4

The French Connection

Neither Cardano nor Galileo realized that he was on the verge of articulating the most powerful tool of risk management ever to be invented: the laws of probability. Cardano had proceeded from a series of experiments to some important generalizations, but he was interested only in developing a theory of gambling, not a theory of probability. Galileo was not even interested in developing a theory of gambling.

Galileo died in 1642. Twelve years later, three Frenchmen took a great leap forward into probability analysis, an event that is the subject of this chapter. And less than ten years after that, what had been just a rudimentary idea became a fully developed theory that opened the way to significant practical applications. A Dutchman named Huygens published a widely read textbook about probability in 1657 (carefully read and noted by Newton in 1664); at about the same time, Leibniz was thinking about the possibility of applying probability to legal problems; and in 1662 the members of a Paris monastery named Port-Royal produced a pioneering work in philosophy and probability to which they gave the title of *Logic*. In 1660, an Englishman named John Graunt published the results of his effort to generalize demographic data from a statistical sample of mortality records kept by local churches. By the late 1660s, Dutch towns that had traditionally financed themselves by selling annuities were able to put these policies on a sound actuarial footing. By 1700, as we mentioned earlier, the English government was financing its budget deficits through the sale of life annuities.

The story of the three Frenchmen begins with an unlikely trio who saw beyond the gaming tables and fashioned the systematic and theoretical foundations for measuring probability. The first, Blaise Pascal, was a brilliant young dissolute who subsequently became a religious zealot and ended up rejecting the use of reason. The second, Pierre de Fermat, was a successful lawyer for whom mathematics was a sideline. The third member of the group was a nobleman, the Chevalier de Méré, who combined his taste for mathematics with an irresistible urge to play games of chance; his fame rests simply on his having posed the question that set the other two on the road to discovery.

Neither the young dissolute nor the lawyer had any need to experiment in order to confirm their hypotheses. Unlike Cardano, they worked inductively in creating for the first time a *theory* of probability. The theory provided a measure of probability in terms of hard numbers, a climactic break from making decisions on the basis of degrees of belief.

Pascal, who became a celebrated mathematician and occasional philosopher, was born in 1623, just about the time Galileo was putting the finishing touches on *Sopra le Scoperte dei Dadi*. Born in the wake of the religious wars of the sixteenth century, Pascal spent half his life torn between pursuing a career in mathematics and yielding to religious convictions that were essentially anti-intellectual. Although he was a brilliant mathematician and proud of his accomplishments as a "geomaster," his religious passion ultimately came to dominate his life.[1]

Pascal began life as a child prodigy. He was fascinated with shapes and figures and discovered most of Euclidean geometry on his own by drawing diagrams on the tiles of his playroom floor. At the age of 16, he wrote a paper on the mathematics of the cone; the paper was so advanced that even the great Descartes was impressed with it.

This enthusiasm for mathematics was a convenient asset for Pascal's father, who was a mathematician in his own right and earned a comfortable living as a tax collector, a functionary known at the time as a tax farmer. The tax farmer would advance money to the monarch—the equivalent of planting his seeds—and then go about collecting it from the citizenry—the equivalent of gathering in a harvest whose ultimate value, as with all farmers, he hoped would exceed the cost of the seeds.

While Pascal was still in his early teens, he invented and patented a calculating machine to ease the dreary task of adding up M. Pascal's daily accounts. This contraption, with gears and wheels that went forward and backward to add and subtract, was similar to the mechanical calculating machines that served as precursors to today's electronic calculators. The young Pascal managed to multiply and divide on his machine as well and even started work on a method to extract square roots. Unfortunately for the clerks and bookkeepers of the next 250 years, he was unable to market his invention commercially because of prohibitively high production costs.

