Overview of Social Accounting Matrices

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Contents

Introduction: General Motivation

- Detailed and rigorous accounting practices always have been at the foundation of sound and sustainable economic policy.
- A consistent set of real data on the economy is likewise a prerequisite to serious empirical work with economic simulation model.
- For this reason, a complete general equilibrium modeling facility stands on two legs: a consistent economywide database and modeling methodology.

Multi-Sectoral Development Analysis

- Macro policy is important, but so are economic structure and economic interactions.
- Indeed, linkages and indirect effects are often more important than the direct targets of policy.
- To improve visibility for policy makers and make appropriate recommendations, we need to understand these interactions.

What is needed?

To successfully develop a detailed, consistent, and up-to-date SAM, four ingredients are needed:

1. Official commitment
2. Component data resources
3. Methodology
4. Expertise and, where this is lacking, talent
5. Computer hardware and software

Fortunately, we are in a strong position in all these areas.

What is a SAM?

- An economy-wide accounting device to capture detailed interdependencies between institutions and sectors/regions. An extension of input-output analysis.
- A SAM is a form of double entry book keeping that itemizes detailed income and expenditure linkages across the economy.
- It is a closed form accounting system, reflecting the general equilibrium structure of the underlying economic relationships.

SAM Concepts

- A SAM is a square matrix that builds on the input-output table - but it goes further.
- A SAM considers not only production linkages, but tracks income-expenditure feedbacks (institutions are introduced).
- Each transactor (such as factors of production, households, enterprises, the government and the ROW) has a row (income sources) and a column (expenditures) – double entry national income accounting.
- A SAM is consistent data system that provides a snapshot of the economy – note that the SAM reconciles data from different sources.
- Detail is on the the biggest virtues of the SAM approach, but we actually build SAMs from the top down.

I/O to SAM

- At a basic level, the SAM extends the I/O by adding income and transfer accounts, thereby closing the flow of income, i.e.,

I/O

L	F
V	

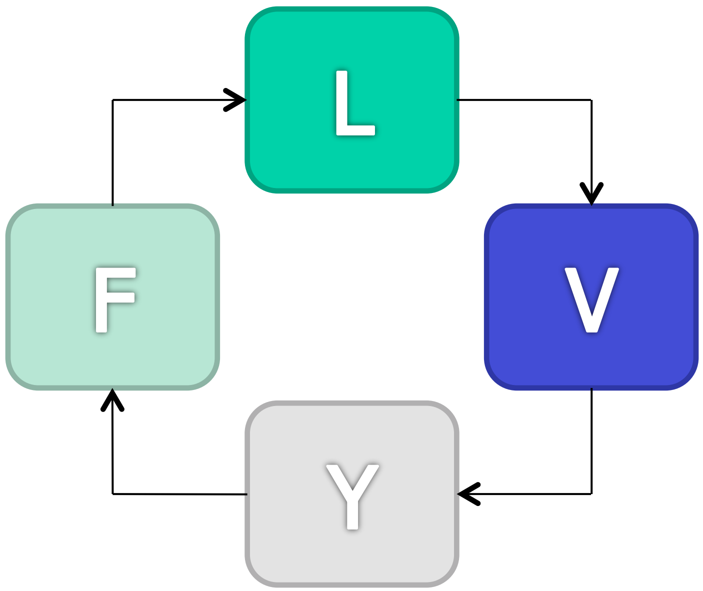
→ **SAM**

	L		F
	V		
		Y	T

where L is the matrix of I/O intermediate transactions, V is value added, F is final demand expenditure, Y is the domestic income, and T represents institutional transfers.

SAM Circular Flow of Income

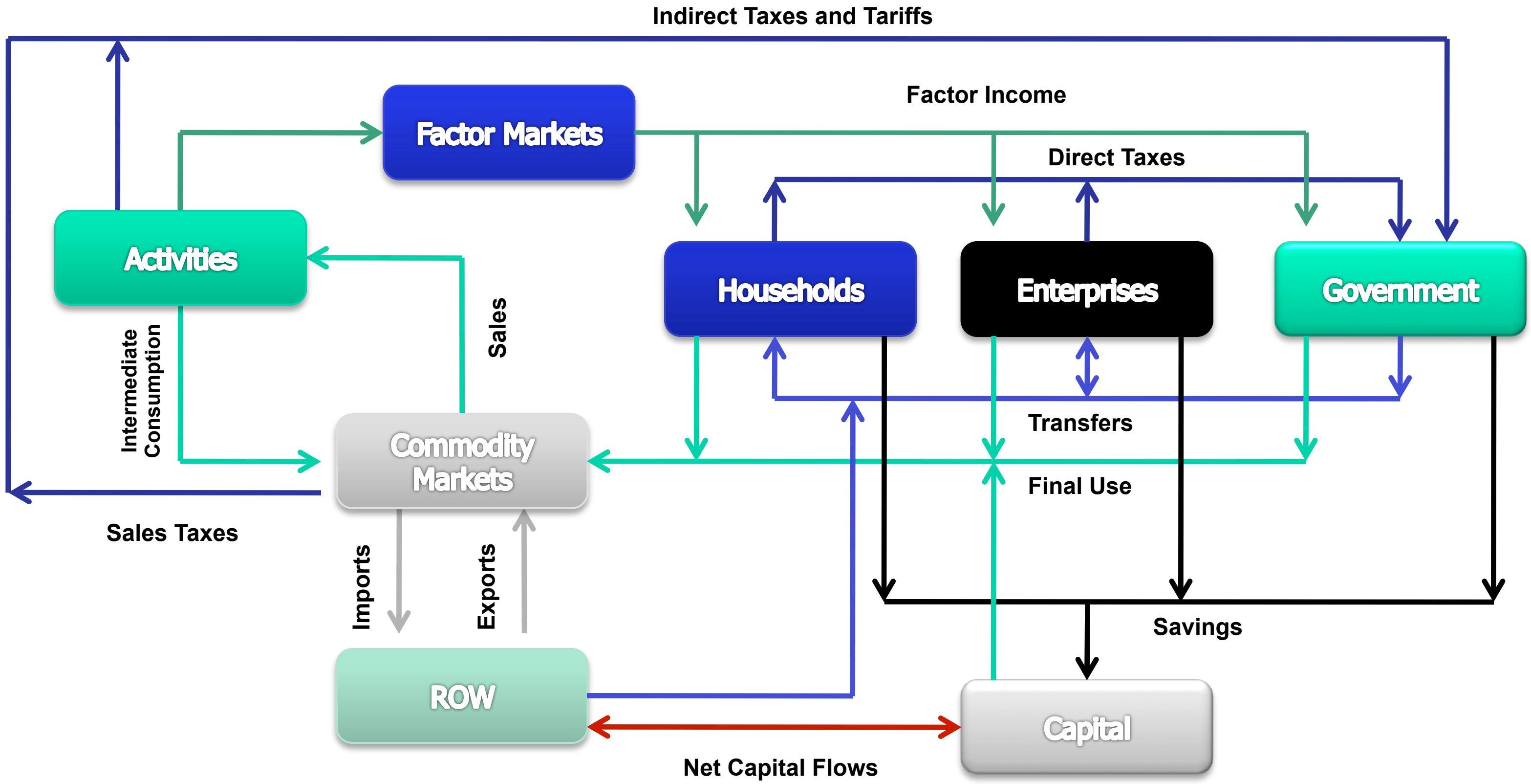
- A simplified circular flow of income is clearly visible from the SAM



- V maps income to factors, Y maps factors to institutions, F maps institutional income to A, A pays V.

