

smol_case

April 1, 2019

```
In [1]: import ingestor, modeller, fitter
import numpy as np
import matplotlib.pyplot as plt

In [2]: plt.style.use('seaborn-notebook')
plt.rc('text', usetex=True)
plt.rc('font', family='serif')
plt.rcParams['figure.figsize'] = [15, 10]

In [3]: from cycler import cycler

new_color = cycler(color=["k"])
plt.rcParams['axes.prop_cycle'] = plt.rcParams['axes.prop_cycle'].concat(new_color)

In [4]: from importlib import reload

reload(fitter)
reload(modeller)
reload(ingestor)

Out[4]: <module 'ingestor' from '/media/dwu402/Data/wrap-mad/ingestor.py'>

In [5]: context = ingestor.Context("runs/minimal4.3.run")

In [6]: model = modeller.Model(context)

In [7]: solver = fitter.Fitter()
solver.construct_objectives(context, model)

In [8]: solver.construct_problems()

In [9]: solver.solve(10**-10)
for rhoi in np.logspace(-7, 4.5, num=51):
    solver.solve(rhoi)
    solver.problems[0].initial_guess = solver.solutions[str(rhoi)][-1].x

In [10]: solver.solutions
```

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Out[10]: {'1e-10': [ fun: 0.00868834096457184
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-3.50933678e-10, 3.59923619e-10, 2.05593958e-10, -4.14272215e-10,
-2.71847096e-09, 1.05101691e-08, 4.55110808e-09, 1.60101734e-09,
-7.18124869e-09, 2.02081755e-09, 1.25097863e-09])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 3
nit: 2
status: 0
success: True
x: array([1.00000004, 0.99999999, 0.99999995, 1.00000009, 0.99999989,
1.00000047, 1.00000022, 0.99999969, 0.99999986, 1.00000036,
0.99999959])), '1e-07': [ fun: 0.0086883409645745
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-4.06575281e-10, 4.28837960e-10, 2.65944162e-10, -5.17101958e-10,
-3.12378219e-09, 1.20417235e-08, 5.14151851e-09, 1.85733257e-09,
-8.39520141e-09, 2.49635655e-09, 1.36385105e-09])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 3
nit: 2
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.0000001 , 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
0.99999951])), '1.6982436524617461e-07': [ fun: 0.0086883409645745
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-4.06971170e-10, 4.29015792e-10, 2.64769240e-10, -5.14592001e-10,
-3.12562259e-09, 1.20461082e-08, 5.14538707e-09, 1.84875520e-09,
-8.38550447e-09, 2.49935375e-09, 1.36927372e-09])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 1
nit: 0
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.0000001 , 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
0.99999951])), '2.8840315031266057e-07': [ fun: 0.0086883409645745
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-4.07522054e-10, 4.29255160e-10, 2.62653438e-10, -5.10106295e-10,
-3.12869322e-09, 1.20534349e-08, 5.15193506e-09, 1.83340494e-09,
-8.36859897e-09, 2.50500358e-09, 1.37753204e-09])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 1
nit: 0
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.0000001 , 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
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0.99999951]]], '4.073802778041131e-06': [      fun: 0.008688297106187952
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-5.01733835e-10,  4.60944190e-10,  3.61532018e-10, -7.06234594e-10,
-3.20428678e-09,  1.22913704e-08,  5.35598643e-09,  2.37679094e-09,
-8.53644474e-09,  2.58166094e-09,  2.18148080e-09])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 1
nit: 0
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.00000001, 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
0.99999951]]], '6.9183097091893625e-06': [      fun: 0.008688297106187952
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-5.75376945e-10,  4.94012568e-10,  3.82963791e-10, -7.47908850e-10,
-3.21915366e-09,  1.23287147e-08,  5.42210270e-09,  2.52218701e-09,
-8.45820849e-09,  2.59834062e-09,  2.28946932e-09])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 1
nit: 0
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.00000001, 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
0.99999951]]], '1.1748975549395302e-05': [      fun: 0.008688325158236086
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-1.91497577e-10,  1.52637228e-10,  7.32274917e-10, -1.47144143e-09,
-3.16358102e-09,  1.24297919e-08,  5.33399381e-09,  5.09543686e-09,
-1.11956003e-08,  2.33102594e-09,  5.23253438e-09])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 1
nit: 0
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.00000001, 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
0.99999951]]], '1.995262314968883e-05': [      fun: 0.008688325158236086
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-7.39775491e-11,  1.33501327e-11,  8.31406817e-10, -1.68195215e-09,
-3.00321872e-09,  1.20403982e-08,  4.93665742e-09,  6.24589233e-09,
-1.22049152e-08,  1.94997651e-09,  5.52165474e-09])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 1
nit: 0
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.00000001, 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,

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0.99999951]]], '0.0002818382931264455': [      fun: 0.008688391747133699
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 2.04134899e-09,  7.36824168e-09,  2.78431709e-08, -4.87702946e-08,
-2.50958315e-08,  5.59780129e-08,  6.34435188e-08, -3.37771135e-08,
 3.05279168e-08,  1.36697358e-08, -3.98382340e-08])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 1
nit: 0
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.00000001 , 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
0.99999951]]], '0.00047863009232263854': [      fun: 0.008688523347585877
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-3.37774209e-09,  3.57391843e-08,  7.99376118e-08, -1.44254418e-07,
-8.23290891e-08,  1.71016712e-07,  2.18630507e-07, -1.00128806e-07,
 9.68853479e-08,  2.36281204e-08, -1.06648042e-07])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 1
nit: 0
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.00000001 , 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
0.99999951]]], '0.0008128305161640995': [      fun: 0.008688928992809175
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-2.17323675e-09,  8.69515220e-08,  2.28507965e-07, -4.05773585e-07,
-2.29732184e-07,  4.68244807e-07,  6.11696629e-07, -2.53441287e-07,
 2.69899970e-07,  4.58488813e-08, -2.59417718e-07])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 1
nit: 0
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.00000001 , 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
0.99999951]]], '0.0013803842646028866': [      fun: 0.008690050977242224
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-4.26488435e-08,  2.84259347e-07,  6.87690329e-07, -1.24921370e-06,
-6.72297465e-07,  1.36192522e-06,  1.77874811e-06, -7.27966294e-07,
 7.35450896e-07,  9.65707787e-08, -7.25963950e-07])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 1
nit: 0
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.00000001 , 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,

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0.99999951]]], '0.0023442288153199225': [      fun: 0.008693247148447842
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-8.07705185e-08,  7.07935965e-07,  2.05015741e-06, -3.74796066e-06,
-1.94038309e-06,  3.95761125e-06,  5.24925833e-06, -1.95207821e-06,
 2.13795489e-06,  3.28845620e-07, -2.04988412e-06])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 1
nit: 0
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.00000001, 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
0.99999951]]], '0.003981071705534978': [      fun: 0.00870220495341063
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-1.76089015e-07,  1.96868197e-06,  5.71643287e-06, -1.03942150e-05,
-5.41442782e-06,  1.10229337e-05,  1.45491500e-05, -5.41381463e-06,
 5.77019773e-06,  8.47927870e-07, -5.55841321e-06])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 14
nit: 1
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.00000001, 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
0.99999951]]], '0.006760829753919818': [      fun: 0.008726809548970737
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-3.38484947e-07,  5.16187953e-06,  1.58300173e-05, -2.87412044e-05,
-1.53319732e-05,  3.11523852e-05,  4.09032108e-05, -1.45307618e-05,
 1.54364266e-05,  2.44172057e-06, -1.48301071e-05])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 16
nit: 1
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.00000001, 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
0.99999951]]], '0.01148153621496884': [      fun: 0.008791398960061128
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-1.63916075e-06,  1.40141861e-05,  4.26611203e-05, -7.73420593e-05,
-4.21488840e-05,  8.58203268e-05,  1.12393349e-04, -3.76211703e-05,
 4.06283922e-05,  6.06441724e-06, -3.82826281e-05])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 45
nit: 1
status: 0
success: True
x: array([1.00000005, 0.99999999, 0.99999994, 1.00000001, 0.99999988,
1.00000053, 1.00000025, 0.99999964, 0.99999983, 1.00000043,
0.99999951]]], '0.01148153621496884': [      fun: 0.008791398960061128

