Skeleton text for the algorithm testing section of the EWRI 2011 conference paper

The algorithm testing used a hierarchical building block approach that tests key processes individually and then tests the processes in combinations of increasing complexity (Figure XX). For the example presented here, the tests were developed for a one-dimensional transport code that will be applied to an estuary. Thus the key processes tested are advection, dispersion and reaction (e.g. growth or decay, and other sources/sinks). The tests are qualitatively described below. Detailed descriptions of the tests are beyond the scope of this paper and will appear in a planned journal article.

The hierarchical tests mostly use a Gaussian distribution of mass. For the advection only tests, the mass distribution is transported forwards, backwards, and back and forth. The test is passed if the mass moves to the appropriate location without changing the distribution. For the dispersion tests, the mass spreads out based on specified dispersion coefficients. The test is passed if the center of mass remains in the same location and the mass distribution matches the analytical solution for a back-and-forth motion. For the reaction test, the mass decays according to specified decay coefficients. The test is passed if the center of mass remains in the same location and the distribution matches the analytical solution. The processes were then tested in combinations appropriate for an estuary: advection-dispersion, advection-reaction and finally advection-dispersion-reaction. These tests were conducted for a range of parameter values. Typically the Courant number (a measure of numerical stability of the algorithm), domain length, and dispersion and decay coefficients were fixed, and the grid spacing and time steps were adjusted to maintain the same Courant number. In addition to these hierarchical tests, selected analytical tests were also included in the test suite.

Add brief description of diffusion hump and other tests here.

Kaveh: Maybe you can fit it somewhere in the above paragraph

Water quality phenomena in rivers and estuaries are characterized by spatial and temporal variability. This variability may appear in the form of a variable flow field, spatially and time-varying dispersion coefficient, and/or nonlinear source terms, which depend on the variable velocity field and concentrations (Equation 1 ADR). This is the reason why it is preferable to select a test case for the solver involving highly-variable conditions for complete verification. To this end, there are methods such as Manufactured Solutions (MMS) which are able to verify a code in the most general form. One may ask here: why the above set of tests should be done when MMS individually is able to detect any issue with the code? The answer is that if the MMS test fails, it is hardly unlikely for the developers to identify in which process the problem is with this general test. Hence, small-block/simpler tests should be checked before conducting the MMS (see Fig. XX) . Each of the above tests should not be skipped and these tests should not be substituted with each other. While one test clears some processes, others still do so regarding focusing on different processes.

**Labeled Transport Equation**



Unsteady

Advective

Dispersive

Sink/Source

**SCALING OF THE PROBLEM FOR AN ESTUARY**

**Short version**

The test suit is designed in the matching ranges of the physical phenomena in the nature to be more rigorous. The important parameters in tidal river system are Peclet Number and Damkohler Number, former is the ratio of characteristic scale of advection to the characteristic scale of diffusion, and the latter is defined as the characteristic scale of reaction to the characteristic scale of dominant transport process (usually advection in the case of an estuary)

**Long version**

The ADR solver is tested in the feasible ranges of dimensionless numbers, i.e., the Peclet and Damkohler numbers associated with the problem at hand. (Whereas the former is the ratio of characteristic scale of advection to the characteristic scale of diffusion, the latter is defined as the characteristic scale of reaction to the characteristic scale of dominant transport process; see Steefel and MacQuarrie, 1996). The assumed scales and ranges are as follows: Area~ 1000 [m2], C ~ (0 – 0.05) [vol/vol], u ~ (±0.2-2) [m/s], Ks ~ [2-150 m2/s]. Based on these values, the following ranges for the Peclet and Damkohler numbers are: . The computations associated with the reaction term are possible assuming an entrainment term for the sediment transport problem; we have used the Garcia and Parker (1991) formulation for such a function (Zamani, 2011). It is worth to mention that the length scale in the scaling process assumed the same length of spatial discretization in the numerical solution where needed.

Finally based on the formula suggested by Garcia and Parker (1991) reaction variation range for non-cohesive sediment in an estuary will be ( +1.0×10-3 to -3.7×10-4 [1/s]).

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**What is a grid convergence test for?**

Grid-convergence tests are well-accepted methods of verifying numerical solvers in computational fluid mechanics. They consist in reducing the mesh size (both spatial and time steps) and in checking the evolution of the ratio of error metrics. Grid-convergence tests check if the *formal grid accuracy* of the numerical scheme is reproduced by the code. Coding bugs and implementation errors could detected via this vehicle.

**Zoppou test**

Zoppou and Knight (1997) developed an analytical solution of the advection-dispersion equation with both a spatially-variable flow field, and dispersion coefficient. We implemented this test in our algorithm testing framework. We modified the solution in order to satisfy mass continuity of water. At this point, we are not obtaining the desired second-order accuracy of the solution.

**ADR test**

In order to test the advection-dispersion-reaction equation including all processes, we selected the well-known solution regarding the transformation of a Gaussian mass as it moves due to advection, spreads due to dispersion and decays due to the sink of mass (see for example Khan and Liu 1994). The solution includes a constant area, a constant velocity, and constant linear decay. The solution was implemented through a two-level approach. First, we treated the problem with remote boundaries, where the machine precision exceeds the numerical truncation error. Then, active boundaries were used, where the same test was run but the values of the concentration at the boundary were large enough to influence the solution.