Technical Documentation

A Prototype CGE Model for Mongolia

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This note presents the technical details of a prototype dynamic CGE model of Mongolia. The prototype has some key features for assessing structural and poverty impacts:

- Labor markets disaggregated by skill level
- Land and capital markets disaggregated by type of capital/land
- A production structure which differentiates the substitutability of unskilled labor on the one hand, and skilled labor and capital on the other hand
- Differentiation of production of like-goods (e.g. small- and large-scale farms, or public versus private production)
- Detailed income distribution
- Intra-household transfers (e.g. urban to rural), transfers from government, and remittances
- Multiple households
- A tiered structure of trade (differentiating across various trading partners)
- Possibility of influencing export prices
- Internal domestic trade and transport margins
- Various potential factor mobility assumptions

The rest of the document proceeds to describe all of the model details using the standard circular flow description of the economy. It starts with production (*P*), income distribution (*Y*), demand (*D*), trade (*T*), domestic trade and transport margins (*M*), goods market equilibrium (*E*), macro closure (*C*), factor market equilibrium (*F*), macroeconomic identities (*I*), and growth (*G*).

Table 1 describes the indices used in the equations. Note that the model differentiates between production activities, denoted by the index *i*, and commodities, denoted by the index *k*. In many models, the two will overlap exactly. However, this differentiation allows for the same commodity to be produced by one or more sectors, and to differentiate these commodities by source of production. For example, it could be used in a model of economies in transition where commodities produced by the public sector have a different cost structure than commodities produced by the private sector, and the commodities themselves could be differentiated by consumers.[1] Another example, could be small- versus large-scale agricultural producers.

[1] The model allows for perfect substitution, in which case consumers are indifferent regarding who produces the good. An example might be electricity.

Table 1: Indices used in the model

i	Production activities
k	Commodities
l	Labor skills
ul	Unskilled labor
sl	Skilled labor[a]
kt	Capital types
lt	Land types
e	Corporations
h	Households
f	Final demand accounts[b]
m	Trade and transport margin accounts[c]
r	Trading partners

Notes: a. The unskilled and skilled labor indices, *ul* and *sl*, are subsets of *l*, and their union composes the set indexed by *l*.
b. The standard final demand accounts are '*Gov*' for government current expenditures, '*ZIp*' for private investment, '*ZIg*' for public investment, '*TMG*' for international export of trade and transport services, and '*DST*' for changes in stocks.
c. The standard trade and transport margin accounts are '*D*' for domestic goods, '*M*' for imported goods, and '*X*' for exported goods.

Production

Production, like in most CGE models, relies on the substitution relations across factors of production and intermediate goods. The simplest production structure has a single constant-elasticity-of-substitution (CES) relation between capital and labor, with intermediate goods being used in fixed proportion to output. In the production structure described below, there are multiple types of capital, land and labor, and they are combined in a nested-CES structure which is intended to represent the various substitution possibilities across these different factors of production. Typically, intermediate goods will enter in fixed proportion to output, though at the aggregate level, the model allows for a degree of substitutability between aggregate intermediate demand and value added.[2] The decomposition of value added has several components (see *figure* 1 for a representation of the multiple nests). First, land is assumed to be a substitute for an aggregate capital labor bundle.[3] The latter is then decomposed into unskilled labor on the one hand, and skilled labor cum capital on the other hand. This conforms to recent observations which suggest that capital and skilled labor are complements which can substitute for unskilled labor. The four aggregate factors—unskilled and skilled labor, land and capital, are decomposed by type in a final CES nest.

Top-level nest and producer price

The top-level nest has output, *XP*, produced as a combination of value added, *VA*, and an aggregate demand for goods and non-factor services, *ND*. In most cases, the substitution elasticity will be assumed to be zero, in which case the top-level CES nest is a fixed-coefficient Leontief production function. Equations (P-1) and (P-2) represent the optimal demand conditions for the generic CES

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2 Deviations from this structure might include isolating some key inputs, for example energy, or agricultural chemicals in the case of crops, and feed in the case of livestock.

3 In some sectors the model also allows for a sector-specific factor of production, for example, coal mining and oil production require reserves which cannot be used for any other activity. In this case, the nesting follows the same general structure as depicted in Figure 1.

production function, where *PND* is the price of the *ND* bundle, *PVA* is the aggregate price of value added, *PX* is the unit cost of production, and σ^p is the substitution elasticity. If the latter is zero, both *ND* and *VA* are used in fixed proportions to output, irrespective of relative prices. Equation (P-3) represents the unit cost function, *PX*. It is derived from the CES dual price formula. The model assumes constant-returns-to-scale and perfect competition in all sectors. Hence, the producer price, *PP*, is equal to the unit cost, adjusted for a producer tax/subsidy, τ^p, equation (P-4).

$$ND_i = \alpha_i^{nd}\left(\frac{PX_i}{PND_i}\right)^{\sigma_i^p} XP_i \qquad \text{(P-1)}$$

$$VA_i = \alpha_i^{va}\left(\frac{PX_i}{PVA_i}\right)^{\sigma_i^p} XP_i \qquad \text{(P-2)}$$

$$PX_i = \left[\alpha_i^{nd} PND_i^{1-\sigma_i^p} + \alpha_i^{va} PVA_i^{1-\sigma_i^p}\right]^{1/(1-\sigma_i^p)} \qquad \text{(P-3)}$$

$$PP_i = \left(1+\tau_i^p\right) PX_i \qquad \text{(P-4)}$$

Second-level production nests

The second-level nest has two branches. The first decomposes aggregate intermediate demand, *ND*, into sectoral demand for goods and services, *XAp*. The model explicitly assumes a Leontief structure. Thus equation (P-5) describes the demand for good *k* by sector *j*, where the coefficient *a* represents the proportion between *XAp* and *ND*. The price of the *ND* bundle, *PND*, is the weighted average of the price of goods and services, *PA*, using the technology coefficients as weights, equation (P-6). The so-called Armington price is multiplied by a sector and commodity specific indirect tax, τ^{cp}.

$$XAp_{k,j} = a_{k,j} ND_j \qquad \text{(P-5)}$$

$$PND_j = \sum_k a_{k,j}\left(1+\tau_{k,j}^{cp}\right) PA_k \qquad \text{(P-6)}$$

The second branch decomposes the aggregate value added bundle, *VA*, into three components: aggregate demand for capital and labor, *KL*, aggregate land demand, TT^d, and a sector-specific resource, *NR*,[4] see equations (P-7) through (P-9). The relevant component prices are *PKL*, *PTT* and *PR*, respectively, and the substitution elasticity is given by σ^v. Equation (P-9) allows for the possibility of factor productivity changes as represented by the λ parameter. The price of value added, *PVA*, is the CES aggregation of the three component prices, as defined by equation (P-10).

