

Temporality of the Victorian Hospital Patient Transfer Network

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Introduction

Antimicrobial resistance poses a great threat to human health and development. [1] Globally, it poses a large burden, operationally and economically, on hospital systems. In Australia, we see [significant] economic burden from AMR. this impact could become more significant if we had incursion and subsequent establishment of new [strains] of AMR. One X of interest, for example, is carbapenemase-producing enterobacteriaceae (CPE). Australia has an endemic strain of CPE – IMP4 – but this is at a relatively low level. This already causes some level of burden, and introduced strains would further multiply this.

The spread of AMR in these systems has been studied by modelling the hospital system as a network or interconnected healthcare facilities. For example, [2] examines hospitals in the Netherlands to study the spread of MRSA, followed by a similar analysis of the UK hospital system in [3]. Ultimately, the aim of studying these systems as networks is leveraging the network framework when designing surveillance and control protocols for emerging and recurring AMR outbreaks. This has been analysed to some extent in [4], for example; but such analyses use

There are existing results that show that accounting for temporality in network models results in lower control costs [5]. In the public health space, a lot of attention has been spent on patient-staff and staff-staff contacts within a hospital, for example in [6, 7].

Perhaps most pertinently, [?] explores patient referral networks using a temporal network methodology, and also consider the effect of community stays by augmenting their temporal network with additional edges corresponding to readmissions that occur within a period of k days (representative of the infectious period). Here, we will show that

Methods

Setting

Victoria, Australia, has a population of 7.0 million people [8]. Its hospital system consists of $N_x = 338$ healthcare facilities that vary in size, case mix and services. Admissions data was sourced from the Victorian Admitted Episodes Dataset (VAED) [9]. In particular, data from 1 January 2011 to 9 December 2020 (3631 days) was used in this study. All Victorian hospital admissions during this study period are included. The total number of admission records is 26876787. This data was aggregated into time-discretised temporal networks (described below), and exported from the database; further analysis as presented in this paper is performed on external machines with no access to identifiable protected health information.

Some exploratory analysis of this data was done in [10] using a static network framework.

Network models of patient transfers for temporal fidelity

In this paper, we investigate these differences using the Victorian hospital system as a baseline for comparison. Qualitatively, there are mechanisms implicit in the data that are not previously well-modelled, and may be of importance to constructing robust control measures. Critically, we notice that the distribution of readmission times (the time from a discharge of a given patient from a facility to next admission of that patient at any facility), as plotted in Fig 1, are not well modelled by an exponential distribution, i.e. poorly approximated by Markov processes.

A weighted, directed network can be used to construct a Markov chain that in turn can be analysed or simulated to understand the dynamics of a spreading infectious disease. For each edge e_{ij} with a number of observed movements w_{ij} over some time period τ , we can construct a reaction that has a hazard function $\lambda_{ij} = w_{ij}/\tau$. This implicitly constructs a process that has inter-event times (i.e. times between distinct patient movements) that are exponentially distributed $\text{Exp}(\lambda_{ij})$.

Here, u and v are locations (potentially identical locations), s and t are times. We take that $t \geq s$. \emptyset represents patients that are not observed to return to the hospital

