Sediment Transport, Part III: Bed Forms and Alluvial Roughness

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Abstract: A method is presented that makes the classification of bed forms, the prediction of the bed-form dimensions and the effective hydraulic roughness of the bed forms feasible. The proposed relationships are based on the analysis of reliable flume and field data. A verification analysis using about 1,500 (alternative) reliable flume and field data shows good results in predicting the hydraulic roughness (friction factor). For field conditions, the proposed method yields considerably better results than previously proposed methods, which are reviewed here. The proposed method has also been used to predict the flow depth and the total bed-material load.

Introduction

In two companion papers (see Refs. 37 and 38), bed-load and suspended-load transport are investigated, resulting in new relationships that have been extensively verified. However, in those papers the proposed relationships are not really used in a predictive sense because the applied Chézy-coefficient is based on measured variables (flow velocity, depth and energy gradient). Even in uniform flow conditions the morphological behavior of an alluvial channel is rather complicated. The fundamental difficulty is that the channel bed characteristics (bed forms), and thus, hydraulic roughness, depend on flow conditions (flow velocity and depth) and sediment transport rate. These flow conditions are, in turn, strongly dependent on the channel bed configuration and its hydraulic roughness. Therefore, additional information is required for morphological predictions. Firstly, the relationship between the flow conditions and sediment transport rate should be known, while the relationship between hydraulic roughness and the flow conditions should also be available. The importance of the application of a reliable roughness prediction method for morphological computations has been pointed out very clearly by De Vries (42).

In the present analysis only the hydraulic roughness due to small-scale bed forms is considered; other geometrical characteristics such as breading and meandering are not taken into account. Basically, a one-dimensional morphological system can be described by the following 5 equations:

fluid continuity: $Q = \bar{u}\, d\, b$ . . . . (1)

fluid motion: $\bar{u}\,\frac{\delta \bar{u}}{\delta x} + g\,\frac{\delta d}{\delta x} + g\,\frac{\delta z_b}{\delta x} = -g\,\frac{\bar{u}|\bar{u}|}{C^2 d}$ . . . . (2)

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sediment transport: $Q_t = F(\mu, \rho, \rho_s, g, D_{50}, \sigma_s, \bar{u}, C, d, b)$ . . . . (3)

sediment continuity: $$\frac{\delta z_b}{\delta t} + \frac{1}{(1-p)b}\frac{\delta Q_t}{\delta x} = 0 \quad (4)$$

alluvial roughness: $C = F(\mu, \rho, \rho_s, g, D_{50}, \sigma_s, \bar{u}, d)$ . . . . (5a)

known variables: $\mu, \rho, \rho_s, g, D_{50}, p, \sigma_s, Q, b$ . . . . (5b)

unknown variables: $Q_t, C, \bar{u}, d, z_b$ . . . . (5c)

in which Q = discharge; Q_t = sediment transport rate; C = Chézy-coefficient; $\bar{u}$ = mean flow velocity; d = mean flow depth; b = mean flow width; z_b = bed level; D_{50} = particle diameter; σ_s = geometrical standard deviation of bed material; μ = dynamic viscosity coefficient; ρ = density of fluid; ρ_s = density of sediment; g = acceleration of gravity; p = porosity factor; and t = time.

The system is completely closed if a relationship to predict the bed-roughness, C, can be specified. In the two companion papers (37,38), such a relationship was supposed to be unknown and, therefore, the measured Chézy-coefficient was used to predict the sediment transport rate, Q_t.

In this analysis the attention is focused on the prediction of the bed-form characteristics and the Chézy-coefficient as a function of the flow variables (mean flow velocity, depth) and sediment properties (size, gradation). The aim of the analysis is to propose simple functions based on reliable flume and field data that have a good predicting ability for engineering purposes.

Firstly, a classification diagram for determing the type of bed forms in the lower and transitional flow regime is proposed. Thereafter, new relationships for the dune height and length are derived by analyzing flume and field data. Further, the new relationships for the grain- and form roughness which have been proposed recently (36) are now verified and compared with the existing methods of Engelund-Hansen (9), White et al. (43) and Brownlie (5).

Finally, the proposed method for the hydraulic roughness will be used in predicting the total bed-material load for field data.

Characteristic Parameters

It is assumed that the dimensions of the bed forms are controlled mainly by the bed-load transport. In one companion paper (37), it is shown that the bed-load transport can be described by a dimensionless particle parameter, D_*, and a transport stage parameter, T, as follows:

a. particle diameter, $$D_* = D_{50}\left[\frac{(s-1)g}{\nu^2}\right]^{1/3} \quad (6)$$

in which D_{50} = particle diameter of bed material; s = specific density; and ν = kinematic viscosity coefficient. And

b. transport stage parameter, $$T = \frac{(u'_*)^2 - (u_{*,cr})^2}{(u_{*,cr})^2} \quad (7)$$

in which $u'_* = (g^{0.5}/C')\bar{u}$ = bed-shear velocity related to grains; $C' = 18 \log(12R_b/3D_{90})$ = Chézy-coefficient related to grains; R_b = hydraulic radius related to the bed according to Vanoni-Brooks (32); D_{90} = particle diameter of bed material; $\bar{u}$ = mean flow velocity; and $u_{*,cr}$ = critical bed-shear velocity according to Shields (37).

Bed-Form Classification

Usually, the flow conditions in an alluvial channel are classified into (26):

1. Lower flow regime with plane bed, ripples and dunes.
2. Transitional flow regime with washed-out dunes.
3. Upper flow regime with plane bed and anti-dunes.

In the literature, roughly two groups of classification methods are described. Engelund (9) and Garde-Albertson (11) use the Froude number as a classification parameter, while Liu (19) and Simons-Richardson (26) describe the type of bed forms in terms of a suspension parameter and a particle-related Reynolds number.

In this analysis, attention is focused on the lower and transitional flow regimes only because these regimes are the most important for field conditions. As will be shown, these regimes can be quite well-defined without the use of the Froude number, since the sediment transport is not related to the Froude number in this regime (see the previous section). This may also be indicated by the fact that the transitional stage with washed-out dunes is generated for a Froude number of about 0.6 in flume conditions, and of about 0.2–0.3 in field conditions, as observed in the Missouri River (25). Only in the upper flow regime with anti-dunes is the Froude number of importance, since the generation of anti-dunes is mainly governed by free-surface phenomena, as indicated by the fact that the length of the anti-dunes is equal to the wave length of the free surface (16,46).