4 The latter will typically be zero in most sectors.

$$KL_i = \alpha_i^{kl}\left(\frac{PVA_i}{PKL_i}\right)^{\sigma_i^v} VA_i \qquad \text{(P-7)}$$

$$TT_i^d = \alpha_i^{tt}\left(\frac{PVA_i}{PTT_i}\right)^{\sigma_i^v} VA_i \qquad \text{(P-8)}$$

$$NR_i^d = \alpha_i^{nr}\left(\lambda_i^{nr}\right)^{\sigma_i^v-1}\left(\frac{PVA_i}{PR_i}\right)^{\sigma_i^v} VA_i \qquad \text{(P-9)}$$

$$PVA_i = \left[\alpha_i^{kl} PKL_i^{1-\sigma_i^v} + \alpha_i^{tt} PTT_i^{1-\sigma_i^v} + \alpha_i^{nr}\left(\frac{PR_i}{\lambda_i^{nr}}\right)^{1-\sigma_i^v}\right]^{1/(1-\sigma_i^v)} \qquad \text{(P-10)}$$

Third-level production nest

The third-level nest decomposes the aggregate capital-labor bundle, *KL*, into two components. The first is the aggregate demand for unskilled labor, *UL*, with an associated price of *PUL*. The second is a bundle composed of skilled labor and capital, *KSK*, with a price of *PKSK*. Equations (P-11) and (P-12) reflect the standard CES optimality conditions for the demand for these two components, with a substitution elasticity given by σ^{kl}. The price of capital-labor bundle, *PKL*, is defined in equation (P-13).

$$UL_i = \alpha_i^{u}\left(\frac{PKL_i}{PUL_i}\right)^{\sigma_i^{kl}} KL_i \qquad \text{(P-11)}$$

$$KSK_i = \alpha_i^{ksk}\left(\frac{PKL_i}{PKSK_i}\right)^{\sigma_i^{kl}} KL_i \qquad \text{(P-12)}$$

$$PKL_i = \left[\alpha_i^{u} PUL_i^{1-\sigma_i^{kl}} + \alpha_i^{ksk} PKSK_i^{1-\sigma_i^{kl}}\right]^{1/(1-\sigma_i^{kl})} \qquad \text{(P-13)}$$

Fourth-level production nest

The fourth-level nest decomposes the capital-skilled labor bundle into a capital component, KT^d, and a skilled labor component, *SKL*. Equations (P-14) and (P-15) represent the optimality conditions where the relevant component prices are *PKT* and *PSKL*, and the substitution elasticity is given by σ^{ks}. Equation (P-16) determines the price of the *KSK* bundle, *PKSK*.

$$SKL_i = \alpha_i^s \left(\frac{PKSK_i}{PSKL_i} \right)^{\sigma_i^{ks}} KSK_i \qquad \text{(P-14)}$$

$$KT_i^d = \alpha_i^{kt} \left(\frac{PKSK_i}{PKT_i} \right)^{\sigma_i^{ks}} KSK_i \qquad \text{(P-15)}$$

$$PKSK_i = \left[\alpha_i^s PSKL_i^{1-\sigma_i^{ks}} + \alpha_i^{kt} PKT_i^{1-\sigma_i^{ks}} \right]^{1/(1-\sigma_i^{ks})} \qquad \text{(P-16)}$$

Demand for labor by sector and skill

Equations (P-17) and (P-18) decompose the demands for aggregate unskilled and skilled labor, respectively, across their different components. The variable L^d represents labor demand in sector i for labor of skill level l. The relevant wage is given by W which is allowed to be both sector and skill-specific. The respective cross-skill substitution elasticities are σ^u and σ^s. Both equations (P-17) and (P-18) incorporate sector and skill specific labor productivity, represented by the variable λ^l. The aggregate unskilled and skilled price indices are determined in equations (P-19) and (P-20), respectively *PUL* and *PSKL*.

$$L_{i,ul}^d = \alpha_{i,ul}^l \left(\lambda_{i,ul}^l \right)^{\sigma_i^u - 1} \left(\frac{PUL_i}{W_{i,ul}} \right)^{\sigma_i^u} UL_i \qquad \text{for } ul \in \{\text{Unskilled labor}\} \qquad \text{(P-17)}$$

$$L_{i,sl}^d = \alpha_{i,sl}^l \left(\lambda_{i,sl}^l \right)^{\sigma_i^s - 1} \left(\frac{PSKL_i}{W_{i,sl}} \right)^{\sigma_i^s} SKL_i \qquad \text{for } sl \in \{\text{Skilled labor}\} \qquad \text{(P-18)}$$

$$PUL_i = \left[\sum_{ul \in \{\text{Unskilled labor}\}} \alpha_{i,ul}^l \left(\frac{W_{i,ul}}{\lambda_{i,ul}^l} \right)^{1-\sigma_i^u} \right]^{1/(1-\sigma_i^u)} \qquad \text{(P-19)}$$

$$PSKL_i = \left[\sum_{sl \in \{\text{Skilled labor}\}} \alpha_{i,sl}^l \left(\frac{W_{i,sl}}{\lambda_{i,sl}^l} \right)^{1-\sigma_i^s} \right]^{1/(1-\sigma_i^s)} \qquad \text{(P-20)}$$

The aggregate land and capital bundles, KT^d and TT^d respectively, are disaggregated across types, leading to type- and sector-specific capital and land demand, K^d and T^d. The decomposition is represented in equations (P-21) and (P-23), where the respective prices are R and PT which are both type- and sector-specific. The equations also incorporate productivity factors. Equations (P-22) and (P-24) represent the price indices for aggregate capital and land, respectively *PKT* and *PTT*.

Demand for capital and land across types

$$K_{i,kt}^{d} = \alpha_{i,kt}^{k}\left(\lambda_{i,kt}^{k}\right)^{\sigma_i^k - 1}\left(\frac{PKT_i}{R_{i,kt}}\right)^{\sigma_i^k} KT_i^d \tag{P-21}$$

$$PKT_i = \left[\sum_{kt}\alpha_{i,kt}^{k}\left(\frac{R_{i,kt}}{\lambda_{i,kt}^{k}}\right)^{1-\sigma_i^k}\right]^{1/(1-\sigma_i^k)} \tag{P-22}$$

$$T_{i,lt}^{d} = \alpha_{i,lt}^{t}\left(\lambda_{i,lt}^{t}\right)^{\sigma_i^t - 1}\left(\frac{PTT_i}{PT_{i,lt}}\right)^{\sigma_i^t} TT_i^d \tag{P-23}$$

$$PTT_i = \left[\sum_{lt}\alpha_{i,lt}^{k}\left(\frac{PT_{i,lt}}{\lambda_{i,lt}^{k}}\right)^{1-\sigma_i^t}\right]^{1/(1-\sigma_i^t)} \tag{P-24}$$

Commodity aggregation

Each activity produces a single commodity, *XP*, indexed by *i*. Consumption goods, indexed by *k*, are a combination of one or more produced goods. Aggregate domestic supply of good *k*, *X*, is a CES combination of one or more produced goods *i*. In many cases, the CES aggregate is of a single commodity, i.e. there is a one-to-one mapping between a consumed good and its relevant production. There are cases, however, where it is useful to have consumed goods be an aggregation of produced goods, for example when combining similar goods with different production characteristics (e.g. public versus private, commercial versus small-scale, etc.) Equation (P-25) represents the optimality condition of the aggregation of produced goods into commodities. The producer price is *PP*, and the price of the aggregate supply is *P*. The degree of substitutability across produced commodities is σ^c. Equation (P-26) determines the aggregate supply price, *P*. The model allows for perfect substitutability, in which case the law of one price holds and the produced commodities are simply aggregated to form aggregate output.[5]