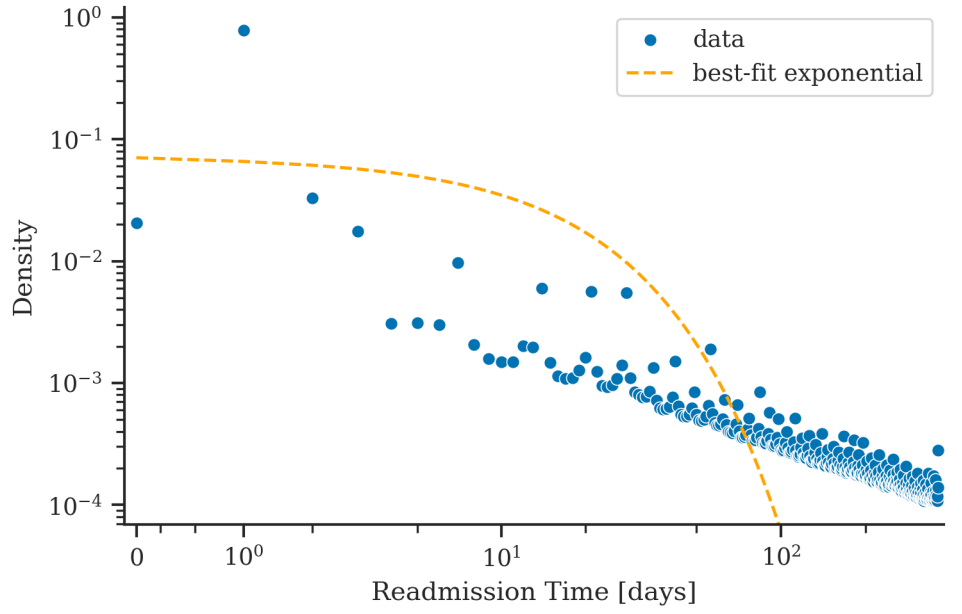


Figure 1. Distribution of empirical readmission times (time from discharge to next admission), truncated at 365 days; and best-fit exponential (via MLE).

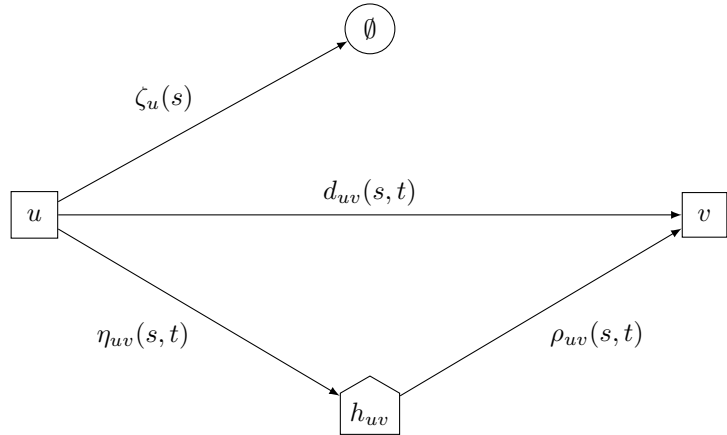


Figure 2. Sketch of the generalised model with processes of (1) permanent discharge ($u \rightarrow \emptyset$), (2) direct transfer ($u \rightarrow v$), (3) return home ($u \rightarrow h_{uv}$), (4) readmission ($h_{uv} \rightarrow v$).

system after their discharge from location u . We assume the associated hazard for this process $(\zeta_u(t))$ to be known for all locations u at all times t : the modelling of this process is the same for all network models, and will be neglected in the following.

Further, we define the function $w_{uv}(s, t)$ to be the number of individuals that are observed to discharge from u at time s and readmit at time t . We allow a maximum of one of the pairs u, v and s, t to be reflexive, i.e. $u = v$ or $s = t$, but not both.

We will also assume that we will have quantities N_u that represent the “size” of a location u . We assume that all locations u will have N_u individuals in them at all times, and that the observed movements $w_{uv}(s, t)$ can be thought of as being scaled by N_u .

The governing equations can be roughly expressed as a function of the hazards of each process.

$$\lambda(u(s) \rightarrow \emptyset) = \zeta_u(s)$$

$$\lambda(u(s) \rightarrow v(t)) = d_{uv}(s, t)$$

$$\lambda(u(s) \rightarrow z_{uv}(s, t)) = \eta_{uv}(s, t)$$

$$\lambda(z_{uv}(s, t) \rightarrow v(t)) = \rho_{uv}(s, t)$$

We note that as a consequence of the model structure, the hazards can also be interpreted as modelling the holding time an individual patient spends in each state.

Naïve Static Model

We can begin with a simple naive static model. This model will have the edge weights between healthcare facilities that represent the number of transfers from the source node to the target node over a certain period of observation time (T_Σ) , regardless of how long the patient is absent from the healthcare facilities between initial discharge and subsequent readmission.