SAM Circular Flow of Income

- A more detailed mapping of income flows:



SAM Feedbacks

- The circular flow of income is a very important concept in SAMs. Whereas I/O tables capture indirect linkages through inter-industry structure, SAMs also capture feedback effects because they include the induced effects of circular income flows on production.
- Induced effects refer to the new demand for goods and services caused by institutions spending their new income that results from new output induced by an exogenous shock.

SAM Interdependency

- By bringing together all economic accounts, SAMs contain the full range of interdependencies in a socioeconomic system:
- The SAM connects:
  - Production of goods and services
  - Generation of factor incomes
  - Levels and distributions of income available to institutions
  - Transfer payments and savings by institutions
  - Expenditures on goods and services

Main Features of a SAM

There are three main features of a SAM (Round, 2003)

- Square. SAM accounts are represented as a square matrix (note that the I/O table is typically not), where inflows-outflows for each account are rows-columns; this structure shows interconnections between agents in an explicit way.
- Comprehensive. SAMs portray all economic activities: production, consumption, accumulation, distribution.
- Flexible. SAMs are flexible in aggregation and emphasis.

SAM Uses

SAMs are useful for:

- Data Reconciliation. SAMs provide a coherent and consistent framework for bringing together data from many disparate sources, highlighting potential inconsistencies in data and thus improving data quality.
- Structural Insights. SAMs show clearly the structural interdependencies underlying an economy.
- Modeling. SAMs provide an accounting and analytical framework for fixed price multiplier (FPM) and CGE models.

SAM Construction

- We will begin with a national macro SAM and work our way down to a regional micro SAM.
- Because many of you are working on building sub-national SAMs, this approach is likely the approach that many of you will use in your projects.
- These macro-micro and micro-macro directions are often complementary: We will use the macro SAM as a means to maintain consistency for the micro SAM, and the micro SAM as a means to check the accuracy of our data in the macro SAM.

SAMs from a Macroeconomic Perspective

A macroeconomic SAM is also an extension of basic national income identities:

1. $Y + M = C + G + I + E$ (GNP)
2. $C + T + Sh = Y$ (Income)
3. $G + Sg = T$ (Govt. Budget)
4. $I = Sh + Sg + Sf$ (Savings-Investment)
5. $E + Sf = M$ (Trade Balance)

Schematic Macroeconomic SAM

			Expenditures			
Receipts	1	2	3	4	5	Total
1. Suppliers	-	C	G	I	E	Demand
2. Households	Y	-	-	-	-	Income
3. Government	-	T	-	-	-	Receipts
4. Capital Acct.	-	S_h	S_g	-	S_f	Savings
5. Rest of World	M	-	-	-	-	Imports
Total	Supply	Expenditure	Expenditure	Investment	ROW	

Receipts	Expenditures								
	1. Activities (124)	*2. Commodities (124)*	*3. Factors (13)*	*4. Private Households (5)*	*5. Enterprises (3)*	*6. Recurrent State (1)*	*7. Investment Savings (1)*	*8. Rest of World (94+1)*	*9. Total*
1. Activities (124)		Marketed Production							Total Sales
2. Commodities (124)	Intermediate Consumption			Private Consumptio n		State Consumption	Investment	Exports	Total Commodity Demand
3.Factors (13)	Value Added								Value Added
4. Private Households (5)			Wages, Salaries and Other Benefits		Distributed Profits and Social Security	Social Security and Other Current Transfers to Households		Net Foreign Transfers to Household s	Private Household Income
5. Enterprises (3)			Gross Profits					Net Foreign Transfers	Enterprise Income
6. Recurrent State (1)	Indirect Taxes	Consumption Taxes plus Import Tariffs	Factor Taxes	Income Taxes	Enterprise Income Taxes			Net Foreign Transfers to State	State Revenue
7. Investment Savings (1)				Household Savings	Retained Earnings	State Savings		Net Capital Inflows	Total Savings
8. Rest of World (94+1)		Imports							Imports
9. Total	Total Payments	Total Commodity Supply	Total Factor Payment s	Allocation of Private Household Income	Total Enterprise Expenditur e	Allocation of State Revenue	Total Investment	Total Foreign Exchange	

Sample National SAM (Thailand)

- 180 domestic production activities/commodities
- 4 factors of production
  - Labor: Ag and Non-Ag
  - Capital: Ag and Non-Ag
- 10 household types
- 1 Enterprise
- State (six catagories of fiscal instruments), could be disaggregated by central and regional government accounts
- Consolidated capital account
- Up to 94 international trading partners

Data Sources – Production Accounts

Row	Column	Data source and data compilation
1.Commodities	2.Activities	I/O Table
	4.Households	Final consumption, I/O Table, further disaggregated with , SES data
	6. Recurrent State	Central (and possibly regional) Government Expenditure
	7. Investment/ Savings	Fixed Investment (with our without inventories) I/ O Table
	8. ROW	I/O Table, Customs, and UN COMTRADE
	9. Total	Sum of row
2. Activities	1. Commodities	I/O Table

Data Sources - Factors

3. Labor	**2. Activities**	**I/O Table, Detailed data on wages and employment by occupation**
3. Land	**2. Activities**	**Estimation from independent sources, NBS**
3. Capital	**2. Activities**	**I/O Table**

Data Sources - Households

4. Households	**3. Labor**	**T_{32} in the SAM, <u>SES</u>**
	3. Land	**T_{42} in the SAM, <u>SES</u>**
	3. Capital	**Flow of Funds , <u>SES</u>**
	5. Enterprises	**Row residual , <u>SES</u>**
	6. Government	**Statistical Bureau, detailed transfer/ subsidy data**
	8. ROW	**Remittances, Statistical Bureau**
	9. Total	**Sum of column**

Data Sources – Other Domestic Institutions

5. Enterprises	**3. Capital**	**Distributed operating revenue, Flow of Funds**
6. Government	**1. Commodities**	**Domestic commodity and import taxes,** Statistical Bureau
	2. Activities	**Production taxes, VAT, and subsidies,** Statistical Bureau
	4. Households	**Tax payments,** Statistical Bureau
	5. Enterprises	**Enterprise taxes,** Statistical Bureau
	9. Total	Statistical Bureau

Data Sources – Trade and Capital Accounts

7. Investment/ Savings	**4. Households**	**Savings, household survey data reconciled with macro aggregates**
	5. Enterprise	**Retained and reinvested operating revenue**
	6. Government	**Net government budget balances**
	7. Inventories	**Input/output table**
	8. ROW	**Net foreign capital flows,** Statistical Bureau
8. ROW	**1. Commodities**	**Import flows, COMTRADE, I/O, Customs**
	4. Households	**Outbound remittances**
	5. Enterprises	**Profits repatriated by foreigners**
	6. Government	**New public foreign borrowing**
	7. Investment/savings	**New private foreign borrowing**

Data Reconciliation

- A quick note on data reconciliation, which is one of the more unsexy but often very valuable uses of SAMs.
- Economic data is often collected by different government ministries, and often there is little attempt to reconcile it even though the individual data is used without question.
- At two ends of the spectrum, national income accounts data is usually based on production surveys, while household survey data often show results that conflict with national data.

