Principles of Mathematical Analysis
Chapter Problems, Theorems, and Examples

Dustin Smith

Contents

1 The Real and Complex Number Systems

Prove that there is no rational p such that $p^2 = 2$.

Suppose p is rational. Then $p = m/n$ where $m, n \in \mathbb{Z}$ such that both m and n are not even.

$$\begin{aligned} p^2 &= \frac{m^2}{n^2} = 2 \\ &= m^2 \ = 2n^2 \end{aligned} \tag{1.1}$$

Therefore, since $m^2 = 2n^2$, m^2 is even and m is even. Since m is even, we can write $m = 2a$ so $m^2 = 4a^2$ and m^2 is divisible by 4. Now that the left hand side of equation (1.1) is divisible by 4 so is the right hand side. Thus, we have reached a contradiction since this would imply both m and n are even.