Recognizing his son's genius, Blaise's father introduced him at the age of 14 into a select weekly discussion group that met at the home of a Jesuit priest named Marin Mersenne, located near the Place Royal in Paris. Abbé Mersenne had made himself the center of the world of science and mathematics during the first half of the 1600s. In addition to bringing major scholars together at his home each week, he reported by mail to all and sundry, in his cramped handwriting, on what was new and significant.[2]

In the absence of learned societies, professional journals, and other means for the exchange of ideas and information, Mersenne made a valuable contribution to the development and dissemination of new scientific theories. The Académie des Sciences in Paris and the Royal Society in London, which were founded about twenty years after Mersenne's death, were direct descendants of Mersenne's activities.

Although Blaise Pascal's early papers in advanced geometry and algebra impressed the high-powered mathematicians he met at Abbé Mersenne's, he soon acquired a competing interest. In 1646, his father fell on the ice and broke his hip; the bonesetters called in to take care of M. Pascal happened to be members of a proselytizing Catholic sect called Jansenists. These people believed that the only path to salvation was through aceticism, sacrifice, and unwavering attachment to the strait and narrow. They preached that a person who fails to reach constantly for ever-higher levels of purity will slip back into immorality. Emotion and faith were all that mattered; reason blocked the way to redemption.

After repairing the hip of Pascal *père*, the Jansenists stayed on for three months to work on the soul of Pascal *fils*, who accepted their doctrine with enthusiasm. Now Blaise abandoned both mathematics and science, along with the pleasures of his earlier life as a man about town.

Religion commanded his full attention. All he could offer by way of explanation was to ask, "Who has placed me here? By whose order and warrant was this place and this time ordained for me? The eternal silence of these infinite spaces leaves me in terror."[3]

The terror became so overwhelming that in 1650, at the age of 27, Pascal succumbed to partial paralysis, difficulty in swallowing, and severe headaches. As a cure, his doctors urged him to rouse himself and resume his pleasure-seeking ways. He lost no time in taking their advice. When his father died, Pascal said to his sister: "Let us not grieve like the pagans who have no hope."[4] In his renewed activities he exceeded even his earlier indulgences and became a regular visitor to the gambling tables of Paris.

Pascal also resumed his researches into mathematics and related subjects. In one of his experiments he proved the existence of vacuums, a controversial issue ever since Aristotle had declared that nature abhors a vacuum. In the course of that experiment he demonstrated that barometric pressure could be measured at varying altitudes with the use of mercury in a tube emptied of all air.

About this time, Pascal became acquainted with the Chevalier de Méré, who prided himself on his skill at mathematics and on his ability to figure the odds at the casinos. In a letter to Pascal some time in the late 1650s, he boasted, "I have discovered in mathematics things so rare that the most learned of ancient times have never thought of them and by which the best mathematicians in Europe have been surprised."[5]

Leibniz himself must have been impressed, for he described the Chevalier as "a man of penetrating mind who was both a gambler and a philosopher." But then Leibniz must have had second thoughts, for he went on to say, "I almost laughed at the airs which the Chevalier de Méré takes on in his letter to Pascal."[6]

Pascal agreed with Leibniz. "M. de Méré," he wrote to a colleague, "has good intelligence but he is not a geometer and this, as you realize, is a great defect."[7] Here Pascal sounds like the academic who takes pleasure in putting down a non-academic. In any case, he underestimated de Méré.[8]

Yet Pascal himself is our source of information about de Méré's intuitive sense of probabilities. The Chevalier bet repeatedly on outcomes with just a narrow margin in his favor, outcomes that his opponents regarded as random results. According to Pascal, de Méré knew that the probability of throwing a 6 with one die rises above 50% with four throws—to 51.77469136%. The Chevalier's strategy was to win a tiny amount on a large number of throws in contrast to betting the chateau on just a few. That strategy also required large amounts of capital, because a 6 might fail to show up for many throws in a row before it appeared in a cluster that would bring its average appearance to over 50%.[9]

De Méré tried a variation on his system by betting that *sonnez*—the term for double-six—had a better than 50% probability of showing up on 24 throws of two dice. He lost enough money on these bets to learn that the probability of double-six was in fact only 49.14% on 24 throws. Had he bet on 25 throws, where the probability of throwing *sonnez* breaks through to 50.55%, he would have ended up a richer man. The history of risk management is written in red as well as in black.

