Central Asian Policy Simulation Model (CAPSIM)

GENERAL MODEL OVERVIEW

CAREC
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Model Background

The CAPSIM model is a regional/global computable general equilibrium (CGE) model, a descendant of the LINKAGE model developed by Dominique van der Mensbrugghe while at the World Bank. Its development dates back to the 1980s, when early versions of the model were used to study global trade reform. Since then, the model has been used to analyze a variety of policy questions, including, more recently, energy and climate policies.

CAPSIM is a multi-country, multi-sector model, combining a recursive, dynamic modeling framework with a comprehensive, consistent economic database. CAPSIM is implemented in the GAMS programming language and calibrated to national data and the Global Trade and Analysis Project (GTAP) database.

CAPSIM uses a 20 country/region framework and 10 sector aggregation. To make the model tractable for online use, CAPSIM is a comparative static model.

The remainder of this document provides a generic overview of the CAPSIM model, focusing on basic model structure and function. For more detailed documentation, contact David Roland-Holst at dwrh@berkeley.edu.

Model Background

CAPSIM divides national economies into five main, interactive components:

- Production
- Consumption
- Government
- Savings-Investment
- Trade

Because these components are all interlinked — for instance, how much producers produce depends on demand, but how much consumers demand depends on producer prices — CAPSIM must find a solution that simultaneously satisfies the requirements of each component. The remainder of this document describes the mathematical notation used to represent each component and its requirements.

Production

The producer's problem is to choose inputs to minimize total costs subject to production constraints. For n inputs, this problem can be represented mathematically as

$$\min_{X_i} \sum_{i}^{n} P_i X_i \quad \text{s.t.} \quad V = f(X_i)$$

X_i	Input I
P_i	Price i
V	Production function

The Langragean for the producer's cost minimization problem is

$$L = \sum_{i}^{n} (P_i X_i + P[V - f(X_i)])$$

Production

CAPSIM uses a constant elasticity of substitution (CES) production. This CES production function has the generic form

$$V = A\left[\sum_{i}^{n} a_i(\lambda_i X_i)^{\rho}\right]^{1/\rho}$$

V	Production function
A	Shift parameter (all inputs)
a_i	Share parameter (input i)
i	Shift parameter (input i)
X_i	Input i
	CES exponent

Production

$$V = A\left[\sum_{i}^{n} a_i(\lambda_i X_i)^{\rho}\right]^{1/\rho}$$

The shift parameters A and λ_i capture uniform and input-specific changes in productivity. For instance, an A value of 0.99 would mean that good V could be produced with 1% less of *all* inputs, whereas a λ value of 0.99 for petroleum inputs would mean that producing good V would require 1% less petroleum.

The share parameter a_i is the share of input i used to produce V. For instance, if A and λ_i are 1, a_i is equal to

$$a_i = \frac{X_i}{V}$$

Production

The CES exponent [illegible] is related to the CES substitution elasticity [illegible] by

$$\rho = \frac{\sigma - 1}{\sigma} \quad \text{or} \quad \sigma = \frac{1}{1-\rho}$$

Substitution elasticities determine how much the use of one input changes with a change in the price of another. As its name suggests, the CES function assumes that the elasticity of substitution ([illegible]) between inputs is constant.

When raised to the power of [illegible], the share parameter a_i can be rewritten as

$$a_i^{\sigma} = a_i^{1/(1-\rho)} = \alpha_i$$

Production

Substituting the CES production function into the Lagrangean from page 5 gives us

$$L = \sum_{i}^{n} \left(P_i X_i + P \left[V - A \left(\sum_{i}^{n} a_i (\lambda_i X_i)^{\rho} \right)^{1/\rho} \right] \right)$$

Finding the partial derivatives of L with respect to P and X_i gives us unit prices

