smol_case

March 28, 2019

In [1]:
```
import ingestor, modeller, fitter
import numpy as np
import matplotlib.pyplot as plt
```

In [2]:
```
plt.style.use('seaborn-notebook')
plt.rc('text', usetex=True)
plt.rc('font', family='serif')
plt.rcParams['figure.figsize'] = [15, 10]
```

In [3]:
```
from importlib import reload

reload(fitter)
reload(modeller)
reload(ingestor)
```

Out[3]:
```
<module 'ingestor' from '/media/dwu402/Data/wrap-mad/ingestor.py'>
```

In [4]:
```
context = ingestor.initialise_context()
ingestor.read_run_file(context, "runs/minimal3.3.run")
```

In [5]:
```
model = modeller.Model(context)
```

In [6]:
```
solver = fitter.Fitter()
solver.construct_objectives(context, model)
```

In [7]:
```
solver.construct_problems()
```

In [8]:
```
for rhoi in np.logspace(-10, 5, num=97):
    solver.solve(rhoi)
    solver.problems[0].initial_guess = solver.solutions[str(rhoi)][-1].x
```

In [9]:
```
solver.solutions
```

Out[9]:
```
{'1e-10': [      fun: 0.11219850428670723
   hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
        jac: array([-5.84809635e-17,  5.17955255e-17, -7.25900421e-18,  3.32651190e-17,
         -3.72594511e-16,  1.75046486e-15,  6.78072659e-16, -5.31301865e-15,
         -1.86077748e-15,  2.93492727e-16, -1.79622906e-15])
    message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
```

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   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'1.4330125702369628e-10': [      fun: 0.11219850428670723
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       jac: array([-8.38039558e-17,  7.42236392e-17, -1.04022442e-17,  4.76693336e-17,
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        -2.66651752e-15,  4.20578767e-16, -2.57401882e-15])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'2.053525026457146e-10': [      fun: 0.11219850428670723
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.20092122e-16,  1.06363408e-16, -1.49065465e-17,  6.83107540e-17,
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        -3.82115313e-15,  6.02694661e-16, -3.68860133e-15])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'2.942727176209282e-10': [      fun: 0.11219850428670723
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       jac: array([-1.72093521e-16,  1.52420101e-16, -2.13612682e-17,  9.78901688e-17,
        -1.09644399e-15,  5.15114051e-15,  1.99538284e-15, -1.56347644e-14,
        -5.47576047e-15,  8.63669027e-16, -5.28581207e-15])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'4.2169650342858224e-10': [      fun: 0.11219850428670723
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       jac: array([-2.46612178e-16,  2.18419920e-16, -3.06109652e-17,  1.40277841e-16,
        -1.57121803e-15,  7.38164910e-15,  2.85940870e-15, -2.24048139e-14,
        -7.84683359e-15,  1.23764857e-15, -7.57463514e-15])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
```

```
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    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
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        -1.12446112e-14,  1.77356597e-15, -1.08545474e-14])
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       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
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       jac: array([-5.06424280e-16,  4.48530774e-16, -6.28603782e-17,  2.88064051e-16,
        -3.22653554e-15,  1.51584012e-14,  5.87186733e-15, -4.60088461e-14,
        -1.61136692e-14,  2.54154234e-15, -1.55547029e-14])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'1.2409377607517196e-09': [      fun: 0.11219850428670723
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-7.25712360e-16,  6.42750239e-16, -9.00797063e-17,  4.12799399e-16,
        -4.62366599e-15,  2.17221795e-14,  8.41445971e-15, -6.59312548e-14,
        -2.30910905e-14,  3.64206215e-15, -2.22900848e-14])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
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    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
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       jac: array([-1.03995494e-15,  9.21069174e-16, -1.29085339e-16,  5.91546710e-16,
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        -3.30898230e-14,  5.21912090e-15, -3.19419717e-14])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
```

```
      nfev: 1
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'2.5482967479793464e-09': [      fun: 0.11219850428670723
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        -4.74181323e-14,  7.47906597e-15, -4.57732471e-14])
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      nfev: 1
       nit: 0
    status: 0
   success: True
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       1.e+00, 1.e+00, 1.e-03])],
'3.651741272548377e-09': [      fun: 0.11219850428670723
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-2.13557349e-15,  1.89143862e-15, -2.65079887e-16,  1.21475586e-15,
        -1.36061877e-14,  6.39224482e-14,  2.47614595e-14, -1.94017696e-13,
        -6.79507798e-14,  1.07175958e-14, -6.55936388e-14])
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'5.232991146814947e-09': [      fun: 0.11219850428670723
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        -9.73743219e-14,  1.53584500e-14, -9.39965095e-14])
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   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'7.498942093324558e-09': [      fun: 0.11219850428670723
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        -1.39538628e-13,  2.20088528e-14, -1.34698181e-13])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
```

