



Lecture 2.2

1-2-Sim?

Easy CGE Modeling

David Roland-Holst and Enkhbayar Shagdar
UC Berkeley and ERINA

Training Workshop
A Prototype CGE Model for Mongolia
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National Statistical Office of Mongolia, Ulaan Baatar



1-2-3 CGE Model

- 1 country, 2 activities, 3 commodities
- 2 activities, producing D and E.
 - E not consumed domestically.
- Additional commodity, M, consumed domestically but not produced.



Structural Assumptions

- Aggregate GDP (X) is fixed.
 - Full employment model.
- Trade balance set exogenously.
- World prices of M and E are fixed.
- Total absorption (Q) is endogenous.

Analytical 1-2-3 Model

Flows

$$1. \bar{X} = G(E, D^S; \Omega)$$

$$2. Q^S = F(M, D^D; \sigma)$$

$$3. Q^D = \frac{Y}{P^q}$$

$$4. \frac{E}{D^S} = g_2(P^e, P^d)$$

$$5. \frac{M}{D^D} = f_2(P^m, P^d)$$

$$6. Y = P^x \cdot \bar{X} + R \cdot B$$

Prices

$$7. P^m = R \cdot pw^m$$

$$8. P^e = R \cdot pw^e$$

$$9. P^x = g_1(P^e, P^d)$$

$$10. P^q = f_1(P^m, P^d)$$

$$11. R \equiv 1$$

Equilibrium Conditions

$$12. D^D - D^S = 0$$

$$13. Q^D - Q^S = 0$$

$$14. pw^m \cdot M - pw^e \cdot E = B$$

Identities

$$15. P^x \cdot X \equiv P^e \cdot E + P^d \cdot D^S$$

$$16. P^q \cdot Q^S \equiv P^m \cdot M + P^d \cdot D^D$$

$$17. Y \equiv P^q \cdot Q^D$$

Definitions for the 1-2-3 CGE Model

Endogenous Variables

E: Export good

M: Import good

D^S : Supply of domestic good

D^D : Demand for domestic good

Q^S : Supply of composite good

Q^D : Demand for composite good

Y: Total income

P^e : Domestic price of export good

P^m : Domestic price of import good

P^d : Domestic price of domestic good

P^x : Price of aggregate output

P^q : Price of composite good

R: Exchange rate

Exogenous Variables

pw^e : world price of export good

pw^m : world price of import good

B: Balance of trade

σ : Import substitution elasticity

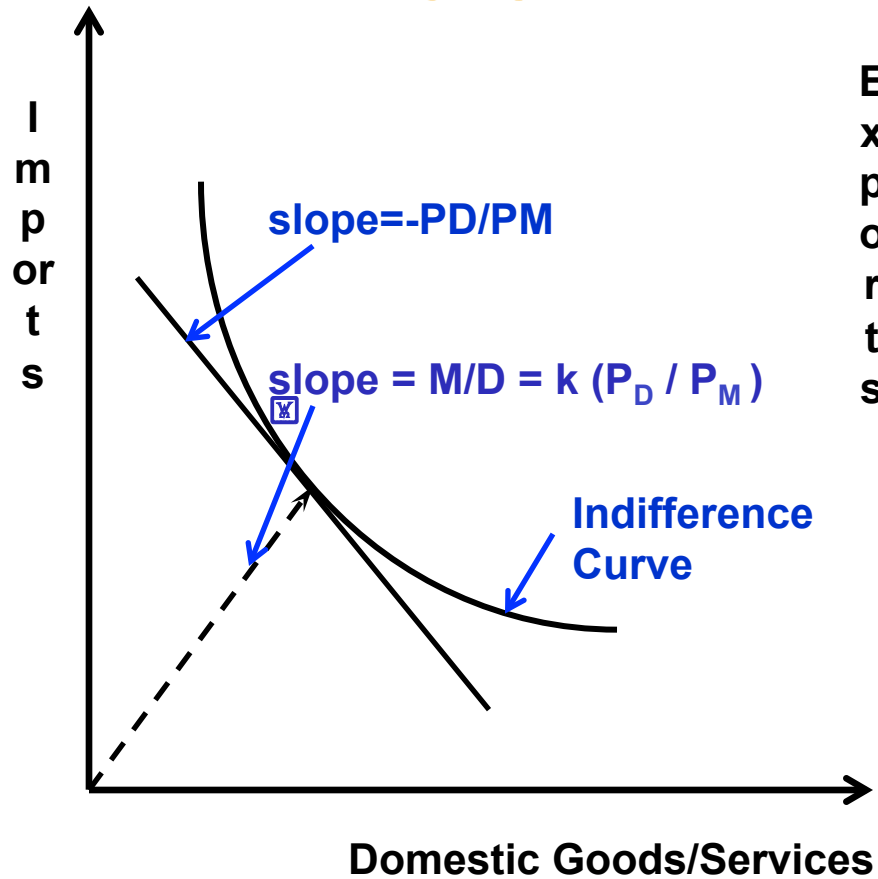
Ω : Export transformation elasticity

SAM 1-2-3

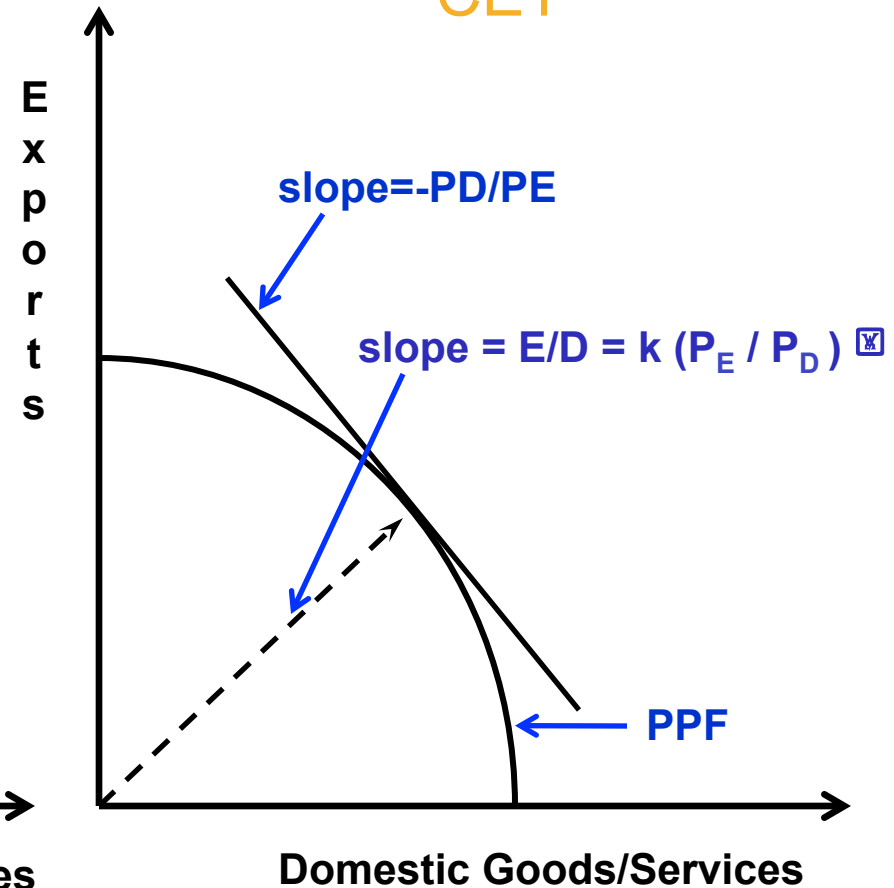
	Activities	Commodity	Hshld	World
Activities		$P^d \cdot D^D$		$P^e \cdot E$
Commodities			$P^q \cdot Q^D$	
Households	$P^x \cdot \bar{X}$			$R \cdot B$
World		$P^m \cdot M$		
Total	$P^d \cdot D^S + P^e \cdot E$	$P^q \cdot Q^S$	Y	