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0.99999951]]], '0.019498445997580455': [      fun: 0.008949042959087166
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-1.70171872e-07, -1.18875830e-06,  1.20980815e-06, -2.86291955e-06,
-1.98283542e-06,  3.30403454e-06,  5.90065525e-07,  3.22288175e-06,
-2.82404373e-06, -9.01279653e-07,  3.72184293e-06])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 9
nit: 3
status: 0
success: True
x: array([1.00028531, 0.99814702, 0.99507325, 1.00867903, 1.00517726,
0.98923274, 0.98610925, 1.00438237, 0.99536268, 0.99931943,
1.00469075]]], '0.03311311214825914': [      fun: 0.009308857823425018
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 4.78235213e-07,  2.48388311e-07, -9.02687278e-07,  1.10185934e-06,
 9.80071846e-07, -1.03818735e-06, -4.85383188e-06,  3.80534472e-07,
-2.13306354e-06, -1.09437476e-06, -1.17097237e-06])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 3
nit: 2
status: 0
success: True
x: array([1.00098286, 0.99570009, 0.98931382, 1.01823497, 1.01260299,
0.97339987, 0.96740344, 1.00834522, 0.99128774, 0.99892371,
1.00962179]]], '0.05623413251903491': [      fun: 0.01004152260978744
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 4.40171625e-06,  1.24431387e-06, -7.32850676e-06,  1.51741806e-05,
-1.45668023e-05,  3.36363255e-05,  7.09583006e-06,  7.68542145e-06,
-2.16695537e-05, -2.82977814e-06, -5.37877701e-06])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 50
nit: 3
status: 0
success: True
x: array([1.00287161, 0.99070752, 0.98093471, 1.03060261, 1.02630432,
0.9427322 , 0.93392874, 1.01280696, 0.98714427, 0.9991205 ,
1.01815164]]], '0.09549925860214369': [      fun: 0.01135717676806062
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-5.08234085e-06,  3.70241979e-05, -5.60822085e-05,  1.29404399e-04,
-2.85504473e-05,  7.41886448e-05, -3.45245280e-05,  1.77656223e-05,
-6.71031748e-05, -1.78074263e-05, -2.63247497e-05])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 85
nit: 4
status: 0
success: True
x: array([1.00633293, 0.98327689, 0.97349805, 1.0378791 , 1.04967791,
0.8888363 , 0.88407472, 1.01201781, 0.98987875, 1.00165427,

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1.02987251]]], '0.16218100973589297': [      fun: 0.013453002444476298
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-1.94703980e-05,  2.15485388e-05, -5.51144805e-08,  1.15536405e-06,
  1.09831143e-06,  1.84872783e-06,  6.13012812e-06, -2.14805788e-05,
  1.78436197e-05, -2.41111916e-06, -4.78504996e-06])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 27
nit: 7
status: 0
success: True
x: array([1.01291313, 0.96831371, 0.98549675, 0.99352795, 1.08050218,
  0.81792888, 0.83801732, 1.00361302, 1.00529433, 1.01268159,
  1.05192412]))], '0.2754228703338169': [      fun: 0.016546858288458186
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-4.39524075e-06,  2.64663879e-05,  2.35704938e-05,  1.07129119e-05,
  1.46930256e-05,  9.73151336e-06, -4.15308107e-05, -9.55830645e-06,
 -2.01913900e-06,  6.32093392e-07, -1.95941505e-05])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 93
nit: 10
status: 0
success: True
x: array([1.00282097, 0.96292541, 1.03675056, 0.79028628, 1.09647027,
  0.81806276, 0.78274364, 1.00984989, 0.99238152, 1.04537616,
  1.09234271]))], '0.4677351412871981': [      fun: 0.020500761509583684
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-2.03019426e-05,  9.01138669e-06, -1.11918970e-04,  2.77709354e-04,
 -3.32197923e-05,  1.84268673e-04,  1.45040905e-04,  3.93244185e-05,
 -3.82017816e-05,  1.81325622e-05, -1.38994950e-07])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 58
nit: 10
status: 0
success: True
x: array([0.93867109, 0.97159974, 1.02125495, 0.60229536, 1.09000407,
  0.90978898, 0.79079069, 1.0416525 , 0.94775532, 1.08579521,
  1.13481693]))], '0.7943282347242822': [      fun: 0.026516739516226365
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-2.2252284e-05, -9.93804624e-05, -6.05242216e-05,  1.72252573e-04,
 -4.73422145e-05,  5.62751951e-05,  1.53891220e-04,  5.82000324e-05,
  8.93004568e-07, -5.47921049e-05,  4.67003649e-05])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 84
nit: 10
status: 0
success: True
x: array([0.89188318, 0.99596225, 0.92135989, 0.56264771, 1.11120138,
  0.9539824 , 0.80862548, 1.05592336, 0.94355492, 1.15231087,

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1.18245914]]], '1.3489628825916533': [      fun: 0.032522472454258246
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 1.12883200e-05,  1.42723799e-04,  1.84659051e-04, -1.80205352e-04,
  1.71168857e-04, -1.58434328e-04, -1.13033936e-04,  4.84741626e-04,
 -2.42566184e-04,  1.63620773e-05,  1.98384423e-05])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 108
nit: 29
status: 0
success: True
x: array([1.31987195, 1.06914093, 0.94917679, 1.05089125, 1.14084708,
 0.81787605, 0.81917482, 0.44933799, 0.91818257, 1.42791062,
 1.27543739]))], '2.290867652767775': [      fun: 0.03636694248446844
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 3.23748903e-04, -1.52689148e-04, -9.48677087e-05,  1.29582375e-04,
 -2.63810060e-04,  2.93787770e-04,  5.77560605e-04,  2.13206321e-04,
 -1.28091468e-04,  7.02151372e-05, -6.36180228e-05])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 87
nit: 13
status: 0
success: True
x: array([1.38879953, 1.11994784, 1.07216886, 1.00043238, 1.1671135 ,
 0.84591244, 0.80716409, 0.28943566, 0.80341202, 1.56014678,
 1.30267961]))], '3.890451449942813': [      fun: 0.03953423296369001
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 8.60789621e-05, -1.26742734e-05, -2.25367230e-04,  7.40559705e-05,
 3.30887386e-05, -2.02868939e-04, -1.71108442e-05,  3.89344384e-04,
 -2.89803154e-04,  3.88920687e-05,  4.57811592e-04])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 66
nit: 9
status: 0
success: True
x: array([1.41677147, 1.19168678, 1.14871563, 0.99217573, 1.17759409,
 0.85375873, 0.80777523, 0.2402818 , 0.76532501, 1.65549739,
 1.34106538]))], '6.606934480075965': [      fun: 0.04253406899998817
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 0.00033452, -0.00014927,  0.0001294 , -0.00035818,  0.00044403,
 -0.00037661, -0.0007515 ,  0.00257904, -0.00142873,  0.00021712,
 0.00082288])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 33
nit: 8
status: 0
success: True
x: array([1.45149006, 1.26398801, 1.22165003, 1.00010539, 1.17287229,
 0.87271281, 0.80173003, 0.21614874, 0.74893385, 1.73674335,

```

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1.36337263]]], '11.220184543019652': [      fun: 0.04521979158740129
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 1.31013552e-04, -8.24589267e-05,  1.12878706e-04, -1.47372911e-05,
-3.04753968e-05,  1.80281172e-04, -4.26467991e-05,  5.70063825e-04,
-4.50218612e-04, -7.24278848e-05,  5.86215961e-05])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 107
nit: 34
status: 0
success: True
x: array([1.46695884, 1.28212336, 1.29445476, 0.98276104, 1.22407149,
0.90984868, 0.8062135 , 0.10836299, 0.60229303, 1.79887348,
1.32164339]]], '19.054607179632523': [      fun: 0.047044644118835216
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 0.00022203, -0.00031825, -0.00054337,  0.0008335 , -0.00054619,
0.00107666,  0.00011704,  0.00056985, -0.00057722, -0.00017418,
0.00099425])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 35
nit: 12
status: 0
success: True
x: array([1.48614332, 1.29371527, 1.30376552, 0.99223857, 1.23244558,
0.91159307, 0.79830624, 0.07121531, 0.55915319, 1.8218799 ,
1.32901508]]], '32.35936569296281': [      fun: 0.04893539464049279
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-0.00315737,  0.00070888, -0.00301539,  0.00163523, -0.02081563,
0.03978222,  0.03888307, -0.0025223 , -0.0176072 , -0.00297866,
0.00507592])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 98
nit: 5
status: 0
success: True
x: array([1.48482012, 1.29885628, 1.30422358, 0.99297117, 1.22371516,
0.92194371, 0.79675798, 0.06872599, 0.56132688, 1.82233313,
1.33037656]]], '54.954087385762485': [      fun: 0.05350125708587047
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 9.17699834e-04, -9.83125123e-04, -6.32516984e-04,  9.99411752e-04,
-1.24729772e-04,  5.19034391e-04,  2.39553821e-05,  1.62113800e-03,
-1.07401178e-03,  2.96222309e-04,  2.93194472e-04])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 98
nit: 45
status: 0
success: True
x: array([1.46761001, 1.47705615, 1.28776625, 1.11122956, 1.24417419,
0.67988494, 0.99068937, 0.38039894, 0.88438567, 1.85929103,