SAM Balancing Methods

Obviously, SAMs are built from very diverse data souces. Since these may be partially conflicting, a reconcilation or balancing process is necessary to produce a consistent, reconciled set of unified accounts.

There are two general approaches, algebraic and statistical. To indroduce these concepts, we survey the first approach. For empirical reasons, the more complex latter approch is generally used.

SAM Multipliers

- SAM multipliers are similar to I/O multipliers in both their algebra and economic interpretation.
- However, where the I/O multipliers are "open," SAM multipliers reflect closed circular flow of income effects, so we can look at both:
  - Induced effects through income-expenditure linkages
  - Distribution of income through institutional accounts
- The general idea with most SAM multiplier analyses is to examine two groups of actors (producers and households) interacting in two markets (commodity and factor).

Endogenous and Exogenous Accounts

- To calculate SAM multipliers we need to first separate the SAM into endogenous and exogenous accounts, both for economic and mathematical reasons.
- Economically, the SAM does not describe all of the factors at work in an economy (e.g., government spending habits).
- Mathematically, without exogenizing some accounts we will end up with a singular A matrix and will not be able to calculate multipliers.

Endogenous Accounts

- Endogenous accounts include those accounts where income-expenditure is governed by mechanisms that operate entirely within the SAM framework.
- Typically, endogenous accounts include:
  - Production-commodity accounts
  - Factor accounts
  - Household accounts
  - Capital account (sometimes)

Exogenous Accounts

- Exogenous accounts are those accounts where income and/or expenditure are governed by forces external to the SAM framework.
- Typically, exogenous accounts include the government, ROW, and sometimes the capital account.
- For government and ROW, it should be fairly intuitive why these accounts are exogenous: The SAM tells us nothing about how government will plan expenditures, or what is happening in ROW.

Endogenous and Exogenous Accounts

- In a SAM matrix framework, this endogenous-exogenous division gives us

	Expenditure					
		Endogenous	Sum	Exogenous	Sum	Total
Income	Endogenous	T_{nn}	n	T_{nx} (injections)	x	Y_n
	Exogenous	T_{xn} (leakages)	l	T_{xx} (residual balance)	t	Y_x
	Totals	Yn'		Yx'		

Adapted from Khan, 2007

where we can see that endogenous incomes are equal to incomes generated within endogenous accounts plus injections, or

$$y_n = n + x$$

Injections and Leakages

- Endogenous and exogenous accounts are connected by two mechanisms:
  - Injections (T_{nx}), usually denoted by the letter x. Injections, following the subscript notation, are exogenous account expenditures on endogenous accounts (e.g., agricultural subsidies).
  - Leakages (T_{xn}), which are endogenous account expenditures on exogenous accounts (e.g., income taxes).
  - Residual balances (T_{xx}) consist of transfers between exogenous accounts (e.g., government savings).

SAM A Matrix

- As with the I/O table, for the SAM we can calculate a matrix of average expenditure propensities by dividing SAM entries by their column totals.

The total matrix

$$a_{ij} = \frac{t_{ij}}{T_j}$$

is known as the A matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ a_{n1} & a_{n2} & a_{nn} \end{bmatrix}$$

SAM Multipliers

- We can calculate SAM multipliers using an approach similar to the material balance equation we used for calculating I/O multipliers.
- SAM endogenous incomes

$$y_n = n + x$$

can be rewritten as

$$y_n = A_n y_n + x$$

which is equivalent to

$$y_n = (I-A_n)^{-1}x = M_a x$$

and again

$$dy_n = (I-A_n)^{-1}dx = M_a dx$$

SAM Multipliers

- We can calculate leakage multipliers in a similar fashion. From

$$\Lambda = A_{\Lambda}\, y_n$$

we can substitute

$$y_n = (I\text{-}A_n)^{-1}x = M_a x$$

which gives us

$$\Lambda = A_{\Lambda} M_a x$$

and similarly

$$d\Lambda = A_{\Lambda} M_a dx$$

SAM Multipliers

- As $y_n = M_a x$ suggests, the SAM multiplier M_a captures the multiplier effects of an exogenous shock x on endogenous income y_n, where x is a vector of injections into endogenous (row) accounts.

SAM Multiplier Limitations

- SAM multiplier limitations include:
  - Excess capacity in all sectors and unemployed or underemployed factors of production; multipliers will overstate the total effects if capacity constraints exist.
  - No allowance for substitution effects
  - Fixed prices
  - Limit to the endogenous effects that can be captured (exogenous accounts will be affected by initial shock, leakage from endogenous to exogenous)

Fixed-Price Multiplier Models

- While SAM multipliers can reveal interesting and policy-relevant information about economic structure and living standards, they do not contain information about economic behavior and are still accounting multipliers.
- Fixed-price multiplier (FPM) models add some behavioral characteristics into the SAM accounting framework by converting the SAM A matrix of average expenditure propensities into a matrix of marginal expenditure propensities.

FPM Models

- FPM models operate under the assumption that relative prices do not change as income changes, or correspondingly that supply prices are independent of the scale of production, hence the name 'fixed-price.'

FPM Mathematics

- The basic idea is this: In the SAM accounting framework we had

$$dy_n = dn + dx$$

where

$$dy_n = A_n dy_n + dx$$

- In the FPM model we have

$$dy_n = C_n dy_n + dx$$

where C_n is a matrix of marginal expenditure propensities (MEPs)

FPM Mathematics

- As a matrix of MEPs, C_n can be represented by

$$C_n(i,j) = \frac{\partial n_i}{\partial y_{n,j}}$$

and C can be calculated from A by

$$C = \eta A$$

where η is a matrix of income elasticities and C reflects the change in row inputs with respect to column income.

FPM Mathematics

- We can calculate multipliers for FPM models in the same way that we did for SAMs:

$$dy_n = C_n dy_n + dx$$

$$dy_n = (I-A)^{-1} C_n dx = M_c dx$$

And similarly changes in leakages resulting from injection of:

$$d\Lambda = C_\Lambda dy_n$$

$$d\Lambda = C_\Lambda (I-A)^{-1} C_n dx = C_\Lambda M_c dx$$

FPM in Practice

- Given

$$C(i,j) = \eta A(i,j)$$

when $\eta = 1$, $C = A$ and the FPM A matrix is identical to the SAM A matrix.

- In practice, given both theoretical considerations and the enormous task of calculating income elasticities for every element in the SAM, $\eta = 1$ is assumed for a substantial portion of the SAM — most FPM models replace A_n elements with C_n estimates only for household expenditures.