```
      nfev: 1
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       1.e+00, 1.e+00, 1.e-03])],
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       1.e+00, 1.e+00, 1.e-03])],
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   success: True
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    status: 0
   success: True
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```

```
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    status: 0
   success: True
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       1.e+00, 1.e+00, 1.e-03])],
'9.30572040929699e-08': [      fun: 0.11219850428670723
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        -1.73158780e-12,  2.73116623e-13, -1.67152113e-12])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'1.333521432163324e-07': [      fun: 0.11219850428670723
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       jac: array([-7.79856316e-14,  6.90704907e-14, -9.67981120e-15,  4.43594409e-14,
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        -2.48138728e-12,  3.91379863e-13, -2.39531115e-12])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
```

```
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    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'1.9109529749704404e-07': [      fun: 0.11219850428670723
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        -3.55585954e-12,  5.60852898e-13, -3.43251173e-12])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
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       nit: 0
    status: 0
   success: True
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       1.e+00, 1.e+00, 1.e-03])],
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   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
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        -1.46213294e-12,  6.86916304e-12,  2.66088954e-12, -2.08493148e-11,
        -7.30204932e-12,  1.15173001e-12, -7.04875408e-12])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'5.62341325190349e-07': [      fun: 0.11219850428670723
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       jac: array([-3.28862864e-13,  2.91268486e-13, -4.08163505e-14,  1.87058027e-13,
        -2.09525575e-12,  9.84360120e-12,  3.81309050e-12, -2.98773435e-11,
        -1.04639318e-11,  1.65044907e-12, -1.01009597e-11])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
```

```
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'8.058421877614818e-07': [      fun: 0.11219850428670723
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-4.71264767e-13,  4.17391924e-13, -5.84878422e-14,  2.68053100e-13,
        -3.00252958e-12,  1.41060129e-11,  5.46421141e-12, -4.28146361e-11,
        -1.49949527e-11,  2.36512555e-12, -1.44748154e-11])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'1.1547819846894582e-06': [      fun: 0.11219850428670723
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-6.75328639e-13,  5.98128947e-13, -8.38086771e-14,  3.84116471e-13,
        -4.30266625e-12,  2.02141115e-11,  7.83029347e-12, -6.13539679e-11,
        -2.14879697e-11,  3.38927781e-12, -2.07426197e-11])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'1.6548170999431815e-06': [      fun: 0.11219850428670723
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-9.67755055e-13,  8.57128502e-13, -1.20088341e-13,  5.50429378e-13,
        -6.16578224e-12,  2.89671124e-11,  1.12209292e-11, -8.79211225e-11,
        -3.07925596e-11,  4.85692529e-12, -2.97244907e-11])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'2.3713737056616552e-06': [      fun: 0.11219850428670723
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.38680644e-12,  1.22828044e-12, -1.72066443e-13,  7.88742741e-13,
        -8.83565869e-12,  4.15103110e-11,  1.60797740e-11, -1.25992310e-10,
        -4.41261844e-11,  6.96013270e-12, -4.25956836e-11])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
```

```
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'3.3982083289425593e-06': [      fun: 0.11219850428670723
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.98731370e-12,  1.76015060e-12, -2.46528899e-13,  1.13021773e-12,
        -1.26616413e-11,  5.94849512e-11,  2.30426035e-11, -1.80549051e-10,
        -6.32334988e-11,  9.97415829e-12, -6.10403859e-11])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'4.869675251658631e-06': [      fun: 0.1121985049428704
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 1.88482314e-11, -3.65197477e-11, -3.86711111e-13, -4.77413984e-12,
         1.13304583e-11, -5.65488568e-11, -4.23191871e-11, -1.30541469e-10,
         5.25607896e-12, -1.45239540e-11, -5.39729663e-11])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'6.978305848598663e-06': [      fun: 0.1121985049428704
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 2.70097124e-11, -5.23332395e-11, -5.54044146e-13, -6.84157718e-12,
         1.62365280e-11, -8.10345469e-11, -6.06434938e-11, -1.87067889e-10,
         7.53177527e-12, -2.08129629e-11, -7.73441558e-11])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'1e-05': [      fun: 0.1121985049428704
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 3.87051752e-11, -7.49941531e-11, -7.93710450e-13, -9.80442503e-12,
         2.32668177e-11, -1.16122137e-10, -8.69019990e-11, -2.68071301e-10,
         1.07926112e-11, -2.98251434e-11, -1.10835592e-10])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
```

```
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'1.4330125702369627e-05': [      fun: 0.11219850279396318
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-3.67192281e-11,  9.65171599e-13,  2.21866723e-11, -3.18140417e-11,
        -2.38105847e-11,  7.90513086e-11,  6.16461897e-11, -1.06086502e-10,
         1.75415046e-11,  5.10651475e-11, -7.81041635e-11])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'2.0535250264571462e-05': [      fun: 0.11219850279396318
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-5.26192966e-11,  1.38360260e-12,  3.17936231e-11, -4.55896631e-11,
        -3.41206648e-11,  1.13280682e-10,  8.83392099e-11, -1.52023756e-10,
         2.51378138e-11,  7.31763410e-11, -1.11924340e-10])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'2.9427271762092818e-05': [      fun: 0.11219850279396318
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-7.54044856e-11,  1.98374574e-12,  4.55603387e-11, -6.53300293e-11,
        -4.88949259e-11,  1.62330923e-10,  1.26590059e-10, -2.17852906e-10,
         3.60240704e-11,  1.04861267e-10, -1.60389175e-10])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'4.216965034285822e-05': [      fun: 0.11219850279396318
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.08056340e-10,  2.84483894e-12,  6.52878752e-11, -9.36176628e-11,
        -7.00661901e-11,  2.32618725e-10,  1.81402806e-10, -3.12187911e-10,
         5.16255478e-11,  1.50264742e-10, -2.29840090e-10])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
```

```
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'6.042963902381328e-05': [      fun: 0.11219850279396318
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.54847662e-10,  4.08101485e-12,  9.35569847e-11, -1.34153049e-10,
        -1.00403979e-10,  3.33338311e-10,  2.59947697e-10, -4.47373220e-10,
         7.39854017e-11,  2.15325574e-10, -3.29364533e-10])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'8.659643233600654e-05': [      fun: 0.11219851007680452
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.34670731e-09,  3.70974947e-09,  1.15272632e-09, -5.49242708e-10,
        -2.21272170e-11, -9.26895527e-10, -2.01700307e-10, -4.20684435e-09,
         2.28101517e-09,  1.18927745e-09, -1.78411395e-09])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'0.00012409377607517196': [      fun: 0.11219850877510787
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-2.17277720e-09,  5.06162033e-09,  1.40052183e-09, -5.70952142e-10,
         1.37089385e-10, -1.78636807e-09, -5.93981209e-10, -6.27547279e-09,
         3.61983852e-09,  1.43767987e-09, -2.39951551e-09])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'0.00017782794100389227': [      fun: 0.11219850877510787
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-3.11327572e-09,  7.25265832e-09,  2.00685501e-09, -8.18181079e-10,
         1.96480405e-10, -2.55984665e-09, -8.51243986e-10, -8.99193220e-09,
         5.18683880e-09,  2.06005729e-09, -3.43821432e-09])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
```

```
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'0.00025482967479793463': [      fun: 0.11219851950877388
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-6.48757711e-09,  4.52472682e-09,  5.75028658e-09, -6.80215212e-09,
        -3.12712491e-09,  8.09764036e-09,  5.77673676e-09, -5.23197736e-09,
         4.00450940e-09,  6.35589606e-10, -4.09092152e-09])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'0.0003651741272548377': [      fun: 0.11219852122087162
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.13685595e-08,  8.03729024e-09,  8.18129117e-09, -9.47291276e-09,
        -2.66267806e-09,  5.99012770e-09,  4.32900525e-09, -1.25155943e-08,
         8.99597466e-09,  9.57767864e-10, -7.13461277e-09])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'0.0005232991146814947': [      fun: 0.11219852783090671
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-2.11587109e-08,  1.48965486e-08,  1.20537029e-08, -1.36728970e-08,
        -4.54181956e-09,  1.08385895e-08,  7.36296916e-09, -2.30857636e-08,
         1.56518741e-08,  7.00558939e-10, -1.17436033e-08])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'0.0007498942093324559': [      fun: 0.11219858787808332
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-5.55441000e-08,  2.01287872e-08,  1.17120224e-08, -1.38958428e-08,
        -2.49792794e-08,  6.94188615e-08,  4.82203468e-08, -1.77025282e-08,
         2.96887891e-08, -2.71088450e-08, -9.89601776e-09])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
```

```
      nfev: 1
       nit: 0
    status: 0
   success: True
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       1.e+00, 1.e+00, 1.e-03])],
'0.0010746078283213176': [      fun: 0.11219872679972129
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.35081565e-07,  5.52931022e-08,  2.86195740e-08, -3.41052811e-08,
        -4.44177356e-08,  1.16274196e-07,  8.20232327e-08, -6.75888310e-08,
         8.55892006e-08, -6.26420348e-08, -4.10546117e-08])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'0.0015399265260594 92': [      fun: 0.11219892388287467
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-2.75704019e-07,  1.52079307e-07,  8.62509331e-08, -9.85803839e-08,
        -1.07183972e-07,  2.71783890e-07,  1.98083288e-07, -1.65318215e-07,
         1.69732162e-07, -1.29638548e-07, -8.66687400e-08])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'0.0022067340690845897': [      fun: 0.11219934553321972
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-5.71494225e-07,  2.98893811e-07,  1.82707887e-07, -2.14974278e-07,
        -1.90496321e-07,  4.72520541e-07,  3.43368017e-07, -3.73226965e-07,
         3.65849331e-07, -2.68101656e-07, -2.09240819e-07])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'0.0031622776601683794': [      fun: 0.11220020163248316
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.13342653e-06,  6.27094594e-07,  3.43978853e-07, -3.96491128e-07,
        -4.01316564e-07,  1.00577110e-06,  7.31019497e-07, -7.32533873e-07,
         7.10045473e-07, -5.48794628e-07, -4.32635689e-07])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
```

```
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'0.004531583637600818': [      fun: 0.11220207853478559
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-2.42061543e-06,  1.22076065e-06,  7.50342995e-07, -8.87874820e-07,
        -8.29671697e-07,  2.06263111e-06,  1.49213948e-06, -1.51772316e-06,
         1.52942355e-06, -1.16494285e-06, -8.71380557e-07])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'0.006493816315762113': [      fun: 0.11220577048012075
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-4.92246520e-06,  2.56529165e-06,  1.53821866e-06, -1.80077657e-06,
        -1.64469478e-06,  4.11215575e-06,  2.98528181e-06, -3.12932550e-06,
         3.15904727e-06, -2.37398540e-06, -1.63322103e-06])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([3.e-01, 1.e+00, 7.e-01, 1.e+00, 2.e+00, 1.e+00, 1.e+00, 1.e+00,
       1.e+00, 1.e+00, 1.e-03])],
'0.009305720409296 99': [      fun: 0.11221302441567842
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.00206317e-05,  5.33658018e-06,  3.07545503e-06, -3.58352827e-06,
        -3.58159905e-06,  8.78507066e-06,  6.45447728e-06, -6.39308093e-06,
         6.44976828e-06, -4.89242995e-06, -3.44144048e-06])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 13
       nit: 1
    status: 0
   success: True
         x: array([3.00000000e-01, 1.00000000e+00, 7.00000000e-01, 1.00000000e+00,
       2.00000000e+00, 1.00000000e+00, 1.00000000e+00, 1.00000000e+00,
       1.00000000e+00, 1.00000000e+00, 1.00000006e-03])],
'0.013335214321633 24': [      fun: 0.112227798967004
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-2.06775431e-05,  1.09911921e-05,  6.13510167e-06, -7.12252810e-06,
        -7.05208663e-06,  1.73409198e-05,  1.27395342e-05, -1.30735462e-05,
         1.35519127e-05, -1.04500883e-05, -6.71642111e-06])
```

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   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 14
       nit: 1
    status: 0
   success: True
         x: array([3.00000000e-01, 1.00000000e+00, 7.00000000e-01, 1.00000000e+00,
       2.00000000e+00, 1.00000000e+00, 1.00000000e+00, 1.00000000e+00,
       1.00000000e+00, 1.00000000e+00, 1.00000008e-03])],
'0.019109529749704406': [      fun: 0.11225711210591993
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-4.24695709e-05,  2.31378912e-05,  1.23852479e-05, -1.42836417e-05,
       -1.39045495e-05,  3.40659577e-05,  2.50196166e-05, -2.74973752e-05,
        2.83608467e-05, -2.19272589e-05, -1.34958111e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 19
       nit: 1
    status: 0
   success: True
         x: array([3.0000000e-01, 1.0000000e+00, 7.0000000e-01, 1.0000000e+00,
       2.0000000e+00, 1.0000000e+00, 1.0000000e+00, 1.0000000e+00,
       1.0000000e+00, 1.0000000e+00, 1.0000001e-03])],
'0.027384196342643614': [      fun: 0.11231494941242733
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-8.71086632e-05,  4.73951301e-05,  2.36772227e-05, -2.71315002e-05,
       -2.78781201e-05,  6.83532252e-05,  5.02898504e-05, -5.52583409e-05,
        5.92028214e-05, -4.66095219e-05, -2.53254158e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 70
       nit: 1
    status: 0
   success: True
         x: array([3.0000000e-01, 1.0000000e+00, 7.0000000e-01, 1.0000000e+00,
       2.0000000e+00, 1.0000000e+00, 1.0000000e+00, 1.0000000e+00,
       1.0000000e+00, 1.0000000e+00, 1.0000001e-03])],
'0.03924189758484536': [      fun: 0.11222188600625763
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 1.26803845e-05, -8.65595047e-06,  1.52114157e-07,  0.00000000e+00,
         1.38444987e-05,  1.71892836e-07, -3.78624171e-06,  1.54849841e-05,
       -2.68302016e-05,  1.45403040e-05,  2.55670720e-06])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 66
       nit: 24
    status: 0
   success: True
         x: array([3.1866975 , 5.26576545, 0.        , 2.93311158, 0.0054179 ,
       0.02045667, 0.27754971, 2.03602991, 1.93205538, 1.34580052,
       3.46112585])],
'0.05623413251903491': [      fun: 0.11224415636594046
```

