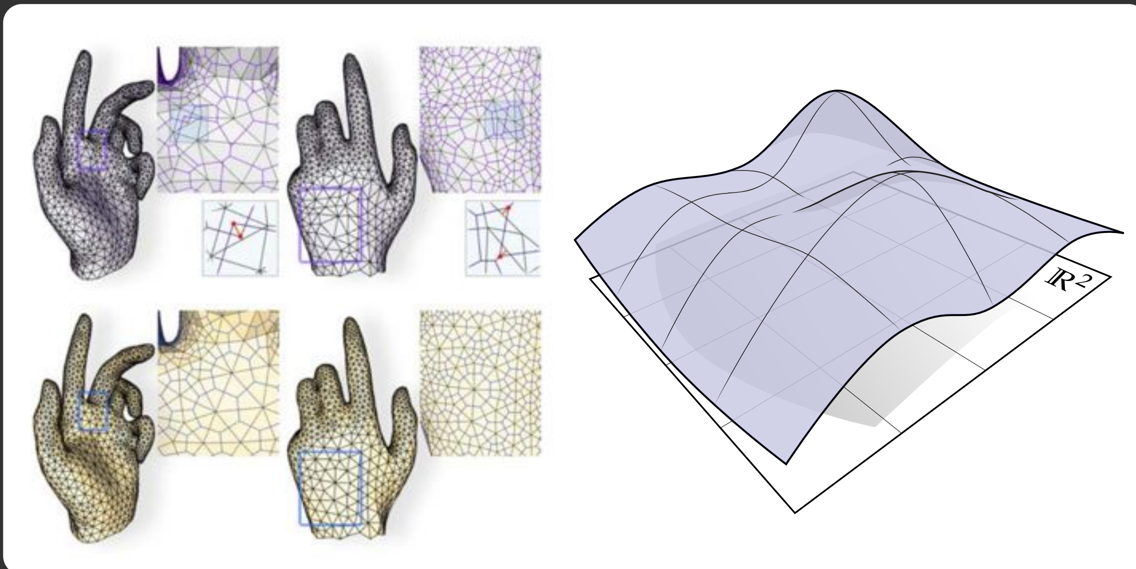
geometry

CS148 / 22 july 2014

thanks!

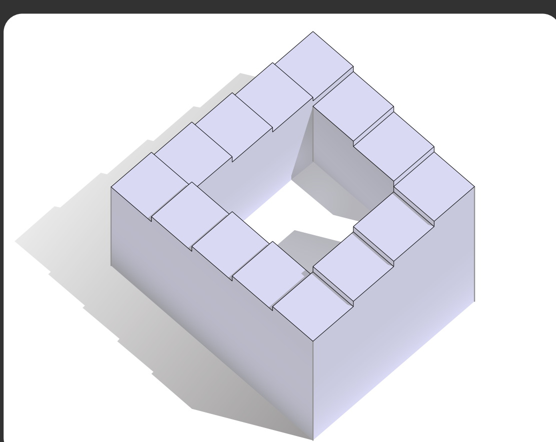
geometry



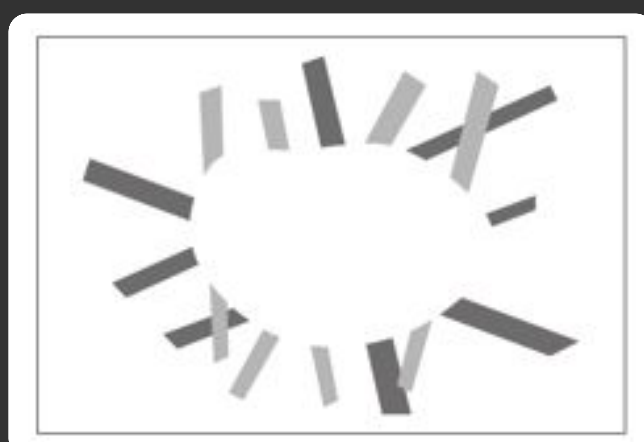
with a little help from my friends:
fernando de goes, keenan crane, etienne vouga, justin solomon, mirela ben-chen

CS148 / 22 july 2014

why?



impossible staircase!



shirley fig. 22.9

why?

http://pixabay.com/p-217062/
http://en.wikipedia.org/wiki/Walt_Disney_Concert_Hall

caveat !

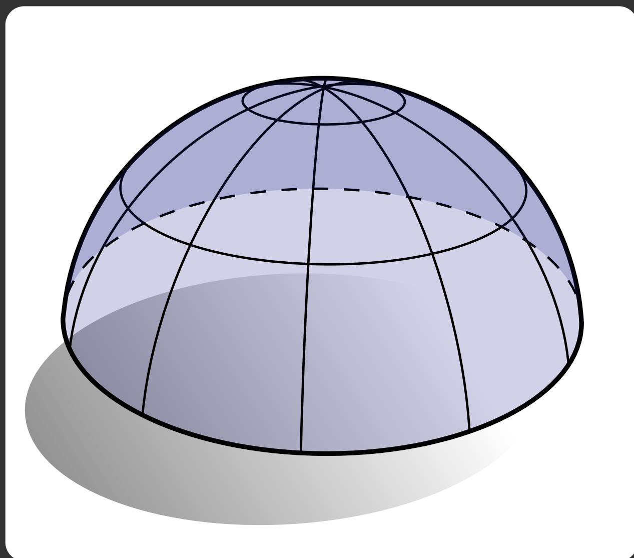
I am a little biased.

discrete differential geometry
& discrete exterior calculus

caveat !

differential geometry

study of smooth surfaces



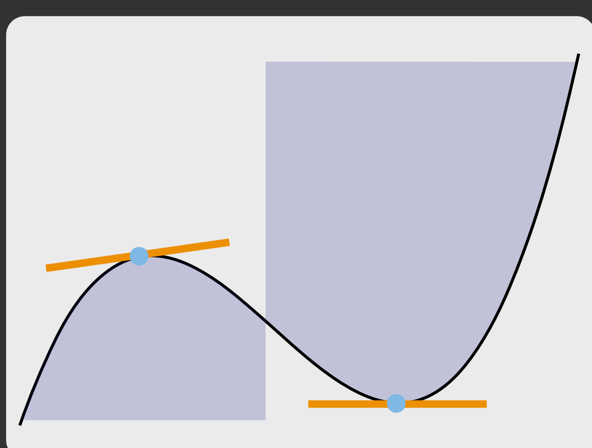
differential geometry

wait:
what is a smooth surface?

caveat !

differential geometry

study of smooth surfaces

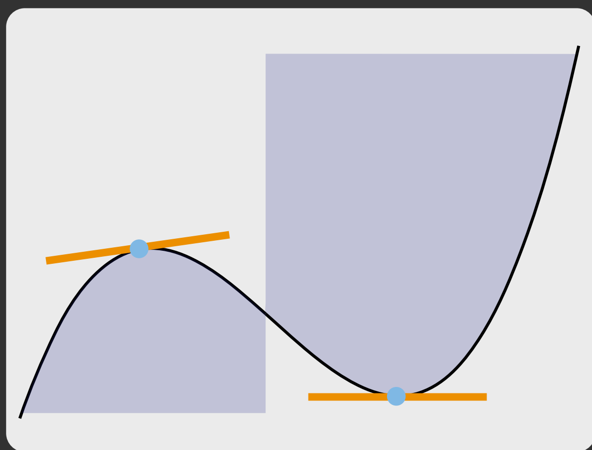


derivatives exist,
and are continuous

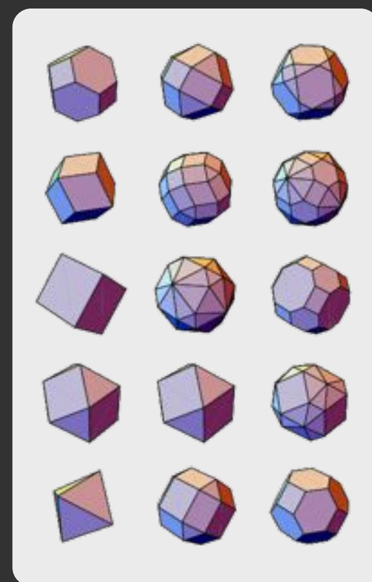
caveat

(discrete) differential geometry

study of smooth surfaces, represented discretely



maybe not the best starting point:

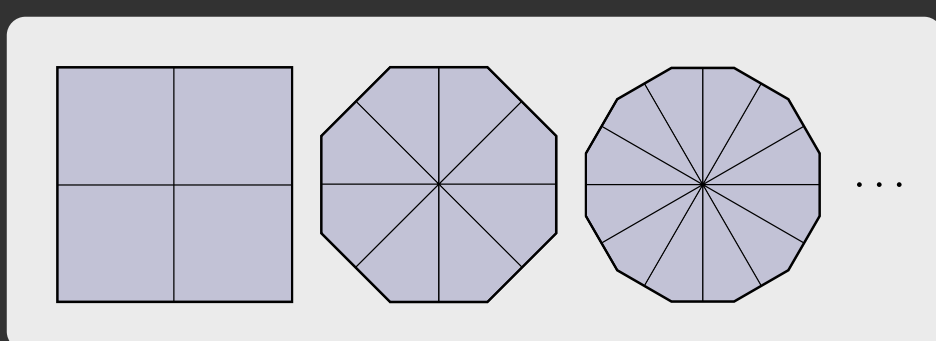


credit:
http://www.watermanpolyhedron.com/images/WatermanPolyhedra_1000.gif

caveat

(discrete) differential geometry

study of smooth surfaces, represented discretely



one big idea:
under refinement, discrete representations
should converge to the continuous (smooth) case

this point of view often leads to useful representations

today:

it matters!

describing.

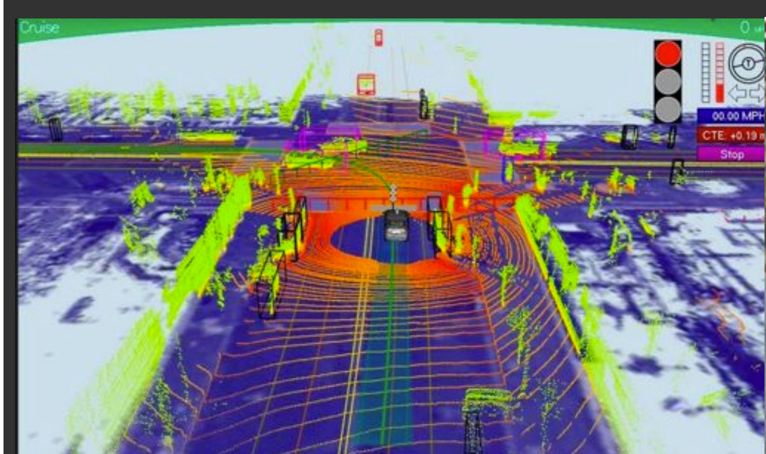
acquiring?

other representations

...special guest!

applications

computer vision



recognition, navigation, reconstruction, segmentation

http://whitenoise.gizmodo.com/how-the-world-looks-to-googles-self-driving-car-486123596
http://recode.net/2014/05/27/googles-new-self-driving-car-ditches-the-steering-wheel/

