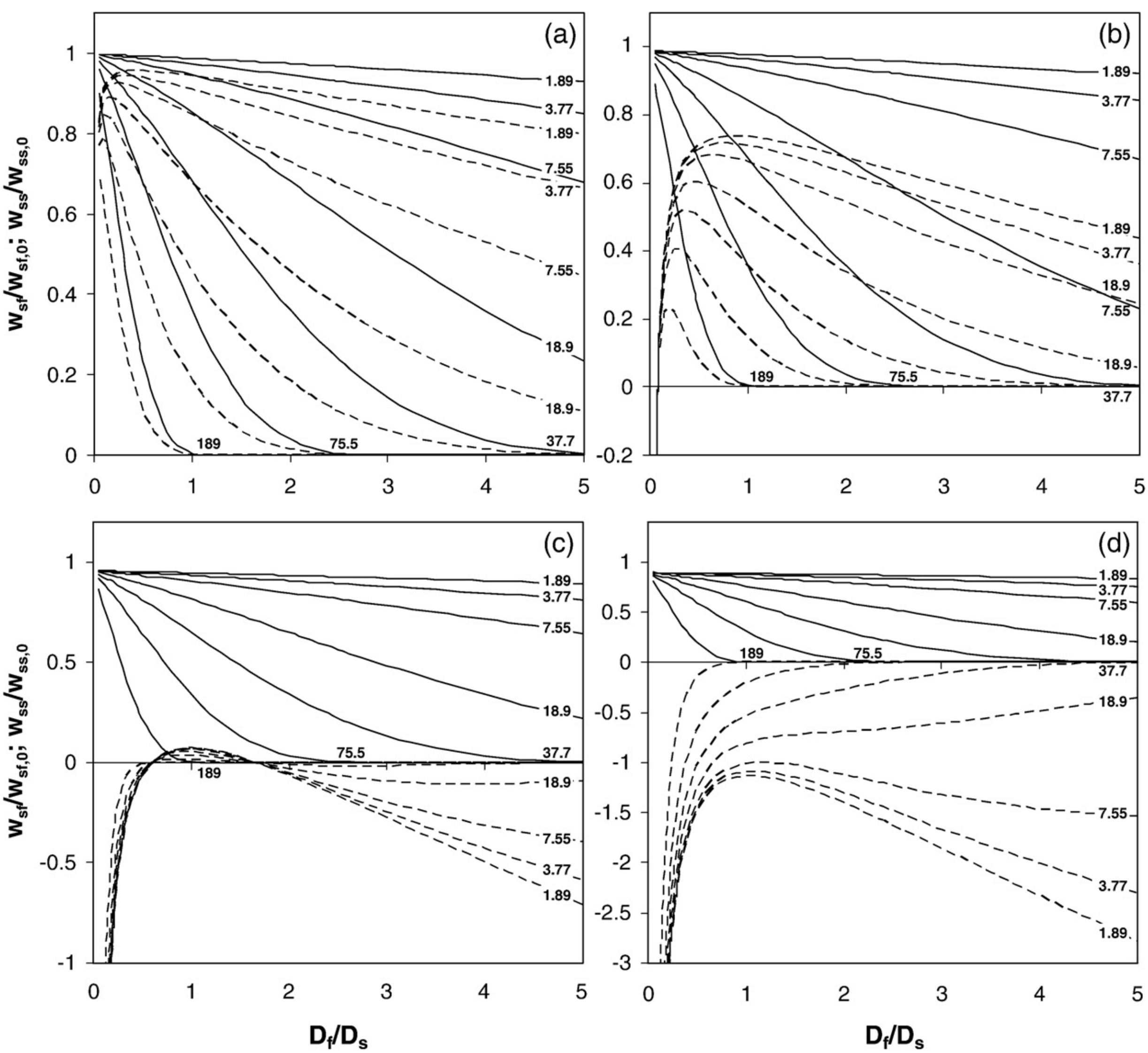

Fig. 7. Hindered settling ratios $w_{sf}/w_{sf,0}$ (− −) and $w_{ss}/w_{ss,0}$ (−) versus size ratio D_f/D_s (D_s=200 μm) showing predictions from the modified Davis and Gecol method for sand content ϕ_s values of (a) 1.89×10^{-4}; (b) 1.89×10^{-3}; (c) 7.55×10^{-3} and (d) 1.89×10^{-2}. Mud content ϕ_p ($\times10^{-4}$) values are as shown.