Trade Schematically

CES



CET



1-2-3 as a Programming Model

Maximize $Q = F(M, D; \sigma)$

with respect to: M, E, D^D, D^S

subject to:

$$1. G(E, D^S; \Omega) \leq \bar{X}$$

technology

Shadow Prices

$$\lambda^x = P^x / P^q$$

$$2. pw^m \cdot M \leq pw^e \cdot E + \bar{B}$$

balance of trade

$$\lambda^b = R / P^q$$

$$3. D^D \leq D^S$$

domestic market

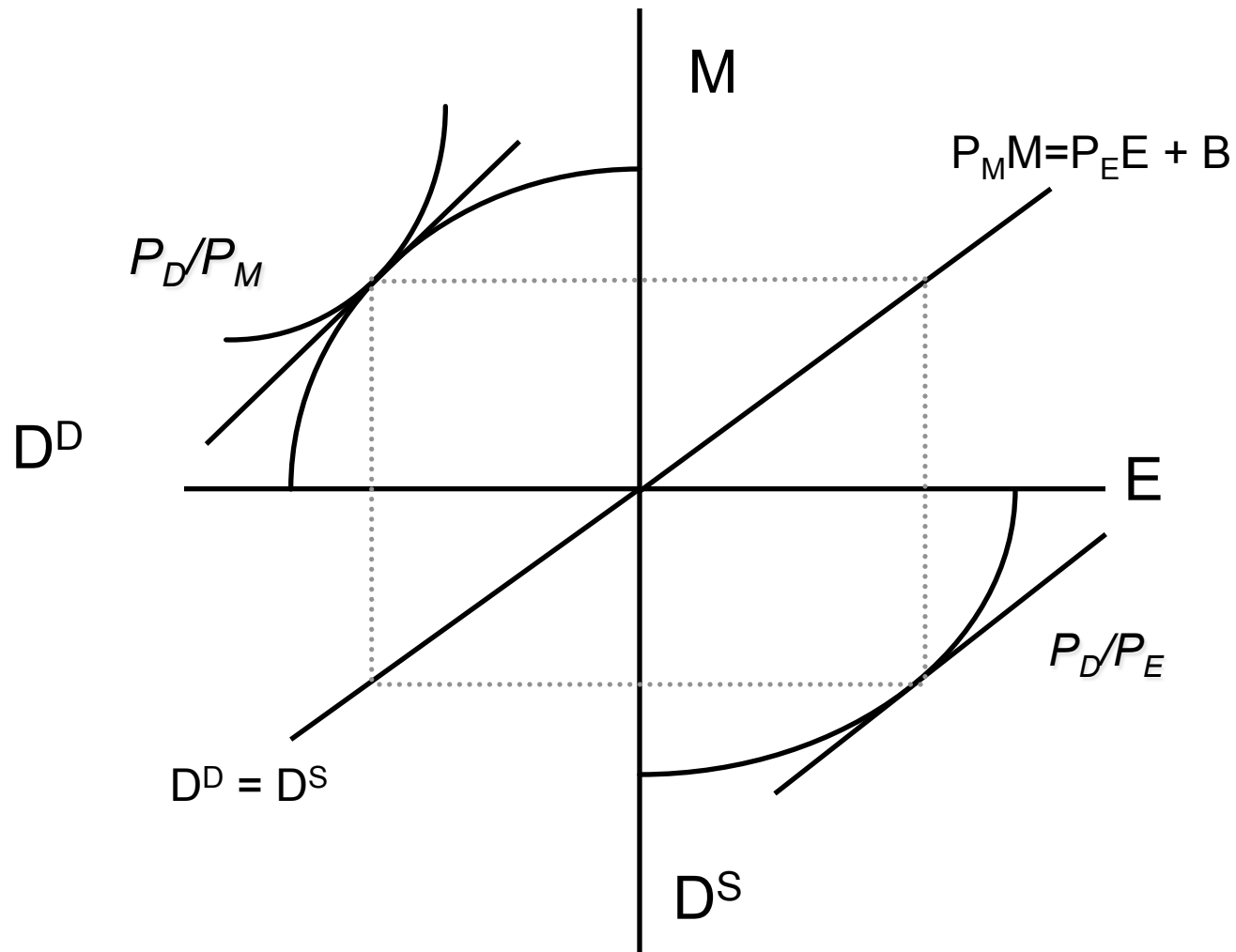
$$\lambda^d = P^d / P^q$$



Assumptions

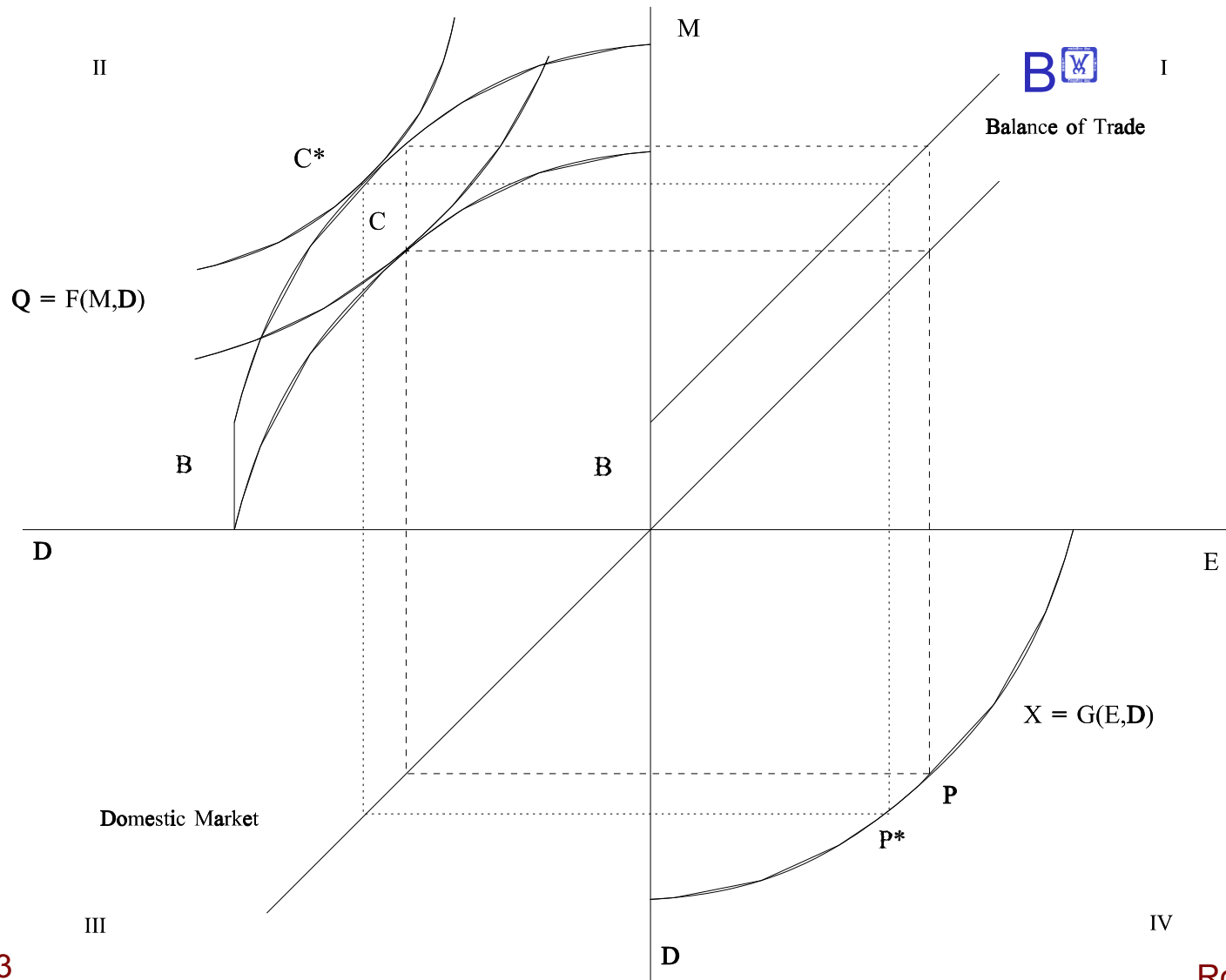
- A CET transformation technology between a domestic good D and an export Good E
- CES preferences in final demand over D and imports M
- A fixed balance of trade
- Fixed government demand and investment (example of “macroeconomic closure”)
- Fixed terms-of-trade (small country assumption)
- Macro identities hold (income constraints, balance of trade, etc.)