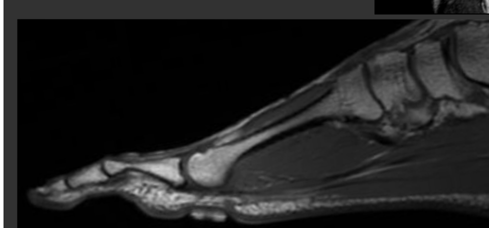
applications
manufacturing

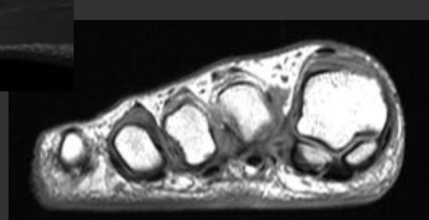
defect detection
simulation of materials
computer-aided design (CAD)

http://grabcad.com/library/quick-release-buckle-19mm
http://grabcad.com/library/mechanical-part-4

applications
medical imaging

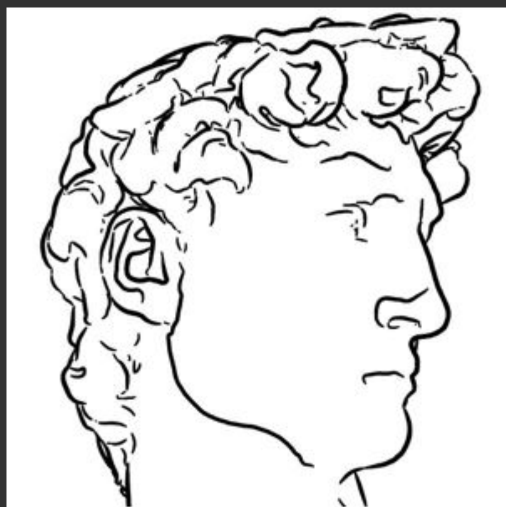
image analysis
segmentation
registration





applications

expressive rendering

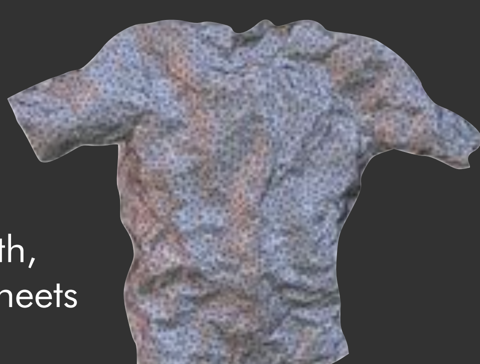


http://gfx.cs.princeton.edu/gfx/pubs/DeCarlo_2003_SCF/index.php
"stylizing animation by example", bénard et al. 2013

applications

simulation

fluids



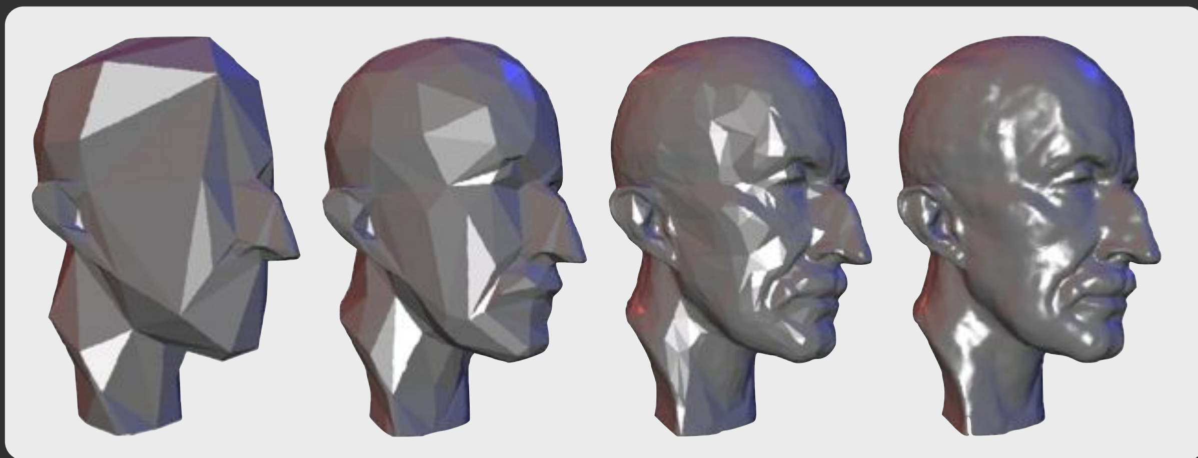
cloth,
thin sheets

"folding and crumpling adaptive sheets"
narain et al. 2013

physBAM/
fedkiw group (stanford)

best representation

depends on context

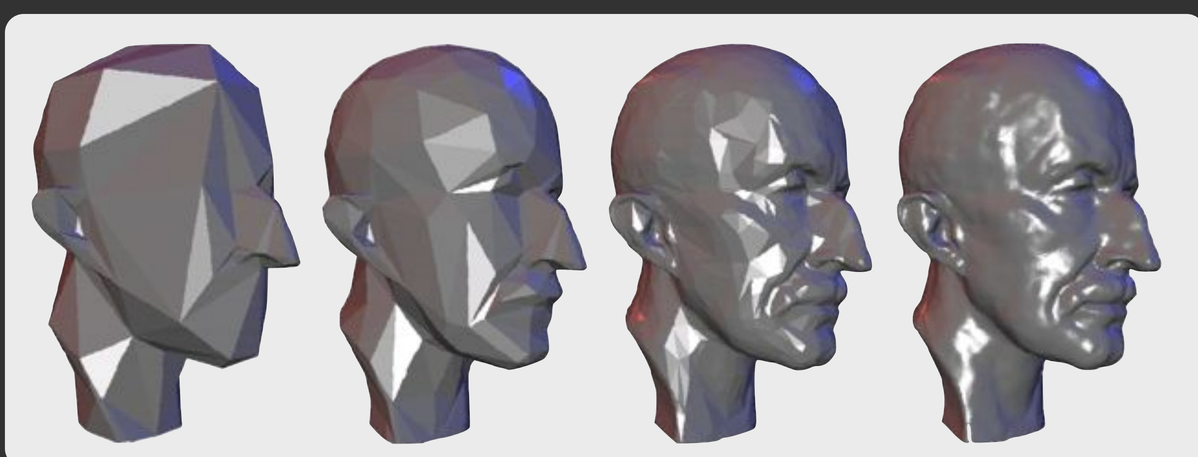


remeshing

multiple resolutions for efficiency, level-of-detail rendering

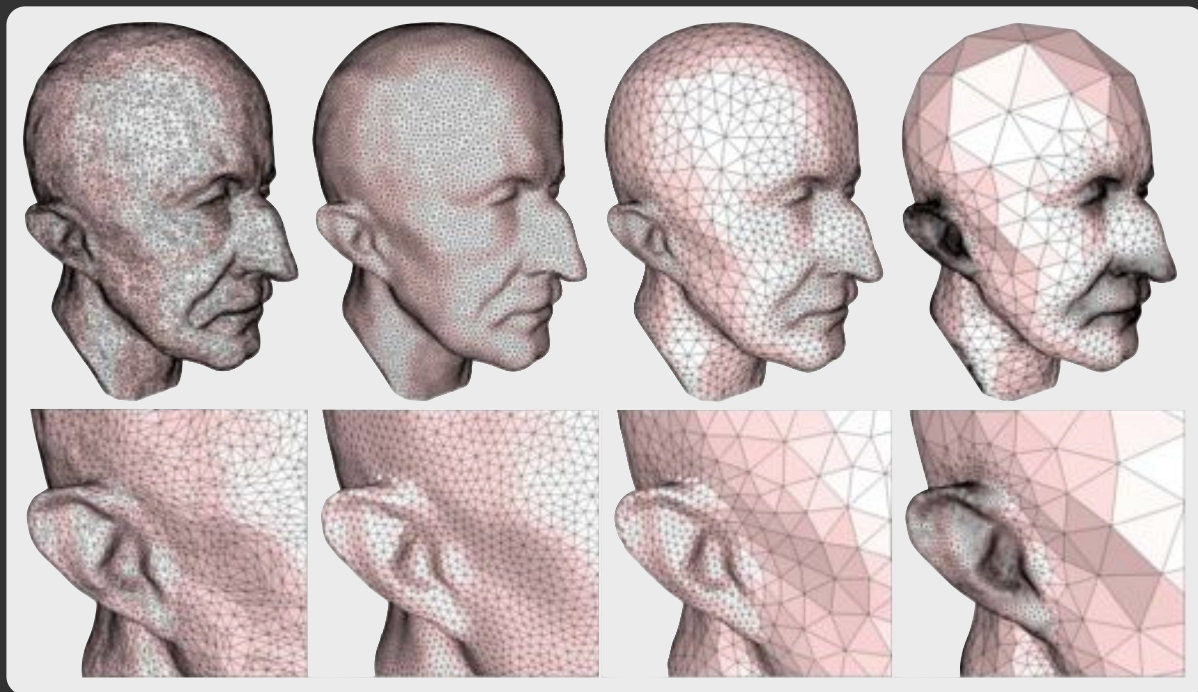
best representation

depends on context



tradeoff:

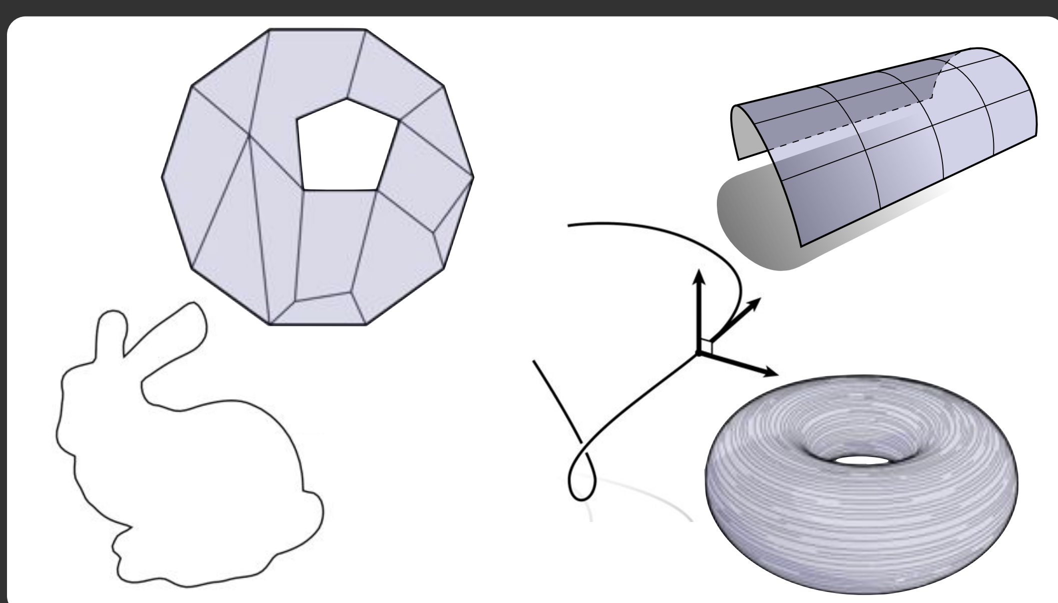
smaller size, higher error — larger size, reduced error



for triangle quality, capturing detail

describing geometry

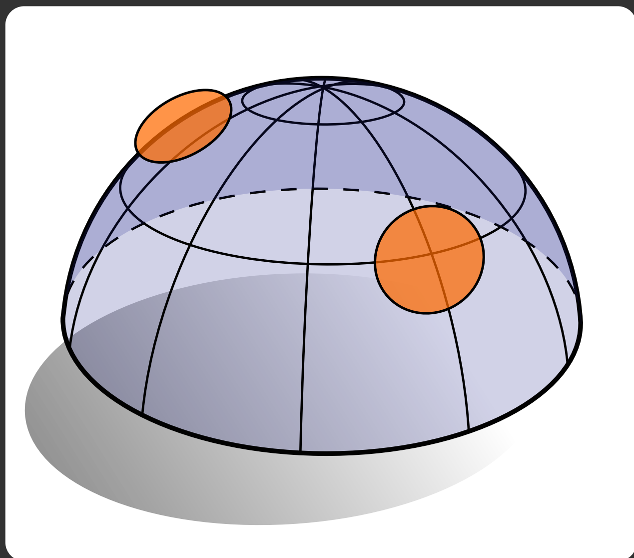
what can we say?

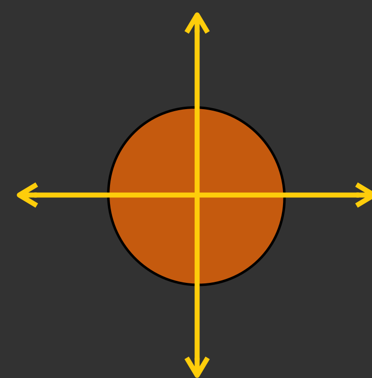


$\subseteq \mathbb{R}^2$ $\subseteq \mathbb{R}^3$

describing surfaces

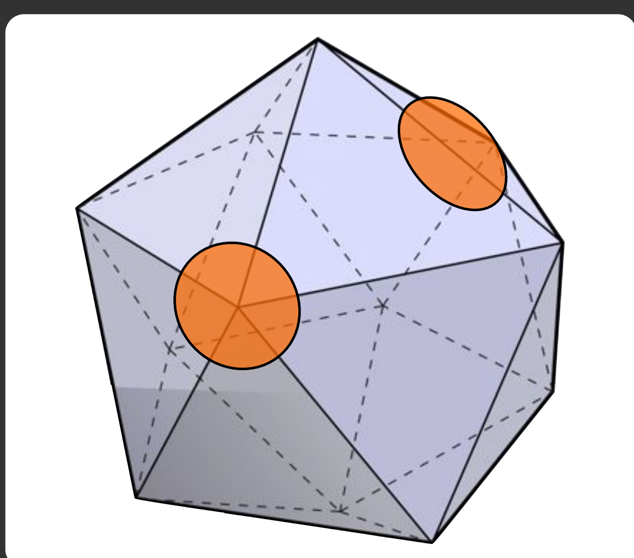
manifold?