At the time he first met Pascal, the Chevalier was discussing with a number of French mathematicians Paccioli's old problem of the points—how should two players in a game of *balla* share the stakes when they leave the game uncompleted? No one had yet come up with an answer.

Although the problem of the points fascinated Pascal, he was reluctant to explore it on his own. In today's world, this would be the topic for a panel at an annual meeting of one of the learned societies. In Pascal's world, no such forum was available. A little group of scholars might discuss the matter in the intimacy of Abbé Mersenne's home, but the accepted procedure was to start up a private correspondence with other mathematicians who might be able to contribute something to the investigation. In 1654, Pascal turned to Pierre de Carcavi, a member of Abbé Mersenne's group, who put him in touch with Pierre de Fermat, a lawyer in Toulouse.

Pascal could not have approached anyone more competent to help him work out a solution to the problem of the points. Fermat's erudition was awesome.[10] He spoke all the principal European languages and even wrote poetry in some of them, and he was a busy commentator on the literature of the Greeks and Romans. Moreover, he was a mathematician of rare power. He was an independent inventor of analytical geometry, he contributed to the early development of calculus, he did research on the weight of the earth, and he worked on light refraction and optics. In the course of what turned out to be an extended correspondence with Pascal, he made a significant contribution to the theory of probability.

But Fermat's crowning achievement was in the theory of numbers—the analysis of the structure that underlies the relationships of each individual number to all the others. These relationships present countless puzzles, not all of which have been resolved to this very day. The Greeks, for example, discovered what they called perfect numbers, numbers that are the sum of all their divisors other than themselves, like $6 = 1 + 2 + 3$. The next-higher perfect number after 6 is $28 = 1 + 2 + 4 + 7 + 14$. The third perfect number is 496, followed by 8,128. The fifth perfect number is 33,550,336.

Pythagoras discovered what he called amicable numbers, "One who is the other I," numbers whose divisors add up to each other. All the divisors of 284, which are 1, 2, 4, 71, and 142, add up to 220; all the divisors of 220, which are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, and 110, add up to 284.

No one has yet devised a rule for finding all the perfect numbers or all the amicable numbers that exist, nor has anyone been able to explain all the varying sequences in which they follow one another. Similar difficulties arise with prime numbers, numbers like 1, 3, or 29, that are divisible only by 1 and by themselves. At one point, Fermat believed he might have discovered a formula that would always produce a prime number as its solution, but he warned that he could not prove *theoretically* that the formula would always do so. His formula produced 5, then 17, then 257, and finally 65,537, all of which were prime numbers; the next number to result from his formula was 4,294,967,297.

Fermat is perhaps most famous for propounding what has come to be known as "Fermat's Last Theorem," a note that he scribbled in the margin of his copy of Diophantus's book *Arithmetic*. The notion is simple to describe despite the complexity of its proof.

The Greek mathematician Pythagoras first demonstrated that the square of the longest side of a right triangle, the hypotenuse, is equal to the sum of the squares of the other two sides. Diophantus, an early explorer into the wonders of quadratic equations, had written a similar expression: $x^4 + y^4 + z^4 = u^2$. "Why," asks Fermat, "did not Diophantus seek two [rather than three] fourth powers such that their sum is square? The problem is, in fact impossible, as by my method I am able to prove with all rigor."[11] Fermat observes that Pythagoras was correct that $a^2 + b^2$

$= c^2$, but $a^3 + b^3$ would not be equal to c^3, nor would any integer higher than 2 fit the bill: the Pythagorean theorem works only for squaring.