```
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 2.17015959e-05, -1.49249721e-05,  3.80398260e-07,  0.00000000e+00,
         2.58254289e-05,  4.17949645e-07, -7.27978910e-06,  2.68049056e-05,
        -4.65781335e-05,  2.65564709e-05,  3.91934241e-06])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 28
       nit: 1
    status: 0
   success: True
         x: array([3.1866975 , 5.26576545, 0.        , 2.93311158, 0.0054179 ,
        0.02045667, 0.27754971, 2.03602991, 1.93205538, 1.34580052,
        3.46112585])],
'0.08058421877614819': [      fun: 0.11228539356791034
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.17773936e-05,  4.75192926e-06,  1.30117696e-06,  0.00000000e+00,
         4.73519284e-05,  8.30963710e-07, -1.37707365e-05, -6.71591138e-06,
         4.85441913e-06,  1.15889728e-05, -8.91672221e-06])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 73
       nit: 3
    status: 0
   success: True
         x: array([3.18522918e+00, 5.26751344e+00, 0.00000000e+00, 2.93311158e+00,
        5.18436315e-03, 2.04560447e-02, 2.77759336e-01, 2.03355152e+00,
        1.93695899e+00, 1.34082717e+00, 3.46148964e+00])],
'0.11547819846894582': [      fun: 0.11236208711351184
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-3.27654418e-05,  1.73064371e-05,  2.72468614e-06,  0.00000000e+00,
         8.49813159e-05,  1.82976683e-06, -2.58126387e-05, -2.83428078e-05,
         3.30408378e-05,  1.76594616e-05, -2.05314984e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 84
       nit: 1
    status: 0
   success: True
         x: array([3.18522918e+00, 5.26751344e+00, 0.00000000e+00, 2.93311158e+00,
        5.18436315e-03, 2.04560447e-02, 2.77759336e-01, 2.03355152e+00,
        1.93695899e+00, 1.34082717e+00, 3.46148964e+00])],
'0.16548170999431813': [      fun: 0.11246496410144084
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-6.08384729e-07,  1.86107008e-06, -2.35756128e-06,  1.84642444e-09,
         1.44410601e-04,  0.00000000e+00, -9.29121263e-06, -1.90476723e-05,
         1.49225122e-06, -4.42000165e-06, -9.76098365e-06])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 66
       nit: 17
    status: 0
   success: True
```

```
        x: array([3.55555201e+00, 5.24204004e+00, 7.85757777e-04, 2.93311158e+00,
       0.00000000e+00, 1.27795200e-02, 5.15877888e-01, 1.97147501e+00,
       1.98970238e+00, 8.78558090e-01, 3.83764295e+00])],
'0.23713737056616552': [      fun: 0.11266867281803106
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-9.15943964e-06,  1.61385042e-05, -2.58575409e-06,  2.40226028e-09,
         2.30437784e-04,  0.00000000e+00, -5.16162795e-06, -6.41959341e-05,
         5.06072976e-05,  1.19458235e-05, -2.25584806e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 42
       nit: 1
    status: 0
   success: True
         x: array([3.55555201e+00, 5.24204004e+00, 7.85757777e-04, 2.93311158e+00,
       0.00000000e+00, 1.27795200e-02, 5.15877888e-01, 1.97147501e+00,
       1.98970238e+00, 8.78558090e-01, 3.83764295e+00])],
'0.33982083289425596': [      fun: 0.1129539316019084
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 2.67226850e-05, -2.48684197e-05,  2.10940837e-05, -7.62486595e-06,
         3.59322272e-04,  0.00000000e+00,  8.60307471e-06,  4.40115716e-06,
        -3.79005572e-05,  3.87281469e-05,  4.12912114e-06])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 52
       nit: 29
    status: 0
   success: True
         x: array([3.24966655, 6.0395248 , 0.68249834, 2.91691352, 0.        ,
       0.01277952, 0.56537171, 2.22479901, 1.92102287, 0.67367398,
       5.91300829])],
'0.4869675251658631': [      fun: 0.11341985967350718
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 7.13914901e-05, -8.47792607e-05,  2.39994151e-05, -8.86867341e-06,
         6.21816723e-04,  0.00000000e+00, -2.46890935e-05,  1.29457798e-04,
        -1.19686205e-04,  9.16602456e-05,  9.54356263e-06])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 86
       nit: 5
    status: 0
   success: True
         x: array([3.22049323, 6.05073882, 0.662571  , 2.92413346, 0.        ,
       0.01277952, 0.5491168 , 2.24958862, 1.87483978, 0.59453109,
       5.91176824])],
'0.6978305848598664': [      fun: 0.11408277617205513
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-9.91063166e-05,  2.55069361e-05,  2.44234253e-06, -8.90328603e-07,
         9.71073352e-04,  0.00000000e+00,  5.18664512e-06, -7.23171186e-05,
         8.48051695e-05, -1.52133452e-05, -4.62942321e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
```

```
      nfev: 74
       nit: 9
    status: 0
   success: True
         x: array([3.21113752, 6.11128859, 0.63423027, 2.93532007, 0.        ,
        0.01277952, 0.59423306, 2.17095937, 1.75081003, 0.45010102,
        5.93277329])],
'1.0': [      fun: 0.11503923435635244
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 3.66737971e-05,  4.23210690e-06, -1.37129638e-05,  2.35876502e-06,
         1.47879873e-03,  0.00000000e+00,  5.66681073e-05, -1.30149474e-04,
         3.10777009e-04,  2.43263175e-04, -4.33316368e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 81
       nit: 1
    status: 0
   success: True
         x: array([3.21113752, 6.11128859, 0.63423027, 2.93532007, 0.        ,
        0.01277952, 0.59423306, 2.17095937, 1.75081003, 0.45010102,
        5.93277329])],
'1.4330125702369627': [      fun: 0.11632602979364111
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 5.01654755e-05, -1.14468584e-04, -1.24337985e-04,  2.52097794e-05,
         2.85859099e-03,  0.00000000e+00, -2.17451913e-04,  3.32518716e-04,
         3.69550517e-05, -2.59834353e-05, -1.08487091e-04])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 87
       nit: 5
    status: 0
   success: True
         x: array([3.15194775, 6.13058443, 0.64483783, 2.93328209, 0.        ,
        0.01277952, 0.57049208, 2.17819059, 1.67301017, 0.34963675,
        5.93348819])],
'2.0535250264571463': [      fun: 0.11729354811551454
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.96014847e-04,  2.96078044e-05, -3.85146899e-06,  1.12808317e-05,
         1.60699167e-03,  0.00000000e+00,  5.70616688e-05, -6.54289975e-05,
         2.84652985e-04,  1.35193920e-04, -1.37330958e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 99
       nit: 57
    status: 0
   success: True
         x: array([3.08836839e+00, 2.23813015e+01, 1.46654906e+01, 9.24987396e+00,
        0.00000000e+00, 1.27795200e-02, 2.41250844e+00, 7.03705401e+00,
        1.58020398e+00, 0.00000000e+00, 2.70774434e+01])],
'2.942727176209282': [      fun: 0.11915737654714845
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
```