With reference to the general model above, the hazards of the naïve static model are:

$$d_{uv}(s, t) = d_{uv} = \frac{\sum_{s, t} w_{uv}(s, t)}{T_\Sigma N_u}$$

$$\eta_{uv}(s, t) = \eta_{uv} = 0$$

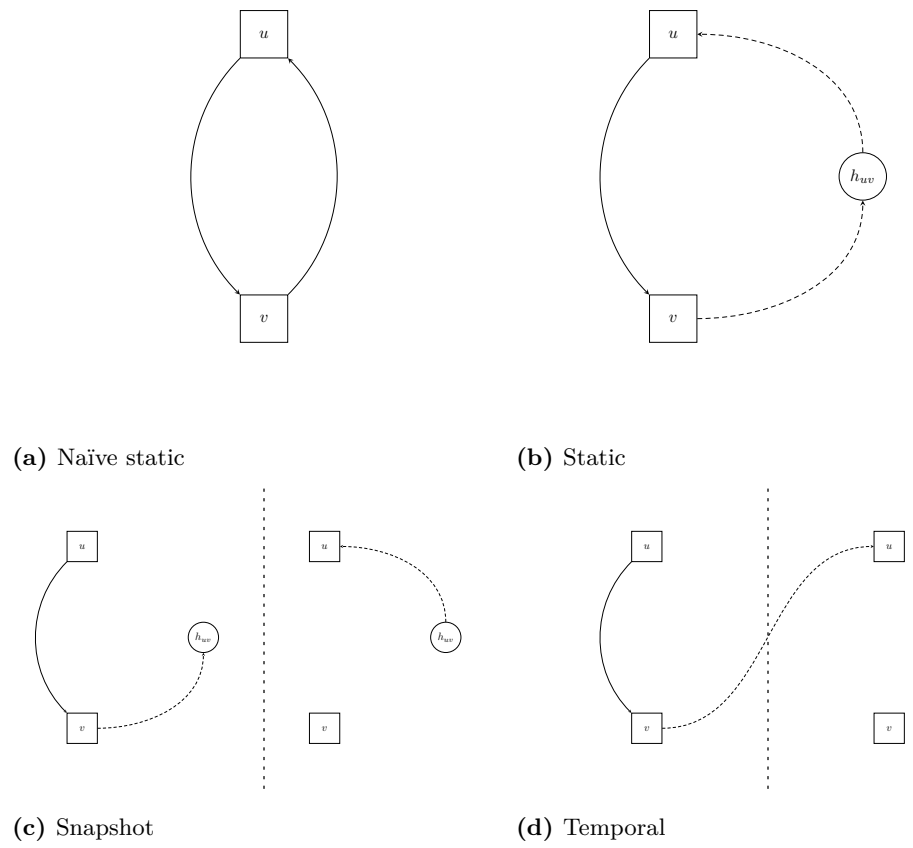


Figure 3. Schematic diagrams of each type of network representation for a two-facility network. Solid arrows represent direct transfers, dashed arrows represent indirect transfers, and wavy lines represent temporal consistency.

$$\rho_{uv}(s, t) = \rho_{uv} = 0$$

That is, all movements are instantaneous, and the rate of movement is the mean rate of movement over the entire observation period. This is an oversimplification of the work done in [?], but represents a “worst” case model of temporal fidelity that can be benchmarked against.

Improved Static Model

We can introduce the concept of “indirectness” by allowing individuals to return home, in an “improved” static model. In this model, we choose some threshold value ω that delineates direct transfers that occur “instantaneously” and indirect transfers that require a patient to first move to an intermediary “home” state before readmitting.

In comparison to the naive model, we will have non-zero hazards η and ρ :

$$\begin{aligned} d_{uv}(s, t) &= d_{uv} = \frac{\sum_{s, t: (t-s) < \omega} w_{uv}(s, t)}{T_{\Sigma} N_u} \\ \eta_{uv}(s, t) &= \eta_{uv} = \frac{\sum_{s, t: (t-s) \geq \omega} w_{uv}(s, t)}{T_{\Sigma} N_u} \\ \rho_{uv}(s, t) &= \rho_{uv} = \left[\frac{\sum_{s, t: (t-s) \geq \omega} (t-s) w_{uv}(s, t)}{\sum_{s, t: (t-s) \geq \omega} w_{uv}(s, t)} \right]^{-1} \end{aligned}$$

Here, we model the rate of indirect transfers η similarly to the rate of direct transfers d , by counting the average rate of observed movements. For the rate of readmission, ρ , we use the inverse of the mean readmission duration/delay.