```

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1.22688867]]], '93.32543007969925': [      fun: 0.05782815899978358
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 7.50027237e-04,  7.40105226e-04,  1.70086939e-03, -1.65360671e-03,
-3.27278348e-04, -1.49734036e-03, -1.12479999e-03, -1.92217854e-03,
 1.45711662e-03,  1.13365349e-03,  9.38400147e-06])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 36
nit: 13
status: 0
success: True
x: array([1.48208453, 1.55514579, 1.32220318, 1.14933665, 1.26018926,
0.6662901 , 1.02778139, 0.41630773, 0.89674117, 1.86990772,
1.22338998]]], '158.48931924611173': [      fun: 0.06193817641174398
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 0.00691001, -0.00566227, -0.00569415,  0.00899188, -0.00160351,
 0.00543714,  0.00035831,  0.00763825, -0.0040989 ,  0.00115316,
 0.00162687])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 47
nit: 30
status: 0
success: True
x: array([1.43323624, 1.63200939, 1.33233716, 1.19462699, 1.37680672,
0.64082198, 1.16357333, 0.50231482, 0.93170266, 1.87321634,
1.19669163]]], '269.1534803926914': [      fun: 0.09775181371615355
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 0.16726645, -0.17695196, -0.01168873,  0.01934376,  0.20580768,
-0.53356474, -0.26671018,  0.26014491, -0.21573363, -0.01131655,
 0.0398933 ])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 57
nit: 1
status: 0
success: True
x: array([1.43323624, 1.63200939, 1.33233716, 1.19462699, 1.37680672,
0.64082198, 1.16357333, 0.50231482, 0.93170266, 1.87321634,
1.19669163]]], '457.0881896148752': [      fun: 0.23368416211153678
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 0.01089291,  0.00439763, -0.0165581 ,  0.08826915, -0.00683596,
 0.03585551,  0.01817272, -0.0350388 , -0.05776663,  0.01003198,
 0.02023876])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 78
nit: 4
status: 0
success: True
x: array([0.50761007, 1.14614948, 1.27647838, 0.58292994, 1.14534153,
0.75142113, 0.5185709 , 0.96920574, 0.          , 1.45308146,

```

```

1.32422899]]], '776.2471166286928': [      fun: 2.6807357293359186
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-0.00856284,  0.00322294, -0.00506569,  0.16522039,  0.00283054,
  0.00944931, -0.00393312,  0.01944915, -0.00843214,  0.01250105,
  0.4159416 ]])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 22
nit: 5
status: 0
success: True
x: array([0.56828951, 1.23929369, 1.29419418, 0.15644509, 1.02351484,
  0.66350763, 0.26675978, 2.00718712, 0.73675804, 1.33499764,
  0.          ]]), '1318.25673855641': [      fun: 2.2023263156409256
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 0.1876139 ,  0.0561913 ,  0.19418536, -0.54945255,  0.15522784,
 -0.37737985, -2.27660252, -0.97036327,  1.23587771, -0.27360968,
 -0.4184058 ]])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 29
nit: 11
status: 0
success: True
x: array([0.25863145, 1.47465438, 0.82660483, 0.92900804, 0.93932802,
  1.85418506, 0.          ,  0.49938446, 0.71062293, 1.31720262,
  0.          ]]), '2238.7211385683377': [      fun: 2.548461735448141
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([ 1.7960097 , -0.67310859, -3.63663625,  1.65533694, -0.17833095,
  0.58790348,  2.15124271, -0.13652247,  0.58348921, -0.48752876,
  0.03877189 ]])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 32
nit: 6
status: 0
success: True
x: array([0.54429076, 1.58625515, 0.37872452, 1.15922257, 1.24423363,
  1.33049065, 0.61076464, 0.9820506 , 0.88729373, 1.04761984,
  0.          ]]), '3801.8939632056126': [      fun: 2.658178645688797
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-0.01257229, -0.0047436 ,  0.01314457, -0.01300222, -0.01890576,
  0.01869975,  0.02013376,  0.00115442, -0.00198604,  0.00863109,
  0.45478322 ]])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 23
nit: 2
status: 0
success: True
x: array([0.55959765, 1.59402179, 0.35900302, 1.17792344, 1.26663866,
  1.31061992, 0.58539951, 0.98172044, 0.88988558, 1.03735077,

```