```
       jac: array([ 5.89585708e-04, -7.50149266e-05, -4.04353125e-06,  1.04433790e-05,
         2.27382619e-03,  0.00000000e+00,  1.82302581e-04,  1.26417662e-04,
         3.71364351e-04,  6.40890384e-04,  1.34935096e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 43
       nit: 1
    status: 0
   success: True
         x: array([3.08836839e+00, 2.23813015e+01, 1.46654906e+01, 9.24987396e+00,
        0.00000000e+00, 1.27795200e-02, 2.41250844e+00, 7.03705401e+00,
        1.58020398e+00, 0.00000000e+00, 2.70774434e+01])],
'4.216965034285822': [      fun: 0.12145240532548511
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 1.24337079e-04,  2.67634889e-05,  1.11154866e-05, -3.74020510e-05,
         5.59292579e-03,  0.00000000e+00, -1.38130570e-05,  2.52517237e-05,
        -1.04908506e-04,  7.11949427e-05, -2.37510135e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 91
       nit: 17
    status: 0
   success: True
         x: array([2.95371082e+00, 2.25008305e+01, 1.46312690e+01, 9.36373992e+00,
        0.00000000e+00, 1.27795200e-02, 2.24210619e+00, 6.52438374e+00,
        1.45028043e+00, 5.47174769e-02, 2.71169378e+01])],
'6.042963902381328': [      fun: 0.12466967425781927
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 9.66424902e-04, -1.07797193e-04,  2.93706359e-05, -9.80427997e-05,
         6.05787716e-03,  0.00000000e+00,  4.54041812e-04,  3.89177975e-04,
         8.22628213e-04, -2.87557921e-04, -2.98063170e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 61
       nit: 2
    status: 0
   success: True
         x: array([2.95194323e+00, 2.25010465e+01, 1.46312392e+01, 9.36383932e+00,
        0.00000000e+00, 1.27795200e-02, 2.24170635e+00, 6.52379500e+00,
        1.45092451e+00, 5.45901461e-02, 2.71169210e+01])],
'8.659643233600654': [      fun: 0.12860381048063774
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 3.61004114e-04, -1.07932535e-04,  1.64687617e-04, -5.06078794e-04,
         1.42574737e-02,  0.00000000e+00, -2.43420652e-05,  4.83632068e-04,
        -7.67253596e-04,  4.99788443e-04, -3.27342499e-07])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 98
       nit: 12
    status: 0
   success: True
         x: array([2.59212332e+00, 2.25636779e+01, 1.46154392e+01, 9.41485192e+00,
```

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       0.00000000e+00, 1.27795200e-02, 2.02538086e+00, 6.33474134e+00,
       1.31463971e+00, 3.22709677e-01, 2.71232616e+01])],
'12.409377607517195': [      fun: 0.13419111747925752
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 8.15437022e-04, -2.28909425e-04,  3.45593597e-04, -1.03657274e-03,
         2.46289668e-02,  0.00000000e+00, -2.51677714e-04,  8.32716510e-04,
        -3.26010684e-03,  1.01539939e-03,  5.66769220e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 64
       nit: 7
    status: 0
   success: True
         x: array([2.37453449e+00, 2.26180356e+01, 1.45729883e+01, 9.54566534e+00,
       0.00000000e+00, 1.27795200e-02, 1.84386942e+00, 6.15694143e+00,
       1.22943553e+00, 4.95972552e-01, 2.71265133e+01])],
'17.78279410038923': [      fun: 0.13960620536478735
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.42600586e-04,  2.20069337e-05,  2.38059910e-05, -5.74003512e-05,
         2.92447384e-02,  0.00000000e+00,  6.90274418e-05,  1.60508791e-04,
         5.17463079e-04,  3.80328388e-05, -5.24347683e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 88
       nit: 40
    status: 0
   success: True
         x: array([2.28913135e+00, 2.25102727e+01, 1.34561413e+01, 1.27224785e+01,
       0.00000000e+00, 1.27795200e-02, 1.37051298e+00, 5.54887739e+00,
       1.16168862e+00, 5.68707138e-01, 2.72906407e+01])],
'25.482967479793466': [      fun: 0.1490476661137105
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 2.56029768e-04, -1.20812352e-05,  1.34244675e-04, -2.86586631e-04,
         4.96217610e-02,  0.00000000e+00, -6.80913317e-04,  4.30038573e-04,
         2.84914566e-04,  1.74510135e-04, -4.91310538e-05])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 93
       nit: 17
    status: 0
   success: True
         x: array([2.17485287e+00, 2.25638462e+01, 1.34248991e+01, 1.27906970e+01,
       0.00000000e+00, 1.27795200e-02, 1.12781618e+00, 5.31698905e+00,
       1.11720892e+00, 7.05371788e-01, 2.73052372e+01])],
'36.51741272548377': [      fun: 0.16201352523081908
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 1.07296621e-04,  7.97554830e-05, -3.81158014e-05,  5.44449829e-05,
         5.64317216e-02,  0.00000000e+00, -8.72792653e-05, -1.66857145e-05,
         7.61893171e-04,  8.09272754e-04, -1.50243087e-04])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 111
```

```
       nit: 43
    status: 0
   success: True
         x: array([2.03765628e+00, 2.18418383e+01, 1.22766431e+01, 1.51610126e+01,
        0.00000000e+00, 1.27795200e-02, 8.40587366e-01, 4.62713564e+00,
        1.06924154e+00, 8.20886273e-01, 2.82309146e+01])],
'52.32991146814947': [      fun: 0.1811345879851342
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.75033511e-05,  2.46323367e-04,  3.53572405e-06, -1.35909275e-05,
         7.19371940e-02,  0.00000000e+00, -1.12139551e-04, -1.74547153e-04,
        -2.30333127e-03,  4.27648394e-04, -3.37182884e-04])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 120
       nit: 27
    status: 0
   success: True
         x: array([1.98187374e+00, 2.19038988e+01, 1.22647837e+01, 1.51834528e+01,
        0.00000000e+00, 1.27795200e-02, 6.85547309e-01, 4.12404921e+00,
        1.01593407e+00, 9.45476874e-01, 2.83470643e+01])],
'74.98942093324558': [      fun: 0.20835589043933417
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-3.07399246e-03,  3.76361040e-04,  8.16976797e-05, -1.37639936e-04,
         7.36275736e-02,  0.00000000e+00,  8.23325249e-04,  4.64207868e-04,
         4.13693839e-03,  1.60887774e-04, -6.79824088e-04])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 82
       nit: 15
    status: 0
   success: True
         x: array([1.82548298e+00, 2.19402192e+01, 1.22636237e+01, 1.51857252e+01,
        0.00000000e+00, 1.27795200e-02, 5.19022378e-01, 3.92247737e+00,
        9.72157962e-01, 1.16413634e+00, 2.83625402e+01])],
'107.46078283213176': [      fun: 0.1992017802341512
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 6.06731583e-02, -2.46354795e-03,  9.94647997e-04, -1.47534106e-03,
         4.85094710e-02, -2.44575233e+00, -9.17036731e-03, -2.21764740e-05,
         1.19661507e-02,  5.17530284e-03, -7.22313107e-04])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 105
       nit: 52
    status: 0
   success: True
         x: array([ 1.49427269, 21.36368008, 11.90820475, 15.73748994,  1.08069455,
         0.05871551,  7.46417758,  2.97513283,  0.80486675,  1.36562317,
        29.03346598])],
'153.9926526059492': [      fun: 0.317664162684332
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 3.94700885e-01, -4.48448244e-02,  1.63146375e-03, -2.38640097e-03,
```

```
        -4.17265397e-01,  1.21451583e+01,  7.84503889e-02,  1.24201599e-01,
        -1.24452679e+00,  2.07747414e-02,  2.47428872e-02])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 64
       nit: 1
    status: 0
   success: True
         x: array([ 1.49427269, 21.36368008, 11.90820475, 15.73748994,  1.08069455,
         0.05871551,  7.46417758,  2.97513283,  0.80486675,  1.36562317,
        29.03346598])],
'220.673406908459': [      fun: 0.6699839721567649
 hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
      jac: array([-3.08223538e-02,  2.72821650e-03,  1.01330278e-03, -1.53318961e-03,
        -8.47138300e-01,  1.84200238e+01,  1.30528666e-01, -4.25802003e-02,
        -2.36828955e-02,  5.30024939e-04,  5.84943508e-03])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 70
       nit: 1
    status: 0
   success: True
         x: array([ 1.49427269, 21.36368008, 11.90820475, 15.73748994,  1.08069455,
         0.05871551,  7.46417758,  2.97513283,  0.80486675,  1.36562317,
        29.03346598])],
'316.22776601683796': [      fun: 0.7365967350726861
 hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
      jac: array([ 1.08950931e-01, -8.29090263e-04,  2.43929624e-04, -3.67370192e-04,
         1.24630122e-01, -6.94806043e+00,  4.75303784e-03, -5.23964039e-02,
         4.38923109e-01,  4.42166983e-03, -8.52446289e-03])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 76
       nit: 1
    status: 0
   success: True
         x: array([ 1.49427269, 21.36368008, 11.90820475, 15.73748994,  1.08069455,
         0.05871551,  7.46417758,  2.97513283,  0.80486675,  1.36562317,
        29.03346598])],
'453.1583637600818': [      fun: 0.7855550013118217
 hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
      jac: array([ 9.28681258e-02,  9.14041430e-04,  5.27173625e-04, -7.84715098e-04,
        -2.49177533e+00,  8.47271151e+00,  2.40801806e-01, -1.64804988e-01,
         1.69307314e+00, -1.08895819e-01, -3.42616908e-02])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 111
       nit: 35
    status: 0
   success: True
         x: array([ 1.43862607, 21.36158526, 11.90776151, 15.73815577,  0.69361086,
         0.04082565,  7.51739723,  2.94964293,  0.78094519,  1.30262462,
```

```
       29.02250461])],
'649.3816315762114': [      fun: 1.025030394521515
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 5.53331724e-01, -3.51886076e-02,  2.96235853e-03, -4.46514259e-03,
         4.64300419e+00, -8.46051366e+01, -2.71488444e-01, -8.79893100e-02,
         2.45780454e-01, -1.65790757e-01, -1.11474984e-02])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 73
       nit: 1
    status: 0
   success: True
         x: array([ 1.43862607, 21.36158526, 11.90776151, 15.73815577,  0.69361086,
         0.04082565,  7.51739723,  2.94964293,  0.78094519,  1.30262462,
        29.02250461])],
'930.572040929699': [      fun: 1.4006246362744286
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 5.90222844e-01, -3.45899137e-02,  6.09454975e-03, -9.19481116e-03,
         7.63662653e-01, -2.27302073e+01,  7.20800686e-02, -9.69958629e-03,
         5.54436302e-01, -1.01561301e-01, -2.13908533e-02])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 9
       nit: 1
    status: 0
   success: True
         x: array([ 1.43862607, 21.36158526, 11.90776151, 15.73815577,  0.69361086,
         0.04082565,  7.51739723,  2.94964293,  0.78094519,  1.30262462,
        29.02250461])],
'1333.521432163324': [      fun: 0.8401184729659011
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-5.42819797e-02,  3.63366886e-03,  2.18804374e-03, -3.31104691e-03,
        -4.88799897e-01, -2.64145477e+01,  6.51523622e-02, -1.54337121e-01,
         3.04480046e+00,  1.65535104e-02, -3.30752610e-02])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 23
       nit: 6
    status: 0
   success: True
         x: array([ 1.41805466, 21.36165678, 11.90808001, 15.73767374,  0.71100714,
         0.03222377,  7.46208868,  2.95196159,  0.45770996,  1.32106214,
        29.03521403])],
'1910.9529749704404': [      fun: 1.422680577247932
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 1.78355134e-02,  2.52584394e-03,  3.21436116e-03, -4.86436152e-03,
        -2.34741204e+00, -3.02634863e+01,  2.67139168e-01, -3.01092913e-01,
         5.45570216e+00, -2.24174442e-02, -7.35965587e-02])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 35
       nit: 1
```

```
    status: 0
   success: True
         x: array([ 1.41805466, 21.36165678, 11.90808001, 15.73767374,  0.71100714,
          0.03222377,  7.46208868,  2.95196159,  0.45770996,  1.32106214,
         29.03521403])],
'2738.4196342643613': [      fun: 5.141537381897458
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 7.96908916e-02, -1.68224429e-03,  6.38832959e-04, -9.66759311e-04,
         5.10313017e+00, -2.41724092e+02, -4.23577218e-01, -2.21272652e-01,
         9.84024023e+00, -6.73438880e-02, -1.35442882e-01])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 40
       nit: 1
    status: 0
   success: True
         x: array([ 1.41805466, 21.36165678, 11.90808001, 15.73767374,  0.71100714,
          0.03222377,  7.46208868,  2.95196159,  0.45770996,  1.32106214,
         29.03521403])],
'3924.189758484536': [      fun: 10.199542213500223
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 4.11111076e-02, -9.43825906e-05, -6.13805330e-04,  9.28884762e-04,
        -1.75052059e-02,  1.15828402e+00,  1.06061839e-01, -4.72522937e-03,
         2.30305324e+00, -3.65040950e-02, -2.54568605e-02])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 54
       nit: 1
    status: 0
   success: True
         x: array([ 1.41805466, 21.36165678, 11.90808001, 15.73767374,  0.71100714,
          0.03222377,  7.46208868,  2.95196159,  0.45770996,  1.32106214,
         29.03521403])],
'5623.413251903491': [      fun: 0.9058466490940724
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 2.73126992e-08, -2.44942766e-10,  6.63080745e-08, -1.05727232e-07,
        -2.92712399e-07,  1.83078988e-06,  1.61948709e-03,  2.45710523e-06,
         2.49867489e-04, -2.13188373e-09,  1.63762826e-04])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 52
       nit: 28
    status: 0
   success: True
         x: array([10.21549239, 22.02092285, 12.1979343 , 15.30015542,  1.1368747 ,
          0.54530038,  0.        , 53.10196082,  0.14929328,  0.        ,
         32.39394119])],
'8058.421877614818': [      fun: 10.727566792293128
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 3.75723700e-14,  3.44969228e-12, -6.56254155e-20,  1.04628219e-19,
         1.70794819e-10,  0.00000000e+00,  9.86678823e-06,  2.20130816e-11,
```