And then Fermat wrote: "I have a truly marvelous demonstration of this proposition which this margin is too narrow to contain."[12] With this simple comment he left mathematicians dumbfounded for over 350 years as they struggled to find a theoretical justification for what a great deal of empirical experimentation proved to be true. In 1993, an English mathematician named Andrew Wiles claimed that he had solved this puzzle after seven years of work in a Princeton attic. Wiles's results were published in the *Annals of Mathematics* in May 1995, but the mathematicians have continued to squabble over exactly what he had achieved.

Fermat's Last Theorem is more of a curiosity than an insight into how the world works. But the solution that Fermat and Pascal worked out to the problem of the points has long since been paying social dividends as the cornerstone of modern insurance and other forms of risk management.

The solution to the problem of the points begins with the recognition that the player who is ahead when the game stops would have the greater probability of winning if the game were to continue. But how much greater are the leading player's chances? How small are the lagging player's chances? How do these riddles ultimately translate into the science of forecasting?

The 1654 correspondence between Pascal and Fermat on this subject signaled an epochal event in the history of mathematics and the theory of probability.* In response to the Chevalier de Méré's curiosity about the old problem, they constructed a systematic method for analyzing future outcomes. When more things can happen than will happen, Pascal and Fermat give us a procedure for determining the likelihood of each of the possible results—assuming always that the outcomes can be measured mathematically.

They approached the problem from different standpoints. Fermat turned to pure algebra. Pascal was more innovative: he used a geomet-

*The full text of this correspondence, translated into English, appears in David, 1962, Appendix 4.

ric format to illuminate the underlying algebraic structure. His methodology is simple and is applicable to a wide variety of problems in probability.

The basic mathematical concept behind this geometric algebra had been recognized long before Fermat and Pascal took it up. Omar Khayyam had considered it some 450 years earlier. In 1303, a Chinese mathematician named Chu Shih-chieh, explicitly denying any originality, approached the problem by means of a device that he called the "Precious Mirror of the Four Elements." Cardano had also mentioned such a device.[13]

