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Considering the Nature of Man

In just a few years the commanding mathematical achievements of Cardano and Pascal had been elevated into domains that neither had dreamed of. First Graunt, Petty, and Halley had applied the concept of probability to the analysis of raw data. At about the same time, the author of the Port-Royal *Logic* had blended measurement and subjective beliefs when he wrote, "Fear of harm ought to be proportional not merely to the gravity of the harm, but also to the probability of the event."

In 1738, the *Papers of the Imperial Academy of Sciences in St. Petersburg* carried an essay with this central theme: "the *value* of an item must not be based on its *price*, but rather on the *utility* that it yields."[1] The paper had originally been presented to the Academy in 1731, with the title *Specimen Theoriae Novae de Mensura Sortis* (*Exposition of a New Theory on the Measurement of Risk*); its author was fond of italics, and all three of the italicized words in the above quotation are his.* So are all those in the quotations that follow.

It is pure conjecture on my part that the author of the 1738 article had read the Port-Royal *Logic*, but the intellectual linkage between the

*As usual, the essay was published in Latin. The Latin title of the publication in which it appeared was *Commentarii Academiae Scientiarum Imperialis Petropolitanae, Tomus V.*

two is striking. Interest in *Logic* was widespread throughout western Europe during the eighteenth century.

Both authors build their arguments on the proposition that any decision relating to risk involves two distinct and yet inseparable elements: the objective facts and a subjective view about the desirability of what is to be gained, or lost, by the decision. Both objective measurement and subjective degrees of belief are essential; neither is sufficient by itself.

Each author has his preferred approach. The Port-Royal author argues that only the pathologically risk-averse make choices based on the consequences without regard to the probability involved. The author of the *New Theory* argues that only the foolhardy make choices based on the probability of an outcome without regard to its consequences.

The author of the St. Petersburg paper was a Swiss mathematician named Daniel Bernoulli, who was then 38 years old.[2] Although Daniel Bernoulli's name is familiar only to scientists, his paper is one of the most profound documents ever written, not just on the subject of risk but on human behavior as well. Bernoulli's emphasis on the complex relationships between measurement and gut touches on almost every aspect of life.

Daniel Bernoulli was a member of a remarkable family. From the late 1600s to the late 1700s, eight Bernoullis had been recognized as celebrated mathematicians. Those men produced what the historian Eric Bell describes as "a swarm of descendants . . . and of this posterity the majority achieved distinction—sometimes amounting to eminence—in the law, scholarship, literature, the learned professions, administration and the arts. None were failures."[3]

The founding father of this tribe was Nicolaus Bernoulli of Basel, a wealthy merchant whose Protestant forebears had fled from Catholic-dominated Antwerp around 1585. Nicolaus lived a long life, from 1623 to 1708, and had three sons, Jacob, Nicolaus (known as Nicolaus I), and Johann. We shall meet Jacob again shortly, as the discoverer of the Law of Large Numbers in his book *Ars Conjectandi* (*The Art of Conjecture*). Jacob was both a great teacher who attracted students from all over Europe and an acclaimed genius in mathematics, engineering, and astronomy. The Victorian statistician Francis Galton describes him as having "a bilious and melancholic temperament . . . sure but slow."[4] His relationship with his father was so poor that he took as his motto *Invito patre sidera verso*—"I am among the stars in spite of my father."[5]

Galton did not limit his caustic observations to Jacob. Despite the evidence that the Bernoulli family provided in confirmation of Galton's theories of eugenics, he depicts them in his book, *Hereditary Genius* as "mostly quarrelsome and jealous."[6]

These traits seem to have run through the family. Jacob's younger brother and fellow-mathematician Johann, the father of Daniel, is described by James Newman, an anthologist of science, as "violent, abusive . . . and, when necessary, dishonest."[*7] When Daniel won a prize from the French Academy of Sciences for his work on planetary orbits, his father, who coveted the prize for himself, threw him out of the house. Newman reports that Johann lived to be 80 years old, "retaining his powers and meanness to the end."

And then there was the son of the middle brother, Nicolaus I, who is known as Nicolaus II. When Nicolaus II's uncle Jacob died in 1705 after a long illness, leaving *The Art of Conjecture* all but complete, Nicolaus II was asked to edit the work for publication even though he was only 18 at the time. He took eight years to finish the task! In his introduction he confesses to the long delay and to frequent prodding by the publishers, but he offers as an excuse of "my absence on travels" and the fact that "I was too young and inexperienced to know how to complete it."[8]

Perhaps he deserves the benefit of the doubt: he spent those eight years seeking out the opinions of the leading mathematicians of his time, including Isaac Newton. In addition to conducting an active correspondence for the exchange of ideas, he traveled to London and Paris to consult with outstanding scholars in person. And he made a number of contributions to mathematics on his own, including an analysis of the use of conjecture and probability theory in applications of the law.