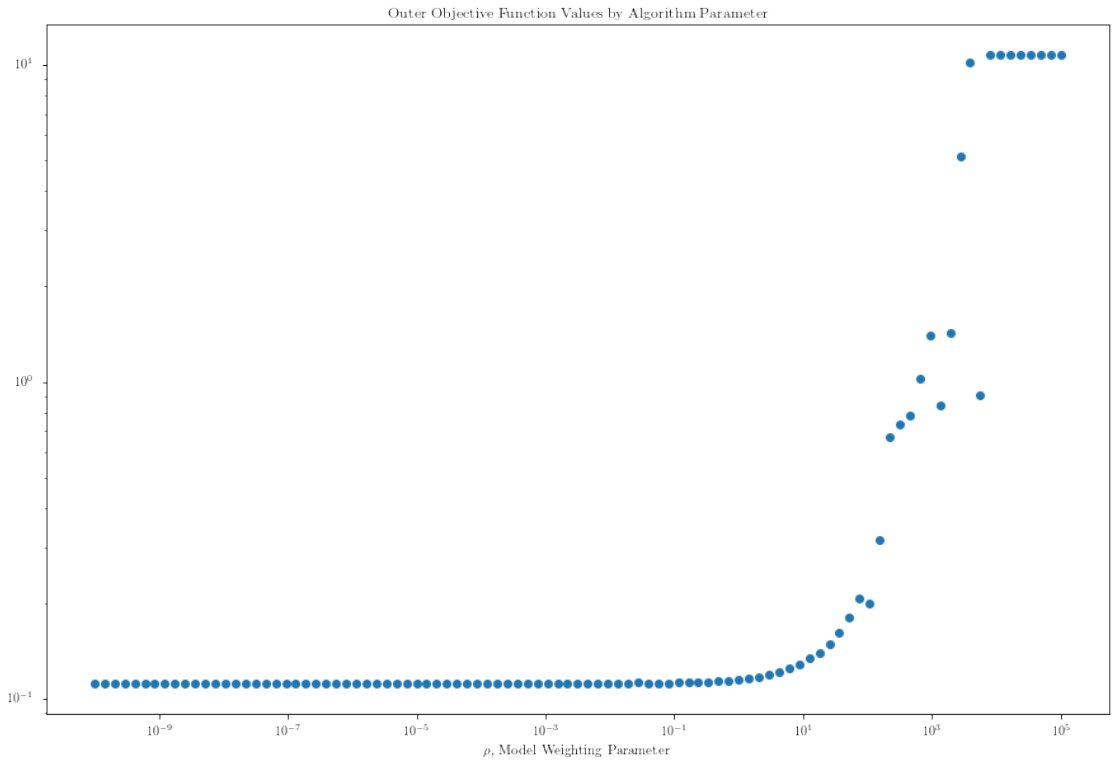
```
        -8.86317766e-15, -3.80006301e-14,  7.29016297e-02])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 3
       nit: 2
    status: 0
   success: True
         x: array([10.21537573, 22.02086508, 12.19803878, 15.29998883,  7.03880851,
         0.        ,  0.        , 70.05095196,  0.        ,  0.        ,
         0.        ])],
'11547.819846894581': [      fun: 10.72756679271465
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 2.45596475e-14,  2.53859785e-12, -7.73833471e-20,  1.23373828e-19,
         8.92984813e-11,  0.00000000e+00,  1.31022794e-05,  1.67989955e-11,
         3.79333988e-14, -3.36051222e-14,  7.28996258e-02])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([10.21537573, 22.02086508, 12.19803878, 15.29998883,  7.03880851,
         0.        ,  0.        , 70.05095196,  0.        ,  0.        ,
         0.        ])],
'16548.170999431815': [      fun: 10.72756679302017
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-1.21244768e-14,  1.02327375e-12, -1.37570737e-19,  2.19378006e-19,
        -4.54734824e-11,  0.00000000e+00,  2.56732251e-05,  8.19147476e-12,
        -1.21059648e-13,  6.11277326e-14,  7.29220352e-02])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([10.21537573, 22.02086508, 12.19803878, 15.29998883,  7.03880851,
         0.        ,  0.        , 70.05095196,  0.        ,  0.        ,
         0.        ])],
'23713.737056616552': [      fun: 10.727566793232082
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 1.06747044e-12,  2.68658515e-11,  2.95584266e-18, -4.71324514e-18,
         4.91636937e-10,  0.00000000e+00, -4.43727500e-04,  9.43214870e-11,
         4.44447564e-12, -2.79943597e-12,  7.22792458e-02])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 17
       nit: 1
    status: 0
   success: True
         x: array([10.21537573, 22.02086508, 12.19803878, 15.29998883,  7.03880851,
         0.        ,  0.        , 70.05095196,  0.        ,  0.        ,
         0.        ])],
```

```
'33982.083289425595': [      fun: 10.72756679337832
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([ 1.79214021e-14,  1.09421948e-12, -3.05391711e-17,  4.86952148e-17,
         2.47562801e-10,  0.00000000e+00, -2.40732584e-02,  6.54083923e-12,
         5.51937748e-14, -3.69846516e-14,  7.29027342e-02])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 20
       nit: 1
    status: 0
   success: True
         x: array([10.21537573, 22.02086508, 12.19803878, 15.29998883,  7.03880851,
         0.        ,  0.        , 70.05095196,  0.        ,  0.        ,
         0.        ])],
'48696.75251658631': [      fun: 10.727566793718644
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-7.54430239e-17,  2.24099139e-14,  7.62280225e-17, -1.21546804e-16,
         3.77287721e-14,  0.00000000e+00,  4.07759848e-04,  2.91543488e-14,
        -9.30190085e-17,  9.18791448e-17,  7.29038676e-02])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([10.21537573, 22.02086508, 12.19803878, 15.29998883,  7.03880851,
         0.        ,  0.        , 70.05095196,  0.        ,  0.        ,
         0.        ])],
'69783.05848598664': [      fun: 10.727566793719031
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-4.74163901e-17,  1.40768846e-14,  7.62276053e-17, -1.21546139e-16,
         3.04087395e-14,  0.00000000e+00,  4.09198541e-04,  2.06673217e-14,
        -6.61092378e-17,  6.32399228e-17,  7.29041771e-02])
   message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
      nfev: 1
       nit: 0
    status: 0
   success: True
         x: array([10.21537573, 22.02086508, 12.19803878, 15.29998883,  7.03880851,
         0.        ,  0.        , 70.05095196,  0.        ,  0.        ,
         0.        ])],
'100000.0': [      fun: 10.727566793719275
  hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
       jac: array([-3.96711399e-17,  1.17743983e-14,  7.62228510e-17, -1.21538558e-16,
         8.83127935e-15,  0.00000000e+00, -3.20897688e-03,  1.45138845e-14,
        -4.64310259e-17,  4.47526288e-17,  7.29071302e-02])
   message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
      nfev: 26
       nit: 1
    status: 0
```