Snapshot Model

Of course, as the hospital system changes over time, the rate of patient transfers, and thus the hazards represented in our model should also change. We see evidence of this in Fig 4 where we plot the overall rates of movement in the network over time.

We approach this using static network snapshots of the hospital system over time.

For the snapshot model, we choose a snapshot duration ω . This defines the threshold duration of an indirect transfer. This introduces an new intermediary state variable z'

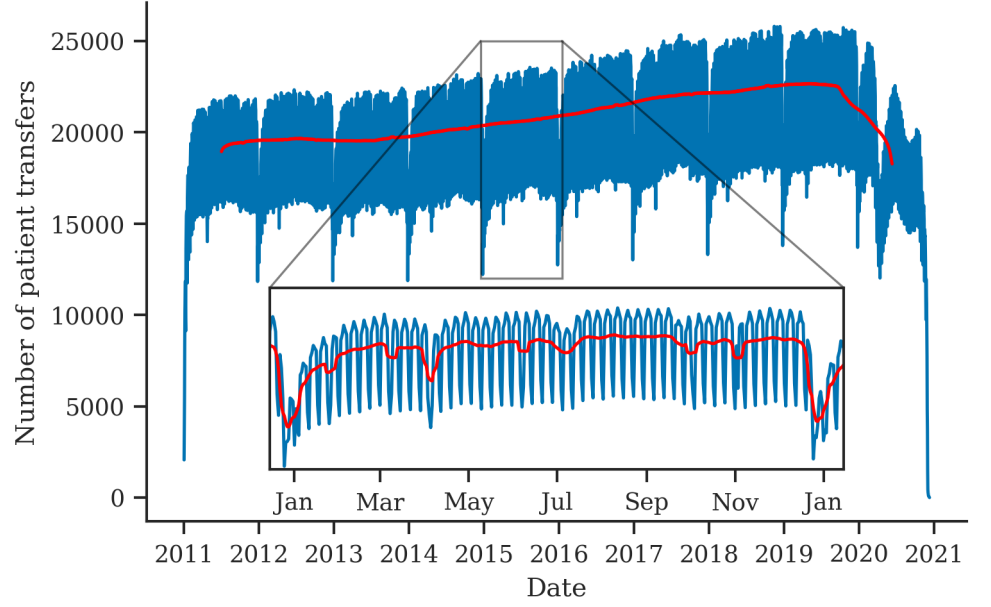


Figure 4. Number of patient transfers throughout the network increases over time. Moving averages plotted in red with periods of 1 year (main plot) and 1 week (inset). Right-hand end of data exhibits censoring behaviour, since we do not see long-terms readmissions beyond the end of the data observation period. We also observe seasonal behaviour (most visually obvious with periods of a year and a week), and consistent decreases of transfers during public holiday periods, especially at the end of the year.

that contains the individuals that would enter z within the duration of a given snapshot 98
 $[t, t + \omega]$, so that they do not immediately readmit at their next healthcare facility. This 99
alters the indirect transfer processes to 100

$$\lambda(u(s) \rightarrow z'_{uv}(s)) = \sum_t \eta_{uv}(s, t)$$

$$\lambda(z'_{uv}(t) \rightarrow z_{uv}(t)) = \delta(t \bmod \omega)$$

where $\delta(\cdot)$ is the Dirac delta function. Alongside this, we also compute from the data, 101
the number of patients “at home” at a given time, in order to inform the denominator 102
for the process from z' to z . We denote this quantity $H_{uv}(t)$ We will have that: 103

$$d_{uv}(s, t) = d_{uv}(s) = \frac{\sum_{t:(t-s) < \omega} w_{uv}(s, t)}{\omega N_u}$$

$$\eta_{uv}(s, t) = \eta_{uv}(s) = \frac{\sum_{t:(t-s) \geq \omega} w_{uv}(s, t)}{\omega N_u}$$

$$\rho_{uv}(s, t) = \rho_{uv}(t) = \frac{\sum_{s:(t-s) \geq \omega} w_{uv}(s, t)}{\omega H_{uv}} \quad 105$$