Chu's precious mirror has since come to be known as Pascal's Triangle. "Let no one say that I have said nothing new," boasts Pascal in his autobiography. "The arrangement of the subject is new. When we play tennis, we both play with the same ball, but one of us places it better."[14]

```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  10  10   5   1
1   6  15  20  15   6   1
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All sorts of patterns greet the eye when we first glance at Pascal's Triangle, but the underlying structure is simple enough: each number is the sum of the two numbers to the right and to the left on the row above.

Probability analysis begins with enumerating the number of different ways a particular event can come about—Cardano's "circuit." That is what the sequence of numbers in each of these expanding rows is designed to provide. The top row shows the probability of an event that cannot fail to happen. Here there is only one possible outcome, with zero uncertainty; it is irrelevant to probability analysis. The next row is the first row that matters. It shows a 50-50 situation: the probability of outcomes like having a boy—or a girl—in a family that is planning to have only one child, or like flipping a head on just one toss of a coin. Add across. With a total of only two possibilities, the result is either one way or the other, a boy or a girl, a head or a tail; the probability of having a boy instead of a girl or of flipping a head instead of a tail is 50%.

The same process applies as we move down the triangle. The third row shows the possible combinations of boys and girls in a family that produces two children. Adding across shows that there are four possible results: one chance of two boys, one chance of two girls, and two chances of one each—a boy followed by a girl or a girl followed by a boy. Now at least one boy (or one girl) appears in three of the four outcomes, setting the probability of at least one boy (or one girl) in a two-child family at 75%; the probability of one boy plus one girl is 50%. The process obviously depends on combinations of numbers in a manner that Cardano had recognized but that had not been published when Pascal took up the subject.

The same line of analysis will produce a solution for the problem of the points. Let us change the setting from Paccioli's game of *balla* to the game of baseball. What is the probability that your team will win the World Series after it has lost the first game? If we assume, as in a game of chance, that the two teams are evenly matched, this problem is identical to the problem of the points tackled by Fermat and Pascal.[15]

As the other team has already won a game, the Series will now be determined by the best of four out of six games instead of four out of seven. How many different sequences of six games are possible, and how many of those victories and losses would result in your team winning the four games it needs for victory? Your team might win the second game, lose the third, and then go on to win the last three. It might lose two in a row and win the next four. Or it might win the necessary four right away, leaving the opponents with only one game to their credit.

Out of six games, how many such combinations of wins and losses are there? The triangle will tell us. All we have to do is find the appropriate row.

Note that the second row of the triangle, the 50-50 row, concerns a family with an only child or a single toss of a coin and adds up to a total of two possible outcomes. The next row shows the distribution of outcomes for a two-child family, or two coin tosses, and adds up to four outcomes, or 2^2. The next row adds up to eight outcomes, or 2^3, and shows what could happen with a three-child family. With six games remaining to settle the outcome of the World Series, we would want to look at the row whose total is 2^6—or two multiplied by itself six times, where there will be 64 possible sequences of wins and losses.* The sequence of numbers in that row reads:

1 6 15 20 15 6 1

Remember that your team still needs four games to win the Series, while the opposing team needs only three. There is just one way your team can win all the games—by winning all the games while the opponents win none; the number 1 at the beginning of the row refers to that possibility. Reading across, the next number is 6. There are six different sequences in which your team (Y) would gain the Series while your opponents (O) win only one more game:

OYYYYY YOYYYY YYOYYY YYYOYY YYYYOY YYYYYO

And there are fifteen different sequences in which your team would win four games while your opponents win two.

All the other combinations would produce at least three games for the opposing team and less than the necessary four for yours. This means that there are $1 + 6 + 15 = 22$ combinations in which your team would come out on top after losing the first game, and 42 combinations in which the opposing team would become the champions. As a result, the probability is 22/64—or a tad better than one out of three—that your team will come from behind to win four games before the other team has won three.

The examples reveal something odd. Why would your team play out all six remaining games in sequences where they would have won the Series before playing six games? Or why would they play out all four games when they could win in fewer games?

Although no team in real life would extend play beyond the minimum necessary to determine the championship, a logically complete solution to the problem would be impossible without *all* of the math-

*Mathematicians will note that what Pascal has really provided here is the binomial expansion, or the coefficients of each successive multiplication of $(a + b)$ by itself. For example, the first row is $(a + b)^0 = 1$, while the fourth row is $(a + b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$.