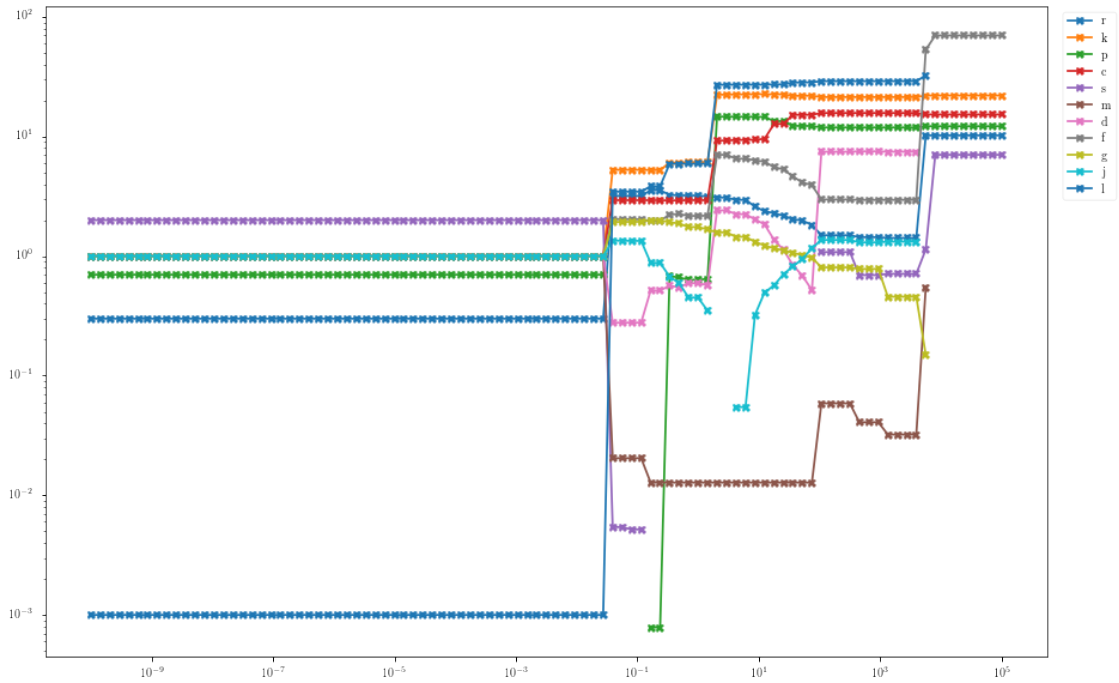
```
    success: True
          x: array([10.21537573, 22.02086508, 12.19803878, 15.29998883,  7.03880851,
          0.        ,  0.        , 70.05095196,  0.        ,  0.        ,
          0.        ])]}
```

In [10]:
```
outer_objective_values = np.array([[float(rho), val[0].fun] for rho, val in solver.solu
plt.loglog(*outer_objective_values.T, 'o')
plt.title("Outer Objective Function Values by Algorithm Parameter")
plt.xlabel(r"$\rho$, Model Weighting Parameter")
```

Out[10]:
```
Text(0.5, 0, '$\\rho$, Model Weighting Parameter')
```



In [11]:
```
rhos = [float(rho) for rho in solver.solutions.keys()]
vals = [val[0].x for val in solver.solutions.values()]
plt.plot(rhos, vals, 'X-')
plt.legend("rkpcsmdfgjl", loc="best", bbox_to_anchor=(1.01, 1))
plt.xscale("log")
plt.yscale("log", nonposy="mask")
```

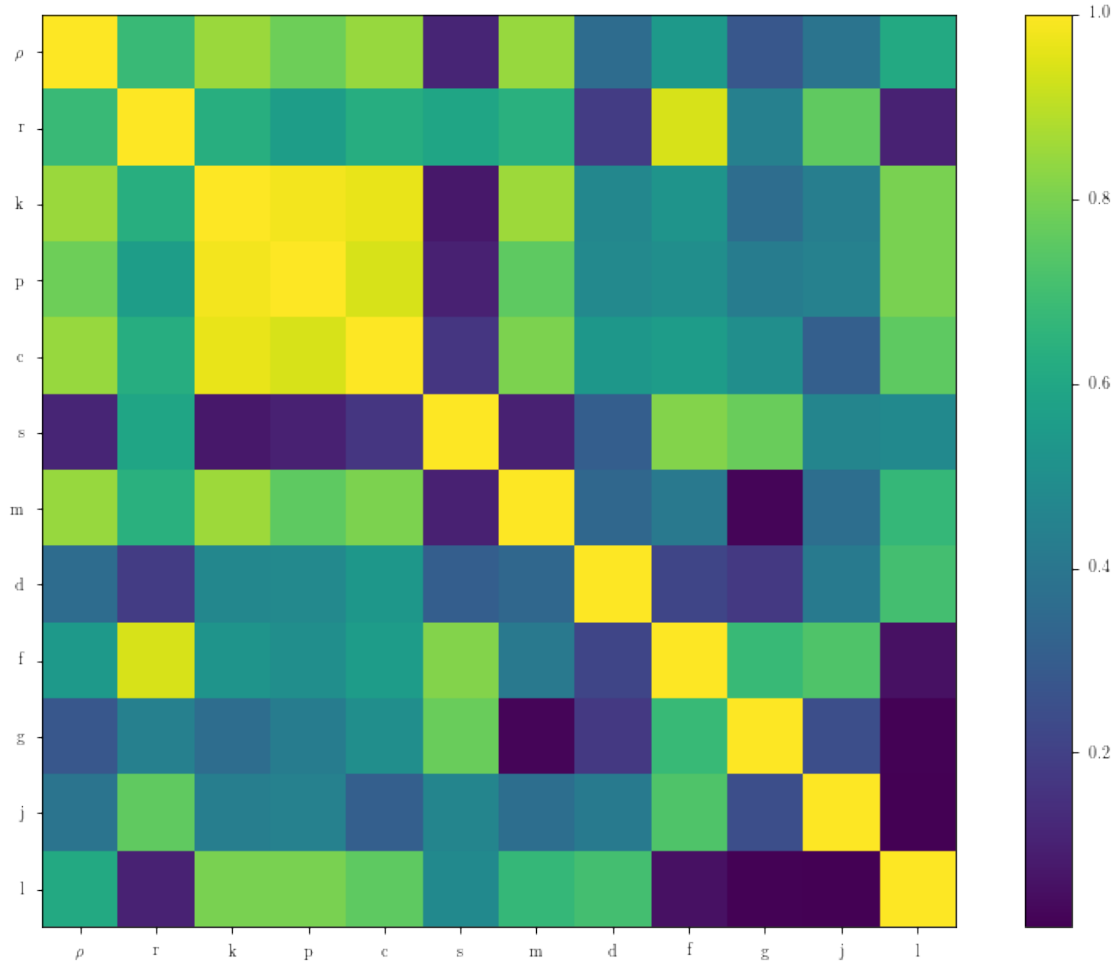


In [12]:
```
# generate a crude correlation plot

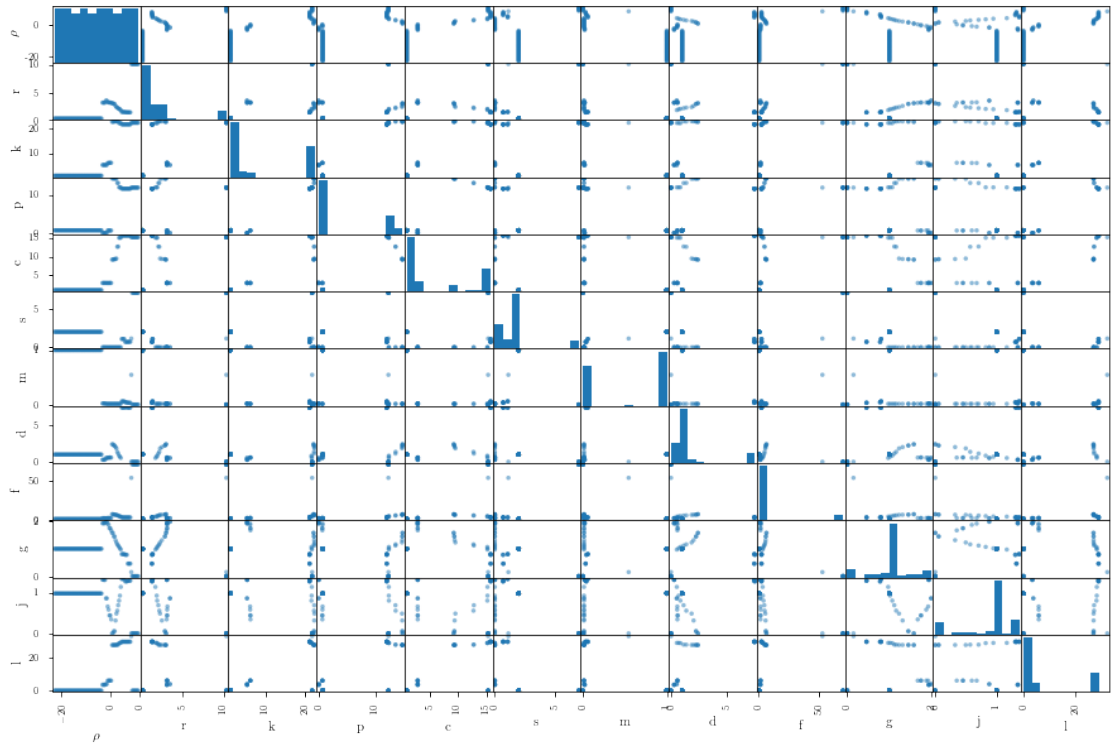
import pandas as pd

results = pd.DataFrame({r"$\rho$": np.log(rhos)})
for idx, name in enumerate("rkpcsmdfgjl"):
    results[name] = [v[idx] for v in vals]
```

In [13]:
```
plt.imshow(np.abs(results.corr()))
plt.colorbar()
plt.xticks(range(12), [r"$\rho$"] + list("rkpcsmdfgjl"))
plt.yticks(range(12), [r"$\rho$"] + list("rkpcsmdfgjl"))
plt.grid(False)
```




```
In [14]: ax = pd.plotting.scatter_matrix(results)
         for a in ax.flatten():
             a.grid(False)
```