Temporal Model 106

For the temporal network, we make a choice of the time discretisation ω . Events that occur within a time window $[\tau, \tau + \omega)$ are collapsed and aggregated to occur at time τ . Over the observation (and simulation) period, we will have a series of time window boundaries $\{\tau_0, \tau_1, \dots\}$. We denote the smallest value in that set larger than a given time t to be $\lceil t \rceil_\omega$. For the readmission process, we assume that individuals that would readmit within a given time window $[\tau, \tau + \omega)$ do so uniformly. This yields: 112

$$d_{uv}(s, t) = \frac{\sum_{t:(t-s) < \omega} w_{uv}(s, t)}{\omega N_u} \quad 113$$

$$\eta_{uv}(s, t) = \frac{\sum_{t:(t-s) \geq \omega} w_{uv}(s, t)}{\omega N_u} \quad 114$$

$$\rho_{uv}(s, t) = \frac{1}{\lceil t \rceil_\omega - t} \quad 115$$

Results 115

Simulation Study 116

Since analytical results are intractable for the more complex network models, we resort to simulation as a statistical proxy. Movements of marked individuals are tracked for each network type, when they are seeded at each facility in the network at $t = 0$. The hitting time k_j to each other healthcare facility is recorded. We mark $N_0 = 30$ individuals at the start of each realisation; we generate $N_r = 20$ realisations for each seed facility. 122

We plot the (empirical) cumulative distribution function of the hitting times for all pairs of seed facility and target facility in Fig 5, with a zoom of the early-time distribution in Fig 6. We see that the naive static network model produces a hitting time distribution that has a large amount of movement of the marked individuals in a short amount of time, and that the full temporal network model produces much less 127

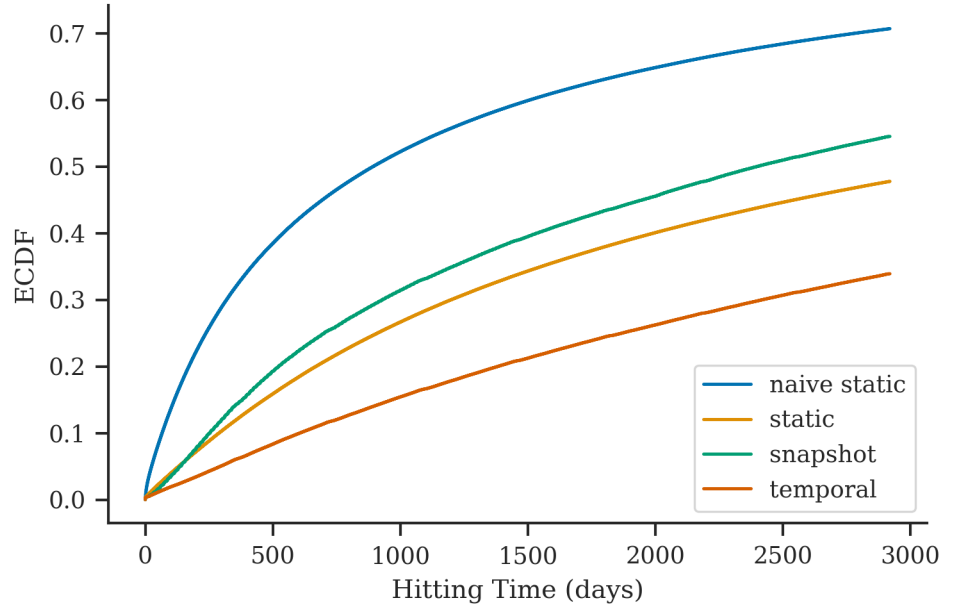


Figure 5. Empirical cumulative distributions of the hitting times between any pair of facilities in the network, for different network models. $N_0 = 30$ individuals are seeded initially, all at one facility, and then allowed to propagate through the network.