A_n – C_n Equivalence

- The rationale for A_n – C_n equivalence is as follows:
  - The fixed price assumption implicitly assumes a Leontief structure on production activities. For instance, if factor prices are fixed then factor costs per unit output are constant.
  - Enterprises are usually assumed to have MEP = AEP as well, though there really is not much economic basis for that assumption.

FPM Models

- As a result of the fixed price assumption, as with SAM multipliers the implicit assumption with FPM models is that the economy is working under capacity.
- In other words, the FPM model is useful for examining quantity based shocks, but not price shocks or price effects.

Limitations of FPM Analysis

- FPM analysis suffers from a number of limitations:
  - Fixed technology. The bulk of empirical evidence suggests that inputs are not fixed, either in time or in scale.
  - Fixed/linear I/O relationships. This implies an economy-wide CRTS, which is unlikely to be the case.
  - Fixed prices. Relative prices are constant and stable.
  - Lack of closure. I/O tables typically do not include the induced effects resulting from income generation (income leaks out of the system rather than being spent).
  - Lack of explicit constraints. I/O analysis typically assumes incomplete capacity utilization.
  - Lack of economic behavior. I/O analysis does not allow for input substitutability or income effects.

Partitioning the SAM_n

- We have been thinking about the endogenous SAM elements as part of one large matrix, but we can separate, or partition, the SAM endogenous A matrix into a 3 x 3 matrix of sub-matrices.

	Expenditures			
Receipts		Activities	Factors	Institutions
	Activities	A_{11}		A_{13}
	Factors	A_{21}		
	Institutions		A_{32}	A_{33}

Partitioning the SAM$_n$

	Expenditures			
Receipts		Activities	Factors	Institutions
	Activities	A_{11}	0	A_{13}
	Factors	A_{21}	0	0
	Institutions	0	A_{32}	A_{33}

- In this partitioned SAM:
  - A_{11} is the I/O transactions table
  - A_{21} represents payments from activities to factors
  - A_{32} represents payments from factors to institutions
  - A_{13} represents payments from institutions to activities
  - A_{33} represents inter-institutional transfers

Partitioning the SAM$_n$

- If we remove inter-industry transfers (transactions) and inter-institutional transfers from the partitioned SAM we can see the circular flow of income

	Expenditures			
Receipts		Activities	Factors	Institutions
	Activities	0	0	A_{13}
	Factors	A_{21}	0	0
	Institutions	0	A_{32}	0

Again, activities pay factors, factor income maps to institutions, and institutions pay activities for goods and services.

SAM Multiplier Decomposition

- Multiplier decomposition techniques allow us to separate multipliers into their component parts to examine different mechanisms within the economy.
- Multiplier components can be additive or multiplicative; in other words, multipliers can be the sum or the product of their component parts.
- We will begin with multiplicative SAM components, examine additive components, and finally demonstrate relationships among all three forms.

Decomposition Algebra

- The mathematics behind multiplier decomposition are fairly intuitive. From our earlier SAM accounting identity we have

$$y_n = A_n y_n + x$$

For any sub-matrix of A_n we can rewrite this as

$$\begin{aligned} y_n &= (A_n - A^o_n)y_n + A^o_n y_n + x \\ &= (I - A^o_n)^{-1} (A_n - A^o_n)y_n + (I - A^o_n)^{-1}x \\ &= A^* y_n + (I - A^o_n)^{-1}x \end{aligned}$$

where $A^* = (I - A^o_n)^{-1} (A_n - A^o_n)$

Decomposition Algebra

- If we multiply both sides of

$$y_n = A^* y_n + (I - A^o_n)^{-1} x$$

by A^* and substitute the $A^* y_n$ term on the LHS with the $A^* y_n = y_n - (I - A^o_n)^{-1} x$ term from the RHS, we get

$$A^* y_n = A^{*2} y_n + A^*(I - A^o_n)^{-1} x$$

$$y_n - (I - A^o_n)^{-1} x = A^{*2} y_n + A^*(I - A^o_n)^{-1} x$$

$$y_n = A^{*2} y_n + (I - A^o_n)^{-1} x + A^*(I - A^o_n)^{-1} x$$

$$y_n = A^{*2} y_n + (I + A^*)\,(I - A^o_n)^{-1} x$$

$$y_n = (I - A^{*2})^{-1}\,(I + A^*)\,(I - A^o_n)^{-1} x$$

Decomposition Algebra

- We can continue to do this indefinitely. For the next round, we multiply both sides of

$$y_n = A^{*}y_n + (I - A^{o}_{n})^{-1}x$$

by A^{*2} and substitute for $A^{*2}y_n$, which gives us

$$\begin{aligned} y_n &= A^{*3}y_n + (I + A^{*} + A^{*2})\,(I - A^{o}_{n})^{-1}x \\ &= (I - A^{*3})^{-1}\,(I + A^{*} + A^{*2})\,(I - A^{o}_{n})^{-1}x \end{aligned}$$

and ultimately to the more general result

$$y_n = (I - A^{*k})^{-1}\,(I + A^{*} + A^{*2} + \ldots + A^{*(k-1)})\,(I - A^{o}_{n})^{-1}x$$

Decomposition Algebra

- While we could do decomposition indefinitely, we typically stop at k = 3 steps because 3 is the number of endogenous accounts within the SAM. In other words, the flow of income around the SAM undergoes 3 steps.

A_n and $A^o{}_n$

- We start by defining three matrices: A_n, $A^o{}_n$, and A*.
- A_n is the A matrix for our complete partitioned SAM

$$A_n = \begin{bmatrix} A_{11} & 0 & A_{13} \\ A_{21} & 0 & 0 \\ 0 & A_{32} & A_{33} \end{bmatrix}$$

- $A^o{}_n$ is the sub-matrix of inter-industry and inter-institutional transfers

$$A^o{}_n = \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_{33} \end{bmatrix}$$

A*

- Remember that $A^* = (I - A_n)^{-1} (A_n - A^o_n)$, where the first term is equivalent to

$$\left(\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} - \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_{33} \end{bmatrix}\right)^{-1} = \begin{bmatrix} (I - A_{11})^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & (I - A_{33})^{-1} \end{bmatrix}$$

and the second term is equivalent to

$$\begin{bmatrix} A_{11} & 0 & A_{13} \\ A_{21} & 0 & 0 \\ 0 & A_{32} & A_{33} \end{bmatrix} - \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & A_{13} \\ A_{21} & 0 & 0 \\ 0 & A_{32} & 0 \end{bmatrix}$$

A*

- Multiplying these two terms gives

$$A^* = \begin{bmatrix} 0 & 0 & (I - A_{11})^{-1} A_{13} \\ A_{21} & 0 & 0 \\ 0 & (I - A_{33})^{-1} A_{32} & 0 \end{bmatrix}$$

Note that we can define the elements of A* as

$(I - A_{11})^{-1}A_{13} = A^*_{13}$ $A_{21} = A^*_{21}$ $(I - A_{33})^{-1}A_{32} = A^*_{32}$

$$A^* = \begin{bmatrix} 0 & 0 & A^*_{13} \\ A^*_{21} & 0 & 0 \\ 0 & A^*_{32} & 0 \end{bmatrix}$$

such that A* follows the circular income flow in the SAM.