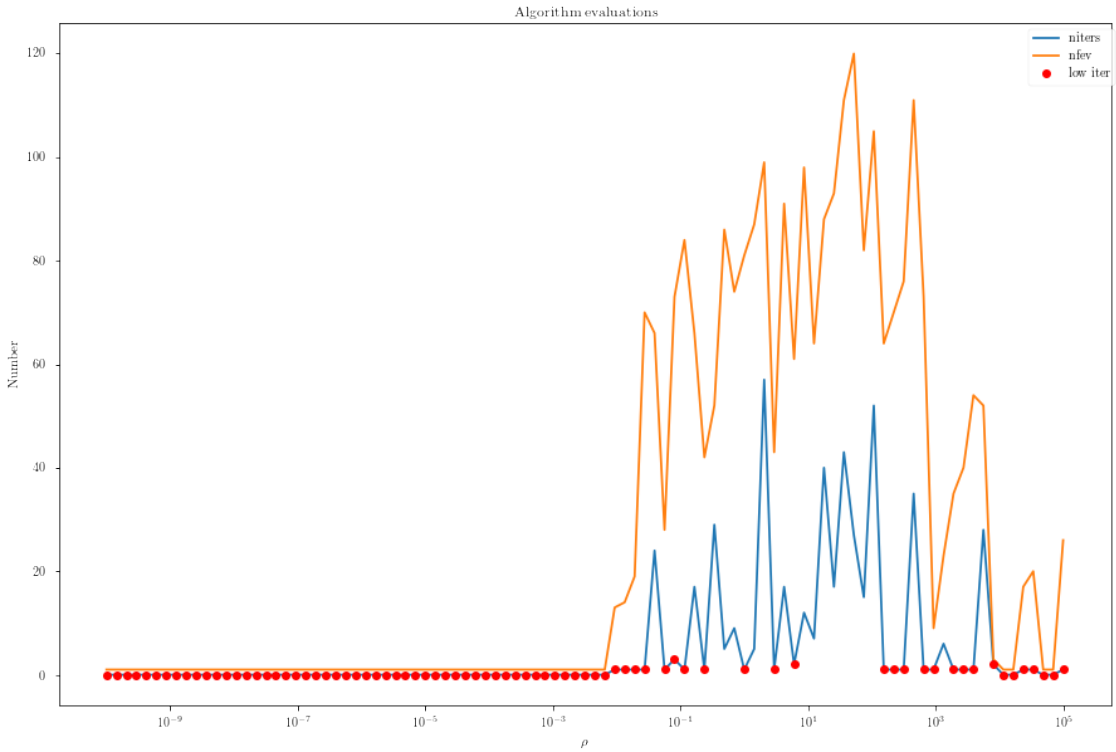


```
In [15]: iters_list = np.array([[float(key), value[0].nit] for key, value in solver.solutions.it
         fevs_list = np.array([[float(key), value[0].nfev] for key, value in solver.solutions.it
         plt.semilogx(*iters_list.T, *fevs_list.T)

         low_iters = np.array([[k,v] for k,v in iters_list if v < 5])
         plt.plot(*low_iters.T, 'ro')

         plt.legend(["niters", "nfev", "low iter"],
                    loc="best", bbox_to_anchor=(1.01, 1))
         plt.title("Algorithm evaluations")
         plt.xlabel(r"$\rho$")
         plt.ylabel("Number")

Out[15]: Text(0, 0.5, 'Number')
```

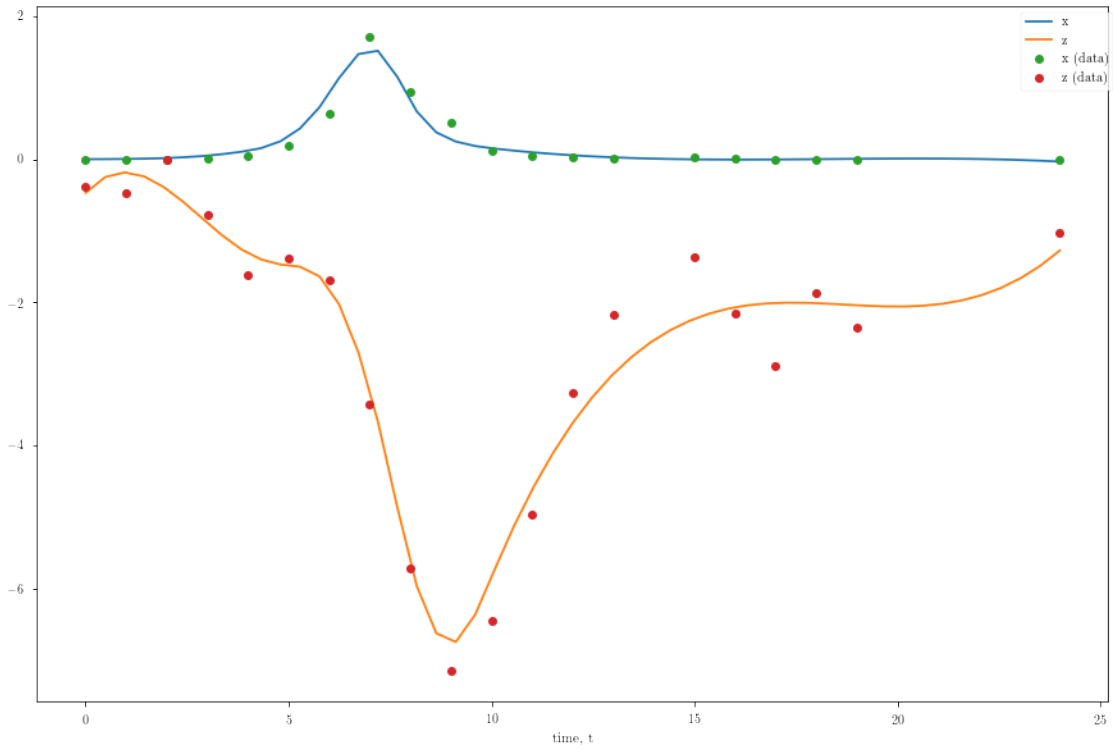



```
In [16]: def tokey(rho, ps):
             return f"{'y'.join(map(str,ps))}r{rho}"
         getx = fitter.ca.Function("getx", [model.ts, *model.cs], model.xs)
```

```
In [17]: target_rho = 74.98942093324558
         c_end = solver.problems[0].cache.results[tokey(target_rho, solver.solutions[str(target_

         xs_end = np.array([np.array(i) for i in getx(model.observation_times,
                                                      *fitter.argsplit(c_end,
                                                                       3)
                                                     )])
         plt.plot(model.observation_times, np.hstack([xs_end[0], xs_end[2]]),
                  context['datasets'][0]['t'], context['datasets'][0]['x'], 'o',
                  context['datasets'][0]['t'], context['datasets'][0]['z'], 'o')
         plt.legend(list("xz") + ["x (data)", "z (data)"], loc="best", bbox_to_anchor=(1.01, 1))
         plt.xlabel("time, t")
```

```
Out[17]: Text(0.5, 0, 'time, t')
```

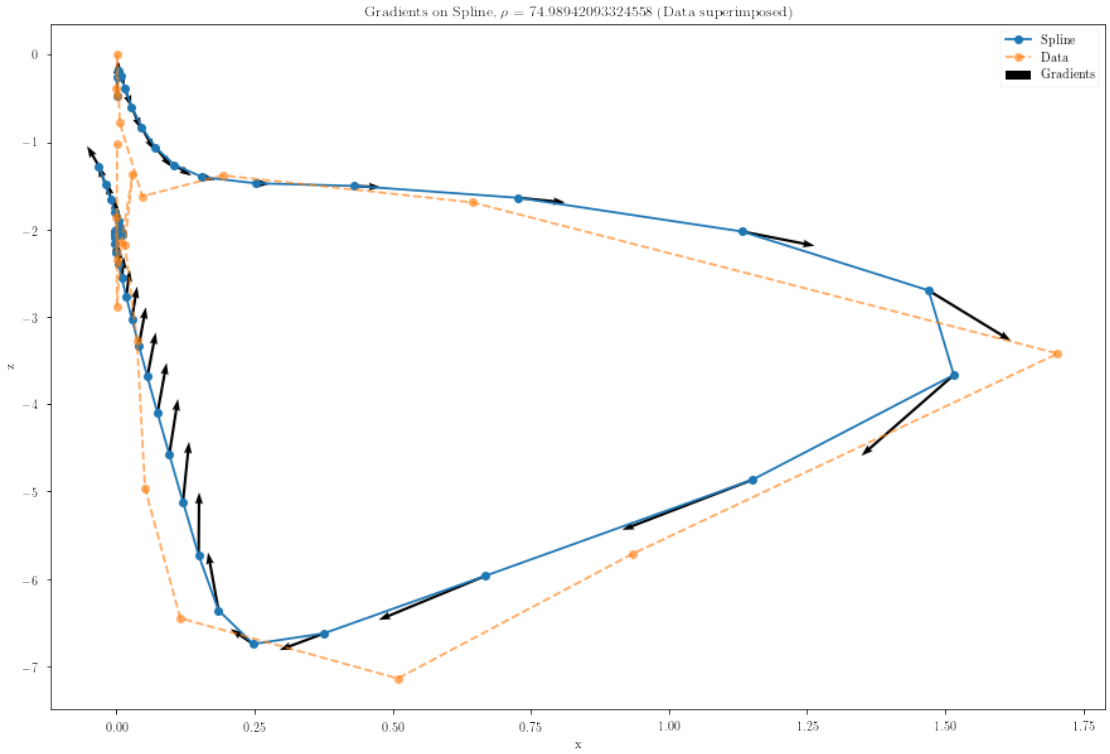


In [30]:
```
spline_dfield = np.array([context['model'](t, xs_end[:,i], solver.solutions[str(target_
                          for i,t in enumerate(model.observation_times)])

spline_dfield = spline_dfield.reshape(spline_dfield.shape[:2])

plt.plot(xs_end[0], xs_end[2], 'o-')
plt.plot(context['datasets'][0]['x'], context['datasets'][0]['z'], 'o--', alpha=0.65)
plt.quiver(xs_end[0], xs_end[2], spline_dfield[:,0], spline_dfield[:,2],
           scale=20, angles='xy', headwidth=3, headlength=4.5, headaxislength=4, width=
plt.title(f"Gradients on Spline, $\\rho$ = {target_rho} (Data superimposed)")
plt.xlabel("x")
plt.ylabel("z")
plt.legend(["Spline", "Data", "Gradients"])
```