1-2-3 Model Descriptively

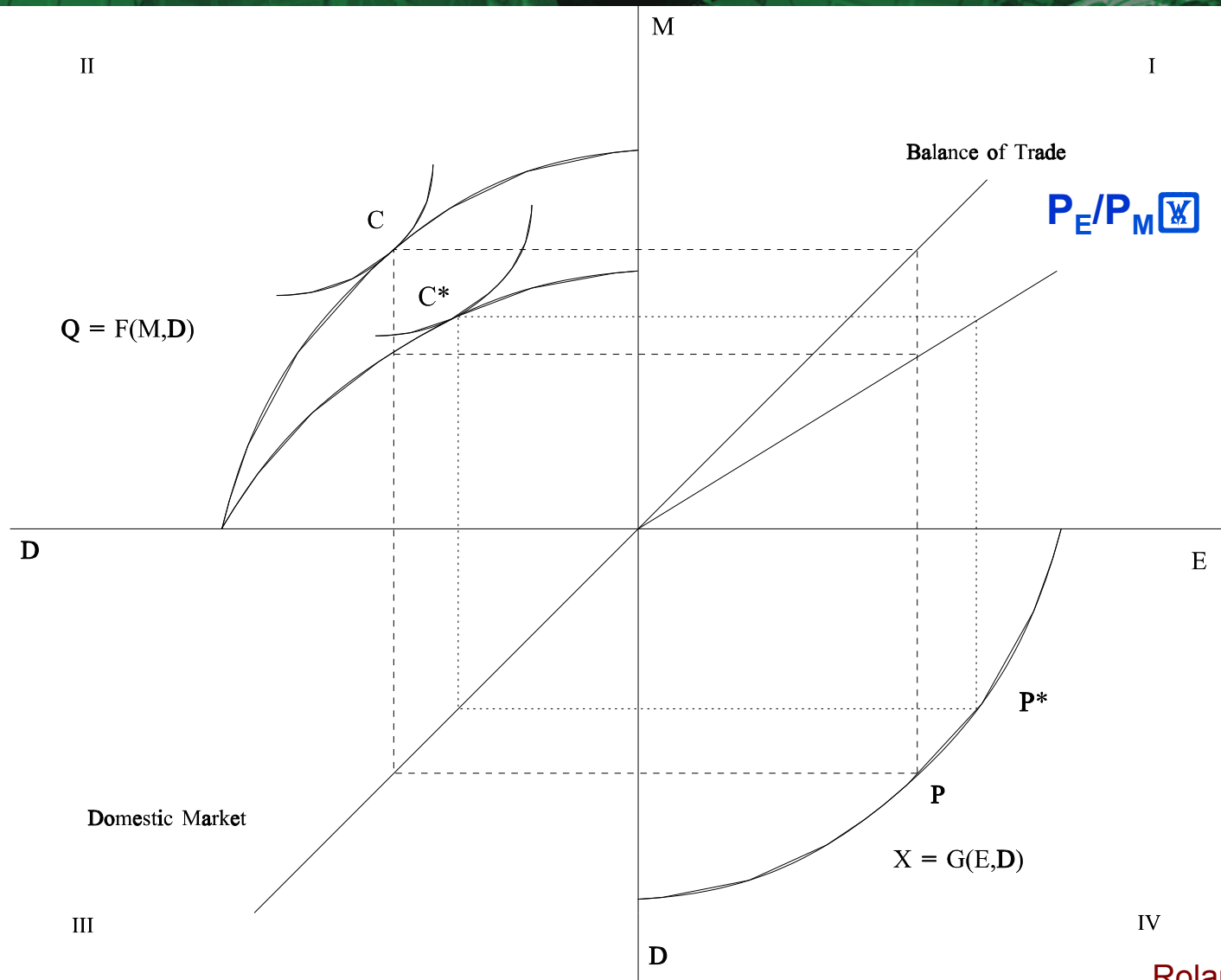




Foreign Capital Inflow



Terms of Trade Deterioration



The 123 model in Excel

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		1-2-3 (CGE) Model for Sri Lanka, 1991											
2		from Devarajan-Go-Lewis-Robnson-Sinko (1997)											
3		note: use "solver", under the "Tools" menu. Solver settings have been set.											
4		Parameters	Exogenous Variables	Base Year	Current	Endogenous Variables	Base Year	Current	Curr/Base	Eq.#	Equations		
5													
6		Elasticity for CET (st)	0.60	World Price of Imports (wm)	0.89	0.89	Export Good (E)	0.33	0.33	1.00	1	CET Transformation (CETEQ)	1.00
7		Elasticity for CES/Q (sq)	0.60	World Price of Exports (we)	1.01	1.01	Import Good (M)	0.50	0.50	1.00	2	Supply of Goods (ARMG)	1.18
8							Supply of Domestic Good (Ds)	0.67	0.67	1.00	3	Domestic Demand (DEM)	1.18
9		Scale for CET (at)	2.22	Import Tariffs (tm)	0.13	0.13	Demand of Domestic Good (Dd)	0.67	0.67	1.00	4	E/D Ratio (EDRAT)	0.49
10		Share for CET (bt)	0.77	Export Duties (te)	0.01	0.01	Supply of Composite Good (Qs)	1.18	1.18	1.00	5	M/D Ratio (MDRAT)	0.75
11		Rho for CET (rt)	2.67	Indirect Taxes (ts)	0.08	0.08	Demand of Composite Good (Qd)	1.18	1.18	1.00			
12				Direct Taxes (ty)	0.03	0.03							
13		Scale for CES/Q (aq)	1.97				Tax Revenue (TAX)	0.20	0.20	1.00	6	Revenue Equation (TAXEQ)	0.20
14		Share for CES/Q (bq)	0.38	Savings rate (sy)	0.17	0.17	Total Income (Y)	1.13	1.13	1.00	7	Total Income Equation (INC)	1.13
15		Rho for CES/Q (rq)	0.67	Govt. Consumption (G)	0.10	0.10	Aggregate Savings (S)	0.27	0.27	1.00	8	Savings Equation (SAV)	0.27
16				Govt. Transfers (tr)	0.12	0.12	Consumption (Cn)	0.83	0.83	1.00	9	Consumption Function (CONS)	0.83
17				Foreign Grants (ft)	0.02	0.02							
18				Net Priv Remittances (re)	0.01	0.01	Import Price (Pm)	1.00	1.00	1.00	10	Import Price Equation (PMEQ)	1.00
19				Foreign Saving (B)	0.08	0.08	Export Price (Pe)	1.00	1.00	1.00	11	Export Price Equation (PEEQ)	1.00
20				Output (X)	1.00	1.00	Sales Price (Pt)	1.08	1.08	1.00	12	Sales Price Equation (PTEQ)	1.08
21							Price of Supply (Pq)	1.00	1.00	1.00	13	Output Price Equation (PXEQ)	1.00
22							Price of Output (Px)	1.00	1.00	1.00	14	Supply Price Equation (PQEQ)	1.00
23							Price of Dom. Good (Pd)	1.00	1.00	1.00	15	Numeraire (REQ)	1.00
24							Exchange Rate (Er)	1.00	1.00	1.00			
25													