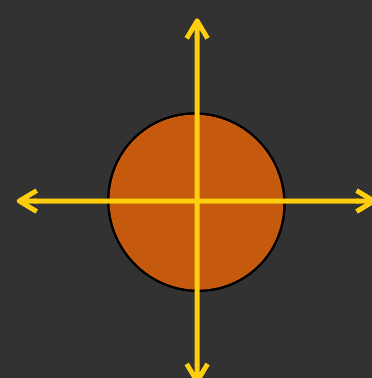


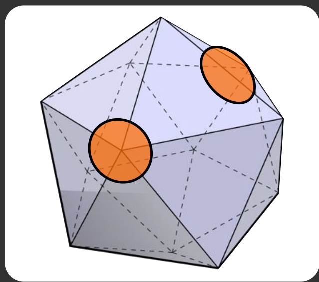


describing surfaces

manifold?





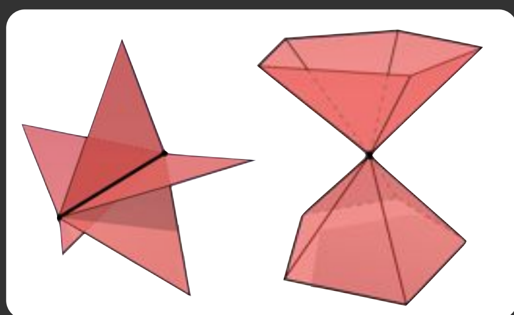


describing surfaces

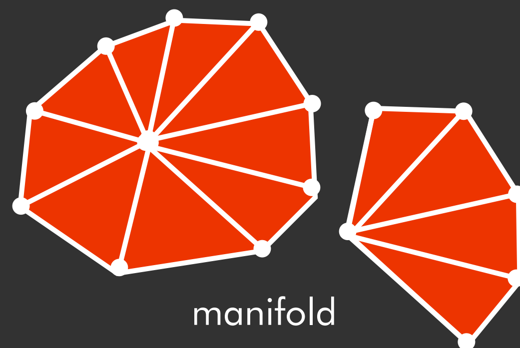
manifold?

faces incident to a vertex
form a closed or open fan

each edge is incident to
one or two faces



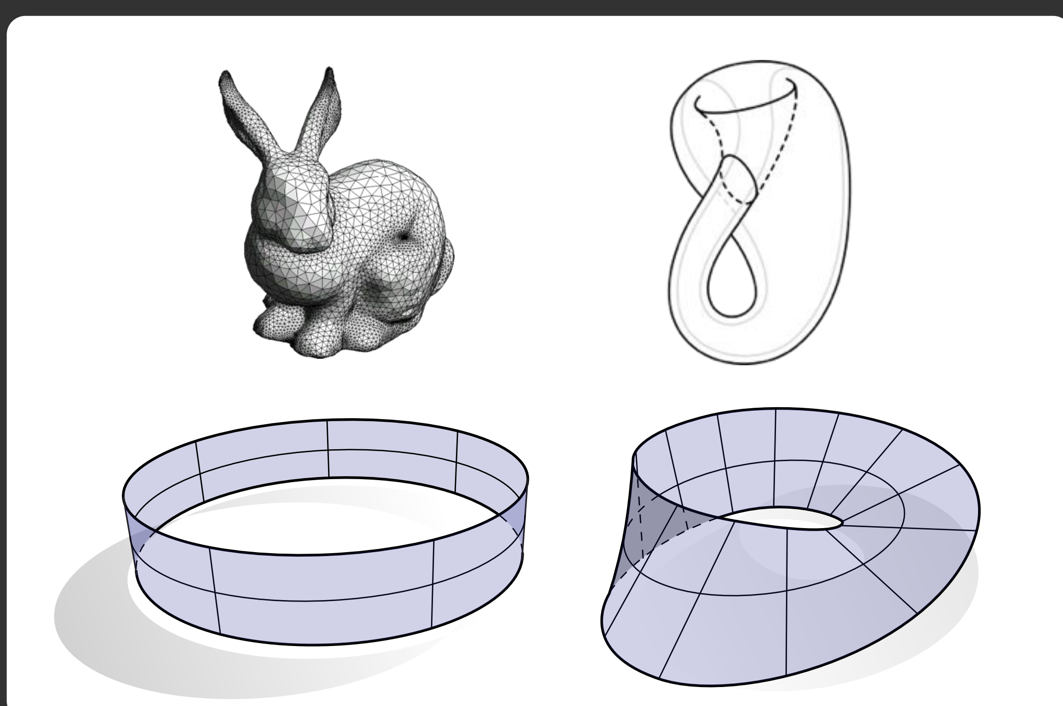
non-manifold



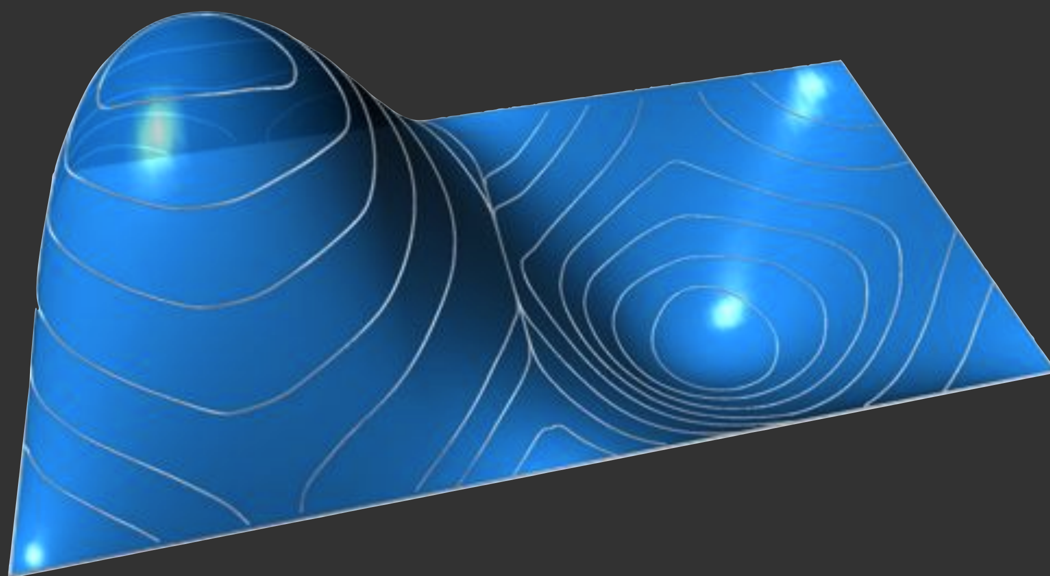
manifold

describing surfaces

orientable?

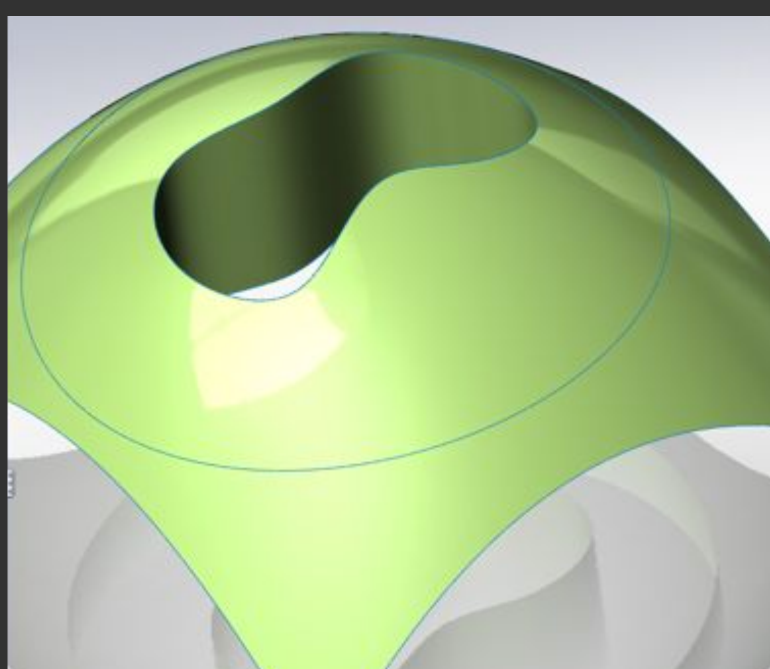


features points, curves



http://www.grasshopper3d.com/forum/topics/principal-curves-on-surface

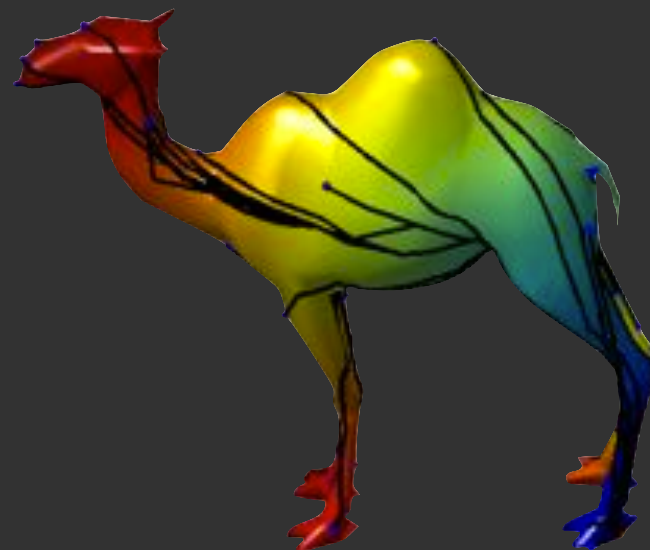
smooth vs. sharp features



boundaries
creases
edges
...

http://www.solidsmack.com/wp-content/uploads/2010/06/solidworks-surface-02a.jpg

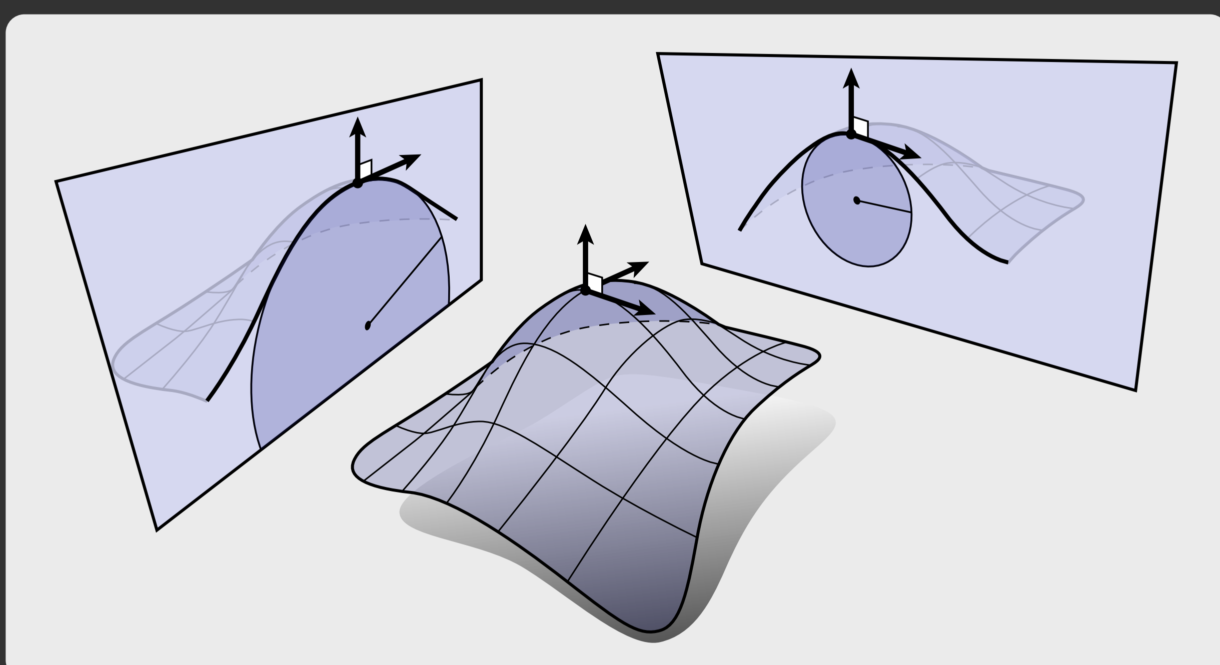
distances along surfaces



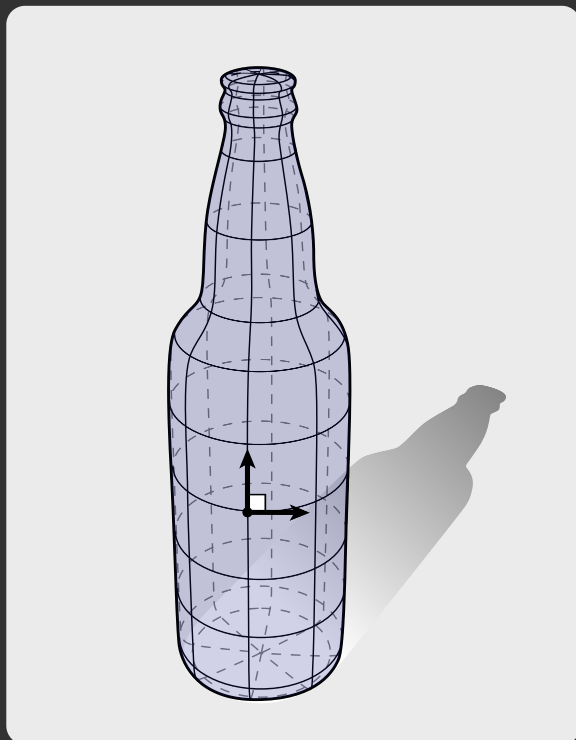
“geodesics in heat”
crane et al. 2012

https://www.ceremade.dauphine.fr/~peyre/numerical-tour/tours/shapes_2_bendinginv_3d/