[5] Electricity is a good example of a homogeneous output but which could be produced by very different production technologies, e.g. hydro-electric, nuclear, thermal, etc.

$$\begin{cases} XP_i = \alpha_{i,k}^c \left(\dfrac{P_k}{PP_i} \right)^{\sigma_k^c} X_k & \text{if} \quad \sigma_k^c \neq \infty \\ PP_i = P_k & \text{if} \quad \sigma_k^c = \infty \end{cases} \qquad \text{(P-25)}$$

$$\begin{cases} P_k = \left[\sum_{i \in K} \alpha_{i,k}^c PP_i^{1-\sigma_k^c} \right]^{1/(1-\sigma_k^c)} & \text{if} \quad \sigma_k^c \neq \infty \\ X_k = \sum_{i \in K} XP_i & \text{if} \quad \sigma_k^c = \infty \end{cases} \qquad \text{(P-26)}$$

Income distribution

The prototype model has a rich menu of income distribution channels—factor income and intra-household, government and foreign transfers (i.e. remittances). The prototype also includes corporations used as a pass-through account for channeling operating surplus.

Factor income

There are four broad factors—a sector specific resource, land, labor and capital—the latter three which can be sub-divided into various types. Equations (Y-1) through (Y-3) determine aggregate net-income from labor, *LY*, capital, *KY*, and land, *TY*, each indexed by its sub-types. The fourth equation determines aggregate income from the sector-specific resource. These are net incomes because the model incorporates factor taxes designated by τ^{fl}, τ^{fk}, τ^{ft} and τ^{fr} respectively.[6]

$$LY_l = \sum_i \frac{W_{i,l} L_{i,l}^d}{1 + \tau_{i,l}^{fl}} \qquad \text{(Y-1)}$$

$$KY_{kt} = \sum_i \frac{R_{i,kt} K_{i,kt}^d}{1 + \tau_{i,kt}^{fk}} \qquad \text{(Y-2)}$$

$$TY_{lt} = \sum_i \frac{PT_{i,lt} T_{i,lt}^d}{1 + \tau_{i,lt}^{ft}} \qquad \text{(Y-3)}$$

$$RY = \sum_i \frac{PR_i R_i^d}{1 + \tau_i^{fr}} \qquad \text{(Y-4)}$$

Distribution of profits

All of labor, land and sector-specific factor income is allocated directly to households.[7] Profits (aggregated with income from the sector-specific resouce), on the other hand, are distributed to

[6] The factor taxes are type- and sector-specific. Note as well that the relevant factor prices represent the perceived cost to employers, not the perceived remuneration of workers.

[7] Depending on the structure of the final SAM, land and or income from the sector-specific resource may also pass through corporate accounts.

three broad accounts, enterprises, households, and the rest of the world (ROW). Equation (Y-5) determines the level of profits distributed to enterprises, TR^E. Equation (Y-6) represents the level of profits distributed directly to households, TR^H. And, equation (Y-7) determines the level of factor income distributed abroad, TR^W. Note that the three share parameters, φ^E, φ^H, and φ^W sum to unity.

$$TR^E_{k,kt} = \varphi^E_{k,kt} KY_{kt} \qquad \text{(Y-5)}$$

$$TR^H_{k,kt} = \varphi^H_{k,kt} KY_{kt} \qquad \text{(Y-6)}$$

$$TR^W_{k,kt} = \varphi^W_{k,kt} KY_{kt} \qquad \text{(Y-7)}$$

Corporate income

Corporate income, TR^E, is split into four accounts. First, the government receives its share through the corporate income tax, κ^c. The residual is split into three: retained earnings, and income distributed to households and the rest of the world. Equation (Y-8) determines corporate income of enterprise e, CY. It is the sum, over possible capital types, of shares of distributed profits (to corporations).[8] Equation (Y-9) determines retained earnings, i.e. corporate savings, S^c, where the rate of retained earnings is given by s^c. Equations (Y-10) and (Y-11) determine the overall transfers to households and to ROW. Note that the two share parameters, φ^H and φ^W, and the retained earnings rate, s^c, sum to unity.

$$CY_e = \sum_{kt} \varphi^e_{kt,e} TR^E_{k,kt} \qquad \text{(Y-8)}$$

$$S^c_e = s^c_e \left(1 - \kappa^c_e\right) CY_e \qquad \text{(Y-9)}$$

$$TR^H_{c,e} = \varphi^H_{c,e} \left(1 - \kappa^c_e\right) CY_e \qquad \text{(Y-10)}$$

$$TR^W_{c,e} = \varphi^W_{c,e} \left(1 - \kappa^c_e\right) CY_e \qquad \text{(Y-11)}$$

Household income

Aggregate household income, YH, is composed of eight elements: labor, land and sector-specific factor remuneration, distributed capital income and corporate profits, transfers from government and households, and foreign remittances, equation (Y-12).[9] Government transfers, in the standard closure, are fixed in real terms and are multiplied by an appropriate price index to preserve model homogeneity. Remittances, are fixed in international currency terms, and are multiplied by the exchange rate, ER, to convert them into local currency terms.[10]

8 The share parameters, φ^e, sum to unity.

9 All share parameters within the summation signs sum to unity.

10 *ER* measures the value of local currency in terms of the international currency.

$$YH_h = \underbrace{\sum_l \varphi^h_{l,l} LY_l}_{\text{Labor}} + \underbrace{\sum_{kt} \varphi^h_{kt,h} TR^H_{k,kt}}_{\text{Capital}} + \underbrace{\sum_{lt} \varphi^h_{lt,h} TY_{lt}}_{\text{Land}} + \underbrace{\varphi^h_{nr,h} RY}_{\text{Sector-specific factor}}$$

$$+ \underbrace{+\sum_e \varphi^h_{e,h} TR^H_{c,e}}_{\text{Enterprise}} + \underbrace{PLEV.TR^h_{g,h}}_{\text{Transfers from government}} + \underbrace{\sum_{h'} TR^h_{h,h'}}_{\text{Intra-household transfers}} + \underbrace{ER\sum_r TR^h_{r,h}}_{\text{Foreign remittances}} \qquad \text{(Y-12)}$$

$$YD_h = \left(1 - \lambda^h \kappa^h_h\right) YH_h - TR^H_h \qquad \text{(Y-13)}$$

$$TR^H_h = \varphi^H_{h,h}\left(1 - \lambda^h \kappa^h_h\right) YH_h \qquad \text{(Y-14)}$$

$$TR^h_{h,h'} = \varphi^h_{h,h'} TR^H_h \qquad \text{(Y-15)}$$

$$TR^w_{h,r} = \varphi^w_{h,r} TR^H_h \qquad \text{(Y-16)}$$

Disposable income, *YD*, is equal to after-tax income, less household transfers, equation (Y-13), where the household tax rate is κ^h. It is multiplied by an adjustment factor, λ^h, which is used for model closure. In the standard closure, government savings (or deficit), is held fixed, and the household tax schedule adjusts (uniformly) to achieve the given government fiscal balance. In other words, under this closure rule, the relative tax rates across households remain constant.[11] Aggregate household transfers, TR^H, is a share of after tax income, equation (Y-14). This is transferred to individual households and abroad, respectively TR^h and TR^w, using constant share equations, (Y-15) and (Y-16).