ematical possibilities. As Pascal remarked in his correspondence with Fermat, the mathematical laws must dominate the wishes of the players themselves, who are only abstractions of a general principle. He declares that "it is absolutely equal and immaterial to them both whether they let the [match] take its natural course."

The correspondence between Pascal and Fermat must have been an exciting exploration of new intellectual territory for both men. Fermat wrote to Carcavi about Pascal that "I believe him to be capable of solving any problem that he undertakes." In one letter to Fermat, Pascal admitted that "your numerical arrangements . . . are far beyond my comprehension." Elsewhere, he also described Fermat as "a man so outstanding in intellect . . . in the highest degree of excellence . . . . [that his works] will make him supreme among the geomasters of Europe."

More than mathematics was involved here for Pascal, who was so deeply involved with religion and morality, and for Fermat the jurist. According to their solutions, there is a matter of *moral right* involved in the division of the stakes in Paccioli's unfinished game of *balla*. Although the players could just as easily split the stakes evenly, that solution would be unacceptable to Pascal and Fermat because it would be unfair to the player who was lucky enough to be ahead when playing ceased.[16]

Pascal is explicit about the moral issues involved and chooses his words with care. In his comments about this work, he points out that "the first thing which we must consider is that the money the players have put into the game no longer belongs to them . . . but they have received in return the right to expect that which luck will bring them, according to the rules upon which they agreed at the outset." In the event that they decide to stop playing before the game is over, they will reenter into their original ownership rights of the money they have put into the pot. At that point, "the rule determining that which will belong to them will be proportional to that which they had the right to expect from fortune . . . . [T]his just distribution is known as the division." The principles of probability theory determine the division, because they determine the just distribution of the stakes.

Seen in these terms, the Pascal–Fermat solution is clearly colored by the notion of risk management, even though they were not thinking explicitly in those terms. Only the foolhardy take risks when the rules are unclear, whether it be *balla*, buying IBM stock, building a factory, or submitting to an appendectomy.

But beyond the moral question, the solutions proposed by Pascal and Fermat lead to precise generalizations and rules for calculating probabilities, including cases involving more than two players, two teams, two genders, two dice, or coins with two sides. Their achievement enabled them to push the limits of theoretical analysis far beyond Cardano's demonstration that two dice of six sides each (or two throws of one die) would produce 6^2 combinations or that three dice would produce 6^3 combinations.

The last letter of the series is dated October 27, 1654. Less than a month later, Pascal underwent some kind of mystical experience. He sewed a description of the event into his coat so that he could wear it next to his heart, claiming "Renunciation, total and sweet." He abandoned mathematics and physics, swore off high living, dropped his old friends, sold all his possessions except for his religious books, and, a short while later, took up residence in the monastery of Port-Royal in Paris.

Yet traces of the old Blaise Pascal lingered on. He established the first commercial bus line in Paris, with all the profits going to the monastery of Port-Royal.

In July 1660, Pascal took a trip to Clermont-Ferrand, not far from Fermat's residence in Toulouse. Fermat proposed a meeting "to embrace you and talk to you for a few days," suggesting a location halfway between the two cities; he claimed bad health as an excuse for not wanting to travel the full distance. Pascal wrote back in August:

> I can scarcely remember that there is such a thing as Geometry [i.e., mathematics]. I recognize Geometry to be so useless that I can find little difference between a man who is a geometrician and a clever craftsman. Although I call it the best craft in the world it is, after all, nothing else but a craft . . . . It is quite possible I shall never think of it again.[17]

Pascal put together his thoughts about life and religion while he was at Port-Royal and published them under the title *Pensées*.[18] In the course of his work on that book, he filled two pieces of paper on both sides with what Ian Hacking describes as "handwriting going in all directions . . . full of erasures, corrections, and seeming afterthoughts." This fragment has come to be known as Pascal's Wager (*le pari de Pascal*), which asks, "God is, or he is not. Which way should we incline? Reason cannot answer."

Here, drawing on his work in analyzing the probable outcomes of the game of *balla*, Pascal frames the question in terms of a game of chance. He postulates a game that ends at an infinite distance in time. At that moment, a coin is tossed. Which way would you bet—heads (God is) or tails (God is not)?

Hacking asserts that Pascal's line of analysis to answer this question is the beginning of the theory of decision-making. "Decision theory," as Hacking describes it, "is the theory of deciding what to do when it is uncertain what will happen."[19] Making that decision is the essential first step in any effort to manage risk.

Sometimes we make decisions on the basis of past experience, out of experiments we or others have conducted in the course of our lifetime. But we cannot conduct experiments that will prove either the existence or the absence of God. Our only alternative is to explore the future *consequences* of believing in God or rejecting God. Nor can we avert the issue, for by the mere act of living we are forced to play this game.

