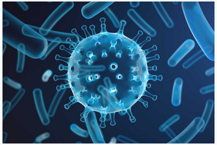
Computational Methods in Epidemic Simulation and Inference

Not-So-Great Models in Complex Situations

David Wu

17 August 2023

About Me



PhD from Dept. Engineering Science at the University of Auckland in New Zealand

Primarily use Python tooling

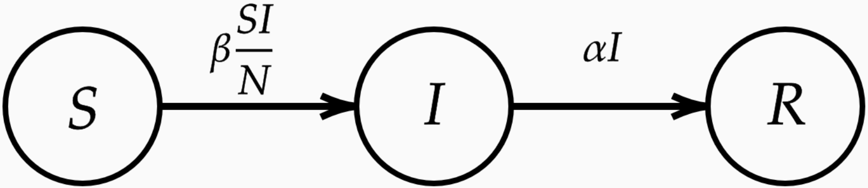
Research is in the field of infectious diseases

Overview

1. Fitting ODE models with generalised profiling
2. Simulating stochastic models on networks (maybe skip)
3. Surrogate modelling techniques for inference

Fitting ODE Models

ODE Models



ODE Models

$$\frac{dS}{dt} = -\beta\frac{SI}{N},$$
$$\frac{dI}{dt} = \beta\frac{SI}{N} - \alpha I,$$
$$\frac{dR}{dt} = \alpha I.$$

ODE Models

$$\frac{d}{dt}\begin{bmatrix} S \\ I \\ R \end{bmatrix} = \begin{bmatrix} -\beta\frac{SI}{N} \\ \beta\frac{SI}{N} - \alpha I \\ \alpha I \end{bmatrix}.$$

ODE Models

$$\frac{d}{dt}(x) = \overbrace{f(x;\theta)}^{\text{RHS}}$$

$$x = \begin{bmatrix} S & I & R \end{bmatrix}^T$$

$$\theta = \begin{bmatrix} \beta & \alpha \end{bmatrix}^T$$

Solving ODE Models

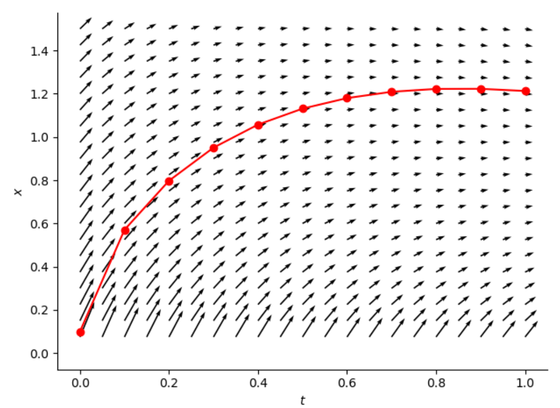
Euler's Method:

- $\frac{dx}{dt} \approx \frac{x(t+\Delta t)-x(t)}{\Delta t}$
- $x(t + \Delta t) = x(t) + (\Delta t) f(x(t); \theta)$

Solving ODE Models

Euler's Method:

- $\frac{dx}{dt} \approx \frac{x(t+\Delta t)-x(t)}{\Delta t}$
- $x(t+\Delta t) = x(t) + (\Delta t) f(x(t);\theta)$



Solving ODE Models

Euler's Method:

- $\frac{dx}{dt} \approx \frac{x(t+\Delta t)-x(t)}{\Delta t}$
- $x(t + \Delta t) = x(t) + (\Delta t) f(x(t); \theta)$

There are methods with higher-order accuracy (wrt Δt) that require evaluation of f at multiple t.

Adaptive methods exist that take smaller steps in regions of time where the behaviour is "faster" than expected, in order to improve accuracy.

These methods can be very slow in particular regions of parameter space. (*Problem 1)

Fitting ODEs

We have some data y, and we want to fit an ODE model to it: find θ that would likely reconstruct y.

We typically work with a simple additive error model:

$$\begin{aligned} y(t) &= \overbrace{g(x(t))}^{\text{observation model}} + \varepsilon, \\ \frac{d}{dt}x(t) &= f(x(t);\theta), \\ \varepsilon &\sim \mathcal{N}(0,\Gamma). \end{aligned}$$

Fitting ODEs: Classical Methods

Non-linear Least Squares (frequentist)

$$\min_{\theta, x(0)} \left\| y - g\left(\int_0^t f(x(t); \theta) \right) \right\|_2^2$$

Fitting ODEs: Classical Methods

Non-linear Least Squares (frequentist)

$$\min_{\theta, x(0)} \left\| y - g\left(\int_0^t f(x(t); \theta) \right) \right\|_2^2$$

Bayesian Analogue

$$p(\theta, x_0 | y) = \underbrace{\left\| y - g\left(\int_0^t f(x(t); \theta) \right) \right\|_{\Gamma^{-1}}^2}_{\text{likelihood: } \cdot p(y|\theta, x_0)} \cdot p(\theta, x_0)$$

Problems

1. Numerical integration can be slow.
2. Solving the ODE implicitly assumes that the model is correct.

A Partial Solution: Collocation on Splines

Instead of integrating the ODE, getting a state, and then comparing against the data, we could go backwards.

We could get a proposal for a vector field from the data*, then compare the vector field with the RHS of the ODE.

Fitting ODEs: Measles in Samoa

Plots of the available data for the 2019 Samoan measles outbreak (current hospitalisations and cumulative discharges available, but not shown). Top-left: cumulative reported cases; top-right: cumulative reported deaths; bottom-left: report incidence (daily rate of new cases since last report); bottom-right: cumulative reported hospitalisations.