When comparing the aggregate number of individuals present or not present in the hospital system between models, we see some striking differences. The naïve static model has no capability for discharging individuals out of the system, so is significantly different to the other network models. For the other models, after some burn-in period, they reach a (pseudo-)steady state. Notable features: - the band at $N=0$ for the snapshot and temporal network models, where some individuals do not discharge - this is usually due to their seed hospital not existing until sometime much later in the simulation. This cannot be modelled with static networks, so instead in the static network, there are bands of lagging that correspond to lower global rates of discharge. - the differing values of steady state for each model. Temporal model has the most at home, followed by the static model, and then the snapshot model. - it seems the mechanisms for readmission in the snapshot model lead to periodic bursty readmissions, which results in a lower number of individuals at home.

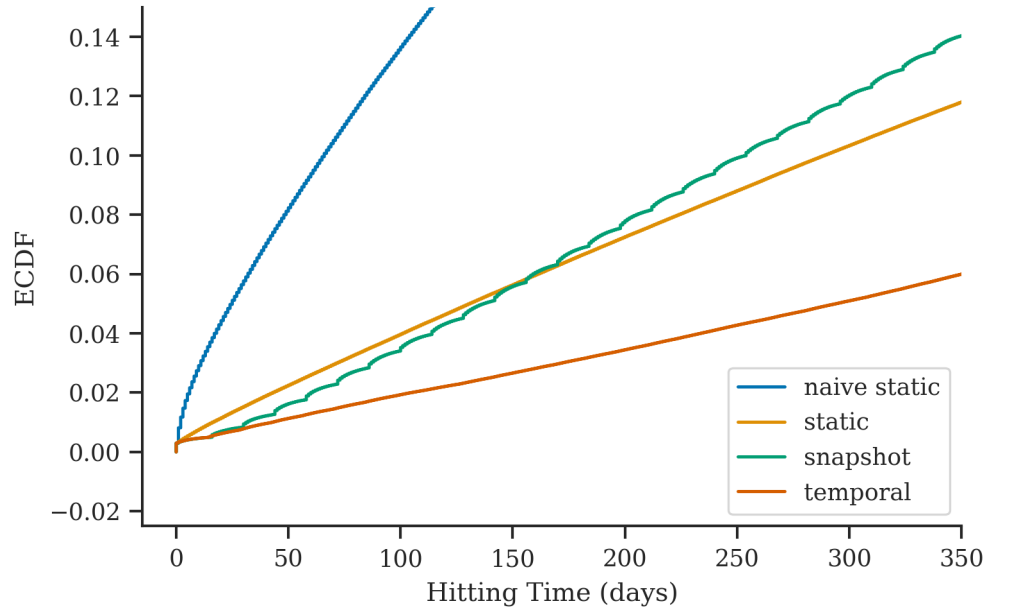


Figure 6. Empirical cumulative distributions of the hitting times between pairs of facilities over approximately the first year, for different network models.