Pascal explained that belief in God is not a decision. You cannot awaken one morning and declare, "Today I think I will decide to believe in God." You believe or you do not believe. The decision, therefore, is whether to choose to act in a manner that will lead to believing in God, like living with pious people and following a life of "holy water and sacraments." The person who follows these precepts is wagering that God is. The person who cannot be bothered with that kind of thing is wagering that God is not.

The only way to choose between a bet that God exists and a bet that there is no God down that infinite distance of Pascal's coin-tossing game is to decide whether an outcome in which God exists is preferable—more valuable in some sense—than an outcome in which God does not exist, even though the probability may be only 50-50. This insight is what conducts Pascal down the path to a decision—a choice in which the value of the outcome and the likelihood that it may occur will differ because the *consequences* of the two outcomes are different.*

If God is not, whether you lead your life piously or sinfully is immaterial. But suppose that God is. Then if you bet against the existence of God by refusing to live a life of piety and sacraments you run the risk of eternal damnation; the winner of the bet that God exists has the possibility of salvation. As salvation is clearly preferable to eternal damnation, the correct decision is to act on the basis that God is. "Which way should we incline?" The answer was obvious to Pascal.

Pascal produced an interesting by-product when he decided to turn over the profits from his bus line to help support the Port-Royal monastery.[20] In 1662, a group of his associates at the monastery published a work of great importance, *La logique, ou l'art de penser* (*Logic, or the Art of Thinking*), a book that ran to five editions between 1662 and 1668.† Although its authorship was not revealed, the primary—but not the sole—author is believed to have been Antoine Arnauld, a man characterized by Hacking as "perhaps the most brilliant theologian of his time."[21] The book was immediately translated into other languages throughout Europe and was still in use as a textbook in the nineteenth century.

The last part of the book contains four chapters on probability that cover the process of developing a hypothesis from a limited set of facts; today, this process is called statistical inference. Among other matters, these chapters contain a "rule for the proper use of reason in determining when to accept human authority," rules for interpreting miracles, a basis of interpreting historical events, and the application of numerical measures to probability.[22]

The final chapter describes a game in which each of ten players risks one coin in the hope of winning the nine coins of his fellow players. The author then points out that there are "nine degrees of probability

*At this point, Pascal anticipates Daniel Bernoulli's epochal breakthrough in decision analysis in 1738, which we explore in detail in Chapter 6.

†The Latin title for this book was *Ars Cogitandi*. See Hacking, 1975, pp. 12 and 24.

of losing a coin for only one of gaining nine."[23] Though the observation is innocuous, the sentence has earned immortality. According to Hacking, this is the first occasion in print "where probability, so called, is measured."[24]

The passage deserves immortality for more reasons than that. The author admits that the games he has described are trivial in character, but he draws an analogy to natural events. For example, the probability of being struck by lightning is tiny but "many people . . . are excessively terrified when they hear thunder."[25] Then he makes a critically important statement: "Fear of harm ought to be proportional not merely to the gravity of the harm, but also to the probability of the event."[26] Here is another major innovation: the idea that both gravity and probability should influence a decision. We could turn this assertion around and state that a decision should involve the strength of our desire for a particular outcome as well as the degree of our belief about the probability of that outcome.

The strength of our desire for something, which came to be known as utility, would soon become more than just the handmaiden of probability. Utility was about to take its place at the center of all theories of decision-making and risk-taking. It will reappear repeatedly in the chapters ahead.

Historians are fond of referring to near-misses—occasions when something of enormous importance almost happened but, for one reason or another, failed to happen. The story of Pascal's Triangle is a striking example of a near-miss. We have seen how to predict the probable number of boys or girls in a multi-child family. We have gone beyond that to predict the probable outcome of a World Series (for evenly matched teams) after part of the Series has been played.

In short, we have been forecasting! Pascal and Fermat held the key to a systematic method for calculating the probabilities of future events. Even though they did not turn it all the way, they inserted the key into the lock. The significance of their pioneering work for business management, for risk management, and, in particular, for insurance was to be seized upon by others—for whom the Port-Royal *Logic* was an important first step. The idea of forecasting economic trends or of using probability to forecast economic losses was too remote for Pascal and Fermat to have recognized what they were missing. It is only with hindsight that we can see how close they came.

The inescapable uncertainty of the future will always prevent us from completely banishing the fates from our hopes and fears, but after 1654 mumbo jumbo would no longer be the forecasting method of choice.