```

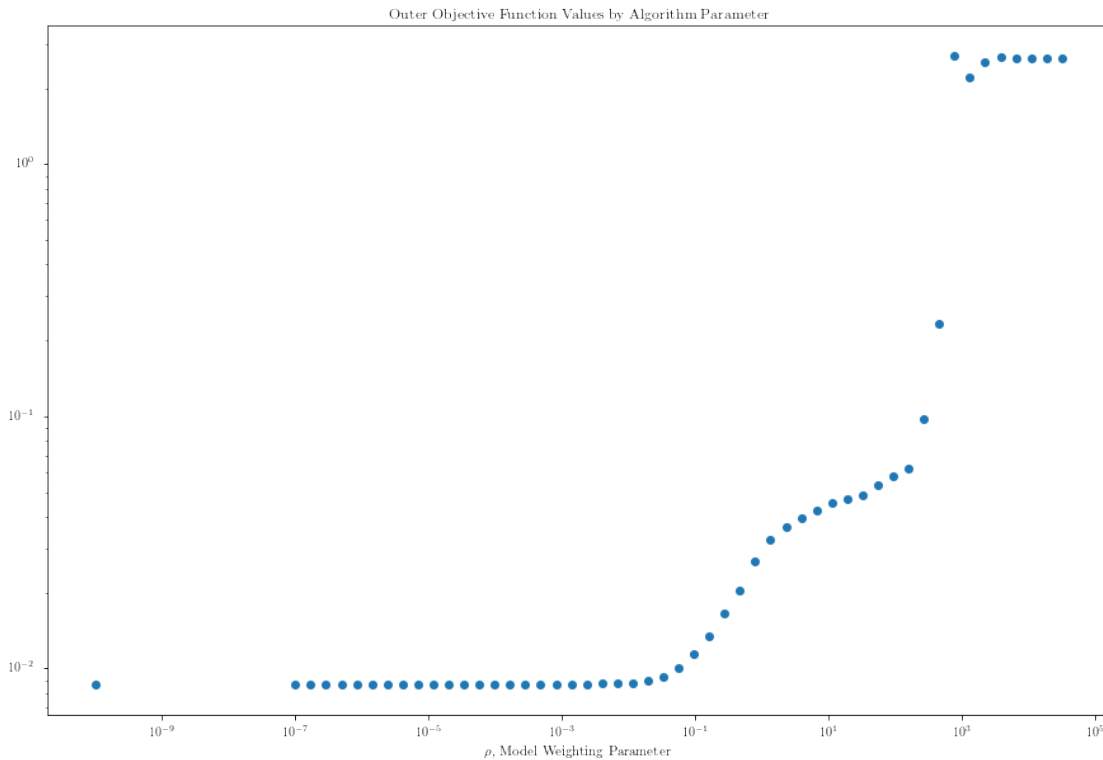
0.      ]]], '6456.542290346562': [      fun: 2.6465018536952174
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-4.35716396e-05,  2.99228093e-05,  1.58643757e-01, -1.14700864e-05,
  3.26789279e-05,  2.49116020e-06,  4.08245694e-06, -4.05569509e-06,
 -1.23073227e-05,  1.04317304e-05,  4.52489765e-01])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 30
nit: 28
status: 0
success: True
x: array([0.99347276, 1.00039614, 0.      , 0.9994265 , 1.00179937,
 0.99562253, 1.0010024 , 0.99656999, 0.99492179, 1.00375256,
 0.      ]]), '10964.781961431874': [      fun: 2.6465484101023327
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-3.50083949e-09, -3.06339900e-07,  1.57122091e-01,  9.60246275e-08,
 -2.40287051e-07, -2.58963628e-07, -3.47535939e-08, -6.44773847e-08,
 -2.21732238e-07,  1.20390704e-07,  4.51557889e-01])
message: b'CONVERGENCE: NORM_OF_PROJECTED_GRADIENT_<=_PGTOL'
nfev: 3
nit: 2
status: 0
success: True
x: array([0.99747235, 0.99934351, 0.      , 1.0000048 , 1.00008319,
 0.99736341, 1.00046562, 0.99811246, 0.99739254, 1.00188712,
 0.      ]]), '18620.871366628733': [      fun: 2.6465756055373495
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-2.02802677e-05, -5.34591048e-06,  1.57116900e-01,  9.41041349e-08,
  6.28201910e-07, -2.10981184e-05,  3.65060233e-06, -1.49742775e-05,
 -2.10452632e-05,  1.51592506e-05,  4.51556674e-01])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 58
nit: 2
status: 0
success: True
x: array([0.99749314, 0.99934908, 0.      , 1.00000471, 1.00008265,
 0.99738524, 1.00046181, 0.99812802, 0.99741411, 1.00187153,
 0.      ]]), '31622.776601683792': [      fun: 2.64659181636465
hess_inv: <11x11 LbfgsInvHessProduct with dtype=float64>
jac: array([-3.18702276e-05, -8.36154434e-06,  1.57104272e-01,  9.22220522e-08,
  9.76197522e-07, -3.33024770e-05,  5.86213232e-06, -2.38038149e-05,
 -3.29646493e-05,  2.38375542e-05,  4.51551665e-01])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
nfev: 71
nit: 2
status: 0
success: True
x: array([0.99752575, 0.99935766, 0.      , 1.00000461, 1.00008165,
 0.99741934, 1.00045582, 0.99815239, 0.99744783, 1.00184714,

```

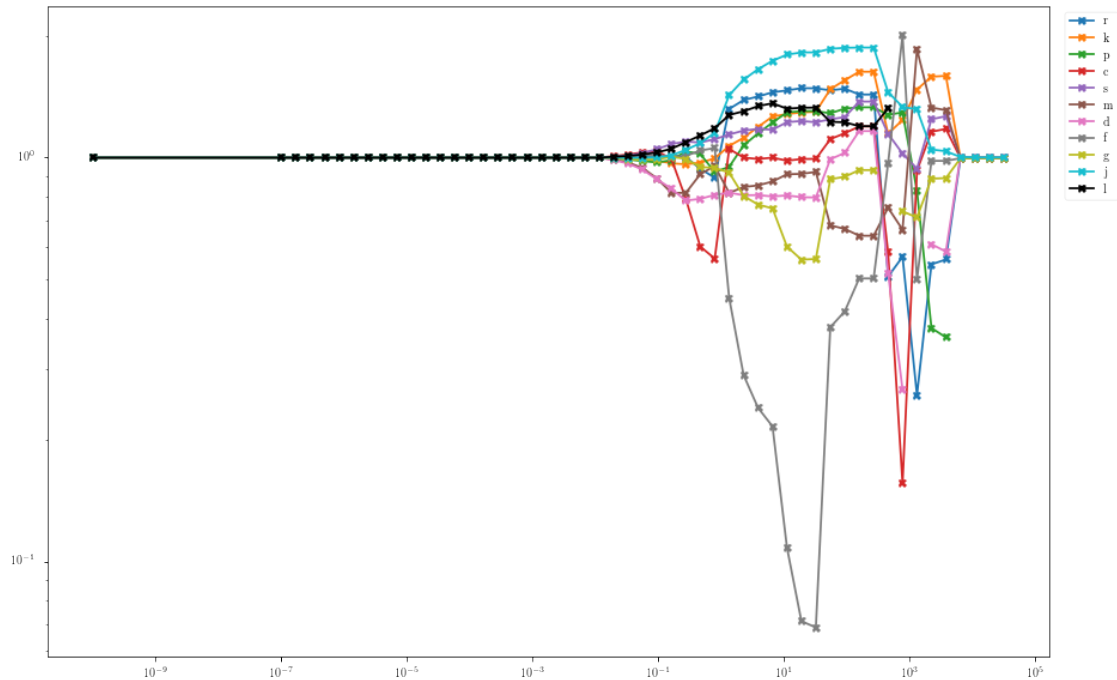
```
0.      ]]]}
```

```
In [11]: outer_objective_values = np.array([[float(rho), val[0].fun] for rho, val in solver.solutions.items()])
plt.loglog(*outer_objective_values.T, 'o')
plt.title("Outer Objective Function Values by Algorithm Parameter")
plt.xlabel(r"$\rho$, Model Weighting Parameter")
```