Domestic final demand

Domestic final demand is composed of two broad agents—households and other domestic final demand. The model incorporates multiple households. Household demand has a uniform specification, however, with household-specific expenditure parameters. The other domestic final demand categories, in the standard model, include government current expenditures, *Gov*, private and public investment expenditures, *ZIp* and *ZIg*, exports of international trade and transport services, *TMG*, and changes in stocks, *DST*. The other domestic final demand categories, indexed by *f*, are also assumed to have a uniform expenditure function, but with agent-specific expenditure parameters. Demand at the top-level, reflects demand for the Armington good. The latter are added up across all activities in the economy and split into domestic and import components at the national level.[12]

Household expenditures

Households have a tiered demand structure, see figure 2. At the top-level, households save a constant share of disposable income, with the savings rate given by s^h. At the next level, residual income is allocated across goods and services, *XAc*, using the linear expenditure system (LES).[13]

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[11] An alternative would be to use an additive factor, which would adjust the average tax rates, not the marginal tax rates.

[12] There are few SAMs, which would allow for agent-specific Armington behavior.

[13] This class of models often uses the so-called extended linear expenditure system, which integrates household savings directly in the utility function. However, this can create calibration problems for households without savings.

Equation (D-1) represents the LES demand function. Household consumption is the sum of two components. The first, θ, is referred to as the subsistence minimum. The second is a share of real supernumerary income. Supernumerary income is equal to residual disposable income, subtracting savings and aggregate expenditures on the subsistence minima from disposable income. The next level, undertaken at the national level, is the decomposition of Armington demand, *XAc*, into its domestic and import components, see below. Equation (D-2) determines household saving, S^h, by residual. The consumer price index, *CPI*, is defined in equation (D-3). Note that the consumer price is equal to the economy-wide Armington price, *PA*, multiplied by a household and commodity specific ad valorem tax, τ^{cc}.

$$XAc_{k,h} = \theta_{k,h} + \frac{\mu_{k,h}}{(1+\tau^{cc}_{k,h})PA_k}\left((1-s^h_h)YD_h - \sum_{k'}(1+\tau^{cc}_{k',h})PA_{k'}\theta_{k'}\right) \quad \text{(D-1)}$$

$$S^h_h = YD_h - \sum_k (1+\tau^{cc}_{k,h})PA_k XAc_{k,h} \quad \text{(D-2)}$$

$$CPI_h = \frac{\sum_k (1+\tau^{cc}_{k,h})PA_k XAc_{k,h,0}}{\sum_k (1+\tau^{cc}_{k,h,0})PA_{k,0} XAc_{k,h,0}} \quad \text{(D-3)}$$

Other domestic demand accounts

The other domestic final demand accounts all use a CES expenditure function (with the option of having fixed volume or value expenditure shares with an elasticity of 0 or 1, respectively). Equation (D-4) determines the expenditure share on goods and services, *XAf*. Equation (D-5) defines the expenditure price index, *PF*. And equation (D-6) defines the value of expenditures, *YF*. Model closure is discussed below.

$$XAf_{k,f} = \alpha^f_{k,f}\left(\frac{PF_f}{(1+\tau^{cf}_{k,f})PA_k}\right)^{\sigma^f_f} XF_f \quad \text{for } f \in \{\text{Other final demand}\} \quad \text{(D-4)}$$

$$PF_f = \left[\sum_k \alpha^f_f\left((1+\tau^{cf}_{k,f})PA_k\right)^{1-\sigma^f_f}\right]^{1/(1-\sigma^f_f)} \quad \text{for } f \in \{\text{Other final demand}\} \quad \text{(D-5)}$$

$$YF_f = PF_f XF_f \quad \text{for } f \in \{\text{Other final demand}\} \quad \text{(D-6)}$$

Trade equations

This section discusses the modeling of trade. There are three sections—import demand, and export supply and demand. The first two use a tiered structure. Import demand is decomposed in two steps. The top tier disaggregates aggregate Armington demand into two components—demand for the domestically produced good and aggregate import demand. At the second tier, the aggregate import demand is allocated across trading partners. Both of these tiers assume that goods indexed by k are differentiated by region of origin, i.e. the so-called Armington assumption. A CES specification is used to model the degree of substitutability across regions of origin. The level of the elasticities will often be determined by the level of aggregation. Finely defined goods, such as wheat, would typically have a higher elasticity than more broadly defined goods, such as clothing. At the same time, non-price barriers may also inhibit the degree of substitutability, for example prohibitive transport barriers (inexistent or few transmission lines for electricity), or product and safety standards. Export supply is similarly modeled using a two-tiered constant-elasticity-of-transformation specification. This permits imperfect supply responses to changes in relative prices. Finally, the small-country assumption is relaxed for exports with the incorporation of export demand functions.

Top-level Armington nest

National demand for the Armington good, XA, is the sum of Armington demand over all domestic agents: intermediate demand, household and other domestic final demand, and demand generated by the internal trade and transport sector, $XAmg$, equation (T-1). Aggregate Armington demand is then allocated between domestic and import goods using a nested CES structure. Equation (T-2) represents demand for the domestically produced good, XD^d, where the top-level Armington elasticity is given by σ^m. Note that the price of the domestic good is equal to the producer price, PD, adjusted by the internal trade and transport margin, τ^{mg}. Demand for aggregate imports, XMT, is determined in equation (T-3). The price of aggregate imports is given by PMT.[14] The Armington price, PA, is defined in equation (T-4), using the familiar CES dual price aggregation formula.

$$XA_k = \sum_j XAp_{k,j} + \sum_h XAc_{k,h} + \sum_f XAf_{k,f} + \sum_m \sum_{k'} XAmg_{k,k',m} \qquad \text{(T-1)}$$

$$XD_k^d = \alpha_k^d \left(\frac{PA_k}{(1+\tau_{k,D}^{mg})\; PD_k} \right)^{\sigma_k^m} XA_k \qquad \text{(T-2)}$$

$$XMT_k = \alpha_k^m \left(\frac{PA_k}{PMT_k} \right)^{\sigma_k^m} XA_k \qquad \text{(T-3)}$$

$$PA_k = \left[\alpha_k^d \left((1+\tau_{k,D}^{mg})\; PD_k \right)^{1-\sigma_k^m} + \alpha_k^m PMT_k^{1-\sigma_k^m} \right]^{1/(1-\sigma_k^m)} \qquad \text{(T-4)}$$

Second-level Armington nest

At the second level, aggregate import demand, XMT, is allocated across trading partners using a CES specification. Equation (T-5) defines the domestic price of imports, PM.[15] It is equal to the world price (in international currency), WPM, multiplied by the exchange rate, and adjusted for by the import tariff, τ^m, i.e. PM represents the port-price of imports, tariff-inclusive. The tariff rate is

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[14] It includes the trade and transport margins, sales tax, and import tariffs.