$M_{a3}M_{a2}M_{a1}$

- With

$$y_n = (I - A^{*3})^{-1} \; (I + A^* + A^{*2}) \; (I - A^o{}_n)^{-1}x = M_a x$$

we can define the SAM multiplier M_a as the product of three matrices:

$$M_a = M_{a3}M_{a2}M_{a1}$$

where

$$M_{a1} = (I - A^o{}_n)^{-1}$$
$$M_{a2} = (I + A^* + A^{*2})$$
$$M_{a3} = (I - A^{*3})^{-1}$$

M_{a1}

- For $M_{a1} = (I - A^o{}_n)^{-1}$

  Remember that in our partitioned SAM

$$A^o{}_n = \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_{33} \end{bmatrix}$$

Thus

$$M_{a1} = \begin{bmatrix} (I - A_{11})^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & (I - A_{33})^{-1} \end{bmatrix}$$

M_{a1}

- From the $(I-A_{11})^{-1}$ and $(I-A_{33})^{-1}$ elements of M_{a1} you can begin to develop some intuition about how to interpret the decomposed multipliers.
- M_{a1} is typically referred to as the transfers, or direct effects, multiplier, because it captures the multiplier effects of transfers within accounts; in this case industries, i.e. $(I-A_{11})^{-1}$, and institutions, i.e. $(I-A_{33})^{-1}$.
- M_{a1} only captures within account effects; it tells us nothing about factors or institutions.

M_{a2}

- Similarly, for $M_{a2} = (I + A^* + A^{*2})$, where A^{*2} is

$$A^{*2} = \begin{bmatrix} 0 & (I-A_{11})^{-1}A_{13}(I-A_{33})^{-1}A_{32} & 0 \\ 0 & 0 & A_{21}(I-A_{11})^{-1}A_{13} \\ (I-A_{33})^{-1}A_{32}A_{21} & 0 & 0 \end{bmatrix}$$

or more simply

$$A^{*2} = \begin{bmatrix} 0 & A^*_{13}A^*_{32} & 0 \\ 0 & 0 & A^*_{21}A^*_{13} \\ A^*_{32}A^*_{21} & 0 & 0 \end{bmatrix}$$

M_{a2}

- Thus $M_{a2} = (I + A^* + A^{*2})$ is

$$\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} + \begin{bmatrix} 0 & 0 & A^*_{13} \\ A^*_{21} & 0 & 0 \\ 0 & A^*_{32} & 0 \end{bmatrix} + \begin{bmatrix} 0 & A^*_{13}A^*_{32} & 0 \\ 0 & 0 & A^*_{21}A^*_{13} \\ A^*_{32}A^*_{21} & 0 & 0 \end{bmatrix}$$

or

$$\begin{bmatrix} I & A^*_{13}A^*_{32} & A^*_{13} \\ A^*_{21} & I & A^*_{21}A^*_{13} \\ A^*_{32}A^*_{21} & A^*_{32} & I \end{bmatrix}$$

M_{a2}

- M_{a2} is the only matrix with off-diagonal elements, and is referred to as the cross-effects, or open-loop, multiplier.
- M_{a2} captures the effects of an injection into the system as it moves through the system without coming back to its origin (hence the name ‘open-loop’). In other words, M_{a2} shows how an external injection travels from endogenous demand to income (“across” institutions), but not from income to demand.

M_{a3}

- $M_{a3} = (I - A^{*3})^{-1}$, where A^{*3} is

$$\begin{bmatrix} A^*{}_{13}A^*{}_{32}A^*{}_{21} & 0 & 0 \\ 0 & A^*{}_{21}A^*{}_{13}A^*{}_{32} & 0 \\ 0 & 0 & A^*{}_{32}A^*{}_{21}A^*{}_{13} \end{bmatrix}$$

and $(I - A^{*3})^{-1}$ is

$$\begin{bmatrix} (I - A^*{}_{13}A^*{}_{32}A^*{}_{21})^{-1} & 0 & 0 \\ 0 & (I - A^*{}_{21}A^*{}_{13}A^*{}_{32})^{-1} & 0 \\ 0 & 0 & (I - A^*{}_{32}A^*{}_{21}A^*{}_{13})^{-1} \end{bmatrix}$$

M_{a3}

- M_{a3} is typically referred to as the circular, or closed loop, multiplier.
- M_{a3} captures the full circular effects of an exogenous income injection on one account, once the circular flow of income returns to the account where the injection took place.
- In other words, M_{a3} represents the full circular multiplier effects net of M_{a1} and M_{a2}.

Additive Multipliers

- All three multiplier forms — aggregate, multiplicative, and additive — are related by

$$M_a = M_3M_2M_1 = I + T + O + C$$

where
  - I = Identity multiplier
  - $T = (M_1 - I)$ = Net transfer multiplier
  - $O = (M_2 - I)M_1 = (M_2M_1 - M_1)$ = Open-loop multiplier
  - $C = (M_3 - I)M_2M_1 = (M_3M_2M_1 - M_2M_1)$ = Closed-loop multiplier

Applications

- Standard multiplier decomposition presents an interesting way of separating out the structural effects of exogenous shocks.
- For instance, in their study of Sri Lanka, Pyatt and Round (1979) found that transfer multipliers were significantly lower than open-loop (between-account) multipliers, suggesting the need for a more comprehensive approach to understanding income flows.

FPM Decomposition

- We can do multiplier decomposition with FPM models in the same way.
- We can also isolate income effects by separating out C_n and A_n

$$dy_n = (C_n - A_n)dy_n + A_n dy_n + dx$$

Regional Multiplier Decomposition

- Another interesting application for multiplier decomposition is the MRSAM trade matrix that we saw in lecture 3.
- For instance, we can create a 3 region transactions matrix where, as we saw previously, bilateral trade flows are on the off-diagonals

T_{11}	T_{12}	T_{13}	F_1
T_{21}	T_{22}	T_{23}	F_2
T_{31}	T_{32}	T_{33}	F_3
V_1	V_2	V_3	X

Regional Multiplier Decomposition

- Using the transactions sub-matrix

T_{11}	T_{12}	T_{13}	F_1
T_{21}	T_{22}	T_{23}	F_2
T_{31}	T_{32}	T_{33}	F_3
V_1	V_2	V_3	X

we can examine regional trade multipliers through the same approach as above, although in this case our $A^o{}_n$ matrix would include T_{11}, T_{22}, and T_{33} along its block diagonal.