```
Out[11]: Text(0.5, 0, '$\rho$, Model Weighting Parameter')
```



```
In [12]: rhos = [float(rho) for rho in solver.solutions.keys()]
vals = [val[0].x for val in solver.solutions.values()]
plt.plot(rhos, vals, 'X-')
plt.legend("rkpcsmdfgjl", loc="best", bbox_to_anchor=(1.01, 1))
plt.xscale("log")
plt.yscale("log", nonposy="mask")
```

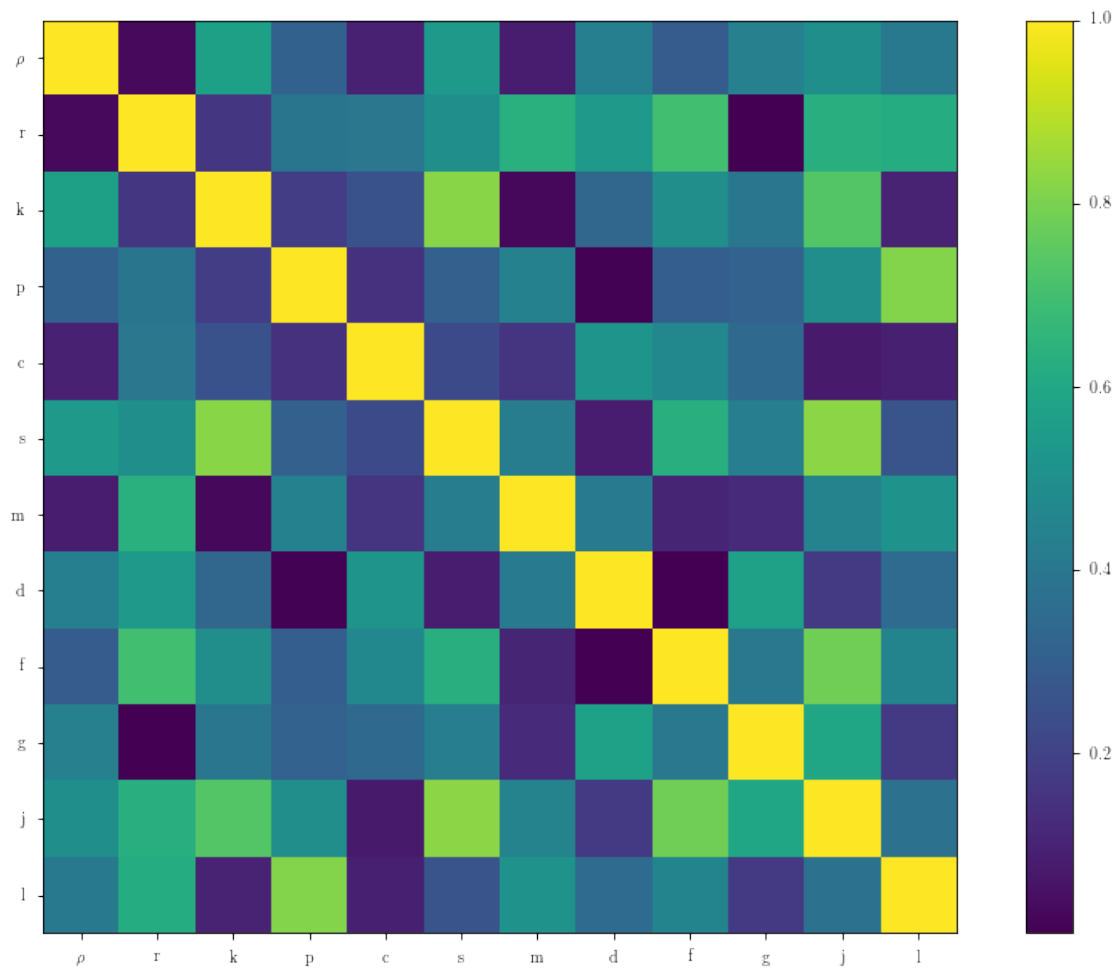


In [13]: # generate a crude correlation plot

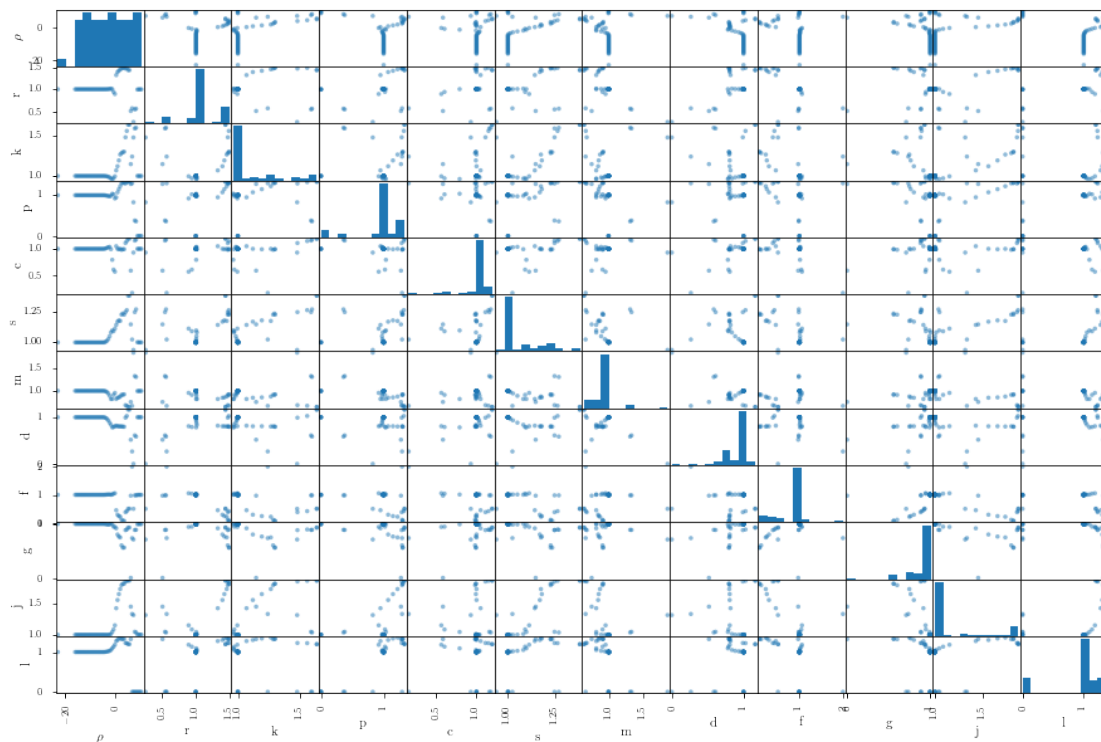
```
import pandas as pd

results = pd.DataFrame({r"$\rho$": np.log(rhos)})
for idx, name in enumerate("rkpcsmdfgjl"):
    results[name] = [v[idx] for v in vals]
```

```
In [14]: plt.imshow(np.abs(results.corr()))
plt.colorbar()
plt.xticks(range(12), [r"$\rho$"] + list("rkpcsmdfgjl"))
plt.yticks(range(12), [r"$\rho$"] + list("rkpcsmdfgjl"))
plt.grid(False)
```

```
In [15]: ax = pd.plotting.scatter_matrix(results)
         for a in ax.flatten():
             a.grid(False)
```

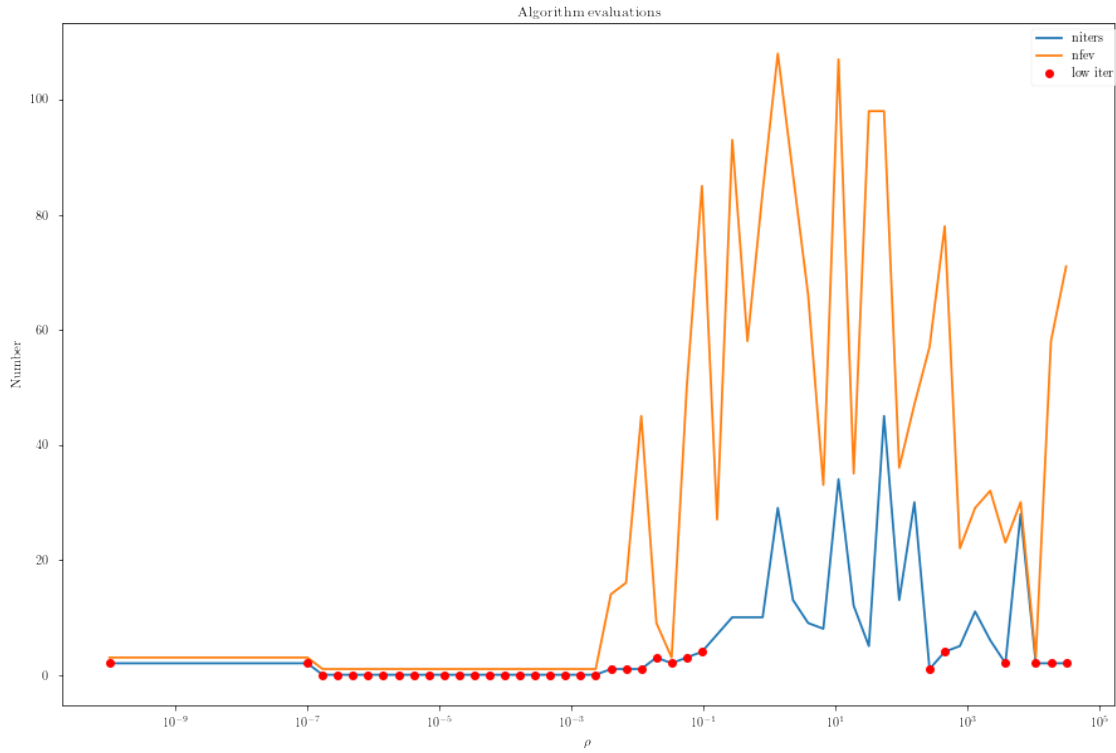


```
In [16]: iters_list = np.array([[float(key), value[0].nit] for key, value in solver.solutions.it
fevs_list = np.array([[float(key), value[0].nfev] for key, value in solver.solutions.it
plt.semilogx(*iters_list.T, *fevs_list.T)

low_iters = np.array([[k,v] for k,v in iters_list if v < 5])
plt.plot(*low_iters.T, 'ro')

plt.legend(["nitters", "nfev", "low iter"],
           loc="best", bbox_to_anchor=(1.01, 1))
plt.title("Algorithm evaluations")
plt.xlabel(r"$\rho$")
plt.ylabel("Number")