curvature



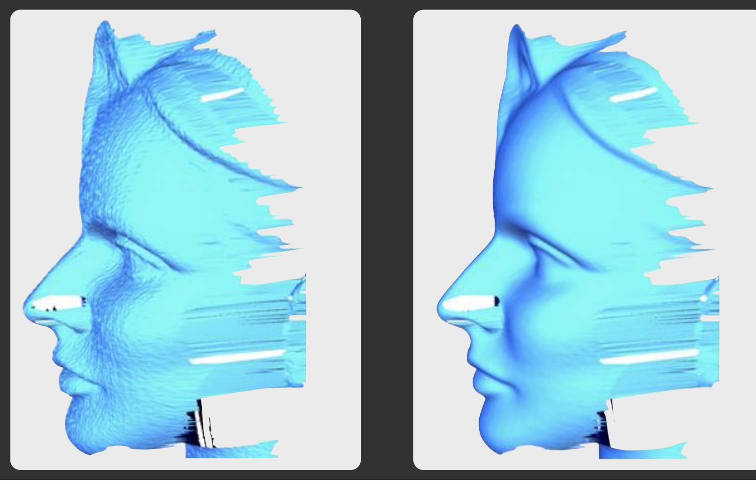
curvature



directions of
principle curvature

mesh fairing

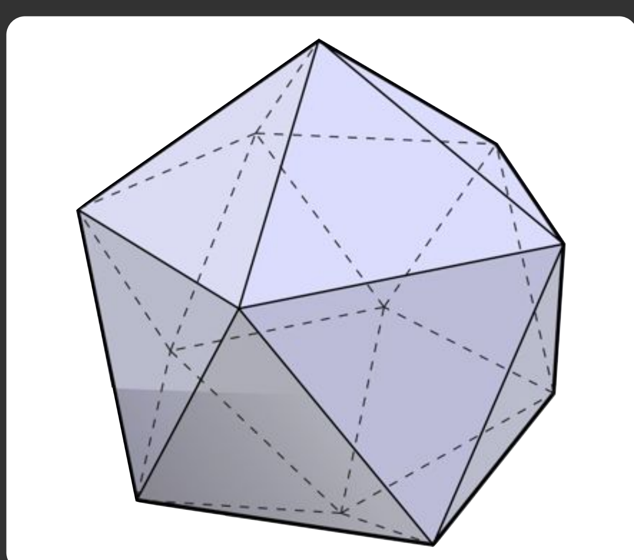
removing high curvature features



"implicit fairing of irregular meshes using diffusion and curvature flow" desbrun et al.

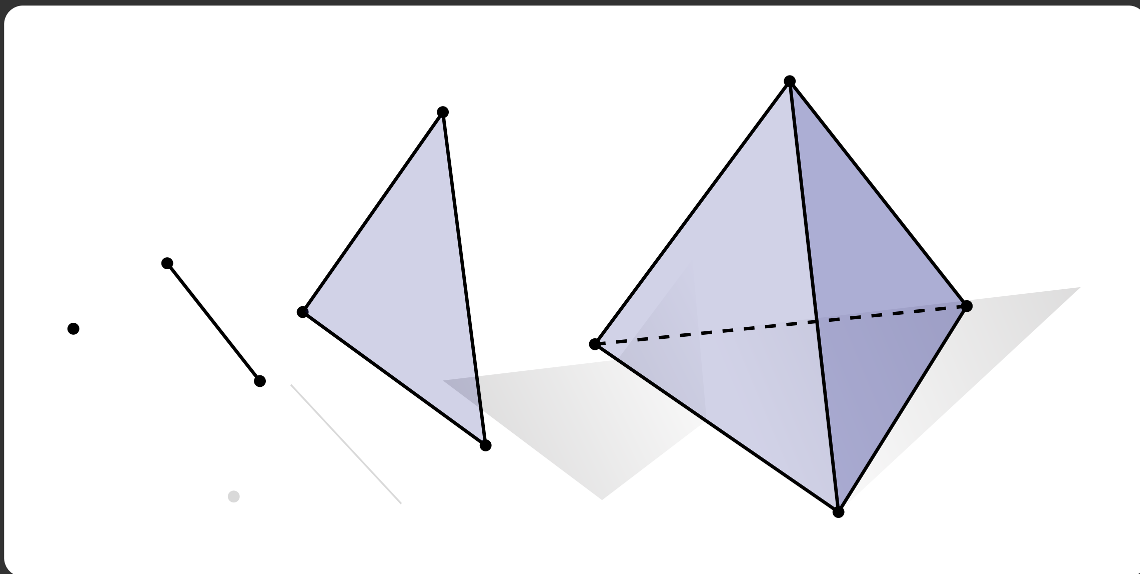
representing surfaces

triangle meshes



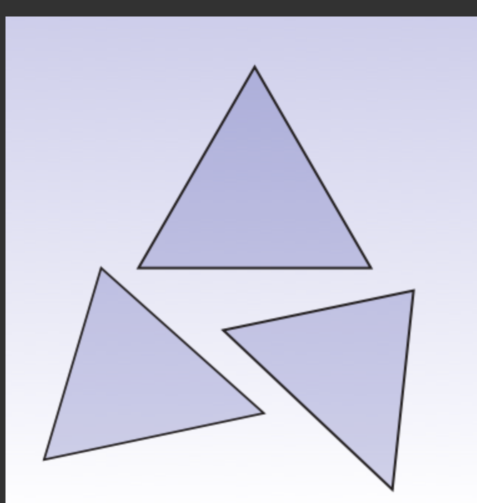
approximate the surface using triangles, "glued" together along edges

many possible data structures!

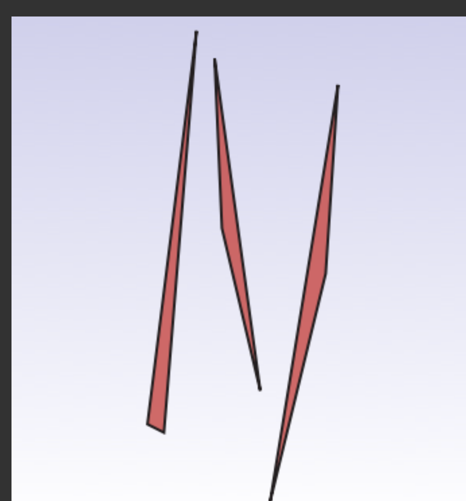


triangle quality

all triangles are good
(but some are better than others)



good triangles

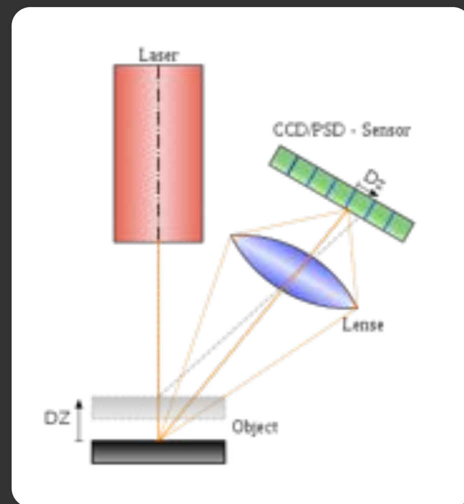


bad triangles

acquiring geometry

laser scanning

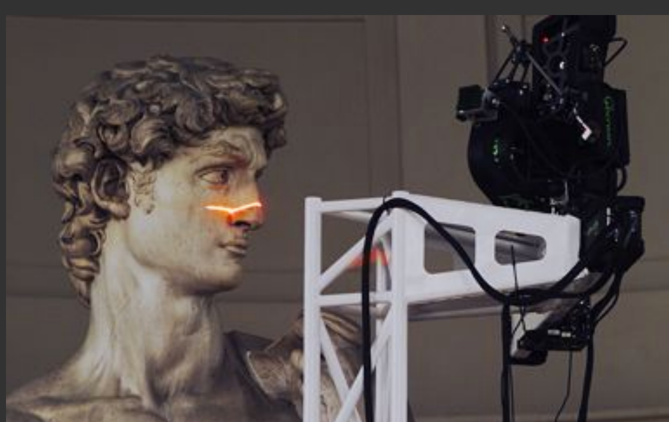
creaform handyscan 3d

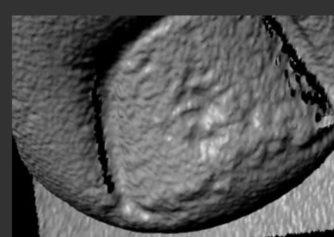


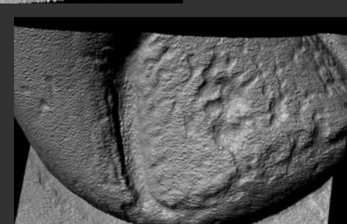
source: wikipedia

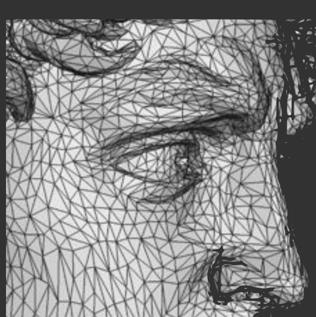
useful:
entertainment, manufacturing industries
design/prototyping, quality control, servicing and repairs

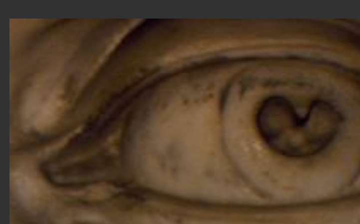
acquiring geometry

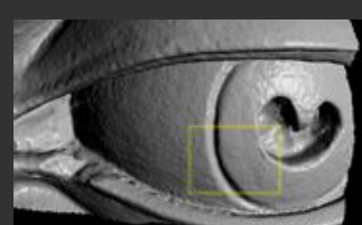








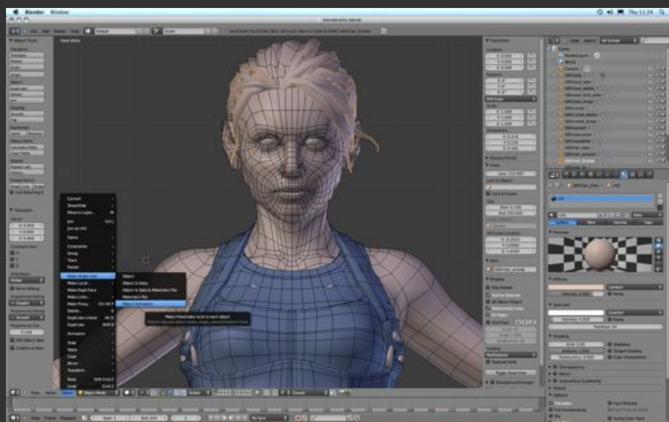




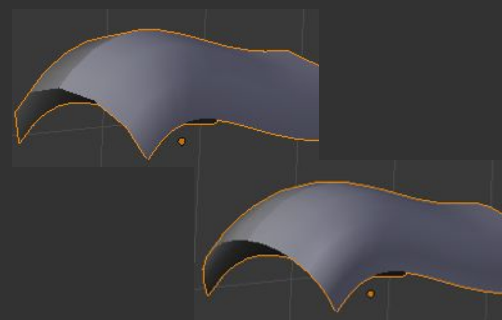
digital michelangelo project
http://graphics.stanford.edu/projects/mich/