Regional Multiplier Decomposition

- The resulting three matrices separate regional linkages into intra-region (M_1), inter-region (M_2), and equilibrium direct (M_3) multipliers:

M_1 =

$(I-A_{11})^{-1}$	0	0
0	$(I-A_{22})^{-1}$	0
0	0	$(I-A_{33})^{-1}$

M_2 =

I	$(I-A_{11})^{-1}A_{12}$	$(I-A_{11})^{-1}A_{13}$
$(I-A_{22})^{-1}A_{21}$	I	$(I-A_{22})^{-1}A_{32}$
$(I-A_{33})^{-1}A_{31}$	$(I-A_{33})^{-1}A_{32}$	I

Regional Multiplier Decomposition

$M_3 =$

$I-D_{12}D_{21}-D_{13}D_{31}$	$D_{21}D_{12}$	$D_{31}D_{13}$
$D_{12}D_{21}$	$I-D_{21}D_{12}-D_{23}D_{32}$	$D_{23}D_{32}$
$D_{13}D_{31}$	$D_{23}D_{32}$	$I-D_{31}D_{13}-D_{23}D_{32}$

where $D = (I-A_{ii})^{-1}A_{ij}$

SAMs to CGE

- While there are many interesting and policy-relevant applications for SAMs, both standard SAM multiplier and FPM models still suffer from some of the deficiencies of I/O tables: fixed coefficients, fixed prices, and spare capacity.
- If structure changes as a result of changes in relative prices, then SAMs are less useful, and we need to look to more complex models, like CGE.

SAM Balancing

- As we have discussed several times, it is normal to encounter inconsistencies when compiling a SAM.
- Inconsistencies can arise from a variety of sources: measurement errors, incompatible data sources, old data, or lack of data (and inconsistent estimations).
- Inconsistencies mean that columns and rows do not balance, and that accounting identities do not hold.

SAM Balancing

- In many cases, in compiling a SAM you will encounter situations in which you have:
  - Comprehensive but outdated data (particularly I/O tables)
  - New macro aggregate data
  - Micro data from (e.g.) household surveys that is inconsistent with macro aggregate data
  - Data that is complete at the aggregate account level, but not disaggregated
- Reconciling these inconsistent data components into a consistent set of SAM accounts is critical for doing any kind of SAM-based modeling.

Data Reconciliation Methods

- In reconciling data sources, there are two extremes:
  - Expert judgment. Determine which data are inconsistent, which data are more likely to be reliable, and make a judgment as to which data to include and which to ignore.
  - Mathematical balancing. Use mathematical techniques to reconcile inconsistencies within tables, and between micro tables and macro aggregate tables.
- These two approaches are not exclusive. For instance, we might use expert judgment to reconcile sources in a micro SAM, and then use mathematical techniques to balance it against macro aggregates.

Updating and Balancing SAMs

- Two primary mathematical techniques have dominated the SAM balancing literature:
  - RAS algorithm (Stone, 1961; Bacharach, 1969)
  - Entropy methods (Judge and Golan, 1994)
- We will work through simple applications with both of these approaches in Lab 4 to give you intuition about their mechanics. SAM balancing typically requires software more powerful than Excel, such as GAMS or MATLAB.

RAS Overview

- RAS is an iterative algorithm of "biproportional adjustment." More simply, RAS is an algorithm that uses row and column scaling factors to iteratively readjust the SAM in light of new row and column data.
- RAS is ideal when we start with a consistent SAM and complete knowledge about new row and column totals.

RAS Mathematics

- If A^0 is an unbalanced SAM A matrix, a balanced (i.e., rows = columns) matrix A^1 can be found by multiplying A^0 by row and column factors r and s, respectively

$$T_j = \sum_{i=1}^{n} t_{ij} \qquad a_{ij}^0 = \frac{t_{ij}}{T_j}$$

$$A^1 = \hat{r} A^0 \hat{s}$$

Hence the name RAS.

RAS Mathematics

- Again, r and s are scaling factors, so

$$A_1 = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} r_1 a_{11} s_1 & r_1 a_{21} s_2 \\ r_2 a_{12} s_1 & r_2 a_{22} s_2 \end{bmatrix}$$

Simple RAS

- It is easiest to understand how RAS works through a step by step example. Consider the following balanced really simple SAM (RSS):

	AG	IND	SVCS	LVA	CVA	UHH	RHH	GOV	INV	Total
AG	25	10	10			10	5	5		65
IND	10	40	10			13	2	10	25	110
SVCS	15	20	30			5				70
LVA	10	10	15							35
CVA	5	30	5							40
UHH				25	35					60
RHH				10	5					15
GOV						15	5			20
INV						17	3	5		25
Total	65	110	70	35	40	60	15	20	25	

Simple RAS Example

- Let's reduce UHH factor income to 20 so that the table is no longer balanced. This throws off both our LVA and UHH row-column totals:

	AG	IND	SVCS	LVA	CVA	UHH	RHH	GOV	INV	Total
AG	25	10	10			10	5	5		65
IND	10	40	10			13	2	10	25	110
SVCS	15	20	30			5				70
LVA	10	10	15							35
CVA	5	30	5							40
UHH				20	35					55
RHH				10	5					15
GOV						15	5			20
INV						17	3	5		25
Total	65	110	70	30	40	60	15	20	25	

Simple RAS Example

- Our new SAM A matrix is:

0.38	0.09	0.14	-	-	0.17	0.33	0.25	-
0.15	0.36	0.14	-	-	0.22	0.13	0.50	1.00
0.23	0.18	0.43	-	-	0.08	-	-	-
0.15	0.09	0.21	-	-	-	-	-	-
0.08	0.27	0.07	-	-	-	-	-	-
-	-	-	0.67	0.88	-	-	-	-
-	-	-	0.33	0.13	-	-	-	-
-	-	-	-	-	0.25	0.33	-	-
-	-	-	-	-	0.28	0.20	0.25	-

- We ar ums should be 35, and the UHH row-column sums should be 60.

Simple RAS Example

- To rebalance the table using RAS techniques, use the following steps:
  - Step 1. Multiply the columns in the A matrix by new column total elements:

0.38	0.09	0.14	-	-	0.17	0.33	0.25	-
0.15	0.36	0.14	-	-	0.22	0.13	0.50	1.00
0.23	0.18	0.43	-	-	0.08	-	-	-
0.15	0.09	0.21	-	-	-	-	-	-
0.08	0.27	0.07	-	-	-	-	-	-
-	-	-	0.67	0.88	-	-	-	-
-	-	-	0.33	0.13	-	-	-	-
-	-	-	-	-	0.25	0.33	-	-
-	-	-	-	-	0.28	0.20	0.25	-

X

65	0	0	0	0	0	0	0	0
0	110	0	0	0	0	0	0	0
0	0	70	0	0	0	0	0	0
0	0	0	35	0	0	0	0	0
0	0	0	0	40	0	0	0	0
0	0	0	0	0	60	0	0	0
0	0	0	0	0	0	15	0	0
0	0	0	0	0	0	0	20	0
0	0	0	0	0	0	0	0	25

Simple RAS Example

- That gives us:

	AG	IND	SVCS	LVA	CVA	UHH	RHH	GOV	INV	
AG	25	10	10	0	0	10	5	5	0	65
IND	10	40	10	0	0	13	2	10	25	110
SVCS	15	20	30	0	0	5	0	0	0	70
LVA	10	10	15	0	0	0	0	0	0	35
CVA	5	30	5	0	0	0	0	0	0	40
UHH	0	0	0	23.3333	35	0	0	0	0	58.3333
RHH	0	0	0	11.6666 7	5	0	0	0	0	16.6667
GOV	0	0	0	0	0	15	5	0	0	20
INV	0	0	0	0	0	17	3	5	0	25
	65	110	70	35	40	60	15	20	25	