Out[16]: Text(0, 0.5, 'Number')
```



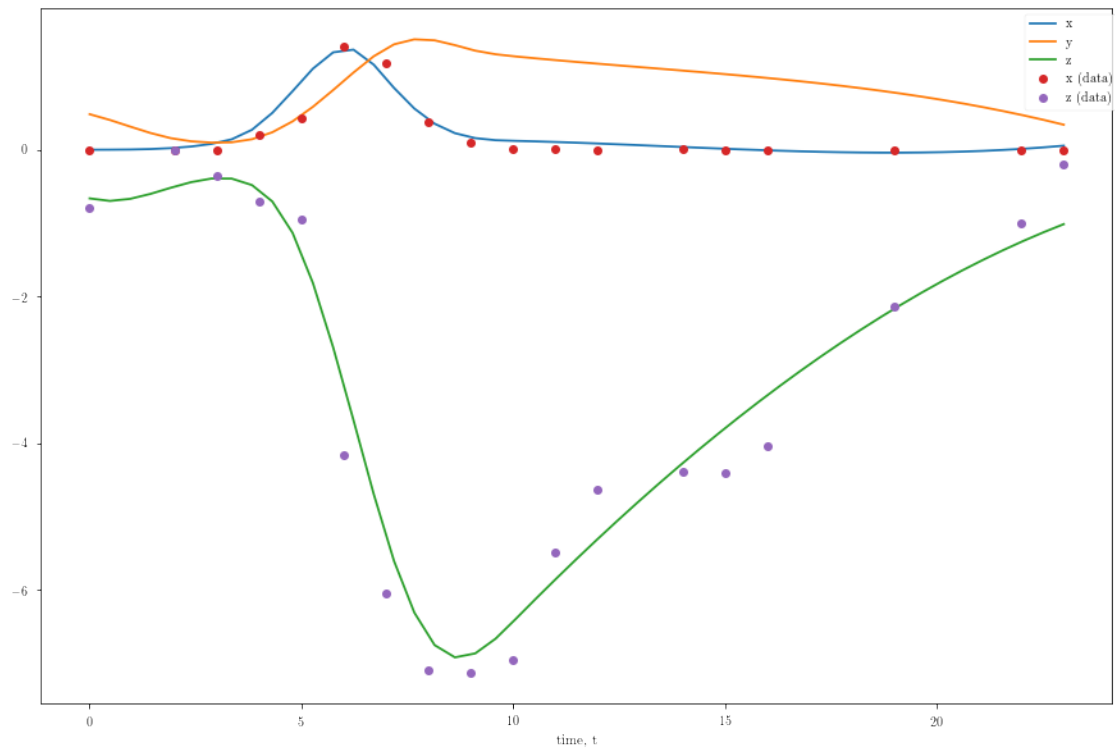
```
In [17]: def tokey(rho, ps):
          return f"{'y'.join(map(str,ps))}r{rho}"
          getx = fitter.ca.Function("getx", [model.ts, *model.cs], model.xs)
```

```
In [18]: target_rho = 93.32543007969925
          c_end = solver.problems[0].cache.results[tokey(target_rho, solver.solutions[str(target_rho)])]
          print(solver.solutions[str(target_rho)][0].x)
          xs_end = np.array([np.array(i) for i in getx(model.observation_times,
                                                         *fitter.argsplit(c_end,
                                                         3)
                                                         )])

          print(xs_end[:,0].T)
          plt.plot(model.observation_times, np.hstack([xs_end[0], xs_end[1], xs_end[2]]),
                    context.datasets[0]['t'], context.datasets[0]['x'], 'o',
                    context.datasets[0]['t'], context.datasets[0]['z'], 'o')
          plt.legend(list("xyz") + ["x (data)", "z (data)"], loc="best", bbox_to_anchor=(1.01, 1))
          plt.xlabel("time, t")
```

```
[1.48208453  1.55514579  1.32220318  1.14933665  1.26018926  0.6662901
 1.02778139  0.41630773  0.89674117  1.86990772  1.22338998]
[[ 0.0039528   0.48856269 -0.65948899]]
```

```
Out[18]: Text(0.5, 0, 'time, t')
```

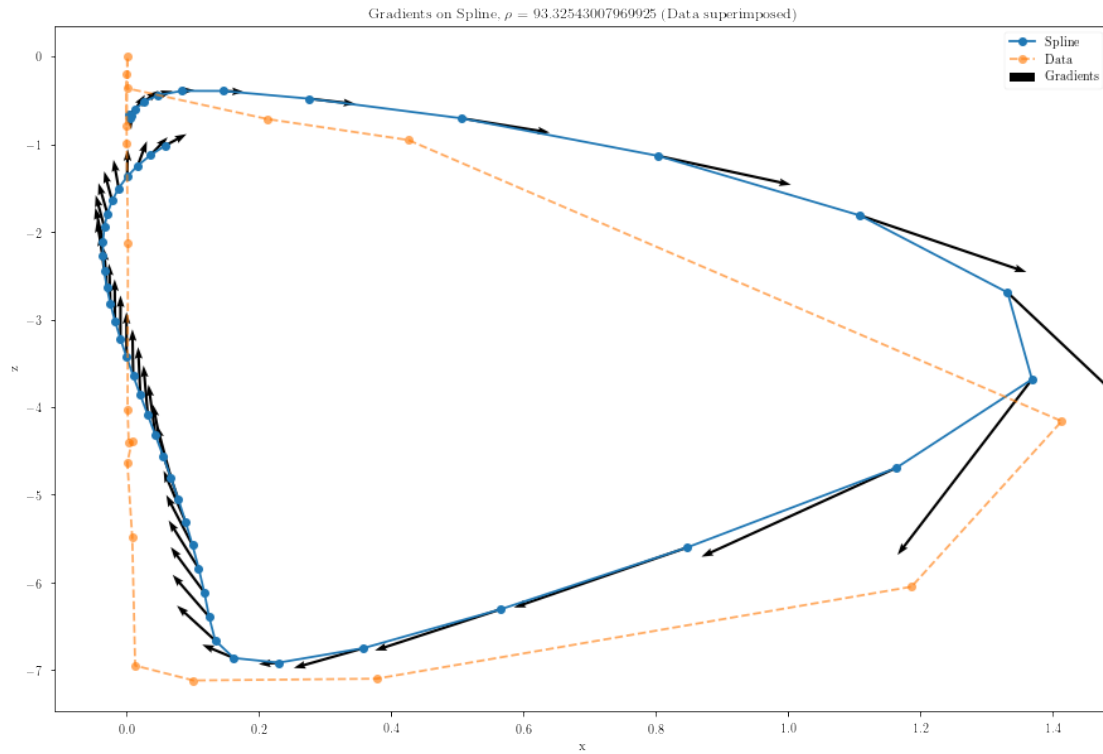


```
In [19]: spline_dfield = np.array([context.model(t, xs_end[:,i], solver.solutions[str(target_rho)
                                             for i,t in enumerate(model.observation_times)])

spline_dfield = spline_dfield.reshape(spline_dfield.shape[:2])

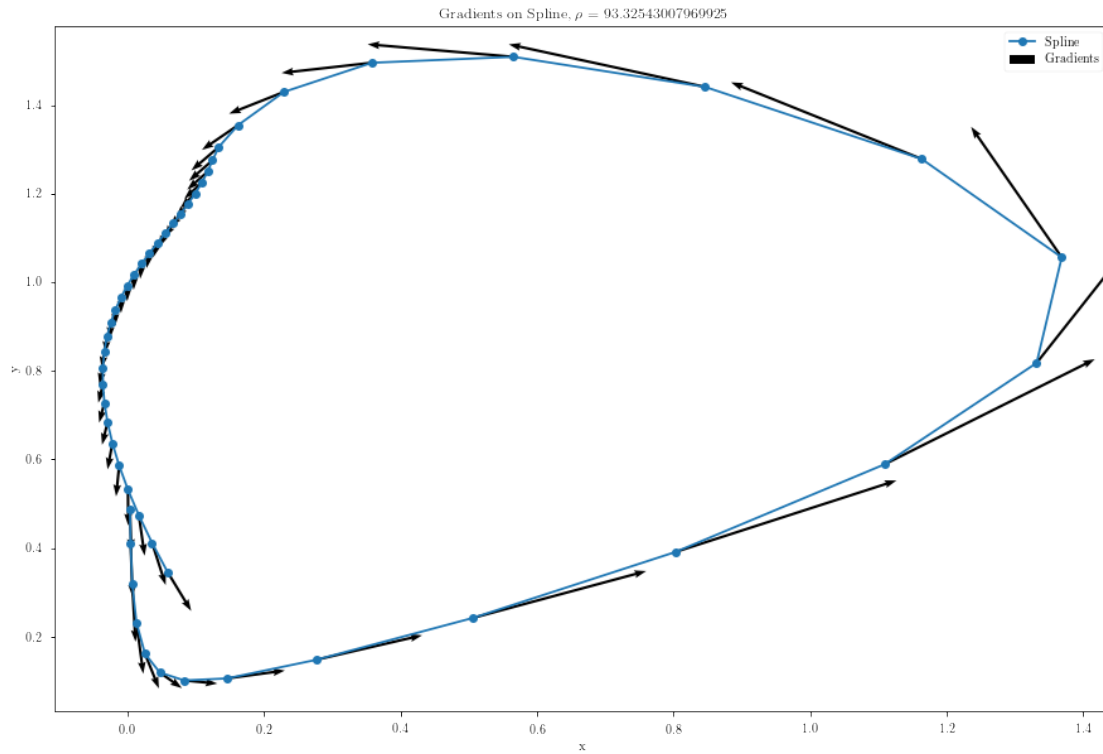
plt.plot(xs_end[0], xs_end[2], 'o-')
plt.plot(context.datasets[0]['x'], context.datasets[0]['z'], 'o--', alpha=0.65)
plt.quiver(xs_end[0], xs_end[2], spline_dfield[:,0], spline_dfield[:,2],
           scale=None, angles='xy', headwidth=3, headlength=4.5, headaxislength=4, width=0.005)
plt.title(f"Gradients on Spline,  $\rho = \{target\_rho\}$  (Data superimposed)")
plt.xlabel("x")
plt.ylabel("z")
plt.legend(["Spline", "Data", "Gradients"])
```