26							Investment (Z)	0.25	0.25	1.00	16	Domestic Good Market (DEQ)	0.00
27							Government Savings (Sg)	-0.01	-0.01	1.00	17	Composite Good Market (QEQ)	0.00
28							Walras Law (Z-S)	0.00	0.00		18	Current Account Balance (CABAL)	0.08
											19	Government Budget (GBUD)	-0.01

- 19 endogenous variables and equations
- variables are "scaled" as a share of GDP
- Basic inputs are macroeconomic accounts data

The 123 model in Excel

	K	L	M	N	O	P	Q	R	S	T	U
1											
2											
3											
4	Eq.# Equations			Data - Sri Lanka, 1991							
5		<i>Real Flows</i>				Rs Billion	Output=1			Rs Billion	Output=1
6	1	CET Transformation (CETEQ)	1.00	National Accounts				3 Fiscal Account			
7	2	Supply of Goods (ARMG)	1.18	1	Output (Value Added)	324.69	1.00	Revenue	76.18	0.23	
8	3	Domestic Demand (DEM)	1.18		Wages	163.32	0.50	NonTax	8.02	0.02	
9	4	E/D Ratio (EDRAT)	0.49					Current Expenditure	83.76	0.26	
10	5	M/D Ratio (MDRAT)	0.75		GDP at market prices	375.34	1.16	Goods & Services	35.58	0.11	
11		<i>Nominal Flows</i>			Private Consumption	291.69	0.90	Interest Payments	22.07	0.07	
12	6	Revenue Equation (TAXEQ)	0.20		Public Consumption	35.58	0.11	Transfers & Subsidies	26.10	0.08	
13	7	Total Income Equation (INC)	1.13		Investment	86.38	0.27	Capital Expenditure	35.77	0.11	
14	8	Savings Equation (SAV)	0.27		Exports	106.39	0.33	Fiscal Balance	-43.35	-0.13	
15	9	Consumption Function (CONS)	0.83		Imports	144.70	0.45				
16		<i>Prices</i>									
17	10	Import Price Equation (PMEQ)	1.00	Tax Revenue				4 Balance of Payments			
18	11	Export Price Equation (PEEQ)	1.00	2	Sales & Excise Tax	32.03	0.10	Exports - Imports	-38.32	-0.12	
19	12	Sales Price Equation (PTEQ)	1.08		Import Tariffs	18.62	0.06	Net Profits & Dividends	-0.78	0.00	
20	13	Output Price Equation (PXEQ)	1.00		Export Duties	1.14	0.00	Interest Payments	-8.82	-0.03	
21	14	Supply Price Equation (PQEQ)	1.00		Payroll Tax	0.00	0.00	Net Private Transfers	11.60	0.04	
22	15	Numeraire (REQ)	1.00		Personal Income Tax	3.54	0.01	Net Official Transfers	7.90	0.02	
23		<i>Equilibrium Conditions</i>			Capital Income Tax	12.84	0.04	Current Account Balance	-28.42	-0.09	
24	16	Domestic Good Market (DEQ)	0.00		Total	68.16	0.21				
25	17	Composite Good Market (QEQ)	0.00					External Debt	260.50	0.80	
26	18	Current Account Balance (CABAL)	0.08					Debt Service Payments	20.21	0.06	
27	19	Government Budget (GBUD)	-0.01								
28											