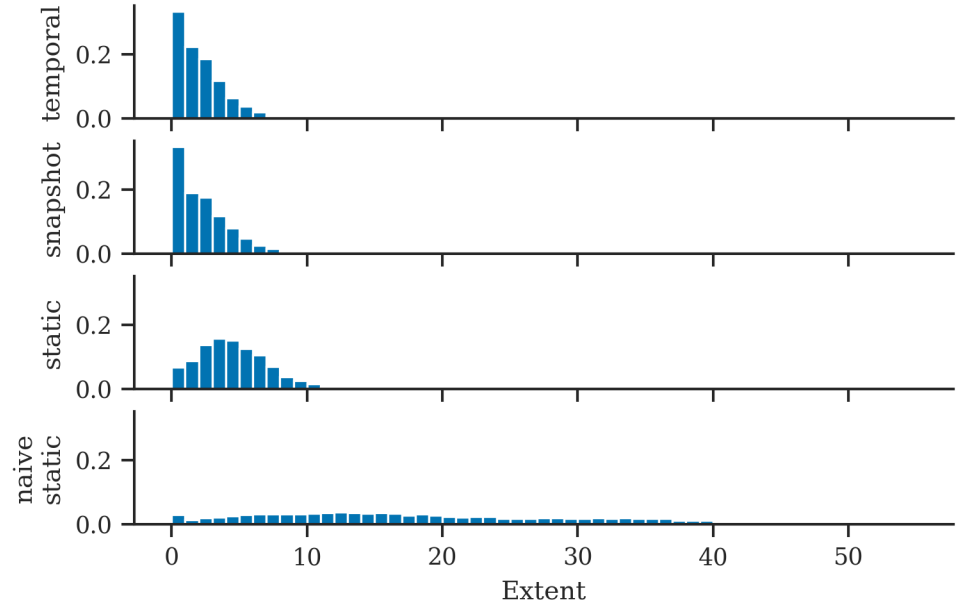


Figure 7. Number of facilities that have had a marked individual admitted after $t = 30$ days. The naïve static model overestimates the range of the spread.

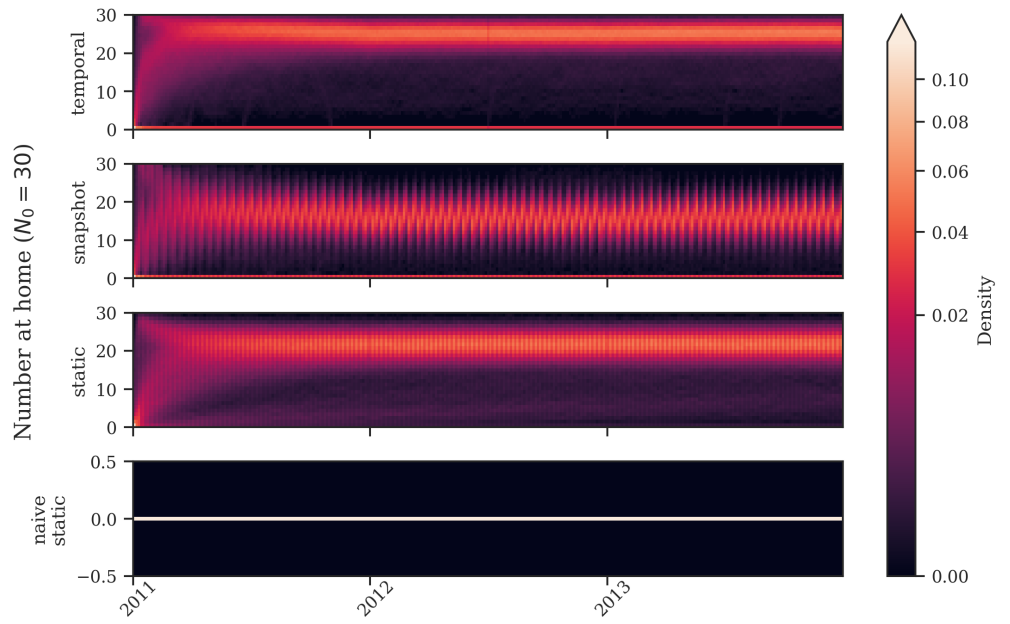


Figure 8. Distribution of the number of individuals at home (not in the hospital system) over the first 3 years of the simulation for each model type.

Discussion

We have investigated the effects of modelling the network representations of patient transfers between healthcare facilities in Victoria with differing levels of temporal fidelity. We see that there are qualitative behavioural differences between the movement patterns induced by these different network representations. One unique temporal property of the networks here is that edges can connect nodes not just in space, but also in time. This leads to a reservoir of individuals of interest that exist outside of the healthcare system that will return at a later time, which leads to rich temporal dynamics. These behaviours are poorly captured by a naïve static network; including this mechanism on top of a static network representation can help significantly in moving the qualitative behaviour fo the

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Supplementary

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Verification of simulation results

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For any continuous-time Markov chain, we can derive the Q-matrix, which represents how ... Importantly, we can use the forward equation to generate a viable representation of the evolution of the probability density function as a system of ODEs that scales with the number of states.