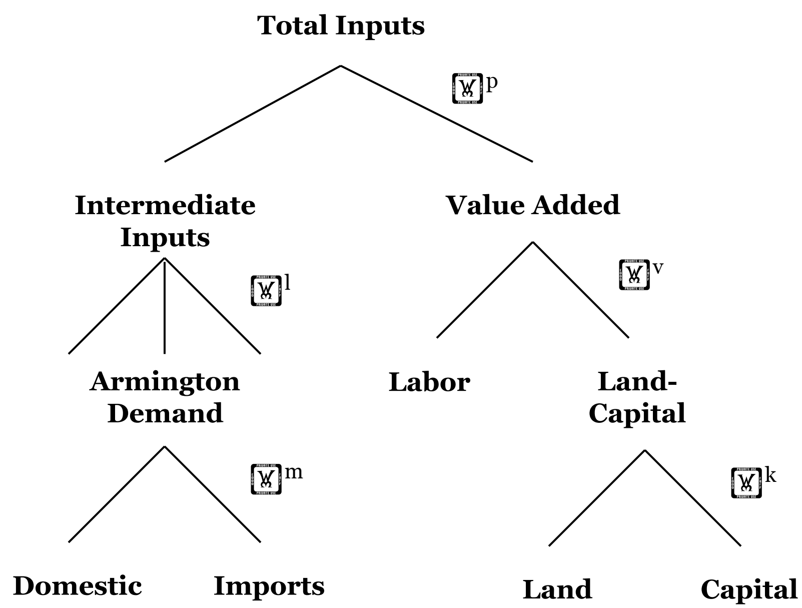
$$P = \left[\sum_{i}^{n} \alpha_i \left(\frac{P_i}{\lambda_i} \right)^{1-\sigma} \right]^{1/(1-\sigma)}$$

and optimal input amounts

$$X_i = \alpha_i {\lambda_i}^{\sigma - 1} \left(\frac{P}{P_i} \right)^{\sigma} V$$

Production

CAPSIM uses a nested CES structure, stratifying input substitution and allowing substitution elasticities to vary over inputs. In the first level, producers choose between intermediate inputs (e.g., steel, fuel, computers) and factor inputs (e.g., labor and capital) with substitution elasticity σ^p. The figure below shows a simplified version of the nested CES structure used in CAPSIM. For the complete structure, refer to the detailed CAPSIM documentation.



Production

Unit prices and demands for the nested components follow the generic CES structure described above. For instance, total unit prices for a given good X are the sum of intermediate and factor prices (PND and PVA, respectively). Using the formula for unit prices from page 9 and the ND-VA substitution elasticity from the previous page (σ^p), unit prices are

$$PX = \left[\alpha^{ND} PND^{1-\sigma^p} + \alpha^{VA} PVA^{1-\sigma^p}\right]^{1/(1-\sigma^p)}$$

Similarly, using the formula for input demands on page 9, demand for total intermediate and factor inputs is

$$ND = \alpha^{ND}\left(\frac{PX}{PVA}\right)^{\sigma^p} XP \quad \text{and} \quad VA = \alpha^{VA}\left(\frac{PX}{PVA}\right)^{\sigma^p} XP$$

where XP is total output of the good.

Consumption

CAPSIM uses an extended linear expenditure system (ELES) function to model household consumption. The ELES function separates household consumption into a subsistence minimum (θ) and discretionary spending (total consumption minus the minimum, or C-θ). In the ELES, households maximize utility from discretionary spending subject to budget and accounting constraints

$$\max_{x_i,S} U = \sum_i^n \mu_i \ln(x_i - \theta_i) + \mu_s \ln\left(\frac{S}{P^s}\right) \quad \text{s.t.} \quad Y = \sum_i^n p_i x_i + S, \qquad \sum_i^n \mu_i + \mu_s = 1$$

U	Household utility	S	Value of savings
μ_i	Income share of good i	P_s	Relative price of savings
x_i	Consumption of good I	Y	Total income
θ_i	Minimum consumption, good i	p_i	Price of good i
μ_s	Savings share of income	x_i	Consumption of good i

Consumption

The Lagrangean for the ELES function is

$$L = \sum_{i}^{n} \mu_i \ln(x_i - \theta_i) + \mu_s \ln\left(\frac{S}{p_S}\right) + \lambda\left(Y - \sum_{i}^{n} p_i x_i - S\right)$$

Finding the partial derivative of L with respect to x_i gives the ELES demand function

$$x_i = \theta_i + \frac{\mu_i}{\lambda p_i}$$

and the partial derivative with respect to S gives household demand for saving

$$S = \frac{\mu_S}{\lambda}$$