- 19 endogenous variables and equations
- variables are “scaled” as a share of GDP
- Basic inputs are macroeconomic accounts data

Variables are identified to the solver by name

Solver Parameters

Set Target Cell:

Equal To: ☒ Max ☐ Min ☐ Value of:

By Changing Cells:

Subject to the Constraints:

- ARMG = Qs
- CABAL = B
- CETEQ = X
- CONS = Cn
- DEM = Qd
- DEQ = 0

Buttons: Solve, Close, Options, Guess, Add, Change, Delete, Reset All, Help

Calibration

- Must run the solver any time parameters or baseline data are changed.

	A	B
3		
4	Parameters	
5		
6	Elasticity for CET (st)	0.6
7	Elasticity for CES/Q (sq)	0.6
8		
9	Scale for CET (at)	$=X0/(bt \cdot E0^{rt} + (1-bt) \cdot Ds0^{rt})^{1/rt}$
10	Share for CET (bt)	$=1/(1 + (Pd0/Pe0) \cdot (E0/Ds0)^{rt-1})$
11	Rho for CET (rt)	$=1/st + 1$
12		
13	Scale for CES/Q (aq)	$=Qs0/(bq \cdot M0^{-rq} + (1-bq) \cdot Dd0^{-rq})^{-1/rq}$
14	Share for CES/Q (bq)	$=((Pm0/Pd0) \cdot (M0/Dd0)^{1+rq}) / (1 + (Pm0/Pd0) \cdot (M0/Dd0)^{1+rq})$
15	Rho for CES/Q (rq)	$=1/sq - 1$
16		



Simulation

- To run a counterfactual experiment
 - Change the Current values of Exogenous Variables (column F of the 1-2-3 Model Sheet)
 - Run the Solver
 - Examine Results Summary and Endogenous Variables
 - NB: Be sure to re-calibrate after each experiment



Moving to GAMS

- The Excel version of 1-2-3 is easily accessible, but must be highly simplified to be tractable.
- Using a higher level programming language enables us to include more economic structure and behavior.
- The Generalized Algebraic Modeling System (GAMS) is the language of choice for this kind of work.

A Model with Consumption, Government, and Investment

Real Flows

- (1) $X = G(E, D^S; \omega)$
- (2) $Q^S = F(M, D^D; \sigma)$
- (3) $Q^D = C + Z + G$
- (4) $E/D^S = g_2(P_e, P_d)$
- (5) $M/D^D = f_2(P_m, P_t)$

Nominal Flows

- (6) $T = t_m \frac{\text{€}}{\text{€}} R \frac{\text{€}}{\text{€}} p_{wm} \frac{\text{€}}{\text{€}} M$
 $+ t_s \frac{\text{€}}{\text{€}} P_q \frac{\text{€}}{\text{€}} Q^D$
 $+ t_y \frac{\text{€}}{\text{€}} Y - t_e \frac{\text{€}}{\text{€}} R \frac{\text{€}}{\text{€}} p_{we} \frac{\text{€}}{\text{€}} E$
- (7) $Y = P_x \frac{\text{€}}{\text{€}} X + t_r \frac{\text{€}}{\text{€}} P_q + r_e \frac{\text{€}}{\text{€}} R$
- (8) $S = s \frac{\text{€}}{\text{€}} Y + R \frac{\text{€}}{\text{€}} B + S_g$
- (9) $C \frac{\text{€}}{\text{€}} P_t = (1 - s - t_y) \frac{\text{€}}{\text{€}} Y$
 $S_g = 0$

Accounting Identities

- (i) $P_x \frac{\text{€}}{\text{€}} X = P_e \frac{\text{€}}{\text{€}} E + P_d \frac{\text{€}}{\text{€}} D^S$
- (ii) $P_q \frac{\text{€}}{\text{€}} Q^S = P_m \frac{\text{€}}{\text{€}} M + P_t \frac{\text{€}}{\text{€}} D^D$

Prices

- (10) $P_m = (1 + t_m) \frac{\text{€}}{\text{€}} R \frac{\text{€}}{\text{€}} p_{wm}$
- (11) $P_e = (1 + t_e) \frac{\text{€}}{\text{€}} R \frac{\text{€}}{\text{€}} p_{we}$
- (12) $P_t = (1 + t_s) \frac{\text{€}}{\text{€}} P_q$
- (13) $P_x = g_1(P_e, P_d)$
- (14) $P_q = f_1(P_m, P_t)$
- (15) $R = 1$

Equilibrium Conditions

- (16) $D^D - D^S = 0$
- (17) $Q^D - Q^S = 0$
- (18) $p_{wm} \frac{\text{€}}{\text{€}} M - p_{we} \frac{\text{€}}{\text{€}} E - ft - re = B$
- (19) $P_t \frac{\text{€}}{\text{€}} Z - S = 0$
- (20) $T - P_q \frac{\text{€}}{\text{€}} G - t_r \frac{\text{€}}{\text{€}} P_q - ft \frac{\text{€}}{\text{€}} R -$



Definitions

Endogenous Variables

E: Export good
M: Import good
DS: Supply of domestic good
DD: Demand for domestic good
QS: Supply of composite good
QD: Demand for composite good
Pe: Domestic price of export good
Pm: Domestic price of import good
Pd: Producer price of domestic good
Pt: Sales price of composite good
Px: Price of aggregate output
Pq: Price of composite good
R: Exchange rate
T: Tax revenue
Sg: Government savings
Y: Total income
C: Aggregate consumption
S: Aggregate savings
Z: Aggregate real investment