[15] *PM* and *WPM* are indexed by both commodity, k, and trading partner, r.

both sector- and region of origin-specific. Equation (T-6) represents the import of commodity k from region r, XM, where the inter-regional substitution elasticity is given by σ^w. The relevant consumer price includes the internal trade and transport margin, τ^{mg}. The aggregate price of imports, PMT, is defined in equation (T-7).

$$PM_{k,r} = ER.WPM_{k,r}(1+\tau^m_{k,r}) \qquad \text{(T-5)}$$

$$XM_{k,r} = \alpha^w_{k,r}\left(\frac{PMT_k}{(1+\tau^{mg}_{k,M})\ PM_{k,r}}\right)^{\sigma^w_k} XMT_k \qquad \text{(T-6)}$$

$$PMT_k = \left[\sum_r \alpha^w_{k,r}\left((1+\tau^{mg}_{k,M})\ PM_{k,r}\right)^{1-\sigma^w_k}\right]^{1/(1-\sigma^w_k)} \qquad \text{(T-7)}$$

Top-level CET nest

Domestic production is allocated across markets using a nested CET specification. At the top nest, producers allocate production between the domestic market and aggregate exports. At the second nest, aggregate exports are allocated across trading partners. The model allows for perfect transformation, i.e. producers perceive no difference across markets. In this case, the law-of-one-price holds. Equation (T-8) represents the link between the domestic producer price, PE, and the world price, WPE. Export prices are both sector- and region-specific. The FOB price, WPE, includes domestic trade and transport margins, τ^{mg}[16], as well as export taxes/subsidies, τ^e. Equations (T-9) and (T-10) represent the CET optimality conditions. The first determines the share of domestic supply, X, allocated to the domestic market, XD^s. The second determines the supply of aggregate exports, XET. PET represents the price of aggregate export supply. The transformation elasticity is given by σ^x. The model allows for perfect transformation. In this case, the optimal supply conditions are replaced by the law-of-one price conditions. Equation (T-11) represents the CET aggregation function. In the case of finite transformation, it is replaced with its equivalent, the CET dual price aggregation function. In the case of infinite transformation, the primal aggregation function is used, where the two components are summed together since there is no product differentiation.

[16] Note that the domestic trade and transport margins are differentiated for three different goods: domestically produced goods sold to the domestic market, exported goods, and imported goods.

$$PE_{k,r}\left(1+\tau^{mg}_{k,X}\right)\left(1+\tau^{e}_{k,r}\right)=ER.WPE_{k,r} \qquad \text{(T-8)}$$

$$\begin{cases} XD^s_k = \gamma^d_k\left(\dfrac{PD_k}{P_k}\right)^{\sigma^x_k} X_k & \text{if} \quad \sigma^x_k \neq \infty \\ PD_k = P_k & \text{if} \quad \sigma^x_k = \infty \end{cases} \qquad \text{(T-9)}$$

$$\begin{cases} XET_k = \gamma^e_k\left(\dfrac{PET_k}{P_k}\right)^{\sigma^x_k} X_k & \text{if} \quad \sigma^x_k \neq \infty \\ PET_k = P_k & \text{if} \quad \sigma^x_k = \infty \end{cases} \qquad \text{(T-10)}$$

$$\begin{cases} P_k = \left[\gamma^d_k PD_k^{1+\sigma^x_k} + \gamma^e_k PET_k^{1+\sigma^x_k}\right]^{1/(1+\sigma^x_k)} & \text{if} \quad \sigma^x_k \neq \infty \\ X_k = XD^s_k + XET_k & \text{if} \quad \sigma^x_k = \infty \end{cases} \qquad \text{(T-11)}$$

Second-level CET nest

The second-level CET nest allocates aggregate export supply, *XET*, across the various export markets, *XE*. Equation (T-12) represents the optimal allocation decision, where σ^z is the transformation elasticity. Equation (T-13) represents the CET aggregation function, where again, the CET dual price formula is used to determine the aggregate export price, *PET*. As above, the model allows the transformation elasticity to be infinite.

$$\begin{cases} XE_{k,r} = \gamma^x_{k,r}\left(\dfrac{PE_{k,r}}{PET_k}\right)^{\sigma^z_k} XET_k & \text{if} \quad \sigma^z_k \neq \infty \\ PE_{k,r} = PET_k & \text{if} \quad \sigma^z_k = \infty \end{cases} \qquad \text{(T-12)}$$

$$\begin{cases} PET_k = \left[\displaystyle\sum_r \gamma^x_{k,r} PE_{k,r}^{1+\sigma^z_k}\right]^{1/(1+\sigma^z_k)} & \text{if} \quad \sigma^z_k \neq \infty \\ XET_k = \displaystyle\sum_r XE_{k,r} & \text{if} \quad \sigma^z_k = \infty \end{cases} \qquad \text{(T-13)}$$

Export demand

Export, *ED*, demand is specified using a constant elasticity function, equation (T-14). If the elasticity, η^e, is finite, demand decreases as the international price of exports, *WPE*, increases. The numerator contains an exogenous export price competitive index. If the latter increases relative to the domestic export price, market share of the domestic exporter would increase. The model allows for infinite demand elasticity. This represents the small-country assumption. In this case, the domestic price of exports (in international currency units) is constant. If the two CET elasticities are likewise infinite, then the domestic producer price is also equal to the world price of exports (adjusted for taxes and trade and transportation margins).

$$\begin{cases} ED_{k,r} = \alpha^e_{k,r}\left(\dfrac{\overline{WPE}_{k,r}}{WPE_{k,r}}\right)^{\eta^e_{k,r}} & \text{if} \quad \eta^e_{k,r} \neq \infty \\ WPE_{k,r} = \overline{WPE}_{k,r} & \text{if} \quad \eta^e_{k,r} = \infty \end{cases} \qquad \text{(T-14)}$$

Domestic trade and transportation margins

The marketing of each good—domestic, imports, and exports—is associated with a commodity specific trade margin.[17] Equations (M-1) through (M-3) define the revenues associated with the domestic trade and transport margins. Domestically produced goods sold domestically generate $Y^{mg}_{.,D}$. Imported goods generate $Y^{mg}_{.,M}$. And exported goods generate $Y^{mg}_{.,X}$. Equation (M-4) defines the volume of margin services. The production of the trade and transport services follows a Leontief technology. Equation (M-5) defines the demand for goods and services. In other words, to deliver commodity k' (in either sector D, M, or X) requires an input from commodity k, the level of which is fixed in proportions to the overall volume of delivering commodity k' in the economy, $XT^{mg}_{k'}$. Equation (M-6) is the expenditure deflator, $PT^{mg}_{k'}$, for individual trade margin activities.

$$YT^{mg}_{k,D} = \tau^{mg}_{k,D} PD_k XD^d_k \qquad \text{(M-1)}$$

$$YT^{mg}_{k,M} = \sum_r \tau^{mg}_{k,M} PM_{k,r} XM_{k,r} \qquad \text{(M-2)}$$

$$YT^{mg}_{k,X} = \sum_r \tau^{mg}_{k,X} PE_{k,r} XE_{k,r} \qquad \text{(M-3)}$$

$$XT^{mg}_{k,m} = YT^{mg}_{k,m} / PT^{mg}_{k,m} \qquad \text{(M-4)}$$

$$XAmg_{k,k',m} = \alpha^{mg}_{k,k',m} XT^{mg}_{k',m} \qquad \text{(M-5)}$$

$$PT^{mg}_{k',m} = \sum_k \alpha^{mg}_{k,k',m} PA_k \qquad \text{(M-6)}$$

Goods market equilibrium

There are three fundamental commodities in the model—domestic goods sold domestically, imports (by region of origin), and exports (by region of destination). All other goods are bundles (i.e. are defined using an aggregation function) and do not require supply/demand balance. The small-country assumption holds for imports, and therefore any import demand can be met by the rest of the world with no impact on the price of imports. Therefore, there is no explicit supply/demand equation for imports.[18] Equation (E-1) represents equilibrium on the domestic goods market, and

[17] The model does not include international trade and transport margins. A change in the latter could be simulated by a change in the relevant world price index, WPM or $\overline{WPE}$.