around 10–12% for the modified Davis and Gecol model (except for highly concentrated sand–mud mixtures close to the gelling point, where $w_{ss}/w_{ss,0} \to 0$ as $\phi \to 1$). This apparent under-prediction of hindered settling conditions by the modified Cheng method again serves to highlight the limitation of "mud-rich" assumption adopted in its formulation, as opposed to the more general polydisperse approach adopted in the modified Davis and Gecol model.

4.2. Modified DG method — predictive capability and sensitivity

In order to further evaluate the predictive capability of the modified DG model, predictions obtained for two monodisperse cases: (i) pure-mud suspensions (with $\phi_s=0$) and (ii) pure-sand suspensions (with $\phi_f=\phi_p=0$) are compared with Winterwerp's (2002) hindered settling formula for a concentrated mud suspension [Eq. (8)] and Cheng's (1997b) expression for concentrated suspensions of sand particles [Eqs. (10) and (11)], respectively. These special cases will allow the sensitivity of the DG model to variations in the monodisperse sedimentation parameters S_{ff} and S_{ss} to be investigated, which for the previous runs (Figs. 4, 6 and 7) were set constant at $S_{ff}=S_{ss}=-5.63$. For the pure-mud suspension case initially, Eq. (27a) of the DG method can be simplified by setting $\phi_s=0$ as

$$w_{sf} = w_{sf,0}\left(1-\phi_f\right)^{-\left(S_{ff}+1\right)}\left(1-\phi_p\right). \tag{28}$$

Fig. 8(a) shows a dimensional plot of predicted hindered settling velocity w_{sf} versus floc size D_f (10–1000 μm) for a range of mud mass concentration c_p values (0.5–20 kg m^{-3}) and three different values of S_{ff} (−5.63, −4.5 and −3.5). Corresponding predictions from Winterwerp's (2002) formula [Eq. (8)] are plotted for comparison. This plot indicates that, at low mud concentration (c_p=0.5 kg m^{-3}), the DG model predictions coincide well with Winterwerp's (2002) formula when S_{ff}=−4.5. The reason for this agreement is that the viscosity factor $1/(1+2.5\phi_f)$ within Winterwerp's equation can be approximated by $(1-\phi_f)^{2.5}$ for low volumetric concentrations ϕ_f, and thus Eq. (8) can be rewritten as

$$w_{sf} \approx w_{sf,0}\left(1-\phi_f\right)\left(1-\phi_p\right)\left(1-\phi_f\right)^{2.5}. \tag{29}$$

Comparing Eqs. (28) and (29), these are equivalent when $-(S_{ff}+1)=3.5$ and hence, $S_{ff}=-4.5$. The viscosity approximation 1/(1+