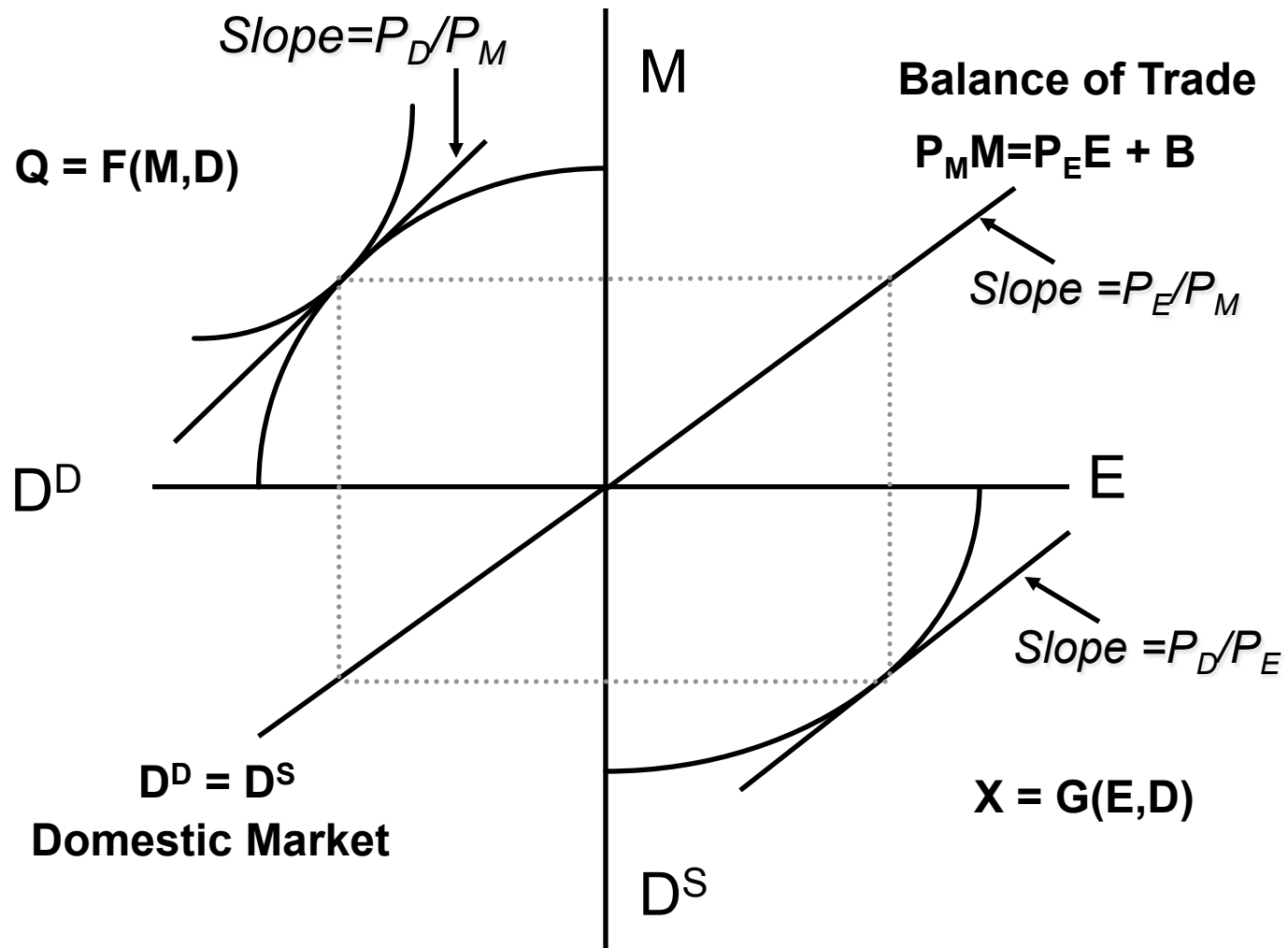
Exogenous Variables

pwm: World price of import good
pwe: World price of export good
tm: Tariff rate
te: Export subsidy rate
ts: sales/excise/value-added tax rate
ty: direct tax rate
tr: government transfers
ft: foreign transfers to government
re: foreign remittances to private sector
 s : Average savings rate
 \bar{X} : Aggregate output
 \bar{G} : Real government demand
 \bar{B} : Balance of trade
rhot: Export transformation elasticity
rhoc: Import substitution elasticity

