1) $\vec{c}$ is the vector to the center of the sphere

$\vec{p}$ is a vector to a point on the sphere

the vector from $\vec{c}$ to $\vec{p}$ is $\vec{r} = \vec{p} - \vec{c}$, the radis

$$r = \sqrt{(\vec{p}-\vec{c})\cdot(\vec{p}-\vec{c})} \therefore$$

$$\boxed{\hat{n}_p = \frac{\vec{p}-\vec{c}}{\sqrt{(\vec{p}-\vec{c})\cdot(\vec{p}-\vec{c})}}}$$

2) Given $\vec{r}(t) = \vec{r}_0 + t\vec{r}_d$

$\vec{r}(t)$ intersects the sphere when $\vec{r}(t) = \vec{p} \therefore$

$$r^2 = (\vec{r}(t) - \vec{c}) \cdot (\vec{r}(t) - \vec{c})$$

$$r^2 = (\vec{r}_0 + t\vec{r}_d - \vec{c}) \cdot (\vec{r}_0 + t\vec{r}_d - \vec{c})$$

$$r^2 = t^2(\vec{r}_d \cdot \vec{r}_d) + 2t\vec{r}_d \cdot (\vec{r}_0 - \vec{c}) + (\vec{r}_0 - \vec{c}) \cdot (\vec{r}_0 - \vec{c})$$

$$\boxed{t^2(\vec{r}_d \cdot \vec{r}_d) + 2t\vec{r}_d \cdot (\vec{r}_0 - \vec{c}) + (\vec{r}_0 - \vec{c}) \cdot (\vec{r}_0 - \vec{c}) - r^2 = 0}$$

3) a) The ray misses the sphere

b) The ray intersectes at only one spot of the sphere. It just skims the surface

c) The ray intersectes at two places on the sphere. Once entering. Once exiting.

4) The smallest positive root since that will be the point at which the ray is entering the lens.

5) This form becomes a problem if $4ac$ is very small so that $b \approx \sqrt{b^2-4ac}$. The alternate method is used to avoid a loss of precision.

6) If the $\sin(\theta_t) > 1$ then the ray is reflected instead of transmitted. The rays from the tree are being reflected off the surface of the water.

7) $\vec{r}_t = \vec{r}_{tp} + \vec{r}_{tn}$ and $\vec{r}_i = \vec{r}_{ip} + \vec{r}_{in}$

$|\vec{r}_t| = |\vec{r}_i| = 1 \;\therefore$

$\vec{r}_t = \vec{r}_t + \cos\theta_t \hat{n} - \cos\theta_t \hat{n}$ and $\vec{r}_i = \vec{r}_i + \cos\theta_i \hat{n} - \cos\theta_i \hat{n}$

$\vec{r}_{tn}$) $\vec{r}_{tn} = -\cos\theta_t \hat{n}$

from Snell's Law: $\sin^2\theta_t = \left(\frac{n_i}{n_t}\right)^2 \sin^2\theta_i$

with: $\sin^2\theta_t = 1 - \cos^2\theta_t$

gives: $1 - \cos^2\theta_t = \left(\frac{n_i}{n_t}\right)^2 \sin^2\theta_i$ or,

$\cos\theta_t = \sqrt{1 - \left(\frac{n_i}{n_t}\right)^2 \sin^2\theta} \;\therefore$

$\vec{r}_{tn} = -\sqrt{1 - \left(\frac{n_i}{n_t}\right)^2 \sin^2\theta}\; \hat{n}$

$\vec{r}_{tp}$) from Snell's Law: $\vec{r}_{tp} = \frac{n_i}{n_t} \vec{r}_{ip}$

$\vec{r}_{tp} = \frac{n_i}{n_t}(\vec{r}_i + \cos\theta_i \hat{n})$

$$\boxed{\vec{r}_t = \left(\frac{n_i}{n_t}\right) \vec{r}_i + \left(\frac{n_i}{n_t}\cos\theta_i - \sqrt{1 - \left(\frac{n_i}{n_t}\right)^2 \sin^2\theta_i}\right) \hat{n}}$$

8) Top Left: Gauss
Top Right: Fisheye
Bottom Left: Wide Angle
Bottom Right: Telephoto

9) User focus distance that gets larger going top to bottom.