[18] One could rather easily add an import supply equation and an equilibrium condition.

essentially determines, PD, the producer price of the domestic good. Equation (E-2) defines the equilibrium condition on the export market. With a finite export demand elasticity, the equation determines WPE, the world price of exports. With an infinite export demand elasticity, the equation trivially equates export demand to the given export supply.

$$XD_k^d = XD_k^s \quad \text{(E-1)}$$

$$ED_{k,r} = XE_{k,r} \quad \text{(E-2)}$$

Macro closure

Macro closure involves determining the exogenous macro elements of the model. The standard closure rules are the following:

- Government fiscal balance is exogenous, achieved with an endogenous direct tax schedule
- Private investment is endogenous and is driven by available savings
- The volume of government current and investment expenditures is exogenous
- The volume of demand for international trade and transport services is exogenous
- The volume of stock changes is exogenous
- The trade balance (i.e. capital flows) is exogenous. The real exchange rate equilibrates the balance of payments.

These are detailed further below.

Government accounts

Equation (C-1) defines total government revenues, GY. There are 10 components: revenues from the production tax, sales tax, import tax, export tax, land, capital and wage tax, corporate and household direct taxes, and transfers from the rest of the world. Equation (C-2) defines the government's current expenditures, $GEXP$. It is the sum of three components: expenditures on goods and services, transfers to households, and transfers to ROW. Government savings (on current operations), S^g, is defined in equation (C-3), as the difference between revenues and current expenditures. Real government savings, RSg, is defined in equation (C-4). It is this latter which essentially determines the level of direct household taxation since RSg is exogenous in the standard closure.

$$
\begin{aligned}
GY \quad = \quad & \underbrace{\sum_k \sum_j \tau^{cp}_{k,j} PA_k XAp_{k,j}}_{\text{Sales tax on intermediate demand}} + \underbrace{\sum_k \sum_h \tau^{cc}_{k,h} PA_k XAc_{k,h}}_{\text{Sales tax on household demand}} + \underbrace{\sum_k \sum_f \tau^{cf}_{k,f} PA_k XAf_{k,f}}_{\text{Sales tax on other final demand}} \\
+ \quad & \underbrace{ER \sum_k \sum_r \tau^{m}_{k,r} WPM_{k,r} XM_{k,r}}_{\text{Import tariff revenues}} + \underbrace{\sum_k \sum_r \tau^{e}_{k,r} (1+\tau^{mg}_{k,X}) PE_{k,r} XE_{k,r}}_{\text{Export tax revenues}} \\
+ \quad & \underbrace{\sum_{lt} \sum_i \frac{\tau^{ft}_{i,lt} PT_{i,lt} T^d_{i,lt}}{1+\tau^{ft}_{i,lt}}}_{\text{Land tax}} + \underbrace{\sum_{kt} \sum_i \frac{\tau^{fk}_{i,kt} R_{i,kt} K^d_{i,kt}}{1+\tau^{fk}_{i,kt}}}_{\text{Capital tax}} + \underbrace{\sum_{l} \sum_i \frac{\tau^{fl}_{i,l} W_{i,l} L^d_{i,l}}{1+\tau^{fl}_{i,l}}}_{\text{Wage tax}} + \underbrace{\sum_i \frac{\tau^{fr}_{i} PR_i NR^d_{i}}{1+\tau^{fr}_{i}}}_{\text{Resource tax}} \qquad \text{(C-1)} \\
+ \quad & \underbrace{\sum_i \tau^p_i PX_i XP_i}_{\text{Production tax}} + \underbrace{\sum_e \kappa^c_e CY_e}_{\text{Corporate tax}} + \underbrace{\lambda^h \sum_h \kappa^h_h YH_h}_{\text{Income tax}} + \underbrace{ER \sum_r TR^g_{W,r}}_{\text{Transfers from ROW}}
\end{aligned}
$$

$$GEXP = YF_{Gov} + PLEV \sum_h TR^H_{g,h} + ER \sum_r TR^W_{g,r} \qquad \text{(C-2)}$$

$$S^g = GY - GEXP \qquad \text{(C-3)}$$

$$RSg = S^g / PLEV \qquad \text{(C-4)}$$

Investment and macro closure

Equation (C-5) defines the investment savings balance. In the standard closure, it determines the level of private investment since public investment and stock changes are exogenous. These three components are financed by aggregate savings defined over corporations, households, and the government, and adjusted by foreign savings. The latter is fixed (in international currency terms). Equations (C-6) through (C-9) define the exogenous volumes of public current and investment expenditures, exports of international trade and transport services and stock changes. The aggregate price level, *PLEV*, is the average absorption (Armington) price, equation (C-10). Equation (C-11) represents the balance of payments (in international currency terms). It can be shown to be redundant, and is dropped from the model specification.

$$YF_{ZIp} + YF_{ZIg} + YF_{DST} = \sum_e S_e^c + \sum_h S_h^h + S^g + ER.\sum_r S_r^f \quad \text{(C-5)}$$

$$XF_{Gov} = \overline{XF}_{Gov} \qquad \text{(C-6)}$$

$$XF_{ZIg} = \overline{XF}_{ZIg} \quad \text{(C-7)}$$

$$XF_{TMG} = \overline{XF}_{TMG} \qquad \text{(C-8)}$$

$$XF_{DST} = \overline{XF}_{DST} \qquad \text{(C-9)}$$

$$PLEV = \frac{\sum_k PA_k XA_{k,0}}{\sum_k PA_{k,0} XA_{k,0}} \qquad \text{(C-10)}$$

$$\begin{aligned} BoP \quad &= \quad \sum_r \sum_k WPE_{k,r} XE_{k,r} + YF_{TMG} + \sum_h TR_{W,h}^h + TR_W^g + S^f \\ &- \quad \sum_r \sum_k WPM_{k,r} XM_{k,r} - \frac{\sum_{kt} TR_{k,kt}^W + \sum_e TR_{c,e}^W + \sum_h TR_h^w}{ER} - TR_g^W \quad \text{(C-11)} \\ &\equiv \quad 0 \end{aligned}$$

Factor market equilibrium

The following sections describe the standard factor market equilibrium conditions.[19]

Labor markets

Labor markets are assumed to clear. Equation (F-1) sets aggregate demand, by skill-level, to aggregate supply, L^s. This equation determines the equilibrium wage, W^e.[20] Equation (F-2) equates sectoral wages to the equilibrium wage, i.e. the model assumes uniform wages across sectors.[21]

$$L_l^s = \sum_i L_{i.l}^d \qquad \text{(F-1)}$$

$$W_{i,l} = W_l^e \qquad \text{(F-2)}$$

[19] More detailed analysis may require more market segmentation, e.g. rural versus urban labor markets, though so of this segmentation can be picked up by the data itself.