Ripples and dunes have different geometrical characteristics. The ripple height is much smaller than and practically independent of the flow depth, while the ripple length may be as large as the flow depth. The generation of ripples seems to depend mainly on the stability of the granular bed surface under the action of turbulent velocity fluctuations. The dune height is strongly dependent on the flow depth, with local maximum values up to the flow depth (26), while the dune length is much larger than the flow depth. The formation of dunes may be caused by large-scale eddies as described by Yalin (46). Due to the presence of large (low frequency) eddies, there will be regions at regular intervals with decreased and increased bed-shear stresses, resulting in the local deposition and erosion of sediment particles. For increasing stages of flow, the particles will go into suspension according to the ratio of the grain-shear stress and the particle-fall velocity resulting in washed-out dunes. The washing-out process can be described quite well by a transport stage parameter as defined by Eq. 7. This parameter expresses the grain-shear velocity, u'_*, which is an estimate for the average shear ve-

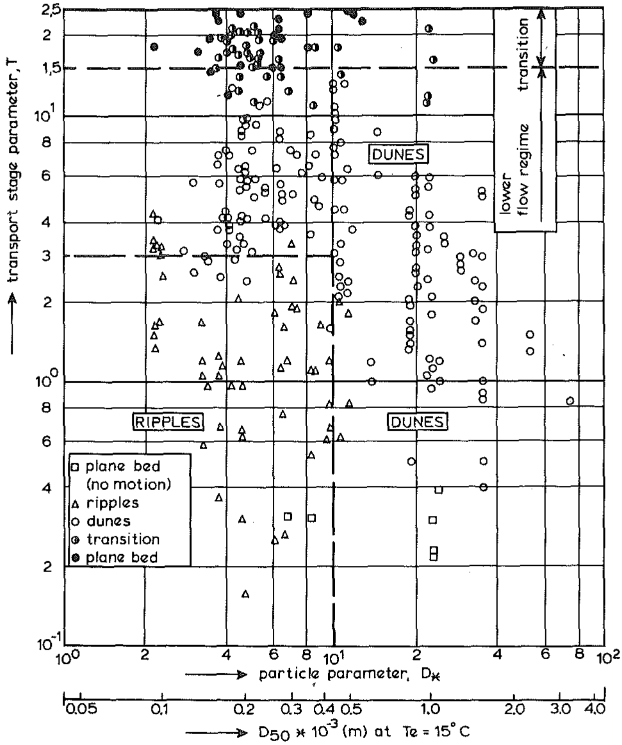

FIG. 1.—Diagram for Bed-Form Classification in Lower and Transitional Flow Regime

locity at the upsloping part of the bed forms, in relation to the critical Shield's value, $u_{*,cr}$. When $u'_* >> u_{*,cr}$ the sediment particles will go directly into suspension and the bed forms will be washed out. Similar reasoning has been done by Yalin (46). The applicability of the T-parameter in characterizing the generation of bed forms in the lower and transitional flow regime is also shown by Fig. 1, which shows distinct zones for ripples, dunes and washed-out dunes. Fig. 1 is based on a large number of reliable flume and field data (1,13,21,26,28–30,40,41,44,48). Dune-type bed forms are present for $T < 15$. However, for particles smaller than about 450 μm, $D_* \simeq 10$ are ripples generated after initiation of motion, but which disappear for $T > 3$. The transitional flow regime with washed-out dunes is present for $15 < T < 25$. For $T > 25$ a flat bed flow will be generated. Finally, it is pointed out that various classification diagrams can be found in the literature (9,11,19,26). According to Simons and Sentürk (27), the diagrams of Liu (19) and Garde-Albertson (11) do not give acceptable results for field conditions because relatively few field data were used. The diagram of Engelund-Hansen (9) is mainly based on flume data. Simons-Richardson (26) use some field data, but only of small-scale rivers. Therefore, the writer's diagram is assumed to be most valid for the lower and transitional flow regime in field conditions because a large number of field data with small and large flow depths have been used.

Bed-Form Dimensions

It is assumed that dimensions and migration of the bed forms are mainly determined by the value of the bed-load transport. In Part I (37) the bed-load transport was described as

$$q_b = \delta_b \, u_b \, c_b \quad (8)$$

in which q_b = bed-load transport rate/unit width; u_b = velocity of bed-load particles; c_b = bed-load concentration; and δ_b = thickness of bed-load layer. Using simple kinematic considerations and the continuity equation, the bed-load transport rate can also be described as (46):

$$q_b = (1 - p)\, \alpha \, \Delta \, u_d \quad (9)$$

in which p = porosity factor; α = shape factor of bed forms; Δ = bed-form height; and u_d = migration velocity of bed forms. From Eqs. 8 and 9 it can be derived that

$$\frac{\Delta}{d} = \frac{c_b}{(1 - p)\,\alpha} \, \frac{u_b}{u_d} \, \frac{\delta_b}{D_{50}} \, \frac{D_{50}}{d} \quad (10)$$

Using the functional relationships as given in Part I (37), it is assumed that

$$c_b, \frac{\delta_b}{D_{50}}, \frac{u_b}{u_d} = F(D_*, T) \quad (11)$$

resulting in

$$\frac{\Delta}{d} = F\left(\frac{D_{50}}{d}, D_*, T\right) \quad (12)$$

Likewise, it is assumed that the ratio of the bed-form height, Δ, and length, λ, can be expressed by a similar functional relationship:

$$\frac{\Delta}{\lambda} = F\left(\frac{D_{50}}{d}, D_*, T\right) \quad (13)$$

To determine these functional relationships, a large quantity of experimental data of bed-form dimensions was analyzed. In all, 84 flume experiments with particle diameters in the range of 190–2,300 μm were used (13,28,41,44,48). Only experiments in the lower and transitional flow regime with dune-type bed forms were considered. Ripple data were not selected because ripples are supposed to be independent of the flow depth. In regard to reliable field data, which are extremely scarce, only 22 data with particle diameters in the range 490–3,600 μm were used from some rivers in the Netherlands: the Ijssel, Waal and Rijn (40); the Rio Parana in Argentina (29); the Mississippi River (17); and some Japanese irrigation channels (30,31). In summary, the selection criteria used were: (1) Dune-type bed forms; (2) width-depth ratio larger than 3; (3) flow-depth larger than 0.1 m; and (4) transport stage parameter, T, smaller

	source	flow velocity $\bar{u}$ (m/s)	flow depth d (m)	particle size D_{50} (μm)	temperature Te (°C)
flume data	o Guy et al	0.34 - 1.17	0.16-0.32	190	8 - 34
	× Guy et al	0.41 -0.65	0.14-0.34	270	8 - 34
	Δ Guy et al	0.47- 1.15	0.16-0.32	280	8 - 34
	Guy et al	0.77-0.98	0.16	330	8 - 34
	□ Guy et al	0.48 - 1.00	0.10 - 0.25	450	8 - 34
	Guy et al	0.53 - 1.15	0.12 - 0.34	930	8 - 34
	Williams	0.54 - 1.06	0.15 - 0.22	1350	25 - 28
	Delft Hydr. Lab.	0.45-0.87	0.26 - 0.49	790	12 - 18
	Stein	0.52 -0.95	0.24 -0.31	400	20 - 26
	Znamenskaya	0.53 -0.80	0.11 - 0.21	800	–
field data	● Dutch Rivers	0.85 - 1.55	4.4 -9.5	490 -3600	5 - 20
	Rio Parana	1.0	12.7	400	–
	Japanese Channels	0.53-0.89	0.25 -0.88	1100 -2300	–
	■ Mississippi River	1.35 - 1.45	6 -16	350 -550	–