Simple RAS Example

- Step 2. Sum the rows in the new table

25	10	10	0	0	10	5	5	0
10	40	10	0	0	13	2	10	25
15	20	30	0	0	5	0	0	0
10	10	15	0	0	0	0	0	0
5	30	5	0	0	0	0	0	0
0	0	0	23.33333	35	0	0	0	0
0	0	0	11.66667	5	0	0	0	0
0	0	0	0	0	15	5	0	0
0	0	0	0	0	17	3	5	0

$\rightarrow$

65
110
70
35
40
58.33333
16.66667
20
25

Simple RAS Example

- Step 3. Divide new row total by resulting column entries

65/65	110/110	70/70	35/35	40/40	60/58.3	15/16.7	20/20	25/25

Simple RAS Example

- Or with matrices

65	110	70	35	40	60	15	20	25

X

0.02	-	-	-	-	-	-	-	-
-	0.01	-	-	-	-	-	-	-
-	-	0.01	-	-	-	-	-	-
-	-	-	0.03	-	-	-	-	-
-	-	-	-	0.03	-	-	-	-
-	-	-	-	-	0.02	-	-	-
-	-	-	-	-	-	0.06	-	-
-	-	-	-	-	-	-	0.05	-
-	-	-	-	-	-	-	-	0.04

Simple RAS Example

- Which gives us

$r^1 =$

1.00	1.00	1.00	1.00	1.00	1.03	0.90	1.00	1.00

Simple RAS Example

- Step 4. Multiply x_{ij}^1 row elements by each element of the resulting row vector

1.00	-	-	-	-	-	-	-	-
-	1.00	-	-	-	-	-	-	-
-	-	1.00	-	-	-	-	-	-
-	-	-	1.00	-	-	-	-	-
-	-	-	-	1.00	-	-	-	-
-	-	-	-	-	1.03	-	-	-
-	-	-	-	-	-	0.90	-	-
-	-	-	-	-	-	-	1.00	-
-	-	-	-	-	-	-	-	1.00

X

25.00	10.00	10.00	-	-	10.00	5.00	5.00	-
10.00	40.00	10.00	-	-	13.00	2.00	10.00	25.00
15.00	20.00	30.00	-	-	5.00	-	-	-
10.00	10.00	15.00	-	-	-	-	-	-
5.00	30.00	5.00	-	-	-	-	-	-
-	-	-	23.33	35.00	-	-	-	-
-	-	-	11.67	5.00	-	-	-	-
-	-	-	-	-	15.00	5.00	-	-
-	-	-	-	-	17.00	3.00	5.00	-
-	-	-	-	-	-	-	-	-

Simple RAS Example

- Which gives us

	AG	IND	SVCS	LVA	CVA	UHH	RHH	GOV	INV	
AG	25	10	10	0	0	10	5	5	0	65
IND	10	40	10	0	0	13	2	10	25	110
SVCS	15	20	30	0	0	5	0	0	0	70
LVA	10	10	15	0	0	0	0	0	0	35
CVA	5	30	5	0	0	0	0	0	0	40
UHH	0	0	0	24	36	0	0	0	0	60
RHH	0	0	0	10.5	4.5	0	0	0	0	15
GOV	0	0	0	0	0	15	5	0	0	20
INV	0	0	0	0	0	17	3	5	0	25
	65	110	70	34.5	40.5	60	15	20	25	

Simple RAS Example

- Step 5. Sum the columns

25	10	10	0	0	10	5	5	0	65
10	40	10	0	0	13	2	10	25	110
15	20	30	0	0	5	0	0	0	70
10	10	15	0	0	0	0	0	0	35
5	30	5	0	0	0	0	0	0	40
0	0	0	24	36	0	0	0	0	60
0	0	0	10.5	4.5	0	0	0	0	15
0	0	0	0	0	15	5	0	0	20
0	0	0	0	0	17	3	5	0	25

$\downarrow$

65	110	70	34.5	40.5	60	15	20	25

Simple RAS Example

- Step 6. Divide new column total by resulting row vector

0.02	-	-	-	-	-	-	-	-
-	0.01	-	-	-	-	-	-	-
-	-	0.01	-	-	-	-	-	-
-	-	-	0.03	-	-	-	-	-
-	-	-	-	0.02	-	-	-	-
-	-	-	-	-	0.02	-	-	-
-	-	-	-	-	-	0.07	-	-
-	-	-	-	-	-	-	0.05	-
-	-	-	-	-	-	-	-	0.04

X

65
110
70
35
40
60
15
20
25

Simple RAS Example

- Which gives us

$$s^1 = \begin{bmatrix} 1.00 \\ 1.00 \\ 1.00 \\ 1.01 \\ 0.99 \\ 1.00 \\ 1.00 \\ 1.00 \\ 1.00 \end{bmatrix}$$

Simple RAS Example

- Step 7. Multiply each value in the column vector by each column in x_{ij}^2

25.00	10.00	10.00	-	-	10.00	5.00	5.00	-
10.00	40.00	10.00	-	-	13.00	2.00	10.00	25.00
15.00	20.00	30.00	-	-	5.00	-	-	-
10.00	10.00	15.00	-	-	-	-	-	-
5.00	30.00	5.00	-	-	-	-	-	-
-	-	-	23.33	35.00	-	-	-	-
-	-	-	11.67	5.00	-	-	-	-
-	-	-	-	-	15.00	5.00	-	-
-	-	-	-	-	17.00	3.00	5.00	-
-	-	-	-	-	-	-	-	-

X

1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1.01	0	0	0	0	0
0	0	0	0	0.99	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1

Simple RAS Example

- Which gives us

	AG	IND	SVCS	LVA	CVA	UHH	RHH	GOV	INV	
AG	25.00	10.00	10.00	-	-	10.00	5.00	5.00	-	65.00
IND	10.00	40.00	10.00	-	-	13.00	2.00	10.00	25.00	110.00
SVCS	15.00	20.00	30.00	-	-	5.00	-	-	-	70.00
LVA	10.00	10.00	15.00	-	-	-	-	-	-	35.00
CVA	5.00	30.00	5.00	-	-	-	-	-	-	40.00
UHH	-	-	-	24.35	35.56	-	-	-	-	59.90
RHH	-	-	-	10.65	4.44	-	-	-	-	15.10
GOV	-	-	-	-	-	15.00	5.00	-	-	20.00
INV	-	-	-	-	-	15.00	5.00	-	-	
	65.00	110.00	70.00	35.00	40.00	43.00	12.00	15.00	25.00	

Simple RAS Example

- Back to Step 2. Sum the rows in the new table, and continue until the rows and columns converge to an acceptable distance.