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Typically, the structure of these models represents the state of the *system*; here we use the model as a representation of the state of an *individual*. Thus, we can envision a simulation with N_0 initial seed individuals as being equivalent to N_0 independent, but identically distributed, Markov chains. Here, then, a state maps onto a facility; for more complex models, a state may map onto either a facility, or a “home” state that could be uniquely identified by a pair of facilities.

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This makes computing the probability density function very computationally expensive when the number of Markov states becomes large. This is due to the fact that the ODE system will tend towards very dense matrices P if the network is connected, even if Q itself is sparse. This makes some representations, in particular the temporal network, completely intractable, since we would have to allocate a $(N_f)^2 \times (N_T)^2$ floating point matrix for each time step. For $N_f \sim O(10^2)$, $N_T \sim O(10^3)$, this yields a roughly $O(10^{10 \sim 11})$ byte (10~100 GB) float32 matrix every time step (in memory).

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Thus, we only solve this problem for the naïve static network case. Below, we plot the analytical solution against the empirical mean hitting time of pairs that have all realisations hitting.

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Uniform dissipation of at-home individuals

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We know that for a random variable t uniformly distributed between $[\tau, \tau + \omega]$, the pdf and survival functions are, respectively:

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$$f(t) = \frac{1}{\omega},$$
$$S(t) = \frac{(\tau + \omega) - t}{\omega},$$

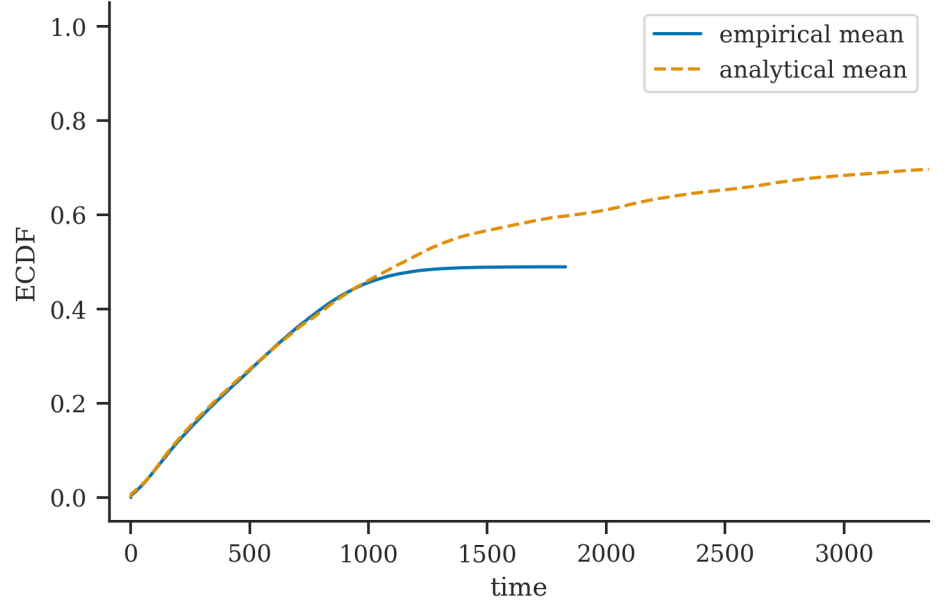


Figure 9. Verification of the empirical simulation results against analytical results. We see that at early time the mean empirical and analytical expected hitting times agree, with a large right-hand censoring effect. When computing the empirical means, we allow 1 nulls (where the target facility is not hit) across all realisations.

which results in a hazard function

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$$\lambda(t) = \frac{1}{(\tau + \omega) - t}.$$

This hazard function can be simulated approximately in our DTMC framework by taking small time steps, and modelling the number of individuals dN that move in a time step dt , where $n(t)$ is the number of individuals still present at time t as

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$$dN(t) \sim \text{Binomial}(n(t), dt\lambda(t)).$$