```
Out[19]: <matplotlib.legend.Legend at 0x7f0ed5448f60>
```



```
In [20]: plt.plot(xs_end[0], xs_end[1], 'o-')
plt.quiver(xs_end[0], xs_end[1], spline_dfield[:,0], spline_dfield[:,1],
           scale=None, angles='xy', headwidth=3, headlength=4.5, headaxislength=4, width=0.001)
plt.title(f"Gradients on Spline,  $\rho = \text{target\_rho}$ ")
plt.xlabel("x")
plt.ylabel("y")
plt.legend(["Spline", "Gradients"])
```

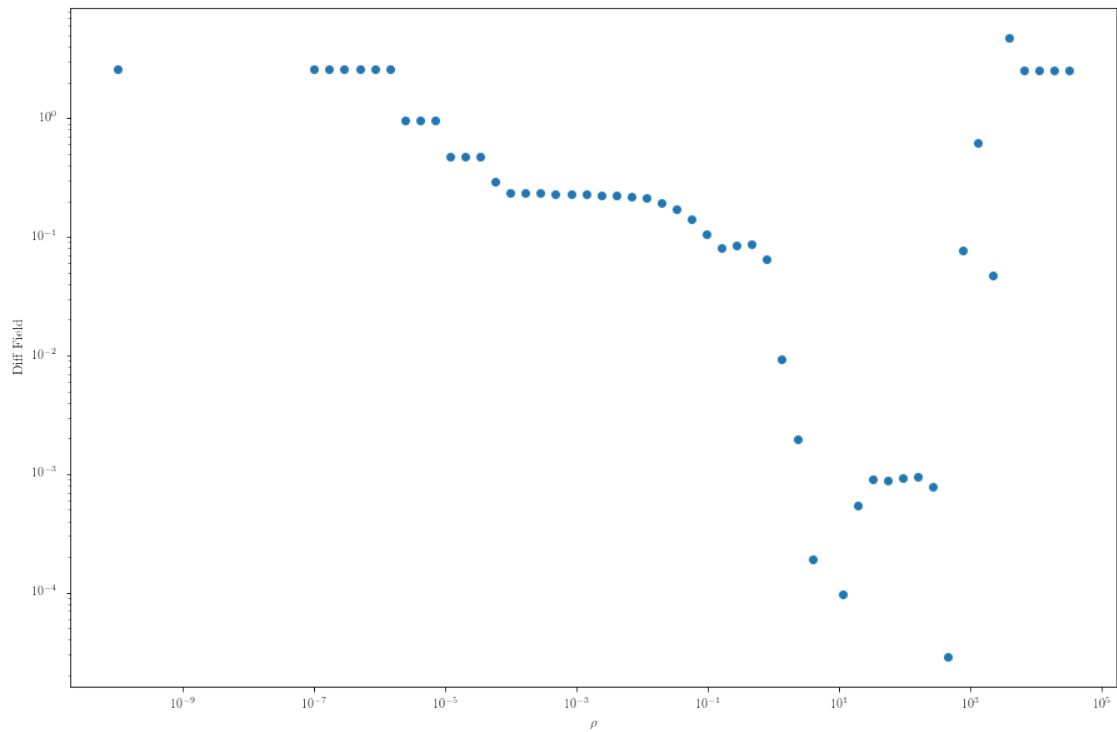
```
Out[20]: <matplotlib.legend.Legend at 0x7f0ed5422f28>
```



```
In [21]: inner_objective_values = np.array([[r, solver.problems[0].cache.results[tokey(r, v)].fu
      dfield = np.hstack([np.array(rhos).reshape(-1,1),
      ((inner_objective_values - outer_objective_values)[:,1] / rhos).res
```

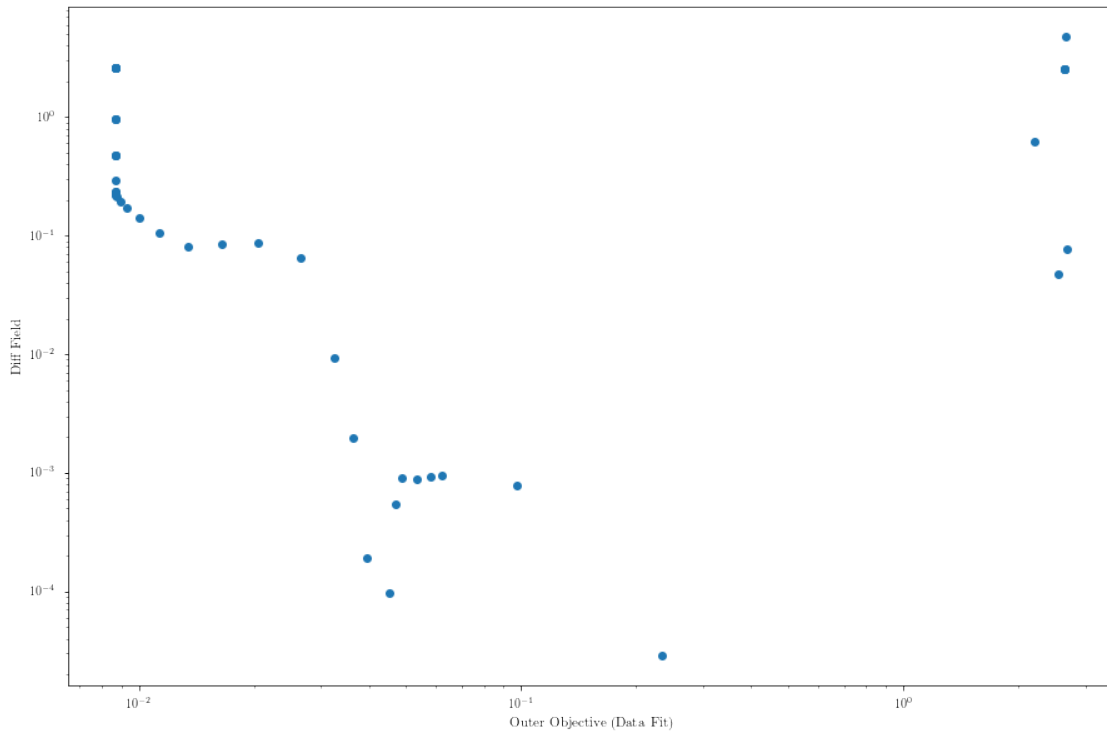
```
In [22]: plt.loglog(*dfield.T, 'o')
      plt.xlabel(r"$\rho$")
      plt.ylabel("Diff Field")
```

```
Out[22]: Text(0, 0.5, 'Diff Field')
```



```
In [23]: plt.loglog(outer_objective_values[:,1], dfield[:,1], 'o')
          plt.xlabel("Outer Objective (Data Fit)")
          plt.ylabel("Diff Field")
```