Simple RAS Example

- A couple things to note from this example.
  - For each row i, the r_i value is the accumulated value of r_{i1} x r_{i2} x r_{i3} … r_{it}. The same applies for the s_j value.
  - As you can see the individual values of LVA are not those that we started with in the balanced table. We have changed the values of RHH as well. To avoid this, we could have subtracted out the values of RHH before starting the algorithm, both from the table and row-column sums, and added them back in at the end of the procedure.
  - Iterative solutions to the RAS algorithm are quite tedious to do by hand; I will be merciful and not ask you to do one.

RAS Notes

- In inter-industry tables r and s do have an economic interpretation (UNSD, 1999), though perhaps not an economic basis:
  - r – substitution – for instance, product i has been replaced by, or used as a substitute for, other products
  - s – fabrication – for instance, industry j uses less inputs
- The RAS procedure assumes uniform substitution and fabrication effects, e.g., in the latter case that commodity i decreases as an input into all industries.

RAS Notes

- RAS can also be applied to non-square I/O tables. The basic approach is the same. The initial matrix is first scaled by gross outputs and subsequently by total intermediate use and value added coefficients. This approach does assume that final demand and value added coefficients are known (in most instances, that you are updating an I/O table). See UNSD (1999) for an overview of this approach.

RAS Limitations

- The most significant shortcoming to RAS is that it is not particularly well-suited to situations in which the SAM compiler has incomplete knowledge of row and column sums, or where the prior SAM is inconsistent.
- Because of their greater flexibility and often efficiency, cross-entropy methods are increasingly preferred to RAS for SAM updating and balancing.

Cross-Entropy Methods

- Cross-entropy methods are an extension of the application of maximum entropy methods to economic accounts.
- In the SAM context, the procedure minimizes the additional information brought into a new SAM vis-à-vis a prior SAM by minimizing the "cross-entropy distance" between the new SAM and the prior SAM.

CE Methods: Roots

- Cross-entropy methods are rooted in the classic Information Theory problem of estimating posterior probabilities (p_1, p_2, p_3,…, p_n) of some series of events (E_1, E_2, E_3,…, E_n) occurring, given new information and prior probabilities (q_1, q_2, q_3,…, q_n).
- For E_1 the new information is equal to $-\ln p_1$, but the additional information provided by p_1 is

$$-(\ln p_1 - \ln q_1) = -\ln(p_1/q_1)$$

CE Methods: Roots

- The expected value of new information is

$$-I(p:q) = -\sum_{i}^{n} p_i \, ln \frac{p_i}{q_i}$$

where I(p:q) is a measure of the "cross entropy" (Kullback-Leibler, 1951) distance between the probability distributions p and q.

CE Application to SAMs

- Golan, Judge, and Robinson (1994) were the first to apply this cross-entropy approach to estimating economic accounts, by taking the above framework and turning it into the constrained minimization problem:

$$\min \left[\sum_i \sum_j A_{ij} \, ln \frac{A_{ij}}{A_{ij}{}^0} \right]$$

where A_{ij} is the new SAM and $A_{ij}{}^0$ is the prior SAM.

Deterministic CE

- This is often rewritten as

$$\min\left[\sum_i \sum_j A_{ij}\, lnA_{ij} - \sum_i \sum_j A_{ij}\, lnA_{ij}{}^0\right]$$

with three primary constraints:

1) $$\sum_i a_{ij} = 1$$

2) $$\sum_j a_{ij} Y_j = Y_i$$

3) $$0 < a_{ij} < 1$$

CE Notes

- The CE problem has no closed form solution and must be solved numerically.
- Excel does not fare particularly well with CE; CE is typically implemented in GAMS.
- $x\ln x = 0$ if $x = 0$, so a small upward adjustment (e.g., 0.0001) is typically made to the values in the CE equation (see the Robinson and El-Said papers).
- Note that it is the distance between SAM A matrices that is being minimized, not the distances between SAMs per se.

CE Notes

- The CE procedure uses logs, which means that negative values in the SAM can derail the procedure. The typical strategy for removing negative values is to “flip” them, i.e., set the negative value to zero and add a corresponding positive value in the appropriate row or column to keep rows and columns balanced (turning a negative expenditure into a positive payment).
- For instance, if T_{ij} is -5, set it to 0 and add 5 to T_{ji}.

CE vs. RAS

- The advantage of the cross-entropy approach is that we can add any number constraints into the minimization problem (e.g., information on output, government revenue and expenditures, value added, etc.).
- Whereas with RAS we need to know both row and column sums, with CE row and column sums are just one possible source of information.

Other Balancing Techniques

- There are a number of other balancing techniques (including other constrained optimization techniques).
- For an overview of balancing approaches other than RAS or CE, see Fofana et al., 2005.

The RAS Procedure

Let $\mathbf{R}_0$ be a known, initial matrix of transactions and let **R** be the unobservable transaction matrix for the year we desire to estimate. Let **p** be a vector whose elements are the ratios of desired period prices to initial period prices. Let $\langle \mathbf{z} \rangle$ denote the diagonal matrix having vector **z** on its main diagonal. The **R** matrix in desired period prices then takes the form:

$$\mathbf{R} = \langle \mathbf{p} \rangle \mathbf{R}_0 \langle \mathbf{p} \rangle^{-1}$$

The next step is to calculate a column vector of intermediate outputs for the desired year as the difference between gross outputs and final demands. Stone and Brown (1965) denote this vector **u**. The row vector **v** of intermediate inputs for the desired year is the difference between gross outputs and value added.

RAS: continued

The following constraints must be satisfied:

$$\mathbf{Ri} = \mathbf{u}$$

$$\mathbf{i'R} = \mathbf{v}$$

where **i** is the conformable unit column vector. The first equation states that the rows of the new transaction matrix must sum to the observed row totals. The second equation states that the columns must sum to the observed column totals.

RAS: continued

The problem is then to adjust **R** to obtain an estimate of **R**. The RAS algorithm proceeds as follows:

Step 0 (Initialization): Set $k = 0$ and $\mathbf{R}_k = \mathbf{R}$.

Step 1 (Row Scaling):

Define $\mathbf{r}_k = \langle\mathbf{u}\rangle(\mathbf{R}_k\mathbf{i})^{-1}$

and update $\mathbf{R}_k$ as $\mathbf{R}^* \square \langle\mathbf{r}_\mathbf{k}\rangle\mathbf{R}_\mathbf{k}$

Step 2 (Column Scaling):

Define $\square_k = (\mathbf{i}'\mathbf{R}^*)^{-1}\langle\mathbf{v}\rangle$

and define $\mathbf{R}_{k+1}$ by $\mathbf{R}_{k+1} = \mathbf{R}^*\langle\square_k\rangle$

Step 3 : Replace $k \square k + 1$ and return to Step 1.

Conclusions

- SAMs are critically important (consistent) data tools
- While they must be consistent with macro information, their biggest virtue is detail.
  - In most cases, indirect effects of economic policy outweigh direct ones, but these are often difficult to ascertain.
  - Data development for SAMs should be correspondingly ambitious.
- Overall goal: Improve visibility for policy makers about the detailed incidence of economic decisions and external events.

DISCUSSION