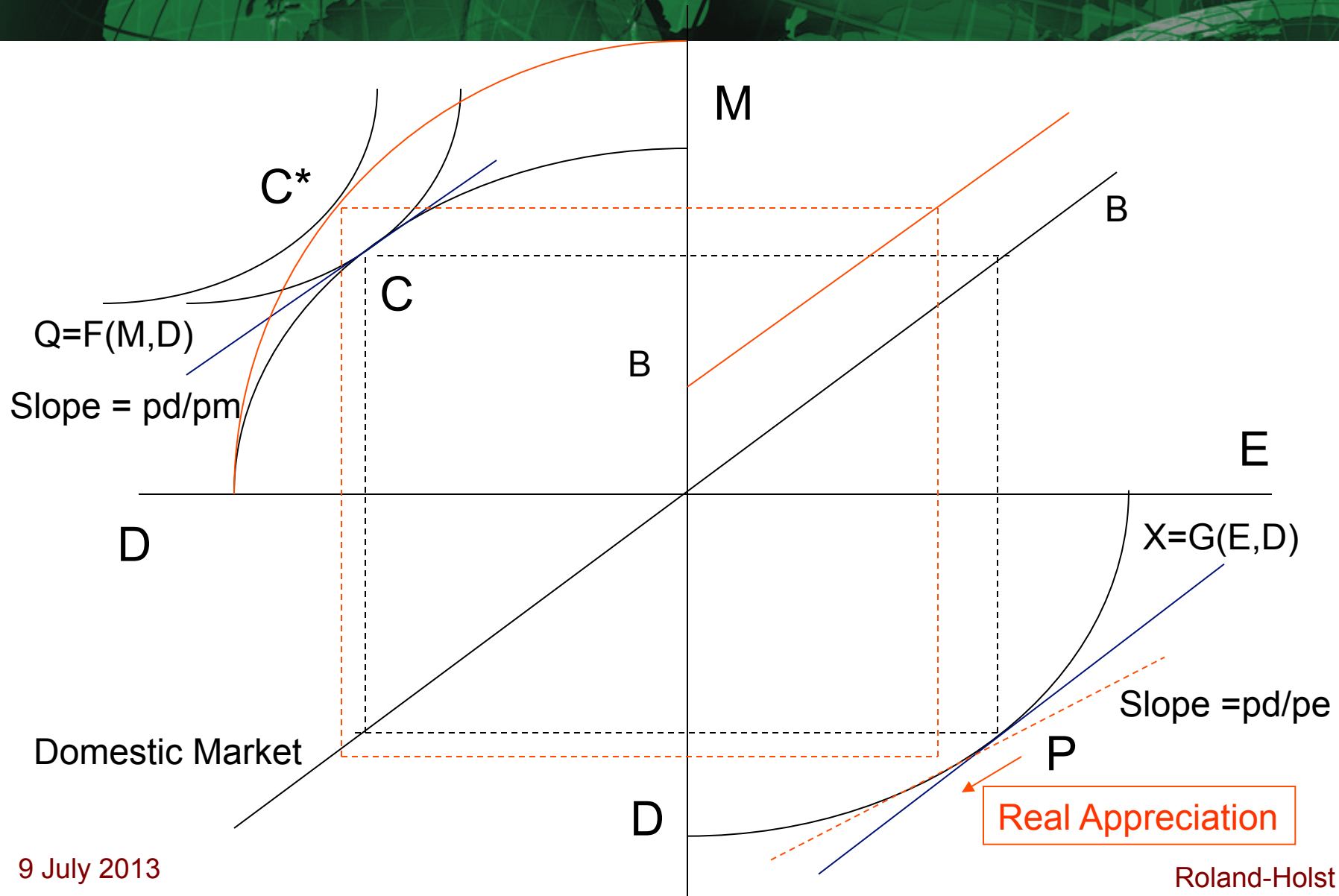
123 SAM

	Act	Com	Hshld	Gov	Cap	Wrld
Act		D				E
Com			C	G	Z	
Hshld	Y					
Govt	T_X		T_H			
Cap			S_H	S_G		S_F
Wrld		M				

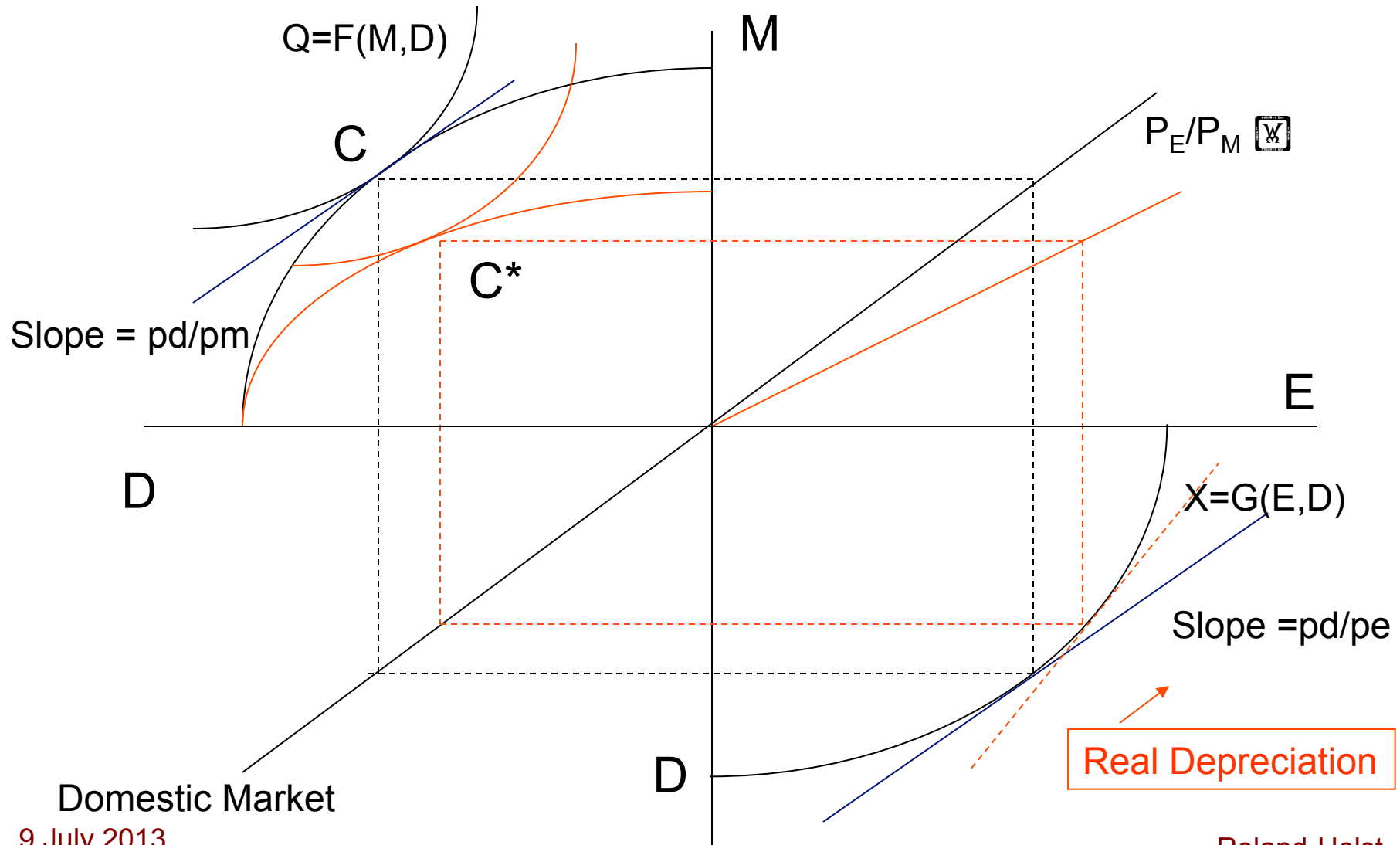
Diagrammatic 1-2-3 model



Foreign Capital Inflow



Adverse Terms of Trade Movement



Parameter declarations

IDE Gamside: D:\Models\Small\small.gam ID:ATEACHU~1\TMPER~1\CEEMOD~1\DISK1\GAMSMD~1\CEE123\CEE123.GMS1

IDE File Edit Search W



cge123.gms

Parameters

** Here we define a mix of policy and functional parameters*

```
PWm      world price of import good
PWe      world price of export good
tm       import tariff
te       export subsidy rate
ts       sales or excise of VAT tax rate
ty       direct income tax rate
tr       government transfers
ft       foreign transfers to government
re       foreign remittances to private sector
sr       average savings rate
X        aggregate output
G        government demand
B        balance of trade
at       technical shift term for CET expression
OMEGA    export transformation elasticity
rt       the CET exponential term
aq       technical shift term for CES expression
sigma    import substitution elasticity
rq       the CES exponential term
bq       the CES weight term in the Armington function
bt       the CET weight term in the national product function
values(vars,exp) a table to hold experiment values
;
```