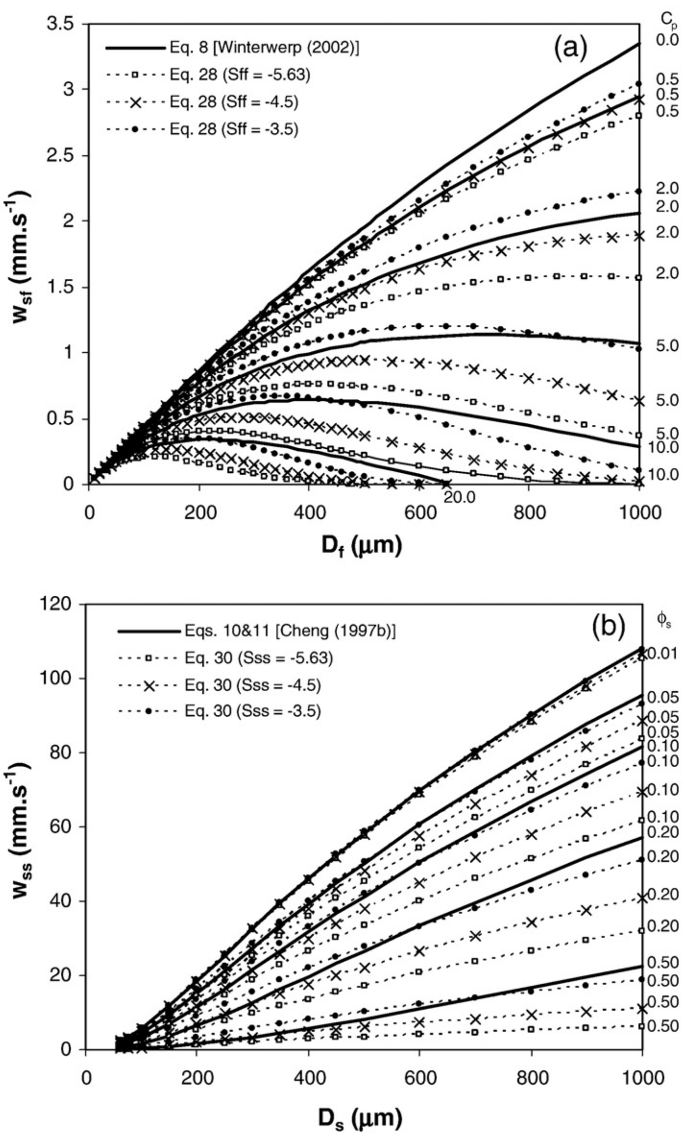

Fig. 8. Hindered mud settling velocity w_{sf} versus floc size D_f for (a) monodisperse "mud-only" suspensions (c_p values as shown) and (b) monodisperse "sand-only" suspensions (ϕ_s values as shown), showing sensitivity to specified S_{ff} and S_{ss} values, respectively.

$2.5\phi_f)\approx(1-\phi_f)^{2.5}$ is, however, clearly not valid for larger ϕ_f values, as indicated by the divergence in Winterwerp (2002) and DG model predictions (with $S_{ff}=-4.5$) at higher mud mass concentration $c_p$values [see Fig. 8(a)]. Similarly, for sand-only concentrations (i.e. $\phi_f=\phi_p=0$), Eq. (27b) of the modified DG method can be simplified to

$$w_{ss}=w_{ss,0}(1-\phi_s)^{-(S_{ss}+1)}(1-\phi_s)=w_{ss,0}(1-\phi_s)^{-S_{ss}} \tag{30}$$

which has the same general form as Richardson and Zaki's (1954) equation for hindered settling. Fig. 8(b) shows a dimensional plot of hindered sand settling velocity w_{ss} versus particle size D_s (65–1000 μm) for a range of volumetric sand concentrations (ϕ_s=0–0.6) and for three different values of sedimentation parameter S_{ss} (−5.63, −4.5 and −3.5). Predictions from Cheng's (1997b) hindered settling formulae [Eqs. (10) and (11)] are also shown in Fig. 8(b) for comparison. At relatively low sand volumetric concentrations (ϕ_s=0.01 — equivalent to c_p=26.5 kg m^{-3} for ρ_s=2650 kg m^{-3}), the DG model predictions are relatively unaffected by the value specified for S_{ss}, as $(1-\phi_s)\rightarrow1$ when $\phi_s\rightarrow0$. In general for all ϕ_s values, the DG predictions obtained for $S_{ss}=-3.5$ show best agreement with Cheng's formulae, although the observed differences between the predictions of the two methods [Fig. 8(b)] arise from the fact that the hindered settling exponent n [≡ $-S_{ss}$ in Eq. (30)] is implicitly varied within Cheng's formulations for different particle Reynolds number (and hence different particle size D_s). Therefore, it may be argued that monodisperse sedimentation parameters S_{ss} and S_{ff} should have different values or value ranges that could vary with sediment particle D_s or mud floc D_f size, as revealed for sand concentrations by Cheng (1997b) and Baldock et al. (2004).

5. Summary and concluding remarks

The hindered settling characteristics of concentrated suspensions containing mixtures of monodisperse mud flocs and sand particles have been examined using three models derived on different approaches to the problem, namely, through consideration of the mixtures as: (i) concentrated mud floc suspensions with relatively low sand contents (e.g. following Winterwerp, 2002; Winterwerp and Van Kesteren, 2004); (ii) monodisperse concentrations of sediment particles, based on the slip velocity formulations of Cheng (1997a,b); and (iii) polydisperse sediment particle concentrations based on the work of Batchelor (1982) and Davis and Gecol (1994).