[20] Market structure can emulate perfect market segmentation by an appropriate definition of labor skills. For example, unskilled rural labor can assume to be only employed in rural sectors, whereas unskilled urban labor is only employed in urban sectors. Perfect market segmentation, as modeled here, does not allow for migration.

[21] Quite a few alternatives could be used allowing for sector-specific wages, for example union wage bargaining models, efficiency wages, etc.

Capital market

Equilibrium on the capital market allows for both limiting cases—perfect capital mobility and perfect capital immobility, or any intermediate case. Aggregate capital, K^s, is allocated across sectors and type according to a nested CET system. At the top-level, the aggregate investor allocates capital across types, according to relative rates of return. Equation (F-3) determines the optimal supply decision, where TK^s is the supply of capital of type kt, with an average return of PTK. PK is the aggregate rate-of-return to capital. If the supply elasticity is infinite, the law-of-one-price holds. Equation (F-4) represents the top-level aggregation function, replaced by the CET dual price function in the case of a finite transformation elasticity. Perfect capital mobility is represented by setting ω^{kt} to infinity. Perfect immobility is modeled by setting the transformation elasticity to 0.

$$\begin{cases} TK_{kt}^{s} = \gamma_{kt}^{tks}\left(\dfrac{PTK_{kt}}{PK}\right)^{\omega^{kt}} K^{s} & \text{if} \quad \omega^{kt} \neq \infty \\ PTK_{kt} = PK & \text{if} \quad \omega^{kt} = \infty \end{cases} \qquad \text{(F-3)}$$

$$\begin{cases} PK = \left[\displaystyle\sum_{kt} \gamma_{kt}^{tks} PTK_{kt}^{1+\omega^{kt}}\right]^{1/(1+\omega^{kt})} & \text{if} \quad \omega^{kt} \neq \infty \\ K^{s} = \displaystyle\sum_{kt} TK_{kt}^{s} & \text{if} \quad \omega^{kt} = \infty \end{cases} \qquad \text{(F-4)}$$

At the second level, capital by type, TK^s, is allocated across sectors using another CET function. Equation (F-5) determines the optimal allocation of capital of type kt to sector i, K^s, where the transformation elasticity is ω^k. Equation (F-6) represents the CET aggregation function. The equilibrium return to capital, R, is determined by equation capital supply to demand, equation (F-7).[22]

$$\begin{cases} K_{i,kt}^{s} = \gamma_{i,kt}^{k}\left(\dfrac{R_{i,kt}}{PTK_{kt}}\right)^{\omega^{k}} TK_{kt}^{s} & \text{if} \quad \omega^{k} \neq \infty \\ R_{i,kt} = PTK_{kt} & \text{if} \quad \omega^{k} = \infty \end{cases} \qquad \text{(F-5)}$$

$$\begin{cases} PTK_{kt} = \left[\displaystyle\sum_{i} \gamma_{i,kt}^{k} R_{i,kt}^{1+\omega^{k}}\right]^{1/(1+\omega^{k})} & \text{if} \quad \omega^{k} \neq \infty \\ TK_{kt} = \displaystyle\sum_{i} K_{i,kt}^{s} & \text{if} \quad \omega^{k} = \infty \end{cases} \qquad \text{(F-6)}$$

$$K_{i,kt}^{d} = K_{i,kt}^{d} \qquad \text{(F-7)}$$

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[22] If the transformation elasticity is infinite, equation (F-5) determines the sector- and type-specific rate of return using the law-of-one price, and equation (F-7) trivially sets capital supply equal to capital demand.

Land market

Land market equilibrium is specified in an analogous way to the capital market with a tiered CET supply system. The first tier allocates total land across types. This could have a zero transformation elasticity if for example land used for rice production could not be used to produce other commodities. Their respective prices are *PLAND* and PTT^s.

$$\begin{cases} TT^s_{lt} = \gamma^{tts}_{lt}\left(\dfrac{PTT^s_{lt}}{PLAND}\right)^{\omega^{tl}} LAND & \text{if} \quad \omega^{tl} \neq \infty \\ PTT^s_{lt} = PLAND & \text{if} \quad \omega^{tl} = \infty \end{cases} \qquad \text{(F-8)}$$

$$\begin{cases} PLAND = \left[\sum_{lt} \gamma^{tts}_{lt}\left(PTT^s_{cl}\right)^{1+\omega^{tl}}\right]^{1/(1+\omega^{tl})} & \text{if} \quad \omega^{tl} \neq \infty \\ LAND = \sum_{lt} TT^s_{lt} & \text{if} \quad \omega^{tl} = \infty \end{cases} \qquad \text{(F-9)}$$

Equations (F-10) and (F-11) determine the optimality conditions at the second and final tier, determining land supply (by type and) by sector of use. Land market equilibrium is represented by equation (F-12).

$$\begin{cases} T^s_{i,lt} = \gamma^t_{i,lt}\left(\dfrac{PT_{i,lt}}{PTT^s_{lt}}\right)^{\omega^t_{lt}} TT^s_{lt} & \text{if} \quad \omega^t_{lt} \neq \infty \\ PT_{i,lt} = PTT^s_{lt} & \text{if} \quad \omega^t_{lt} = \infty \end{cases} \qquad \text{(F-10)}$$

$$\begin{cases} PTT^s_{lt} = \left[\sum_i \gamma^t_{i,lt} PT^{1+\omega^t}_{i,lt}\right]^{1/(1+\omega^t_{lt})} & \text{if} \quad \omega^t_{lt} \neq \infty \\ TT^s_{lt} = \sum_i T^s_{i,lt} & \text{if} \quad \omega^t_{lt} = \infty \end{cases} \qquad \text{(F-11)}$$

$$T^s_{i,lt} = T^d_{i,lt} \qquad \text{(F-12)}$$

Natural resource market

The market for natural resources differs from the others in the sense that there is no inter-sectoral mobility, i.e. this is a sector specific resource. There is therefore a sector specific supply curve (eventually flat).[23] Equation (F-13) describes the sector-specific supply function, or NR^s. Equation (F-14) then determines the equilibrium price, *PR*.

[23] More realistic models allow for kinked supply curves. It is typically easier to take resources out of production than to bring them online—the latter requiring new investments and/or new exploration. Thus a so-called down supply elasticity would be higher than a so-called up supply elasticity.

$$\begin{cases} NR_i^s = \gamma_i^{nr}\left(\dfrac{PR_i}{PLEV}\right)^{\omega^{nr}} & \text{if} \quad \omega^{nr} \neq \infty \\ PR_i = PLEV.PR_{i,0} & \text{if} \quad \omega^{nr} = \infty \end{cases} \qquad \text{(F-13)}$$

$$NR_i^d = NR_i^s \qquad \text{(F-14)}$$

Macroeconomic identities

The macroeconomic identities are not normally needed for the model specification, i.e. they could be calculated at the end of a simulation. In the case of dynamic scenarios, one or more of them could be used to calibrate dynamic parameters to a given set of exogenous assumptions. For example, the growth of GDP could be made exogenous. In this case, a growth parameter, typically a productivity factor, would be endogenous and set to target the given growth path of GDP.