acquiring geometry

polygonal modeling

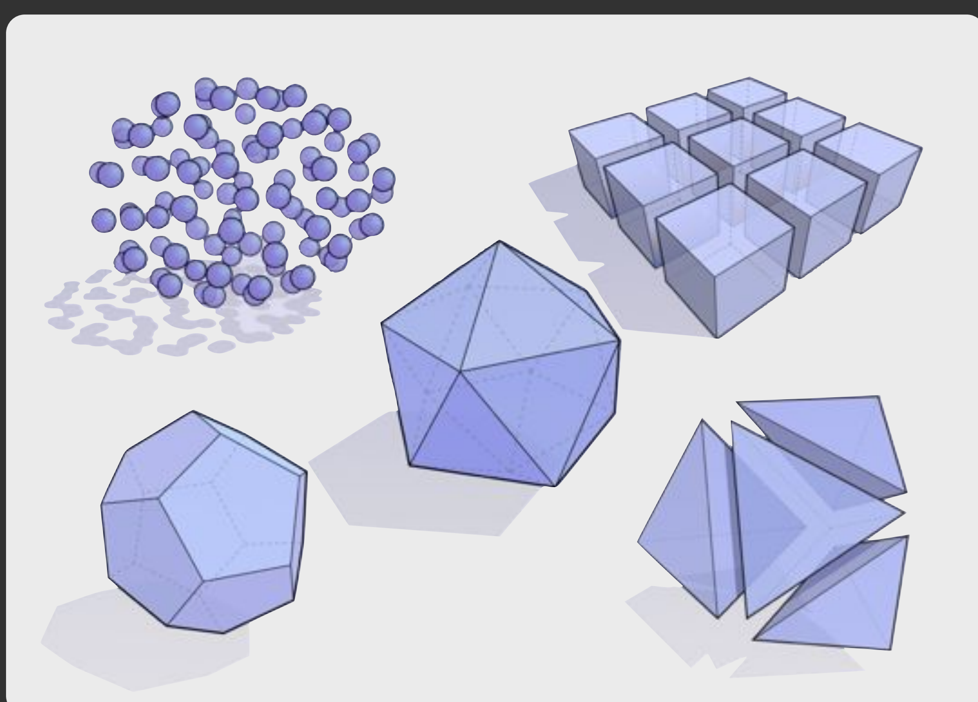


modeling with curves

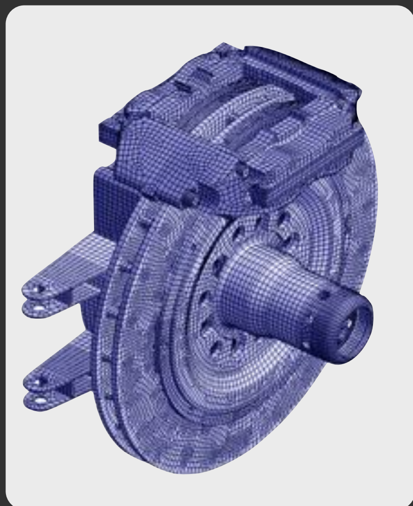


blender: open source modeling & animation; www.blender.org
commercial software: maya, 3ds, houdini, zbrush, autoCAD, ...

other surface discretizations

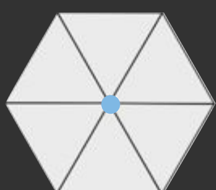


other surface discretizations

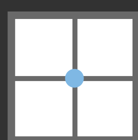


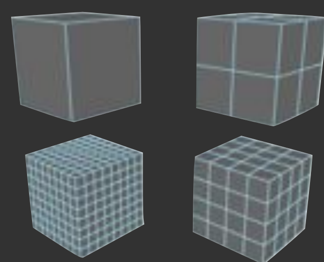
many artists prefer quad meshes:

- regular valence
- possible to follow curvature lines $\Longrightarrow$ easier edge/contour modification
- visualization of subdivision results
- UV coordinate placement
- easily convert to triangles



vs.





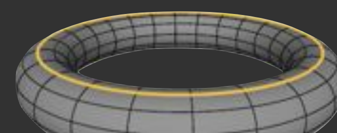


figure:
http://blog.digitaltutors.com/modeling-quads-triangles-use/

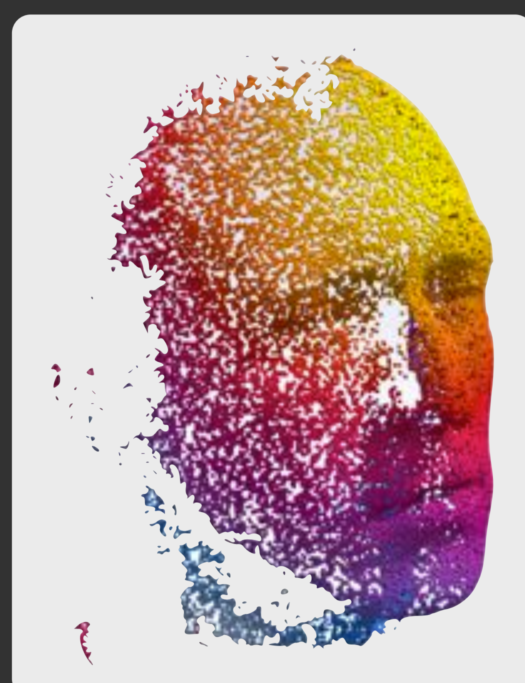
(other meshes may contain quads, triangles, ...)

other surface discretizations

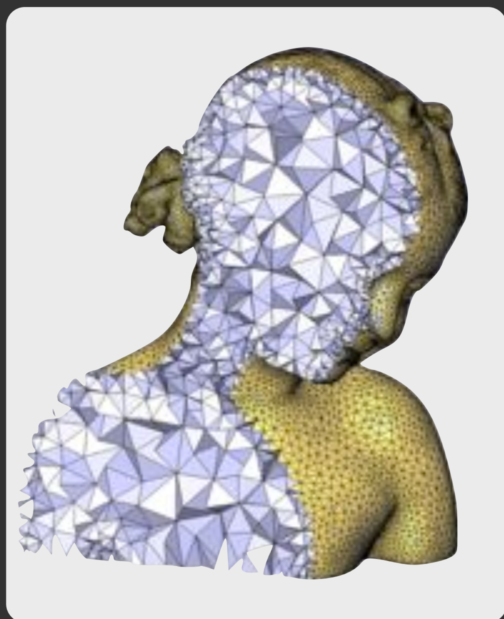
point clouds

real geometry data often originates from sample points with no real connectivity information (e.g., raw scans)

may contain: noise, holes, ...



other surface discretizations

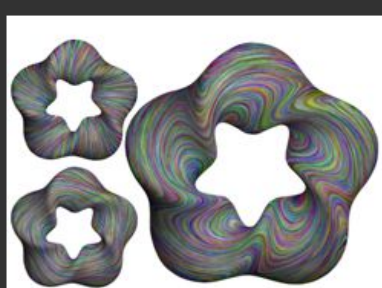


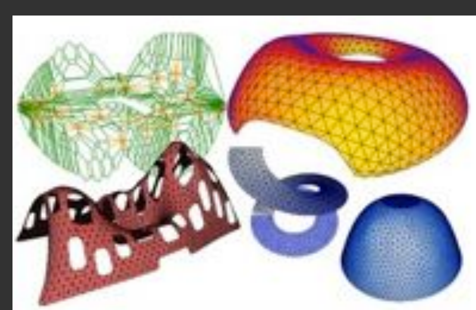
volume / tetrahedral meshes

lots more primitives to consider!

interior resolution is often graded

next up:





Fernando de Goes
PhD @ Caltech & MS @ UNICAMP
next: PDI DreamWorks

DreamWorks Animation SKG

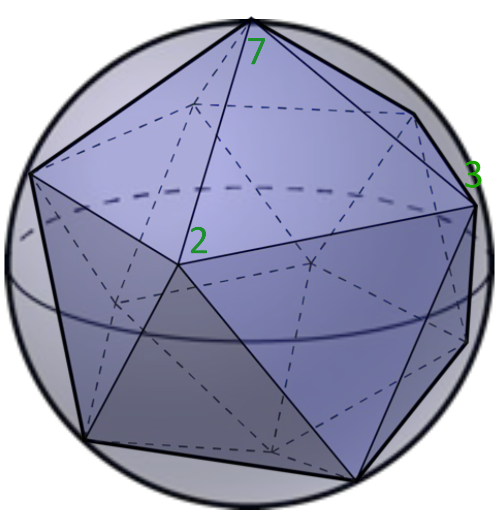
GEOMETRY & SURFACES

INTRO TO COMPUTER GRAPHICS & IMAGING

Fernando de Goes

CALTECH

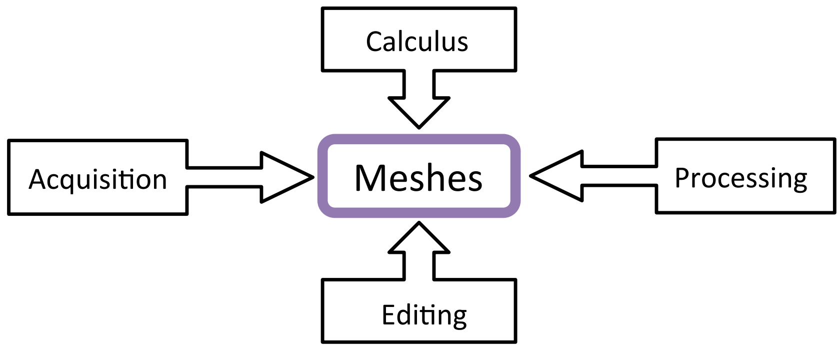
MESHES



v 0 -0.525731 0.850651
v 0.850651 0 0.525731
v 0.850651 0 -0.525731
…

f 2 3 7
f 2 8 3
f 4 5 6
…

OUTLINE

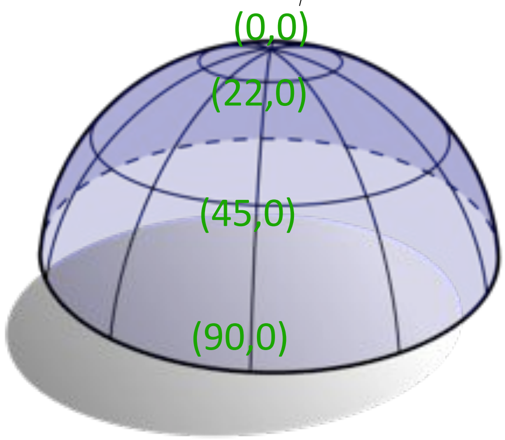


Acquisition

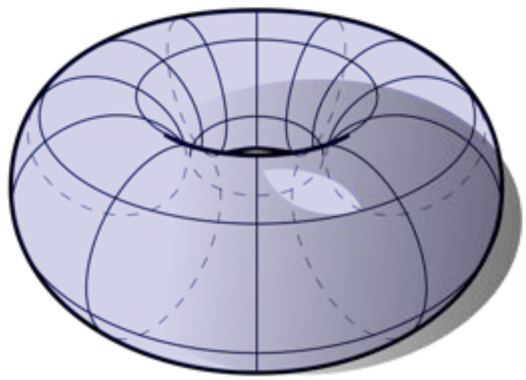
how to create meshes?