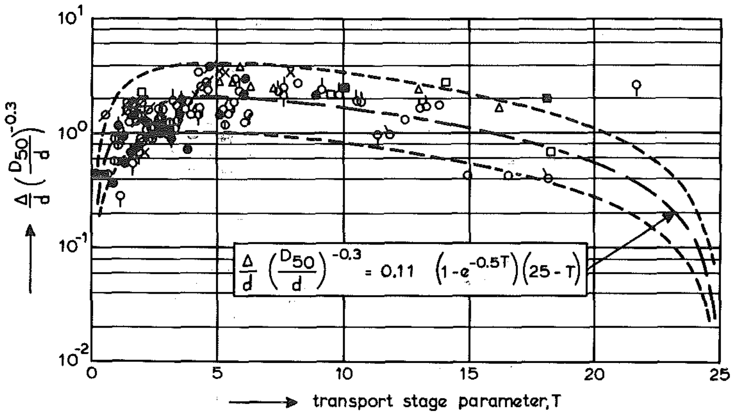

FIG. 2.—Bed-Form Height

than 25. Where the water temperature was not reported, a value of 15° C was assumed and, where the D_{50} of the bed material was not reported, it was computed from the D_{50} and the gradation, σ_s, of the bed material, assuming the latter to be equal to 2. The influence of side-wall roughness was eliminated by using the method of Vanoni-Brooks (32).

In analyzing the data, only for the Missouri River (25) was a clear influence of the particle diameter D_* observed. As reported, a decrease from a temperature of 25° C in early September to 5° C in December (a decrease in D_* of about 30%) caused a remarkable change in the bed configuration, from dune-type bed forms to almost flat beds; although the other hydraulic parameters (discharge, depth, slope and particle size) were essentially unchanged. However, for the large majority of the data, a clear correlation between the bed-form dimensions and the D_*-parameter could not be detected. Therefore, in the present stage of analysis, the D_*-influence was neglected and the bed-form dimensions were related to the particle diameter, D_{50}; the flow depth, d; and the transport stage parameter, T. The best agreement was obtained for (35)

$$\frac{\Delta}{d} = 0.11 \left(\frac{D_{50}}{d}\right)^{0.3} (1 - e^{-0.5T})(25 - T) \quad \ldots\ldots\ldots\ldots \quad (14)$$

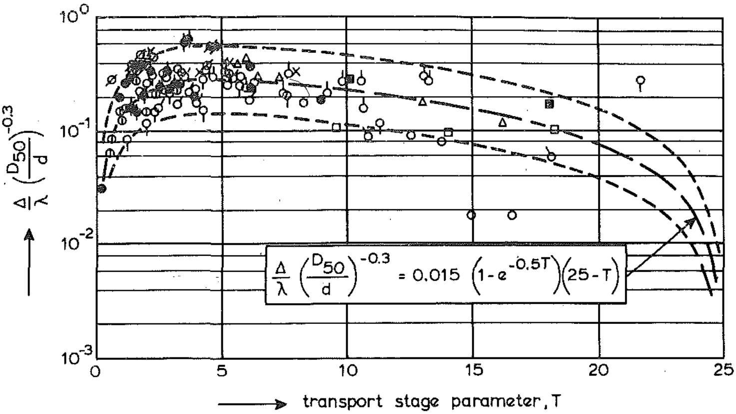

FIG. 3.—Bed-Form Steepness

$$\frac{\Delta}{\lambda} = 0.015 \left(\frac{D_{50}}{d}\right)^{0.3} (1 - e^{-0.5T})(25 - T) \quad (15)$$

It is assumed that for $T \leq 0$ and $T \geq 25$, the bed surface is almost flat (anti-dunes are not considered). Eqs. 14 and 15, as well as the error range of a factor of 2, are shown in Figs. 2 and 3.

Both functions show maximum values for about $T = 5$. From Eqs. 14 and 15 an expression for the bed-form length can also be derived:

$$\lambda = 7.3\, d \quad (16)$$

Eq. 16, based on a large number of reliable flume and field data, indicates that the dune length is related only to the mean flow depth. This has also been reported by Yalin (see Ref. 46, p. 242). Based on theoretical analysis, Yalin derived $\lambda = 2\pi d$, which is close to Eq. 16. Yalin also presented experimental data supporting a constant dimensionless dune length. Physically, it means that the bed-form height is reduced for increasing stages of flow, while the bed-form length remains essentially unchanged. Note finally that Eqs. 14 and 15 are in agreement with the functional relationships for dune characteristics as proposed by Yalin (see Ref. 46, p. 237). To apply Eqs. 14, 15 and 16, the mean flow velocity, flow depth and particle size must be known. The energy gradient need not be known, a fact which may be an additional advantage because the energy gradient may be relatively inaccurate (low flow velocities, rising or falling stages) and because it may be difficult to determine in (isolated) field conditions. Other investigators who have proposed analytical or graphical relationships for the bed-form dimensions are Tsubaki-Shinohara (31), Yalin (46,47), Ranga Raju-Soni (24), Allen (3) and Fredsøe (10). The writer (35) has compared the available methods by computing the bed-form height for a sand bed with a particle diameter of $D_{50} = 600$ μm ($D_{90} = 1{,}500$ μm); a depth of $d = 1$ and 10 m; and a mean flow velocity, $u = 0.5$, 1, 1, 25 and 1.5 m/s. As the methods of Tsubaki-Sinohara, Yalin and Ranga Raju-Soni require the specification of the en-