```
Out[23]: Text(0, 0.5, 'Diff Field')
```



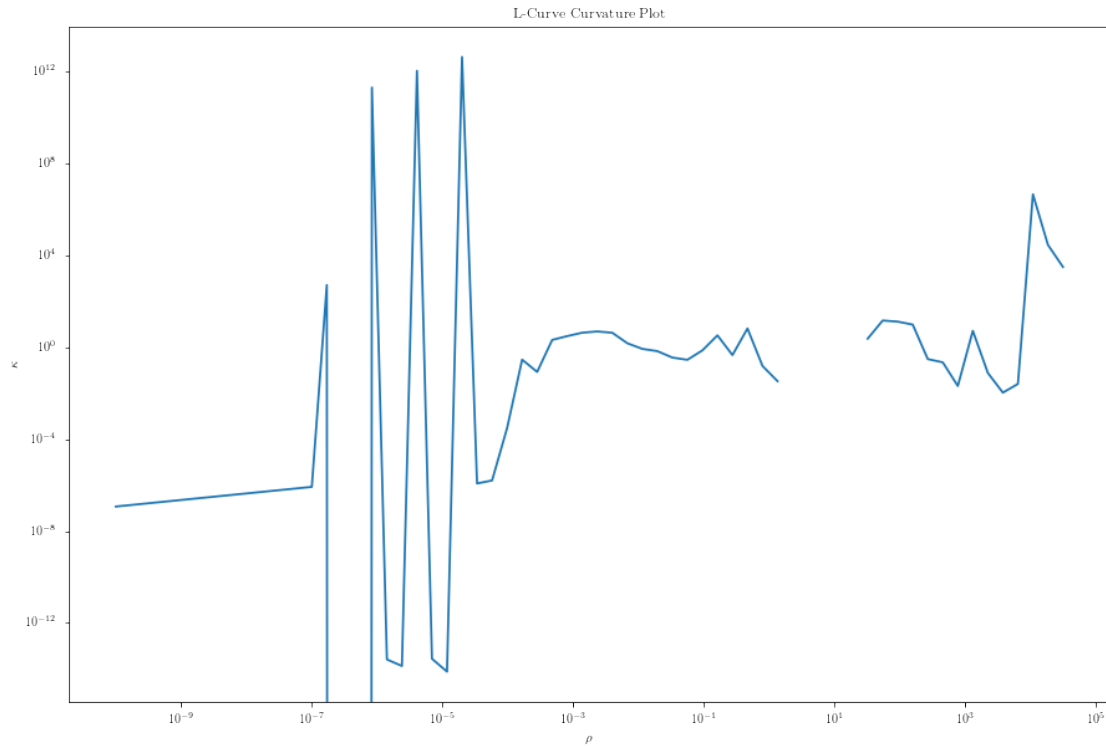
```
In [24]: curv_dx = np.gradient(np.log(outer_objective_values[:,1]))
         curv_dy = np.gradient(np.log(dfield[:,1]))
         curv_dx2 = np.gradient(curv_dx)
         curv_dy2 = np.gradient(curv_dy)

         kappa = np.abs(curv_dx*curv_dy2 - curv_dy*curv_dx2)/(curv_dx**2 + curv_dy**2)**(3/2)

         plt.loglog(rhos, kappa)
         plt.title("L-Curve Curvature Plot")
         plt.xlabel(r"$\rho$")
         plt.ylabel(r"$\kappa$")

/home/dwu402/.virtualenvs/scider/lib/python3.6/site-packages/ipykernel_launcher.py:2: RuntimeWarning:

Out[24]: Text(0, 0.5, '$\kappa$')
```

In [25]: rhos

Out [25]: [1e-10,
 1e-07,
 1.6982436524617461e-07,
 2.8840315031266057e-07,
 4.897788193684466e-07,
 8.317637711026709e-07,
 1.4125375446227554e-06,
 2.3988329190194897e-06,
 4.073802778041131e-06,
 6.9183097091893625e-06,
 1.1748975549395302e-05,
 1.995262314968883e-05,
 3.3884415613920276e-05,
 5.7543993733715664e-05,
 9.772372209558111e-05,
 0.00016595869074375615,
 0.0002818382931264455,
 0.00047863009232263854,
 0.0008128305161640995,
 0.0013803842646028866,
 0.0023442288153199225,

```

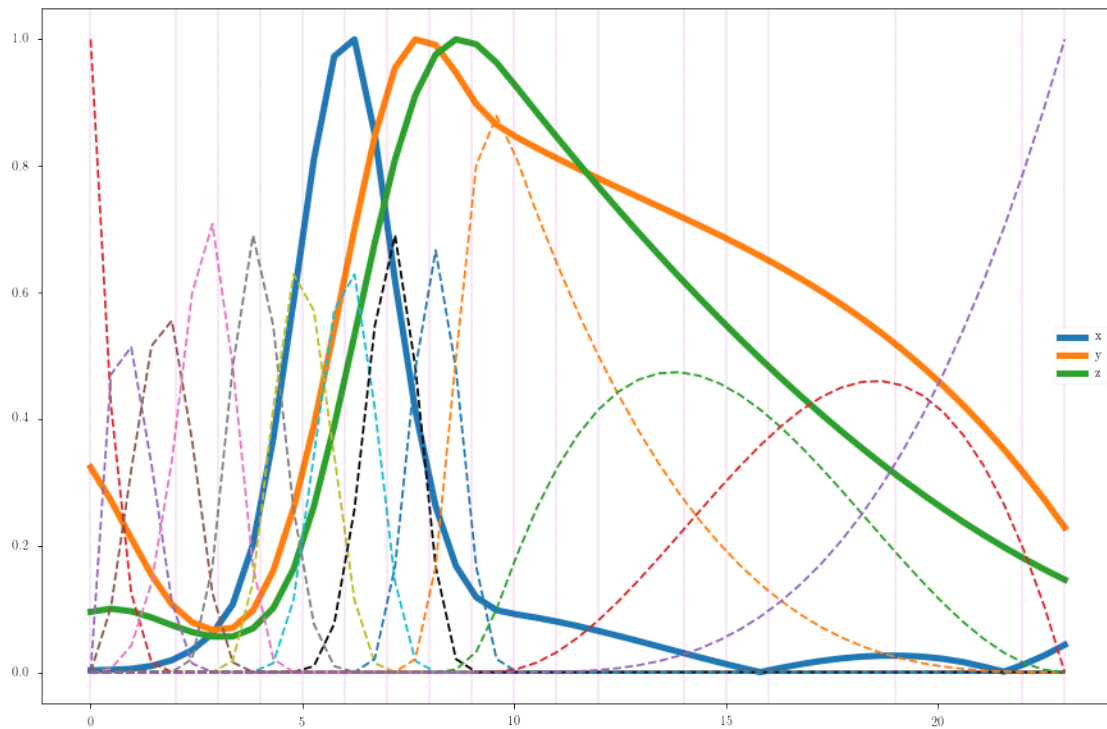
0.003981071705534978,
0.006760829753919818,
0.01148153621496884,
0.019498445997580455,
0.03311311214825914,
0.05623413251903491,
0.09549925860214369,
0.16218100973589297,
0.2754228703338169,
0.4677351412871981,
0.7943282347242822,
1.3489628825916533,
2.290867652767775,
3.890451449942813,
6.606934480075965,
11.220184543019652,
19.054607179632523,
32.35936569296281,
54.954087385762485,
93.32543007969925,
158.48931924611173,
269.1534803926914,
457.0881896148752,
776.2471166286928,
1318.25673855641,
2238.7211385683377,
3801.8939632056126,
6456.542290346562,
10964.781961431874,
18620.871366628733,
31622.776601683792]

```

```

In [29]: bfn = modeller.ca.Function('basis_fns', [model.ts], [model.basis])
plt.plot(model.observation_times, np.abs(np.hstack([xs_end[0]/max(abs(xs_end[0])),
                                                    xs_end[1]/max(abs(xs_end[1])),
                                                    xs_end[2]/max(abs(xs_end[2]))])),
          linewidth=5)
plt.plot(model.observation_times, bfn(model.observation_times), '--')
plt.legend('xyz')
[plt.axvline(x=i, color='m', linewidth=0.25, linestyle='--') for i in context.datasets]
print("")

```



```
In [27]: plt.plot(range(model.K), np.array(fitter.argsplit(c_end, 3)).T, 'o')
plt.legend('xyz')
```

```
Out[27]: <matplotlib.legend.Legend at 0x7f0ed5022550>
```