1. Sampling

$$x = \cos\theta \sin\phi$$
$$y = \sin\theta \sin\phi$$
$$z = \cos\phi$$

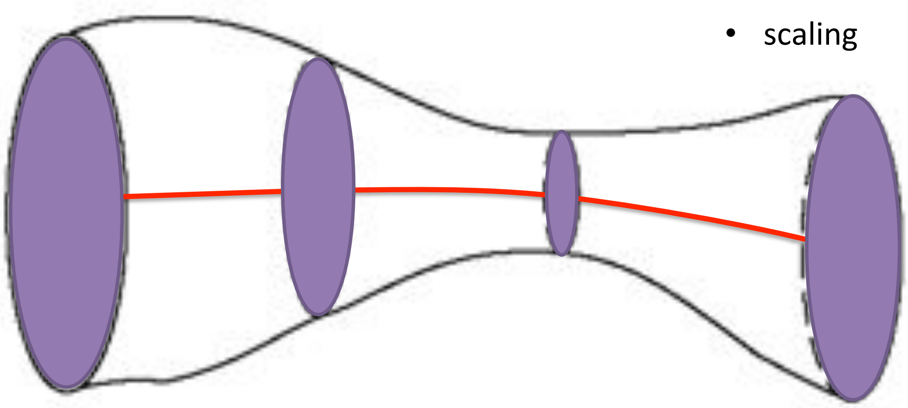


$$x = (R + r\cos\theta)\cos\phi$$
$$y = (R + r\cos\theta)\sin\phi$$
$$z = r\sin\theta$$



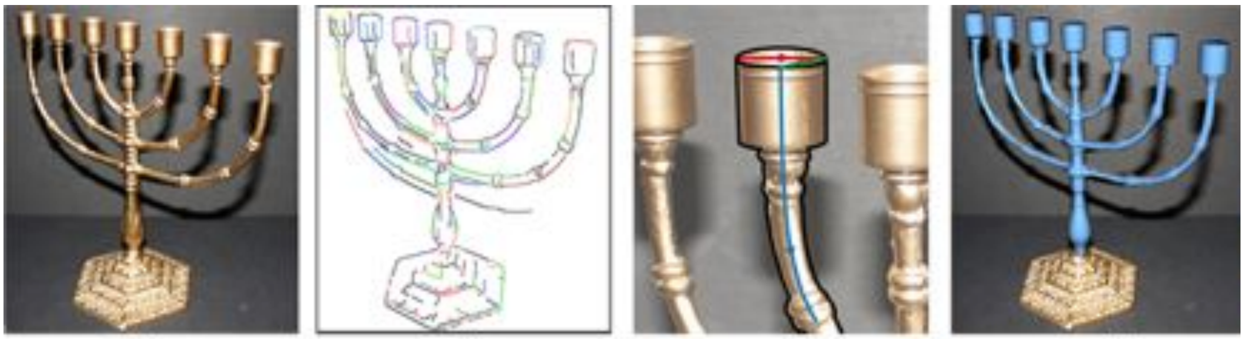
2. Sampling

- centerline
- cross section
- scaling



Example

Chen et al, Siggraph Asia 2013

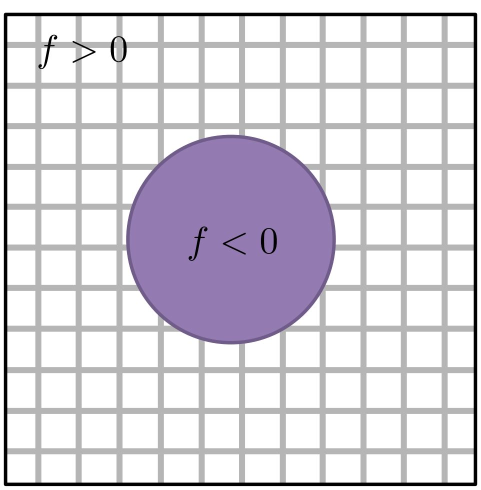


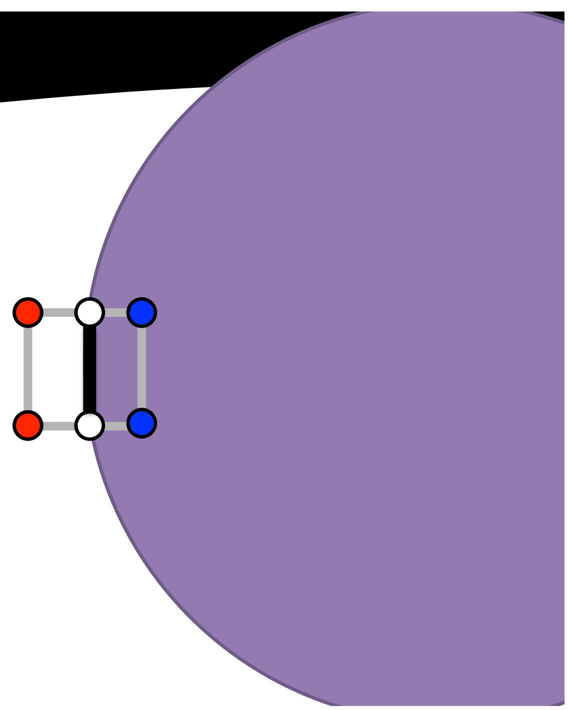
(a) (b) (c) (d)

video1

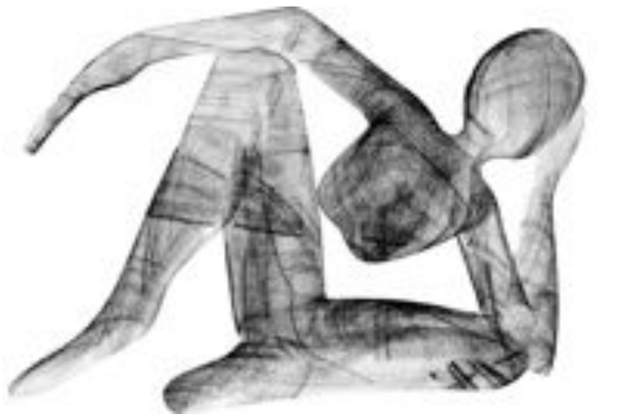
2. Contouring

$$f(x, y) = x^2 + y^2 - r^2$$



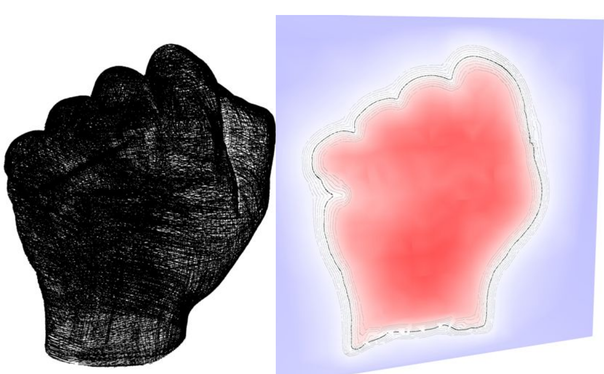


3. Scanning

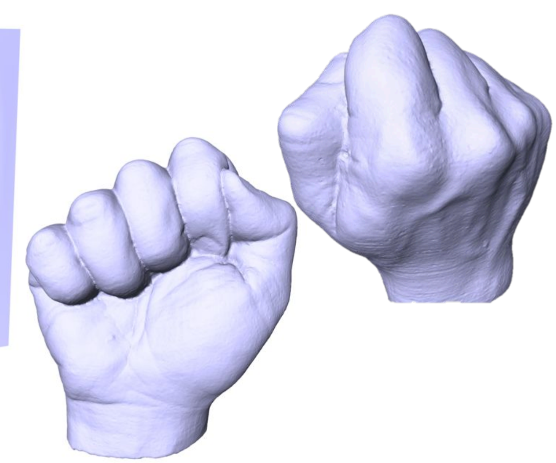


Example

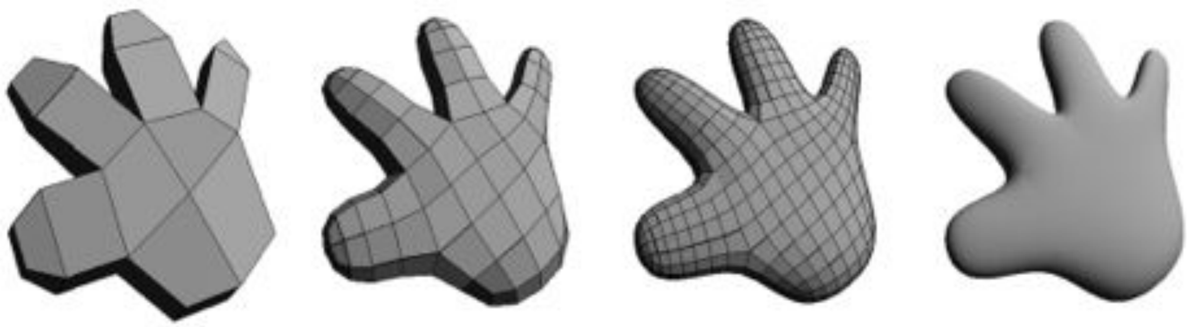
Kazhadan et al, SGP 2006
Mullen et al, SGP 2010



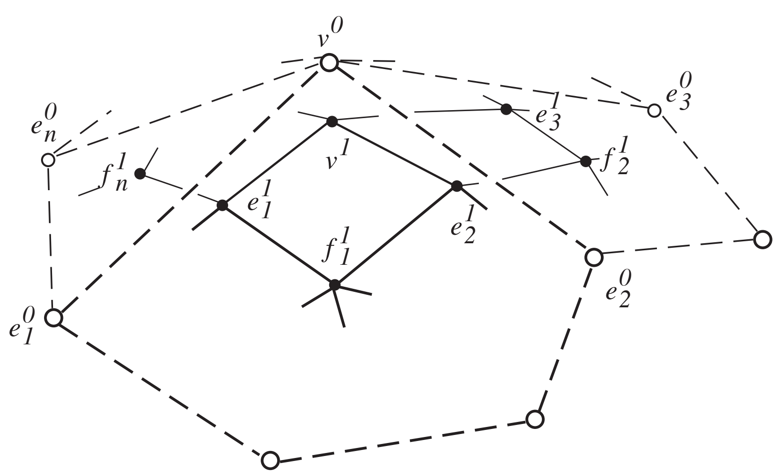
$$\min_f \|\nabla f - \mathbf{n}\|^2$$



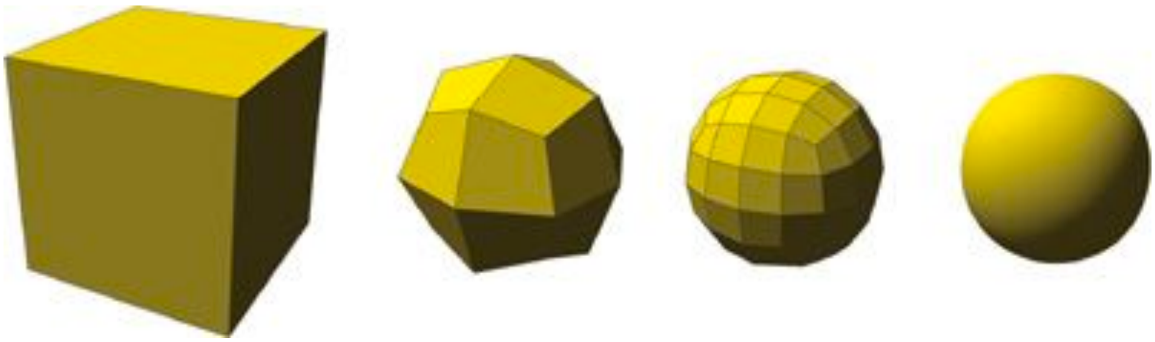
4. Subdividing



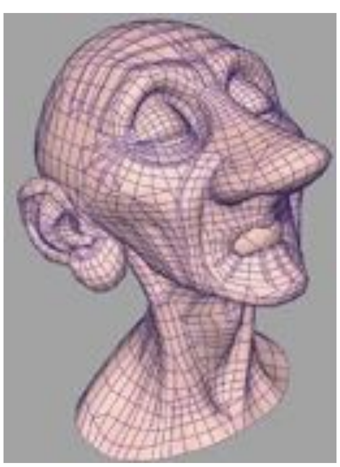
4. Subdividing

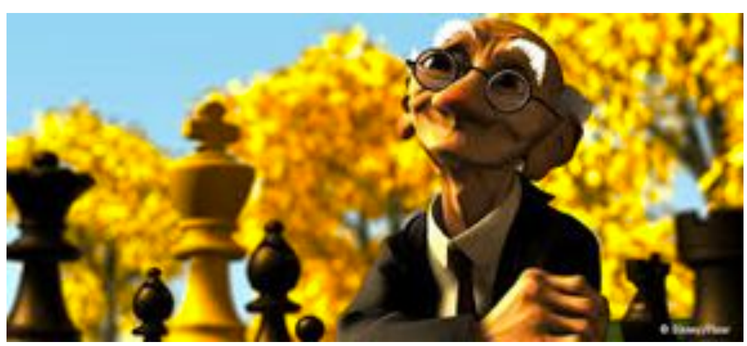


4. SUBDIVIDING



4. SUBDIVIDING



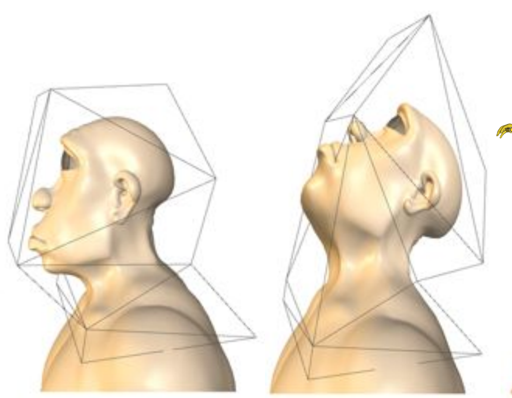


DeRose et al, Siggraph 1998

Editing
how to pose a mesh?