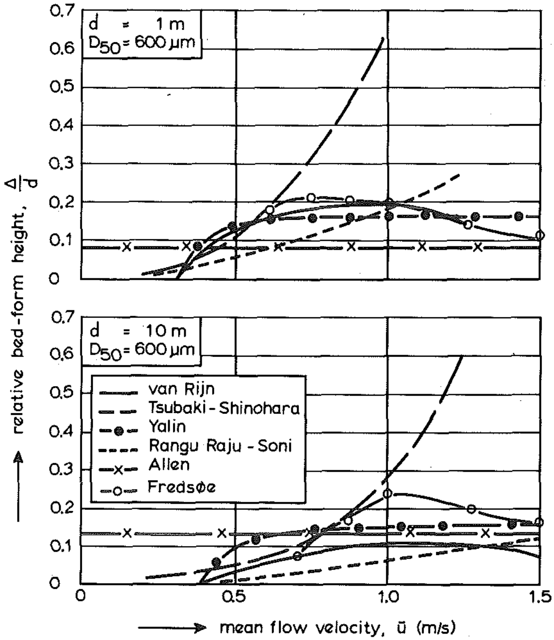

FIG. 4.—Computed Bed-Form Heights

ergy gradient, these methods were used iteratively with an equivalent roughness height equal to half the bed-form height. The results are shown in Fig. 4.

As can be observed, the methods of Ranga Raju-Soni and Tsubaki-Shinohara show an increasing trend. Yalin's method yields values which approach $\Delta/d = 0.167$ for large flow velocities. Only Eq. 14 and the Fredsøe method predict a decreasing dune height for increasing flow velocities (washed-out dunes). For a small depth, $d = 1$ m, the writer's method and the Fredsøe method show good agreement, while for a large depth, $d = 10$ m, the Fredsøe method yields larger values. Finally, note that the method of Ranga Raju-Soni produces remarkably small bed-form heights (and lengths) for large depths, as present in field conditions, probably because only flume data were used for calibration.

Equivalent Roughness of Bed Forms

The hydraulic roughness of a movable bed surface is caused by grain roughness, $k_{s,\text{grain}}$, and by form roughness, $k_{s,\text{form}}$. In an earlier study, the writer (36) has shown that the equivalent or effective grain roughness of a flat bed can be related to the D_{90} of the bed material. From about 100 flume and field data it was derived that

$$k_{s,\text{grain}} = 3\,D_{90} \qquad (17)$$

Similar values have been reported by Kamphuis, $k_s = 2.5\,D_{90}$ (15); Gladki, $k_s = 2.3\,D_{80}$ (12); Hey, $k_s = 3.5\,D_{84}$ (14); and Mahmood, $k_s = 5.1\,D_{84}$ (20).

source	d (m)	ū (m/s)	D_{50} (μm)
Guy et al	0.16-0.32	0.24-0.59	190-930
Znamenskaya	0.08	0.49	800
Ackers	0.17-0.20	0.35-0.43	180
Stein	0.24-0.30	0.88-1.12	400
Laursen	0.17-0.23	0.42-0.71	100

source	d (m)	ū (m/s)	D_{50} (μm)
Barton-Lin	0.14-0.26	0.41-0.58	180
Japanese Channels	0.11-0.43	0.55-0.73	1260-1440
River Lužnic	0.14-0.75	0.35-0.74	2400
Missouri River	4.5	1.5	240

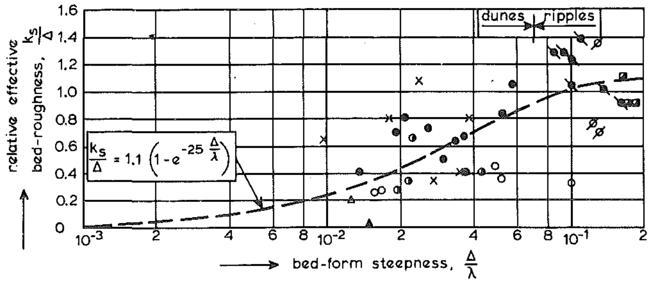

FIG. 5.—Equivalent Roughness of Bed Forms

In regard to the form roughness, the following functional relationship, introduced by Yalin (see Ref. 46, p. 235) is assumed to be valid:

$$k_{s,\text{form}} = F\left(\Delta, \frac{\Delta}{\lambda}\right) \qquad (18)$$

In an earlier study (36), the following expression based on flume and field data was derived:

$$k_{s,\text{form}} = 1.1\,\Delta(1 - e^{-25\psi}) \qquad (19)$$

in which $\psi = \Delta/\lambda$ = bed-form steepness.

Eq. 19, as well as the data, is shown in Fig. 5. Taking into account both the grain and form roughness, it is proposed to compute the effective roughness, k_s, of a movable bed surface in the lower, transitional and upper flow regime (with exception of anti-dunes) by means of

$$k_s = 3\,D_{90} + 1.1\,\Delta(1 - e^{-25\psi}) \qquad (20)$$

It may be noted that for a bed-form steepness, Δ/λ, equal to zero, the flat bed value, $k_{s,\text{grain}} = 3\,D_{90}$, is obtained. Finally, the Chézy-coefficient can be computed by

$$C = 18 \log\left(\frac{12 R_b}{k_s}\right) \qquad (21)$$

in which R_b = hydraulic radius of the bed according to Vanoni-Brooks (32). In summary, the proposed method is as follows:

1. Compute particle diameter, D_*, by Eq. 6.
2. Compute critical bed-shear velocity, $u_{*,\text{cr}}$ (37).
3. Compute transport stage parameter, T, by Eq. 7.
4. Compute bed-form height, Δ, by Eq. 14.
5. Compute bed-form length, λ, by Eq. 16.

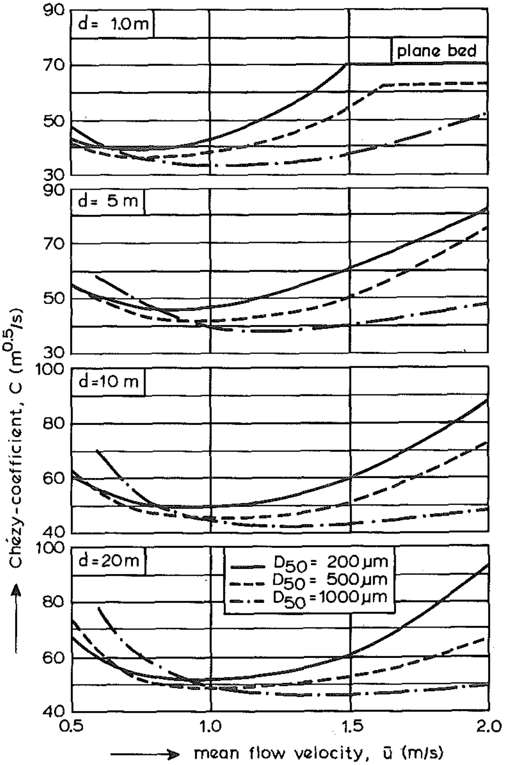

FIG. 6.—Computed Chézy-Coefficients According to van Rijn Method

6. Compute equivalent roughness, k_s, by Eq. 20.
7. Compute Chézy-coefficient, C, by Eq. 21.

The input data are mean flow velocity, $\bar{u}$; mean flow depth, d; mean flow width, b; particle diameter of bed material, D_{50}, D_{90}; fluid and sediment density, ρ, ρ_s; and kinematic viscosity coefficient, ν. As an example, the Chézy-coefficient has been computed for various flow conditions assuming a fluid temperature, Te, of 15° C, and a geometric standard deviation of the bed material, σ_s equal to 2. The results are shown in Fig. 6.