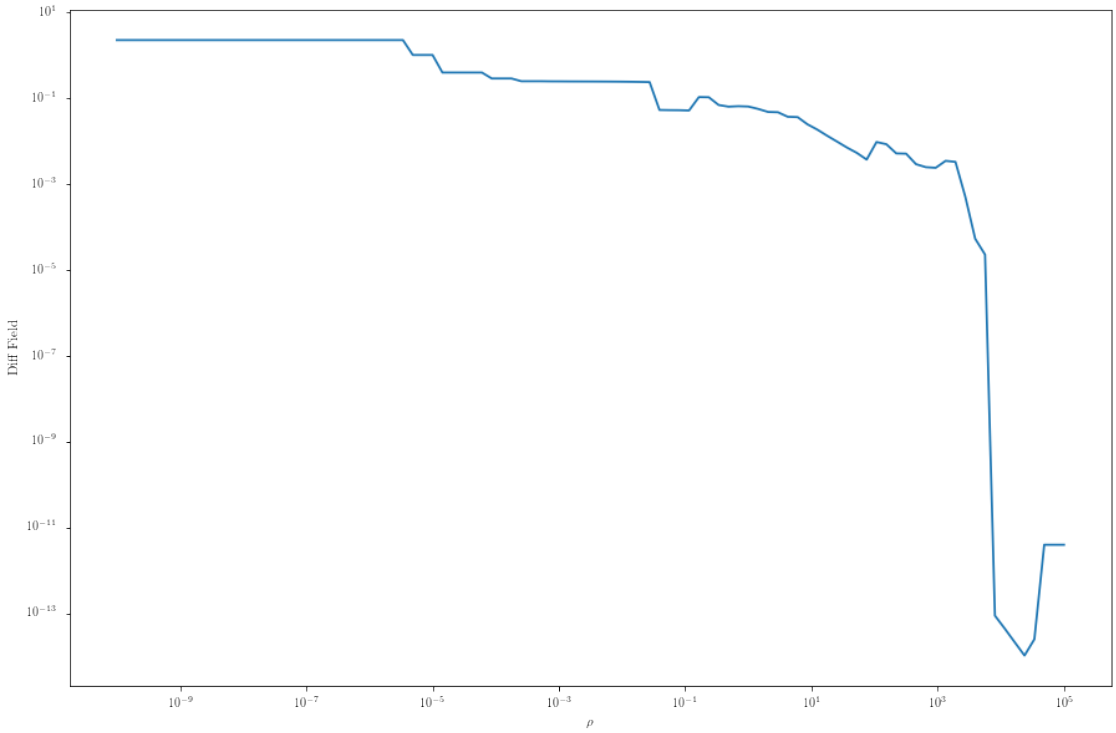
Out[30]:
```
<matplotlib.legend.Legend at 0x7f93eb6c5cc0>
```




```
In [18]: inner_objective_values = np.array([[r, solver.problems[0].cache.results[tokey(r, v)].fu
         dfield = np.hstack([np.array(rhos).reshape(-1,1),
                             ((inner_objective_values - outer_objective_values)[:,1] / rhos).res

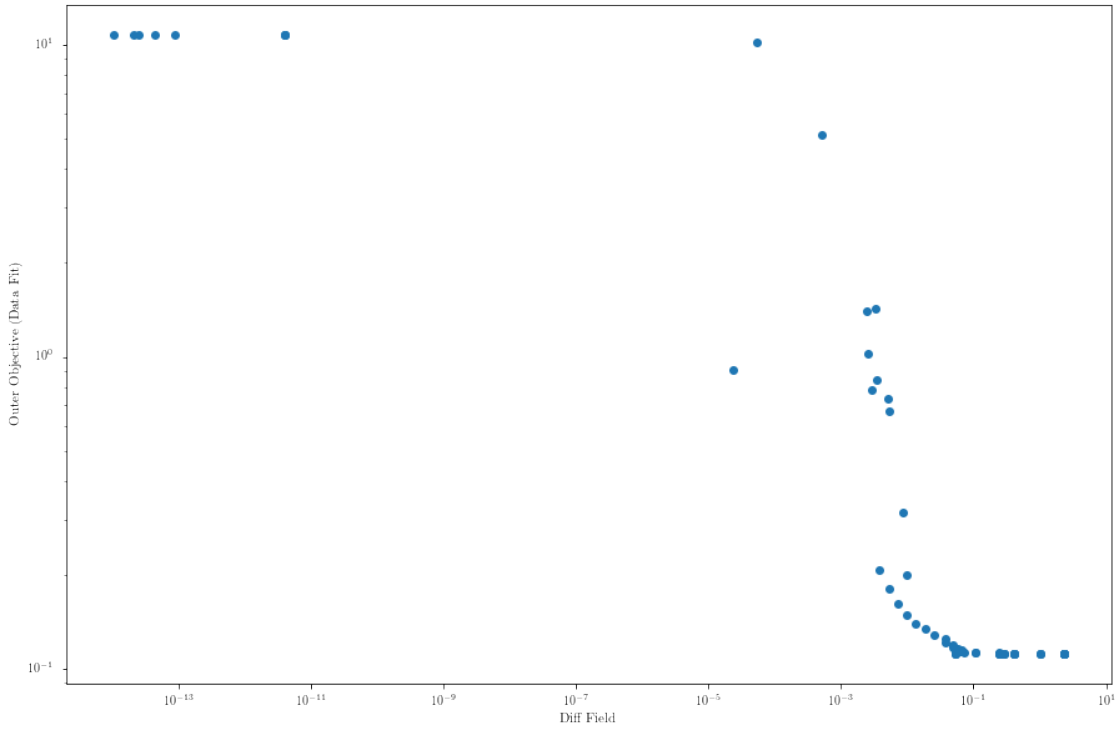
In [19]: plt.loglog(*dfield.T)
         plt.xlabel(r"$\rho$")
         plt.ylabel("Diff Field")

Out[19]: Text(0, 0.5, 'Diff Field')
```




```
In [20]: plt.loglog(dfield[:,1] , outer_objective_values[:,1], 'o')
         plt.xlabel("Diff Field")
         plt.ylabel("Outer Objective (Data Fit)")
```

```
Out[20]: Text(0, 0.5, 'Outer Objective (Data Fit)')
```




```
In [21]: rhos
```

```
Out[21]: [1e-10,
          1.4330125702369628e-10,
          2.053525026457146e-10,
          2.942727176209282e-10,
          4.2169650342858224e-10,
          6.042963902381328e-10,
          8.659643233600653e-10,
          1.2409377607517196e-09,
          1.7782794100389228e-09,
          2.5482967479793464e-09,
          3.651741272548377e-09,
          5.232991146814947e-09,
          7.498942093324558e-09,
          1.0746078283213174e-08,
          1.539926526059492e-08,
          2.20673406908459e-08,
          3.162277660168379e-08,
          4.5315836376008177e-08,
          6.493816315762114e-08,
          9.30572040929699e-08,
          1.333521432163324e-07,
          1.9109529749704404e-07,
```

```
2.7384196342643614e-07,
3.924189758484536e-07,
5.62341325190349e-07,
8.058421877614818e-07,
1.1547819846894582e-06,
1.6548170999431815e-06,
2.3713737056616552e-06,
3.3982083289425593e-06,
4.869675251658631e-06,
6.978305848598663e-06,
1e-05,
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2.0535250264571462e-05,
2.9427271762092818e-05,
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6.042963902381328e-05,
8.659643233600654e-05,
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0.00017782794100389227,
0.00025482967479793463,
0.0003651741272548377,
0.0005232991146814947,
0.0007498942093324559,
0.0010746078283213176,
0.001539926526059492,
0.0022067340690845897,
0.003162276601683794,
0.004531583637600818,
0.006493816315762113,
0.00930572040929699,
0.01333521432163324,
0.019109529749704406,
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0.03924189758484536,
0.05623413251903491,
0.08058421877614819,
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0.16548170999431813,
0.23713737056616552,
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0.6978305848598664,
1.0,
1.4330125702369627,
2.0535250264571463,
2.942727176209282,
4.216965034285822,
6.042963902381328,
```

```
8.659643233600654,
12.409377607517195,
17.78279410038923,
25.482967479793466,
36.51741272548377,
52.32991146814947,
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220.673406908459,
316.22776601683796,
453.1583637600818,
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1333.521432163324,
1910.9529749704404,
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3924.189758484536,
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8058.421877614818,
11547.819846894581,
16548.170999431815,
23713.737056616552,
33982.083289425595,
48696.75251658631,
69783.05848598664,
100000.0]
```