Geometry & Surfaces

1. CAGE / SKELETON ...



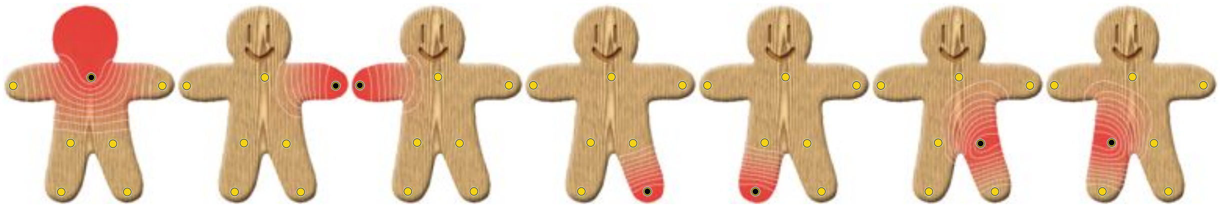
Lipman et al, Siggraph 2008

Jacobson et al, Siggraph 2012

video2

… RIGGING / SKINNING

Jacobson et al, Siggraph 2011



$$\underset{w_j,\ j=1,\ldots,m}{\arg\min} \sum_{j=1}^{m} \frac{1}{2} \int_{\Omega} \|\Delta w_j\|^2 dV \qquad (2)$$

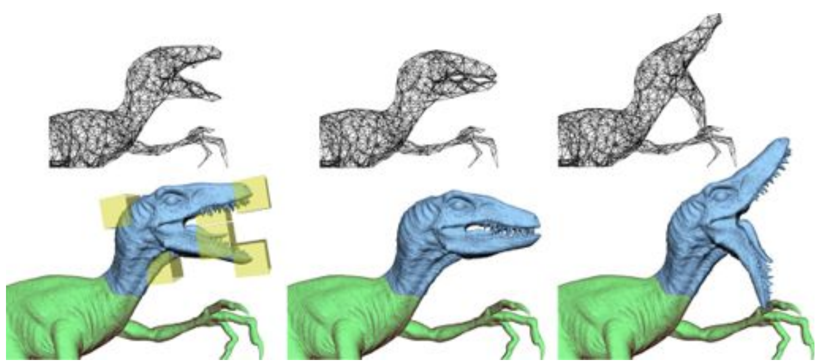
$$\text{subject to: } w_j|_{H_k} = \delta_{jk} \qquad (3)$$

$$w_j|_F \text{ is linear} \qquad \forall F \in \mathcal{F}_C \qquad (4)$$

$$\sum_{j=1}^{m} w_j(\mathbf{p}) = 1 \qquad \forall \mathbf{p} \in \Omega \qquad (5)$$

$$0 \le w_j(\mathbf{p}) \le 1,\ \ j = 1, \ldots, m, \quad \forall \mathbf{p} \in \Omega, \qquad (6)$$

2. AS-RIGID-AS-POSSIBLE



Original Edit 2 Edit 3

Summer et al, Eurographics 2007

video3

3. Structure-aware

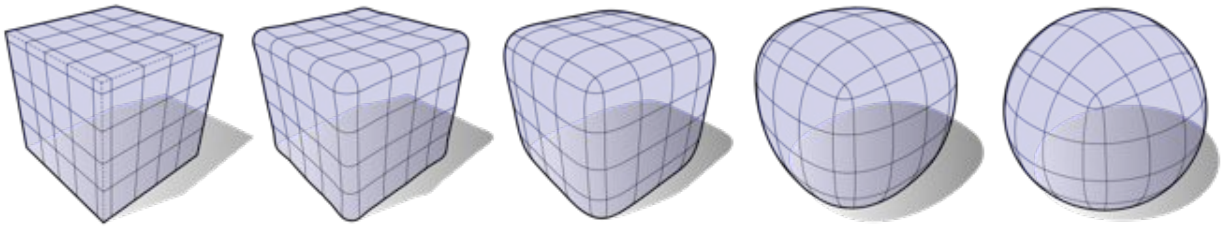
Gal et al, Siggraph 2009

video4

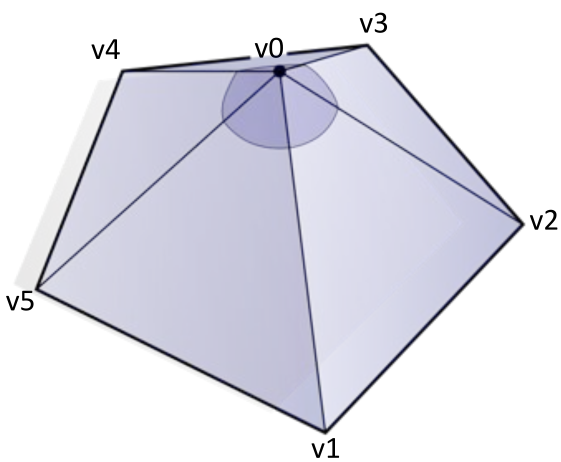
Processing

how to enhance meshes?

1. Smoothing



1. Smoothing



"averaging"

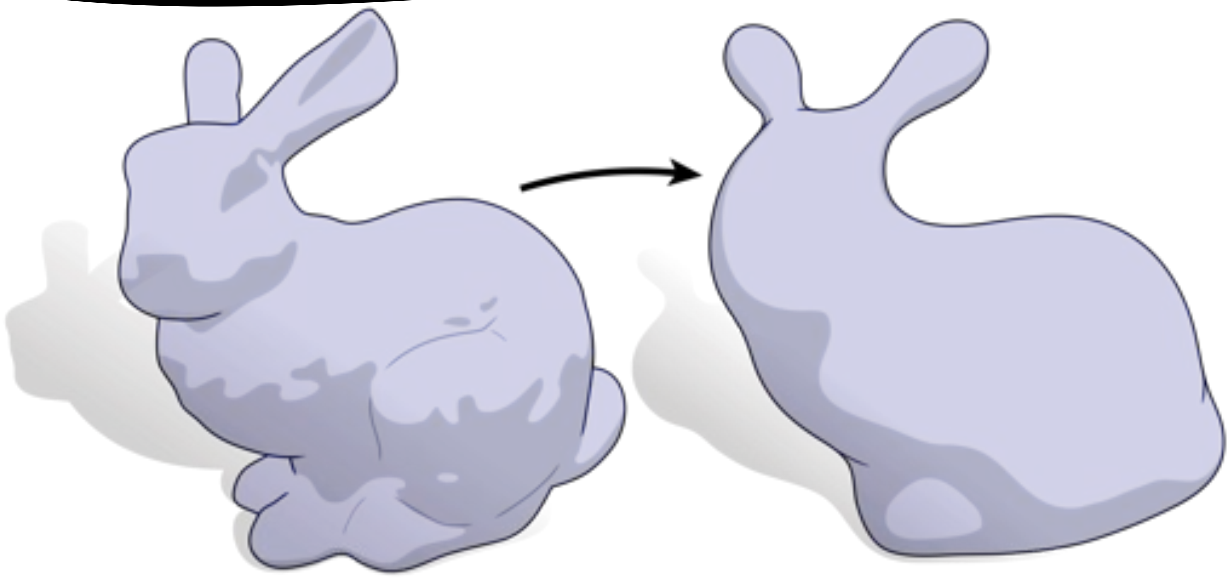
v0 = w0*v0 + w1*v1 + …

Desbrun et al, Siggraph 1999

follow area gradient => closed-form wi

curve regions moves faster

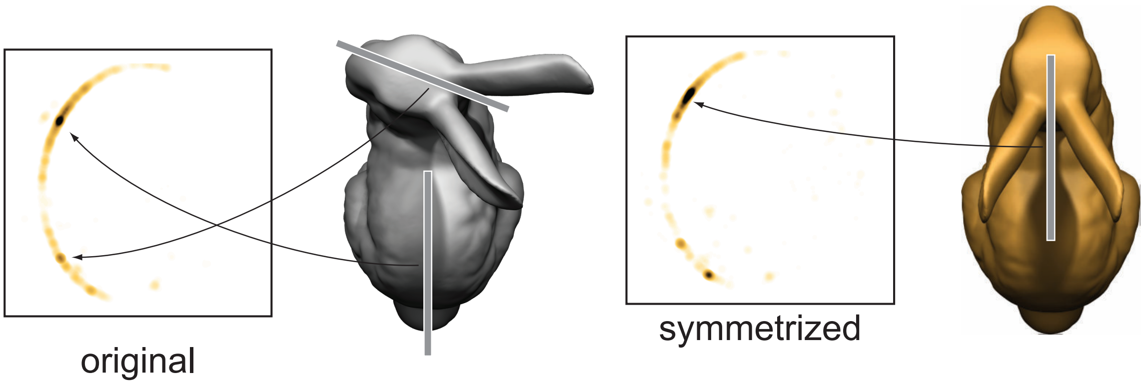
1. Smoothing



2. Symmetrize

2. Symmetrize

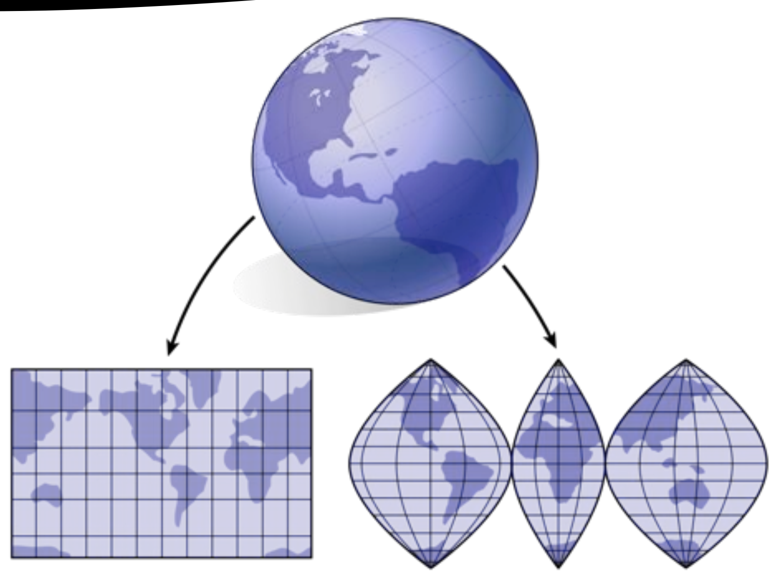
voting strategy



Mitra etl al, Siggraph 2007

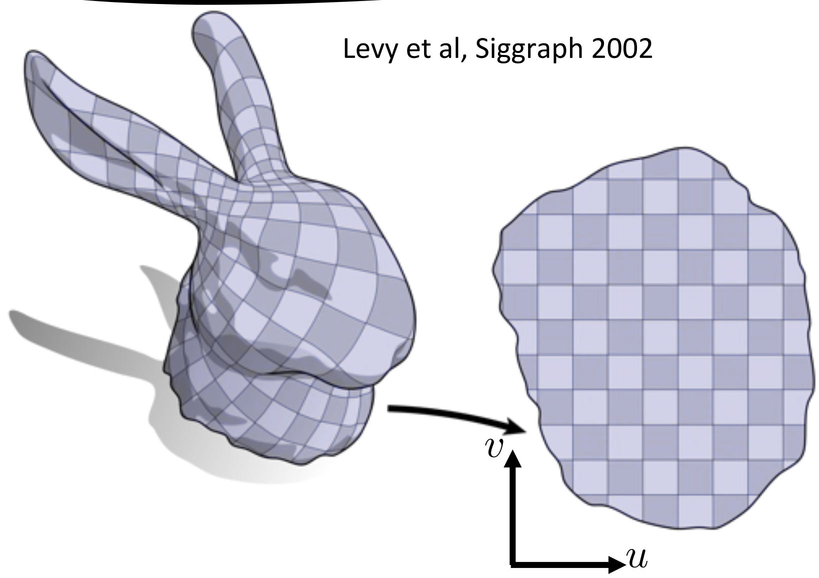
video5

3. Texture mapping



3. TEXTURE MAPPING

Levy et al, Siggraph 2002



as-orthogonal-as-possible

$$\min_{u,v} \|\nabla u - J \nabla v\|^2$$

$$\text{s.t. } u, v|_{\text{boundary}}$$

Calculus

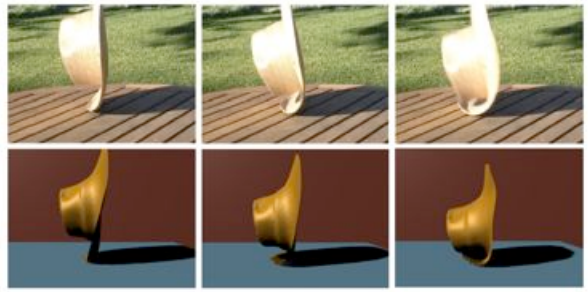
how to compute on meshes?