Independent support for the validity of the proposed method (Eqs. 20 and 21) has been produced by van Urk (39). He used an automatic sounding system to determine the dune dimensions in some branches of the Rhine River, a major river in the Netherlands and Europe. Various hydraulic roughness prediction methods based on the bed-form dimensions were selected from the literature. Using the measured dune dimensions, van Urk computed the Chézy-coefficients for the various methods and compared those values with the Chézy-coefficients derived from the overall hydraulic conditions (mean flow velocity, depth and energy slope). This comparison shows that the writer's prediction method and that of Vanoni-Hwang (33) yield the best results for all data.

Other evidence is presented by van Urk and Klaassen (40). They compared various prediction methods for hydraulic roughness using flume data. This study also supports the validity of the present method.

Finally, a discussion of the form drag expression (Eq. 19) by Engel is reported (8), which shows that Eq. 19 yields a realistic estimate of the equivalent roughness for dunes.

Verification

To verify the proposed method, a large amount of flume and field data were selected from a compendium of solids transport data (23). In addition, about 200 field data reported in the literature were used (4,6,7,18,21,22,25). In all, 786 field data and 758 flume data were selected using the following criteria: (1) Flow depth larger than 0.1 m; (2) width-depth ratio larger than 3; and (3) particle diameter in the range of 100–2,500 μm. To eliminate the influence of side-wall roughness, the method of Vanoni-Brooks (32) was used. Where the water temperature was not reported, a value of 15° C was assumed, and where only the D_{50} of the bed material was reported, the other characteristic particle diameters, D_{35}, D_{65}, and D_{90}, were computed assuming a lognormal particle size distribution with a geometric standard deviation of $\sigma_s = 2$. None of the data were used by the writer to calibrate his proposed relationships! An investigation into the accuracy of the "measured" Chézy-coefficient has shown that errors of up to 20% may be expected (34,35,44,45). These errors may be caused by faulty measuring, non-equilibrium phenomena (rising or falling stages), and three-dimensional effects (alternate bars) in case of a large width-depth ratio. Therefore, it may hardly be possible to predict the Chézy-coefficient with an inaccuracy of less than 20%.

For comparison with the writer's method, the methods of Engelund-Hansen (9) and White et al. (43) were also applied to the selected data. These two methods gave the best results in an extensive verification study carried out by White et al. (43). It must be stressed that Engelund and Hansen used about 100 flume data (13) to calibrate their formulas, while White et al. used about 1,400 flume data and 260 field data for calibration. For each method the score (as a percentage) of the predicted values in the following error ranges were determined: $C_{measured} \pm 10\%$; $C_{measured} \pm 20\%$; and $C_{measured} \pm 30\%$. The results are shown in Table 1.

The Engelund-Hansen method for flume conditions, and the writer's method for field conditions produce the best results. On the other hand, the Engelund-Hansen method produces rather bad results for field conditions, probably because only flume data were used for calibration. The results of the method of White et al. are not as good as they reported in their verification study (43). They gave their results in terms of the ratio of the measured and computed Darcy-Weisbach friction coefficient.

According to White et al., their method scored (for about 2,500 data) 38% in the 0.8–1.25 range (≃ 10% error range for the Chézy-coefficient) and 89% in the 0.5–2.0 range (≃ 40% error range for the Chézy-coefficient). These results are better than those of this analysis, probably because data of the upper flow regime with flat bed conditions were selected, for which the method of White et al. may be invalid. In the

TABLE 1.—Comparison of Computed and

Source (1)	Number of tests (2)	Flow velocity, in meters per second (3)	Flow depth, in meters (4)	Particle diameter, in micrometers (5)	Temperature, in degrees Celsius (6)
Field data:					
U.S.-rivers (Leopold)	55	0.36–1.26	1.50–4.10	140–420	12–26
U.S.-rivers (Corps of Engineers)	300	0.4–2.4	0.3–16.4	120–560	2–35
India canals	31	0.7–1.6	1.3–3.4	90–310	10–30
Pakistan canals	142	0.35–1.3	0.7–4.3	110–290	15–35
Missouri River, U.S.	21	1.35–1.75	2.8–4.3	190–230	3–22
East-Fork River, U.S.	45	0.55–1.4	0.25–2.0	450–1,400	15–20
Elkhorn River, U.S.	43	0.5–2.0	0.4–2.2	240	2–26
Rio Grande, U.S.	79	0.45–2.4	0.3–1.5	180–450	2–29
Mountain Creek, U.S.	47	0.5–1.35	0.15–0.45	290–900	15–25
Japanese channels	23	0.55–0.95	0.10–0.75	1,260–1,440	—
	786				
Flume data:					
Guy et al.	292	0.2–1.9	0.1–0.4	190–930	8–34
Stein	54	0.4–1.65	0.1–0.35	400	20–30
Southampton A	67	0.2–0.8	0.15–0.30	150	15–25
Southampton B	139	0.2–0.7	0.15–0.30	480	21
USWES (sand)	128	0.25–0.55	0.1–0.2	180–950	14–18
Meyer-Peter	22	0.35–0.90	0.1–0.5	400–1,500	—
Barton-Lin	29	0.2–1.1	0.1–0.3	180	15–27
Laursen	14	0.4–1.0	0.1–0.3	110	20–26
Williams	13	0.5–1.15	0.15–0.25	1,350	16–26
	758				
Total	1,544				

verification study of White et al., only data in the lower flow regime were used. To investigate systematic errors, the ratio of the computed and measured Chézy-coefficient were plotted as a function of the transport stage parameter, T. Figs. 7 and 8 show the present method for the field data only. No systematic errors can be detected in relation to the transport stage.

Figs. 9 and 10 show the results for the method of Engelund-Hansen and White et al. Only field data from U.S. rivers and Pakistan canals are presented. Both methods show serious systematic errors for these data. On the average, the predicted values are larger than measured values. In summary, it could be said that the Engelund-Hansen and White et al. methods are not very good for field conditions.