Equations (I-1) and (I-2) define nominal and real GDP, respectively, at market prices. Equation (I-3) is the GDP at market price deflator. Similarly, equations (I-4) and (I-5) define nominal and real GDP at factor cost. Note that real GDP at factor cost is evaluated in efficiency units.[24] Equation (I-6) defines the GDP at factor cost deflator.

$$\begin{aligned} GDPMP \;=\; & \sum_k \sum_h (1+\tau_{k,h}^{cc}) PA_k XAc_{k,h} + \sum_k \sum_f (1+\tau_{k,f}^{cf}) PA_k XAf_{k,f} \\ + \; & ER \sum_k \sum_r WPE_{k,r} XE_{k,r} - \sum_k \sum_r PM_{k,r}(1+\tau_{k,M}^{mg})\; XM_{k,r} \end{aligned} \qquad \text{(I-1)}$$

$$\begin{aligned} RGDPMP \;=\; & \sum_k \sum_h (1+\tau_{k,c,0}^{cc}) PA_{k,0} XAc_{k,h} + \sum_k \sum_f (1+\tau_{k,f,0}^{cf}) PA_{k,0} XAf_{k,f} \\ + \; & ER_0 \sum_k \sum_r WPE_{k,r,0} XE_{k,r} - \sum_k \sum_r PM_{k,r,0}(1+\tau_{k,M,0}^{mg})\; XM_{k,r} \end{aligned} \qquad \text{(I-2)}$$

$$PGDPMP = GDPGMP / RGDPMP \qquad \text{(I-3)}$$

$$GDPFC = \sum_l \sum_i W_{i,l} L_{i,l}^d + \sum_{kt} \sum_i R_{i,kt} K_{i,kt}^d + \sum_{lt} \sum_i PT_{i,lt} T_{i,lt}^d + \sum_i PR_i NR_i^d \qquad \text{(I-4)}$$

$$\begin{aligned} RGDPFC \;=\; & \sum_l \sum_i W_{i,l,0} \lambda_{i,l}^l L_{i,l}^d + \sum_{kt} \sum_i R_{i,kt,0} \lambda_{i,kt}^k K_{i,kt}^d \\ + \; & \sum_{lt} \sum_i PT_{i,lt,0} \lambda_{i,lt}^t T_{i,lt}^d + \sum_i PR_{i,0} \lambda_i^r NR_i^d \end{aligned} \qquad \text{(I-5)}$$

$$PGDPFC = GDPGFC / RGDPFC \qquad \text{(I-6)}$$

---

24 So is nominal GDP at factor cost, but the efficiency factors cancel out in the equation since the nominal wage is divided by the efficiency factor to derive the efficiency wage.

Growth equations

In a simple dynamic framework, equation (G-1) defines the growth rate of GDP at market price. Equation (G-2) determines the growth rate of labor productivity. The growth rate has two components, a uniform factor applied in all sectors to all types of labor, γ^l. In defining a baseline, the growth rate of GDP is exogenous. In this case, equation (G-1) is used to calibrate the gl parameter. In policy simulations, gl is given, and equation (G-1) defines the growth rate of GDP. Other elements of simple dynamics include exogenous growth of labor supply, exogenous growth rates of capital and land productivity (typically 0), and investment driven capital accumulation, equation (G-3).[25]

$RGDPMP = (1+g^y)\ RGDPMP_{-1}$	(G-1)
$\lambda^l_{ip,l} = (1+\gamma^l+\chi^l_{ip,l})\ \lambda^l_{ip,l,-1}$	(G-2)
$K^s = (1-\delta)K^s_{-1} + XF_{ZIp,-1}$	(G-3)

25 Note that public investment, in this version of the model, has no impact on production technology.

Figure 1: Nested structure of production

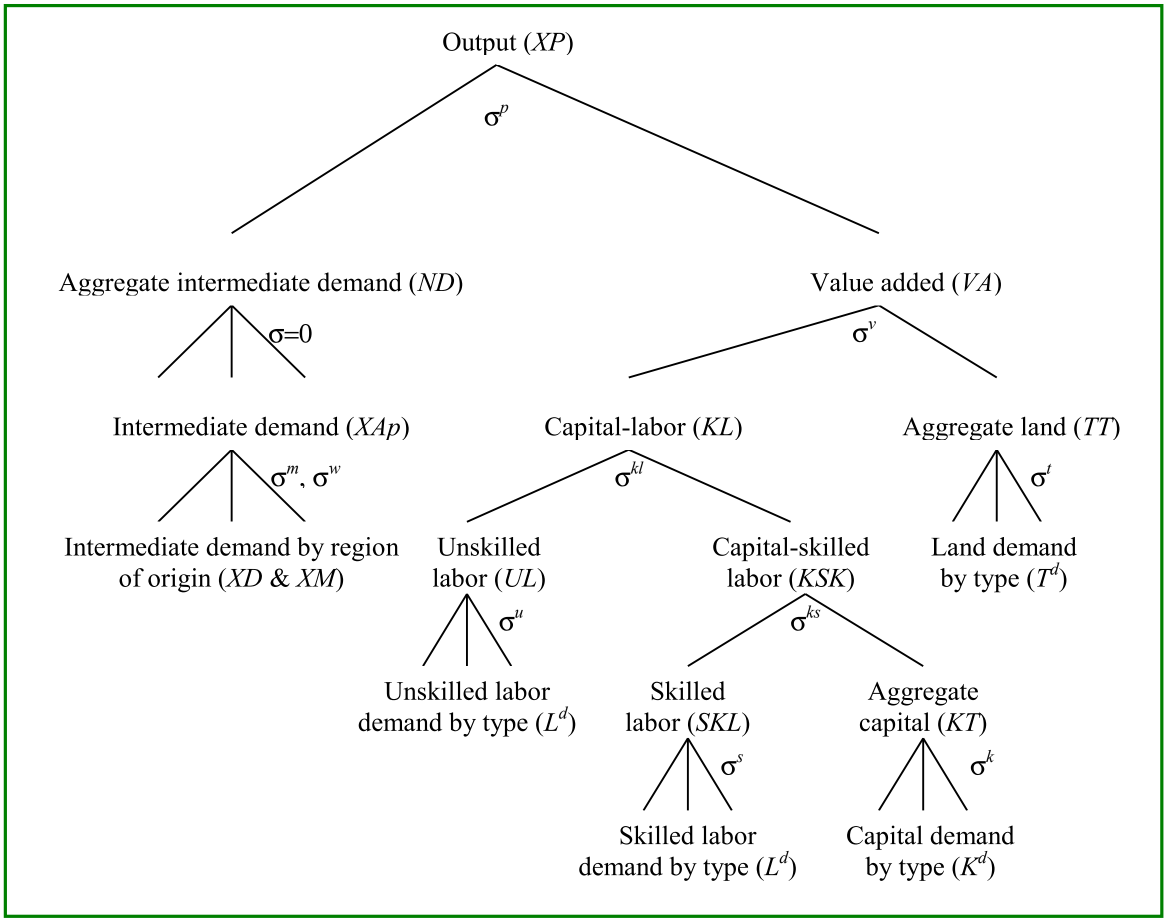

Figure 2: Nested structure of consumer demand