**Initialization of parameters*

```
PWm      = 0.8860;
PWe      = 1.0107;
tm       = 0.1287;
te       = 0.0107;
ts       = 0.0839;
ty       = 0.0350;
tr       = 0.1237;
```



cge123.gms

Equation definitions

\$Stitle equation definitions

Equations

**Real flow equations with numbering following Devarajan et al*

EQ621 definition of the product transformation surface
EQ622 definition of the Armington aggregation function
EQ623 expenditure definition of national income
EQ624 supply ratios based on EQ621
EQ625 supply ratios based on EQ622

**Nominal flows*

EQ626 government tax income definition
EQ627 value added definition of national income
EQ628 savings definition
EQ629 consumption definition

**Price equations*

EQ630 internal import prices
EQ631 internal export prices
EQ632 consumer prices
EQ633 national product price index
EQ634 producer prices for the composite good
EQ635 real exchange rate

**Market clearing Equations*

EQ636 domestic good excess demand
EQ637 Armington composite excess demand
EQ638 capital and current account relationship
EQ639 savings investment linkage

**Objective function*

OBJ This is the fake objective function for GAMS
;

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File Edit Search Winc

Model specification

cge123.gms

```

- endog("Qd")                                     =e= 0;
EQ638..      B
- (PWm*endog("M") -PWe*endog("E") - ft - re)       =e= 0;
EQ639..      endog("Z")
- ((1/endog("Pt")) * endog("S"))                   =e= 0;
OBJ..
          GDP
- endog("Y")                                         =e= 0;

Model model1 / EQ621, EQ622, EQ623, EQ624, EQ625, EQ626, EQ627,
               EQ628, EQ629, EQ630, EQ631, EQ632, EQ633, EQ634,
               EQ635, EQ636, EQ637, EQ638, EQ639, OBJ / ;

option nlp=minos5;
model1.iterlim = 1000; model1.optfile=4;
Solve model1 maximizing GDP using nlp;

values(vars,exp) = endog.l(vars);

$Ontext

    In this next section we implement a free trade experiment,
    along the lines of the Devarajan et al chapter. We set the import
    tariff and export tax to zero

$Offtext

tm = 0;
te = 0;

option nlp=minos5;
model1.iterlim = 1000; model1.optfile=4;
Solve model1 maximizing GDP using nlp;

```

295: 62 Insert

Running in GAMS IDE

IDE Gamside: D:\Models\Small\small.gpr - [D:\TEACHI~1\TINBER~1\CGEMOD~1\DISK\GAMSMO~1\CGE123\CGE123.GMS]

File Edit Search Windows Help

- New Ctrl+N
- Open Ctrl+O
- Reopen Alt+R
- Open in New Window Shift+Ctrl+O
- Model Library
- Project
- Run F9**
- Compile Shift+F9
- Save Ctrl+S
- Save as
- Save All Shift+Ctrl+S
- Close
- Options
- Print
- Previous
- Exit

```
endog("E") - ft - re)

endog("S")

2, EQ623, EQ624, EQ625, EQ626, EQ627,
9, EQ630, EQ631, EQ632, EQ633, EQ634,
6, EQ637, EQ638, EQ639, OBJ / ;

model1.iterlim = 1000; model1.optfile=4;
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tm = 0;
te = 0;

option nlp=minos5;
model1.iterlim = 1000; model1.optfile=4;
Solve model1 maximizing GDP using nlp;
```

295: 62 Insert Run GAMS

Start | qams | villars.doc... | Microsoft P... | IDE Gamside | 1:32 PM

The solution screen

```
GAMS Rev 120 Copyright (C) 1987-2001 GAMS Development. All rights reserved
Licensee: Economics Department A010221:1033CP-WIN
Erasmus Universiteit Rotterdam DC418

--- Starting compilation
--- CGE123.GMS(333) 1 Mb
--- Starting execution
--- CGE123.GMS(276) 1 Mb
--- Generating model model1
--- CGE123.GMS(277) 2 Mb
--- 20 rows, 21 columns, and 65 non-zeroes.
--- CGE123.GMS(277) 2 Mb
--- Executing MINOS5

MINOS5 Feb 14, 2001 WIN.M5.M5 19.8 108.043.039.WAT

The options file could not be found -- using defaults
Work space allocated -- 0.04 Mb
Reading data...
Reading nonlinear code...

Major Minor Ninf Sinf,Objective Viol RG NSB Ncon Penalty Step
1 0 1 0.00000000E+00 8.4E-02 0.0E+00 14 3 1.0E+01 1.0E+00
2 14 0 1.13158934E+00 5.9E-03 0.0E+00 0 4 1.0E+01 1.0E+00
3 0 0 1.12985912E+00 8.9E-06 0.0E+00 0 5 1.0E+01 1.0E+00
4 0 0 1.12986053E+00 1.5E-11 0.0E+00 0 6 1.0E+01 1.0E+00
5 0 0 1.12986053E+00 2.2E-16 0.0E+00 0 7 0.0E+00 1.0E+00

EXIT -- OPTIMAL SOLUTION FOUND

Major, Minor itns 5 14
Objective function 1.1298605309416E+00
```

Close Open Log ☐ Summary only ☐ Update

295: 62 Insert Double-Click to Open File

Some results

```
IDE Gamside: D:\Models\Small\small.gpr - [D:\Models\Small\CGE123.lst]
IDE File Edit Search Windows Help

cge123.gms cge123.lst

Execution

----- 333 PARAMETER values a table to hold experiment values

benchmark  freetrade  revenue_r~

E      0.3276      0.3371      0.3363
M      0.5030      0.5138      0.5129
Ds     0.6724      0.6626      0.6634
Dd     0.6724      0.6626      0.6634
Qs     1.1754      1.1760      1.1760
Qd     1.1754      1.1760      1.1760
Pe     1.0000      1.0107      1.0000
Pm     1.0000      0.8860      0.8860
Pd     0.9999      0.9405      0.9360
Pt     1.0839      0.9939      1.0487
Px     1.0000      0.9639      0.9573
Pq     1.0000      0.9169      0.9144
T      0.1920      0.1284      0.1920
R      1.0000      1.0000      1.0000
Sg     -0.0100     -0.0612      0.0044
Y      1.1299      1.0835      1.0766
C      0.8288      0.8668      0.8163
S      0.2660      0.2069      0.2713
Z      0.2454      0.2082      0.2587
ts     0.0839      0.0839      0.1469

EXECUTION TIME      =      0.050 SECONDS      1.4 Mb      WIN198-120

USER: Economics Department      A010221:1033CP-WIN
      Erasmus Universiteit Rotterdam      DC418
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Questions?