Prediction of Flow Depth

An important objective in river engineering is the prediction of flow depth and flow velocity in uniform flow conditions for a given dis-

Measured Chézy-Coefficients

SCORES, AS PERCENTAGES, OF PREDICTED CHÉZY-COEFFICIENT IN ERROR RANGES								
±10% Error			±20% Error			±30% Error		
van Rijn (7)	Engelund-Hansen (8)	White et al. (9)	R (10)	E-H (11)	W (12)	R (13)	E-H (14)	W (15)
29	25	32	65	34	56	80	45	78
40	20	32	68	38	56	82	50	75
48	29	45	70	38	64	83	45	70
62	24	52	84	42	80	94	66	92
62	43	0	67	62	0	100	100	24
31	22	0	94	67	100	100	100	100
47	23	28	91	42	47	100	72	70
47	29	18	80	44	40	100	58	70
29	27	72	48	91	91	89	91	91
34	34	78	73	91	95	82	100	100
43	25	33	74	47	58	89	62	79
35	30	26	60	61	44	75	70	54
38	9	16	62	35	31	83	51	51
13	52	17	31	82	40	49	86	52
48	45	57	73	73	71	79	80	78
28	50	46	51	81	79	67	91	86
22	22	27	31	36	54	36	63	100
27	27	3	31	51	31	58	62	58
50	35	28	64	50	50	92	64	78
23	46	53	53	61	76	92	69	92
34	37	33	56	65	54	71	75	66
39	31	33	65	56	56	80	68	73

charge, Q; energy gradient, S; width, b; particle diameter, D_{50}; and gradation, σ_s, of bed material, and fluid temperature, Te.

The methods used to predict the flow depth are those of van Rijn, Engelund-Hansen (9) White et al. (45) and Brownlie (5). Only the latter method is straightforward; for the other methods, an iteration technique is necessary, applied as follows:

1. Estimate flow depth, d_j (start value = 1.5 d_{measured}).
2. Compute flow velocity, $\bar{u}_j$, from the discharge.
3. Compute new flow velocity, $\bar{u}_{j+1}$, according to the prediction methods.
4. Compute new flow depth, d_{j+1}, from the discharge.
5. Compare new and old flow depths, if $|d_{j+1} - d_j| > d_{\text{measured}}/100$, then $d_{j+2} = d_{j+1} + d_j/2$, and repeat.

In regard to the selected ata, only those of Guy et al. (13,23), Pakistan canals (21) and U.S. rivers (23) were considered, using the following

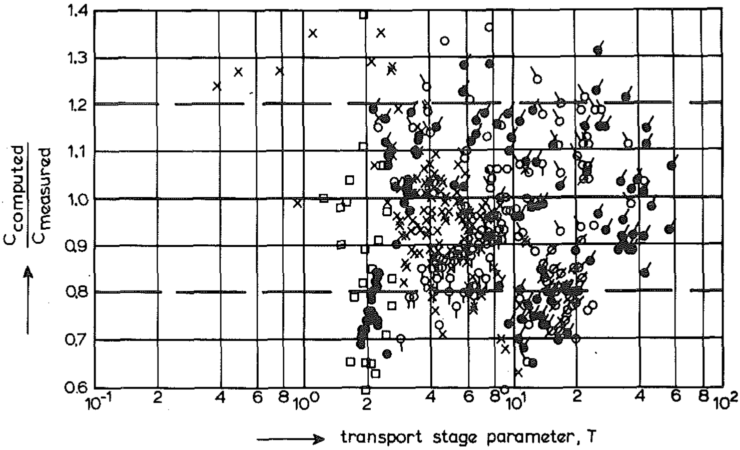

FIG. 7.—Ratio of Computed and Measured Chézy-Coefficients According to van Rijn Method (Small Rivers)

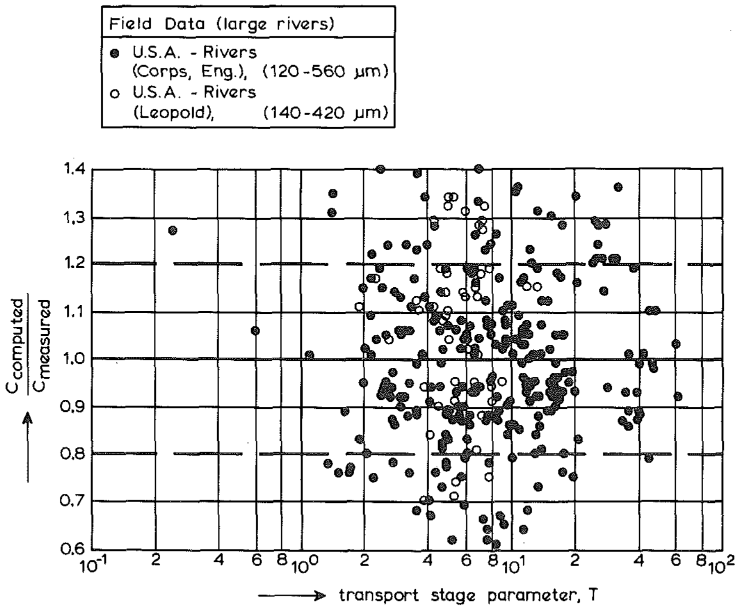

FIG. 8.—Ratio of Computed and Measured Chézy-Coefficients According to van Rijn Method (Large Rivers)

criteria: (1) Guy et al., $d \geq 0.1$ m; (2) Pakistan canals, $d \geq 1.0$ m; and U.S. rivers, $d \geq 5.0$ m. None of these data were used by the writer to calibrate the proposed relationships.