MOTIVATION

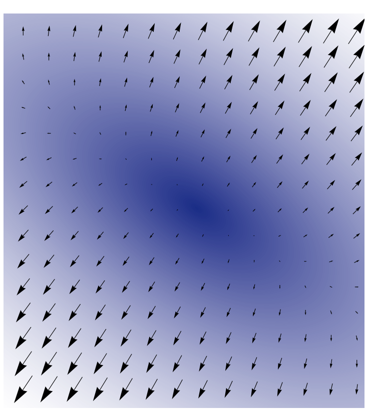
derivatives, integrals, PDEs, …

numerical solutions

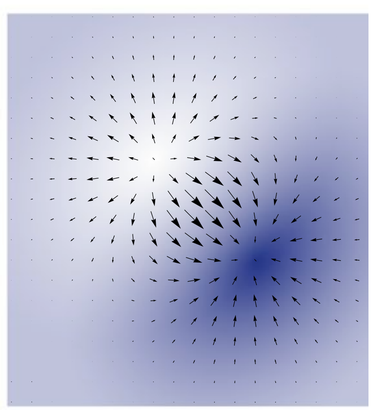
discrete calculus on meshes



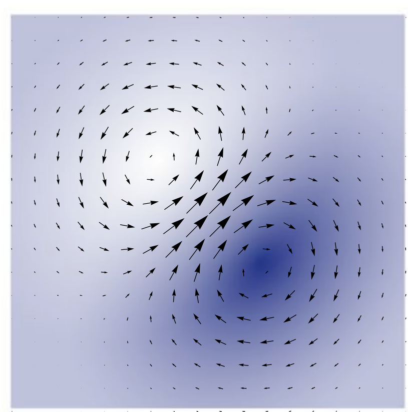
VECTOR CALCULUS



gradient



divergence

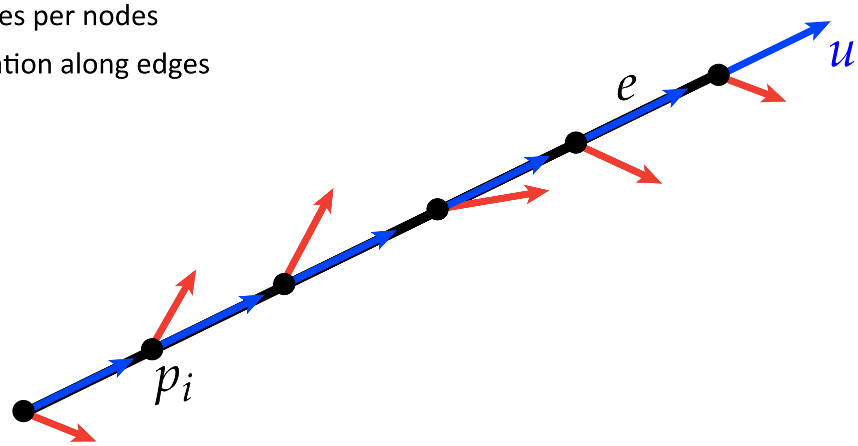


curl

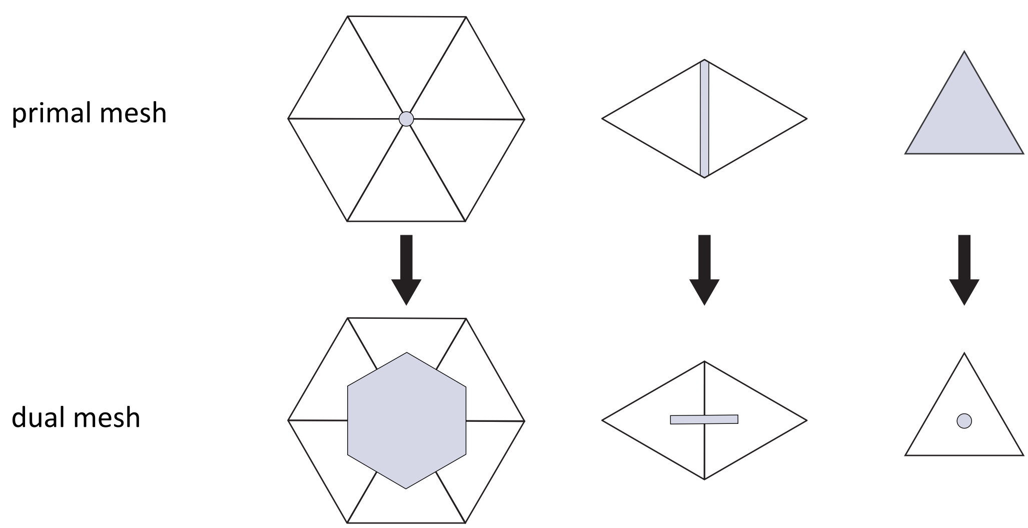
Discrete Functions / Vectors

functions => values per nodes

vectors => circulation along edges



Primal / Dual Meshes



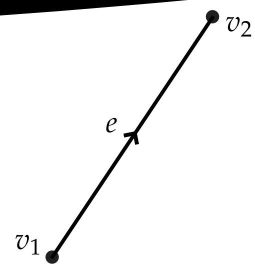
Discrete Operators

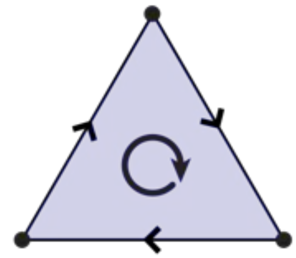
gradient => edge differences

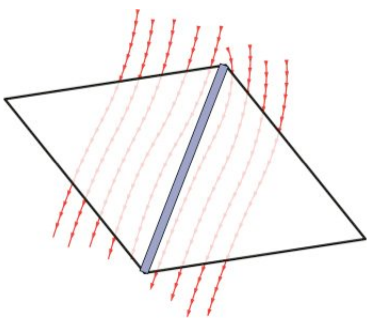
curl => sum of circulations

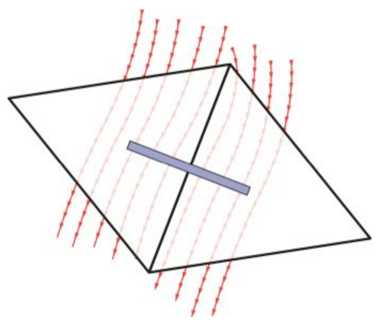
rotate => scale primal to dual

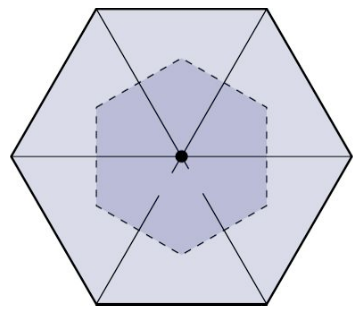
divergence => rotate + curl on dual



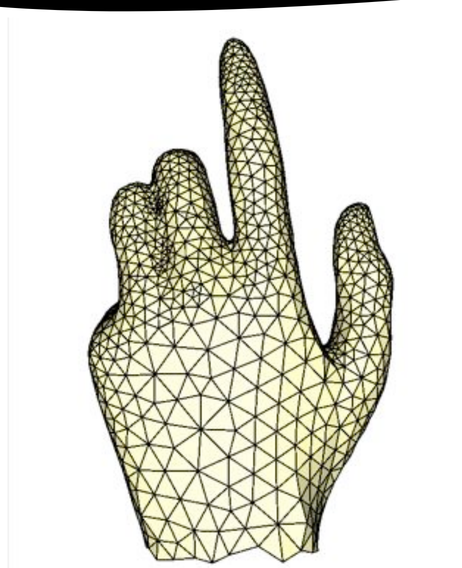


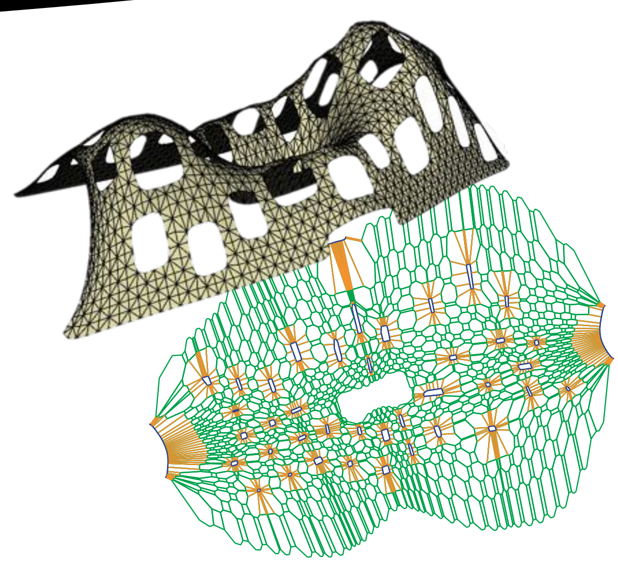




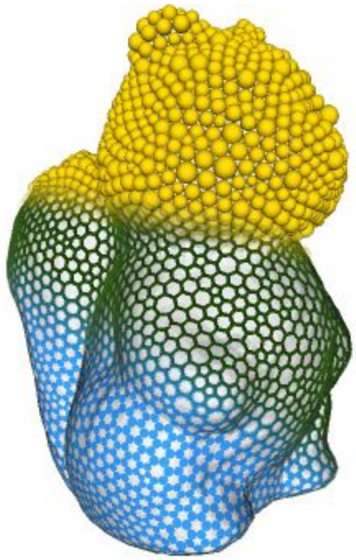


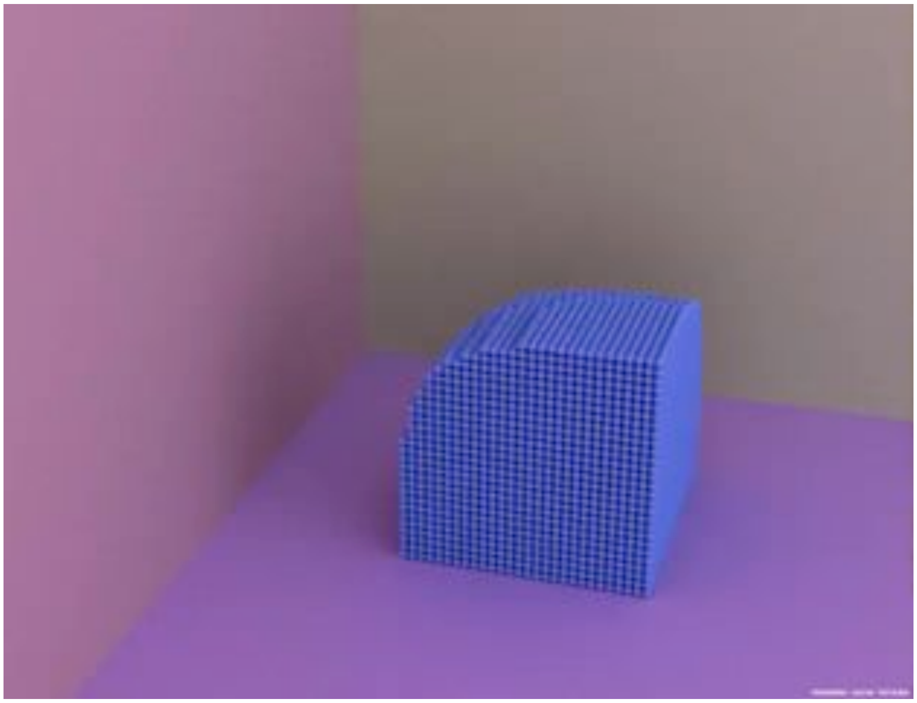
The power of Primal-Dual Meshes

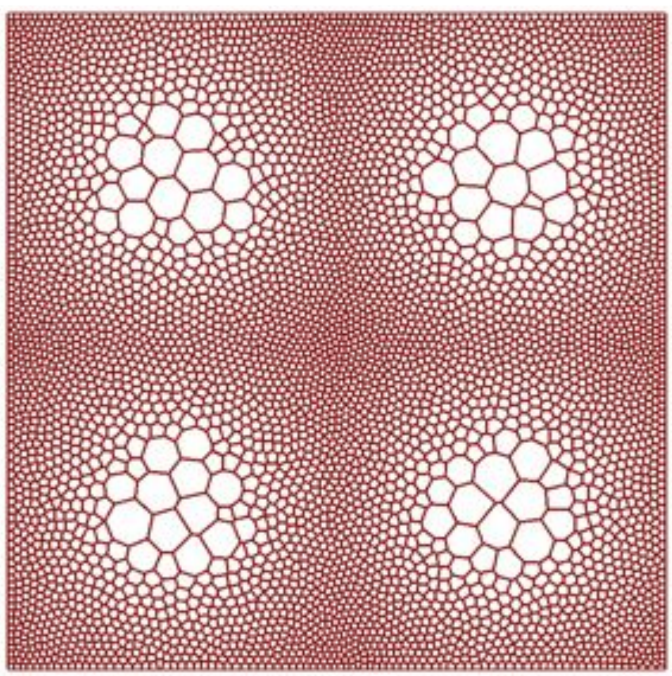




The power of Primal-Dual Meshes







A lot more!

check out Siggraph, SGP, Eurographics, …

Thank you!