The model simulations have indicated, as expected, that the applicability of the Winterwerp and Van Kesteren (WVK) and modified Cheng methods are restricted to simulation of so-called "mud-rich" suspensions in which the volumetric sand content is considerably lower than the mud floc concentration (i.e. $\phi_s<<\phi_f$). As a consequence, these models are unable to account adequately for the enhanced hindered settling effects on the mud fraction that arise from increasing the volumetric sand content ϕ_s within the sand–mud mixture. The results show that the WVK model under-predicted the hindered settling effects on the sand fraction at high volumetric mud contents ϕ_p within the sand–mud mixture. This is a direct consequence of the assumption in the WVK method that the slip velocities of the sand particles ($w_{ss}-w$) and mud flocs ($w_{sf}-w$) are equal to their single particle/floc settling velocities (i.e. $w_{ss,0}$ and w_{ss}, respectively). As noted previously, this assumption has no physical basis for polydisperse suspensions as it implies that the difference between the hindered sand particle and mud floc settling velocities within any suspended sand–mud mixture is equal to the difference in the "single particle" and "single floc" settling velocities [$(w_{ss}-w_{sf})=(w_{ss,0}-w_{sf,0})$]. Therefore, this approach cannot be regarded as being universally valid, even within the "mud-rich" restrictions (i.e. $\phi_s<<\phi_f$) imposed in the WVK method, as indicated by the abrupt reduction in the hindered sand settling velocity w_{ss} predicted as $\phi\rightarrow1$ [see Figs. 2 and 5(i)].

The polydisperse modelling approach developed by modifying the sedimentation formulations of Davis and Gecol (1994) does not suffer from the problems or limitations associated with both the WVK and modified Cheng methods. Specifically, the additional hindering effects resulting from increased volumetric sand concentrations ϕ_s (and/or larger sand particle size D_s) within the sand–mud mixture is shown to have a significant impact on the hindered mud floc settling characteristics. Indeed, at sufficiently high ϕ_s values, this model predicts an upward mud floc movement within the suspension ($w_{sf}/w_{sf,0}<0$), a behaviour which was previously observed experimentally by Dankers (2006) and Amy et al. (2006). This result is considered to be the consequence of the stronger return flow generated by an increased sand content ϕ_s, that will act to negate the settling tendencies of both smaller, denser mud flocs (controlled by size effects) and larger, less dense mud flocs [where density effects due to the fractal floc structure control (i.e. ρ_f decreases as D_f increases)]. The influence of the sand content ϕ_s is also shown to affect the hindered settling of the sand fraction, although to a significantly lesser degree than for the mud flocs. Simulations also indicated that the DG model could be applied to pure-mud and pure-sand suspensions, with the model predictions showing reasonable agreement with Winterwerp's (2002) hindered settling formula for concentrated mud suspensions and Cheng's (1997b) expression for suspended sand concentrations, respectively. Further improvements may be achieved by specifying separate values for S_{ff} and S_{ss} (i.e. $S_{ff}\neq S_{ff}$).

In summary, when considering the hindered settling characteristics of sand–mud mixtures which can be considered "mud-rich" (i.e. $\phi_s<<\phi_f$), this study has demonstrated that the three proposed models provide largely similar predictions for hindered mud settling velocity w_{sf}. However, for mixtures with higher ϕ_s content, the modified Davis and Gecol method provides the only modelling framework within which the additional hindering effects of the sand particles on the hindered settling behaviour of both the sand and mud fractions within the mixture can be accommodated adequately. A key difficulty that remains is the lack of reliable datasets with which to validate these models. As far as the authors are aware, only Dankers (2006) has attempted to measure the hindered settling characteristics of both fractions within a sand–mud suspension simultaneously. However, although these tests were intended for "mud-rich" conditions, the maintenance of flow homogeneity and suspension across the settling tank was found to be difficult. Further research is therefore required to develop advanced laboratory facilities and sophisticated experimental techniques that would enable reliable and repeatable measurements of the hindered settling regime of all types of sand–mud mixtures.

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