Furthermore, only those data with a width-depth ratio larger than 10

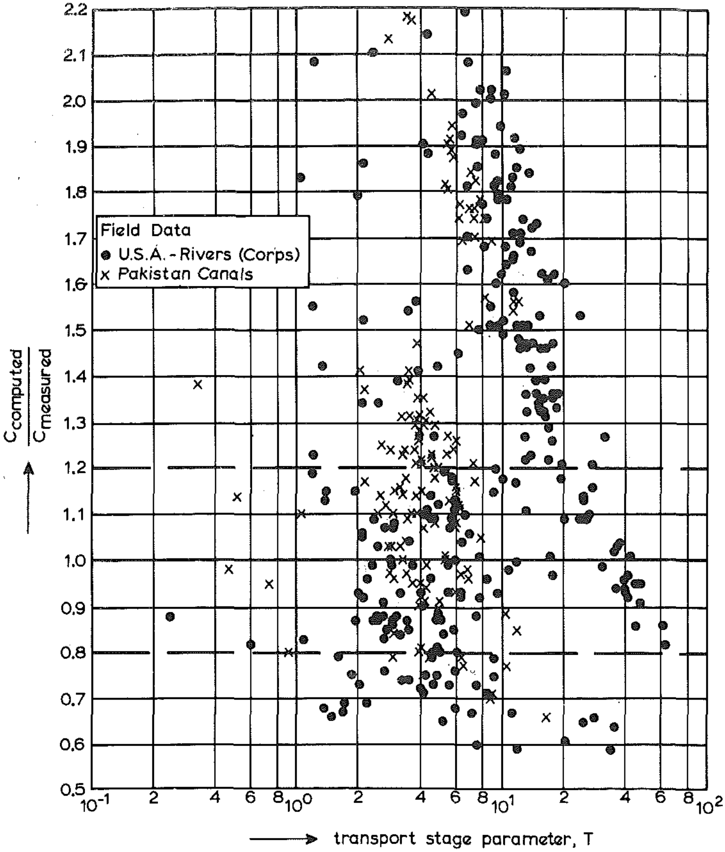

FIG. 9.—Ratio of Computed and Measured Chézy-Coefficients According to Engelund-Hansen Method (Large Rivers)

(to reduce the influence of the side-wall roughness) were selected. No correction method was applied. Where the water temperature, Te, and the geometric standard deviation, σ_s, of the bed material were not given, the values $Te = 15°$ C and $\sigma_s = 2$ were used. The scores (as percentages) of the predicted flow depths in the error ranges ±10%, ±20% and ±30%, are given in Table 2.

In regard to the field data, the method of Brownlie is slightly better than the writer's method and the method of White et al., while the method of Engelund-Hansen gives the least satisfactory results. Compared with the flume data of Guy et al., the method of White et al. does not work well.

Finally, note that the present verification does not give an independent check of the Brownlie method since all data used in the present investigation were used also by Brownlie to calibrate his method. Therefore, it merely is a check of Brownlie's calibration. Additional research with data not used by Brownlie is necessary to verify his method.

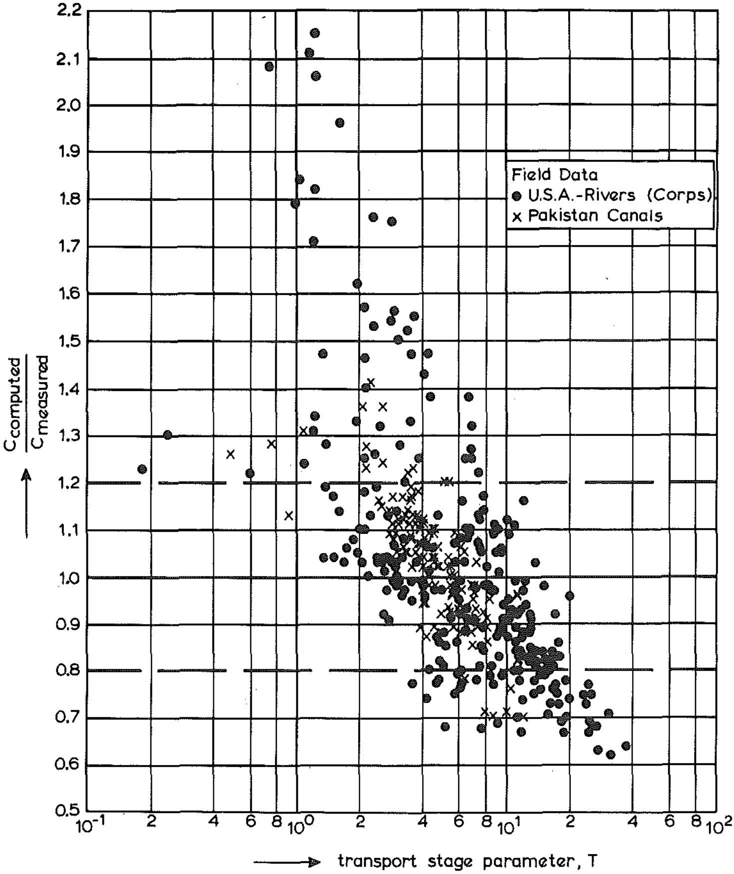

FIG. 10.—Ratio of Computed and Measured Chézy-Coefficients According to Method of White et al. (Large Rivers)

TABLE 2.—Comparison of Computed and Measured Flow Depths

Name and number of data (1)	SCORES, AS PERCENTAGES, OF PREDICTED FLOW DEPTHS IN ERROR-RANGES											
	±10% Error				±20% Error				±30% Error			
	van Rijn (2)	Engelund (3)	White (4)	Brownlie (5)	R (6)	E (7)	W (8)	B (9)	R (10)	E (11)	W (12)	B (13)
USA-rivers 240	53	34	50	60	82	57	83	88	92	72	96	98
Pakistan canals 139	79	49	76	82	95	82	94	95	97	87	99	99
Guy et al. 147	57	47	29	60	79	71	47	89	88	78	60	97

In the two companion papers (37,38), a method to compute the bed and suspended load transport has been proposed. As stated in the introduction, the proposed method was not used in a predictive sense because, in addition to the mean flow velocity, flow depth and width, the energy-gradient must also be known to compute the overall bed-shear velocity. In this Section, the total load transport will be predicted using as input data only the mean flow velocity, depth and width. This is important for the one-dimensional mathematical modeling of river systems with a movable bed, as pointed out by De Vries (42). Usually only the flow discharge is known and the local flow depth and flow velocity are predicted by solving Eqs. 1–5.

The writer's method of predicting the total bed-material load is given in the companion papers, while the Chézy-coefficient is predicted by Eqs. 14, 16, 20 and 21. For comparison with the writer's method, the sediment transport theories of Engelund-Hansen (9) and Ackers-White (2) were applied to the selected data. Basically, the sediment transport theory of Engelund-Hansen is defined in terms of a given flow depth, energy gradient and Chézy-coefficient. Therefore, the two former parameters were assumed to be known, while the Chézy-coefficient was predicted as proposed by Engelund-Hansen (9). The Ackers-White sediment transport theory is defined in terms of a given flow depth, flow velocity and energy-gradient. In the present analysis the flow depth and energy-gradient were assumed to be known while the mean flow velocity was predicted by means of the method of White et al. (43).

In regard to the data, only the 266 field data collected by the Corps of Engineers (23) from U.S. rivers were used, since it represents a wide range of flow conditions for rather large rivers. This data set consists of data from the Rio Grande, Mississippi, Red and Atchafalaya Rivers, which can be considered reliable field data. The accuracy of the three methods is given in terms of a discrepancy ratio, r, defined as the ratio of the computed and measured total load. The scores (as percentages) of the predicted values in three error ranges are given in Table 3. As can be seen, the writer's method is superior to both other methods. The relatively poor results of the Engelund-Hansen and Ackers-White methods stem from the relatively strong dependence of both methods (in their basic form) on the predicted Chézy-coefficient (or mean flow velocity). Finally, a simplified method is given which can be used to compute the total bed-material load when only the mean flow velocity, flow depth

TABLE 3.—Comparison of Computed and Measured Total Load Transport

Researcher (1)	Scores, as Percentages, of Predicted Total Load in Discrepancy Ranges		
	$0.75 \le r \le 1.5$ (2)	$0.5 \le r \le 2$ (3)	$0.33 \le r \le 3$ (4)
van Rijn	52	75	88
Engelund-Hansen	24	44	58
Ackers-White	22	44	62

and the particle size are known. This simplified method is based on the computation of the sediment transport as proposed in the two companion papers in combination with the proposed hydraulic roughness prediction method (38). Using regression analysis, the computational results for various flow and sediment conditions (d = 1–20 m, $\bar{u}$ = 0.5–2.5 m/s and D_{50} = 100–2,000 μm, σ_s = 2, Te = 15° C) were represented by simple power functions

$$\frac{q_b}{\bar{u}\,d} = 0.005\left(\frac{\bar{u} - \bar{u}_{cr}}{[(s-1)\,g\,D_{50}]^{0.5}}\right)^{2.4}\left(\frac{D_{50}}{d}\right)^{1.2} \quad \text{(22)}$$

$$\frac{q_s}{\bar{u}\,d} = 0.012\left\{\frac{\bar{u} - \bar{u}_{cr}}{[(s-1)\,g\,D_{50}]^{0.5}}\right\}^{2.4}\left(\frac{D_{50}}{d}\right)(D_*)^{-0.6} \quad \text{(23)}$$

in which q_b = bed-load transport (volume)/unit width; q_s = suspended load transport (volume)/unit width; and $\bar{u}_{cr}$ = critical mean flow velocity based on Shield's criterion.

For particles in the range of 100–2,000 μm, the critical mean flow velocity can be computed by

$$\bar{u}_{cr} = 0.19\,(D_{50})^{0.1}\log\left(\frac{12\,R_b}{3\,D_{90}}\right), \quad \text{for} \quad 100 \le D_{50} \le 500\ \mu\text{m} \quad \text{(24)}$$

$$\bar{u}_{cr} = 8.5\,(D_{50})^{0.6}\log\left(\frac{12\,R_b}{3\,D_{90}}\right), \quad \text{for} \quad 500 \le D_{50} \le 2{,}000\ \mu\text{m} \quad \text{(25)}$$

in which D_{50}, D_{90} = particle diameters of bed material (in meters). The accuracy of Eqs. 22 and 23 is somewhat less than the original method, as given in the companion papers (37,38). For example, the score of the predicted total loads for the 266 data of the U.S. rivers in the discrepancy ratio of 0.5–2 is 70%, somewhat less than that derived from the original method, which scored 79%. However, the score is considerably better than those of Engelund-Hansen and Ackers-White, which is 44% for both methods. From Eqs. 22 and 23, it follows that the bed-load transport, q_b, and the suspended load transport, q_s, cannot be expressed as unique functions of the Froude number. This seems to confirm the Froude number's lack of importance for the generation of bed forms in the lower flow regime.

Conclusions

The aim of this analysis was to determine simple functions which specify the dimensions of the bed-forms as well as their equivalent roughness. Such a method is not available in literature. From the results of a verification analysis, it can be concluded that the proposed method has a rather good predictive ability for the Chézy-coefficient in the dune and plane bed regimes.

The study led to the following conclusions:

1. The proposed method for the prediction of the Chézy-coefficient and the total load transport is superior to the methods of Engelund-Hansen and Ackers-White et al. in the case of field conditions.

2. The dimensions and the equivalent (or effective) roughness of bed forms in the dune and plane bed regimes can be expressed by simple functions that are valid for particles in the range of 160–3,600 μm and flow depths of up to 20 m, as shown by a verification analysis using about 1,500 flume and field data.

3. The proposed method yields better results in predicting the flow depths than the methods of Ackers-White and Engelund-Hansen, but the Brownlie method gives the best results. However, this verification is not an independent check of Brownlie's method since all verification data were also used by Brownlie for calibration.

4. The type of bed forms in the lower and transitional flow regime can be expressed as a functon of a dimensionless transport stage parameter and a particle parameter.

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Appendix II.—Notation

The following symbols are used in this paper:

a = reference level (L);
b = width (L);
C = Chézy-coefficient ($L^{0.5}T^{-1}$);
C' = Chézy-coefficient related to grains ($L^{0.5}T^{-1}$);
c = concentration;
D = particle diameter (L);
D_* = particle parameter;
d = depth (L);
g = acceleration of gravity (LT^{-2});
k_s = equivalent roughness of Nikuradse (L);
p = porosity factor;
Q = discharge (L^3T^{-1});
Q_t = total load transport (L^3T^{-1});
q = discharge per unit width (L^2T^{-1});
q_b = bed-load transport per unit width (L^2T^{-1});
q_s = suspended load transport per unit width (L^2T^{-1});
R_b = hydraulic radius related to bed (L);
S = energy gradient;
s = specific density;
T = transport stage parameter;
Te = temperature (degrees Celsius);

t = time (T);
$\bar{u}$ = mean flow velocity (LT^{-1});
$\bar{u}_{cr}$ = critical mean flow velocity (LT^{-1});
u_b = velocity of bed-load particles (LT^{-1});
u_d = migration velocity of bed forms (LT^{-1});
u_* = bed-shear velocity (LT^{-1});
u'_* = bed-shear velocity related to grains (LT^{-1});
$u_{*,cr}$ = critical bed-shear velocity for initiation of motion (LT^{-1});
x = longitudinal coordinate (L);
z = vertical coordinate (L);
α = coefficient;
β = ratio of sediment and fluid diffusion coefficient;
Δ = bed-form height (L);
δ_b = thickness of bed-load layer (L);
λ = bed-form length (L);
μ = dynamic viscosity coefficient ($ML^{-1}T^{-1}$);
μ_b = bed-form factor;
ν = kinematic viscosity coefficient (L^2T^{-1});
ρ = density of fluid (ML^{-3});
ρ_s = density of sediment (ML^{-3});
σ_s = geometric standard deviation of bed material;
τ_b = bed-shear stress ($ML^{-1}T^{-2}$); and
ψ = bed-form steepness.