*Newman is not easy to characterize, although his *The World of Mathematics* was a major source for this book. He was a student of philosophy and mathematics who became a highly successful lawyer and public servant. A one-time senior member of the editorial board of *Scientific American*, he was an avid collector of scientific documents of great historical importance. He died in 1966.

To complicate matters further, Daniel Bernoulli had a brother five years older than he, also named Nicolaus; by convention, this Nicolaus is known as Nicolaus III, his grandfather being numberless, his uncle being Nicolaus I, and his elder first cousin being Nicolaus II. It was Nicolaus III, a distinguished scholar himself, who started Daniel off in mathematics when Daniel was only eleven years old. As the oldest son, Nicolaus III had been encouraged by his father to become a mathematician. When he was only eight years old, he was able to speak four languages; he became Doctor of Philosophy at Basel at the age of nineteen; and he was appointed Professor of Mathematics at St. Petersburg in 1725 at the age of thirty. He died of some sort of fever just a year later.

Daniel Bernoulli received an appointment at St. Petersburg in the same year as Nicolaus III and remained there until 1733, when he returned to his hometown of Basel as Professor of Physics and Philosophy. He was among the first of many outstanding scholars whom Peter the Great would invite to Russia in the hope of establishing his new capital as a center of intellectual activity. According to Galton, Daniel was "physician, botanist, and anatomist, writer on hydrodynamics; very precocious."[9] He was also a powerful mathematician and statistician, with a special interest in probability.

Bernoulli was very much a man of his times. The eighteenth century came to embrace rationality in reaction to the passion of the endless religious wars of the past century. As the bloody conflict finally wound down, order and appreciation of classical forms replaced the fervor of the Counter-Reformation and the emotional character of the baroque style in art. A sense of balance and respect for reason were hallmarks of the Enlightenment. It was in this setting that Bernoulli transformed the mysticism of the Port-Royal *Logic* into a logical argument addressed to rational decision-makers.

Daniel Bernoulli's St. Petersburg paper begins with a paragraph that sets forth the thesis that he aims to attack:

> Ever since mathematicians first began to study the measurement of risk, there has been general agreement on the following proposition: *Expected values are computed by multiplying each possible gain by the num-*

ber of ways in which it can occur, and then dividing the sum of these products by the total number of cases.[*10]

Bernoulli finds this hypothesis flawed as a description of how people in real life go about making decisions, because it focuses only on the facts; it ignores the consequences of a probable outcome for a person who has to make a decision when the future is uncertain. Price—and probability—are not enough in determining what something is worth. Although the facts are the same for everyone, "the utility . . . is dependent on the particular circumstances of the person making the estimate . . . . There is no reason to assume that . . . the risks anticipated by each [individual] must be deemed equal in value." To each his own.

The concept of utility is experienced intuitively. It conveys the sense of usefulness, desirability, or satisfaction. The notion that arouses Bernoulli's impatience with mathematicians—"expected value"—is more technical. As Bernoulli points out, expected value equals the sum of the values of each of a number of outcomes multiplied by the probability of each outcome relative to all the other possibilities. On occasion, mathematicians still use the term "mathematical expectation" for expected value.

A coin has two sides, heads and tails, with a 50% chance of landing with one side or the other showing—a coin cannot come up showing both heads and tails at the same time. What is the expected value of a coin toss? We multiply 50% by one for heads and do the same for tails, take the sum—100%—and divide by two. The expected value of betting on a coin toss is 50%. You can expect either heads or tails, with equal likelihood.

What is the expected value of rolling two dice? If we add up the 11 numbers that might come up—2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12—the total works out to 77. The expected value of rolling two dice is 77/11, or exactly 7.

Yet these 11 numbers do not have an equal probability of coming up. As Cardano demonstrated, some outcomes are more likely than others when there are 36 different combinations that produce the 11

*Daniel's uncle Jacob, who will play a major role in the next chapter, once wrote that "the value of our expectation always signifies something in the middle between the best we can hope for and the worst we can fear." (Hacking, 1975, p. 144.)

outcomes ranging from 2 to 12; two can be produced only by double-one, but four can be produced in three ways, by 3 + 1, by 1 + 3, and by 2 + 2. Cardano's useful table (page 52) lists a number of combinations in which each of the 11 outcomes can occur:

Outcome	*Probability*	*Weighted Probability*
2	1/36	$2 \times 1/36 = 0.06$
3	2/36	$3 \times 2/36 = 0.17$
4	3/36	$4 \times 3/36 = 0.33$
5	4/36	$5 \times 4/36 = 0.56$
6	5/36	$6 \times 5/36 = 0.83$
7	6/36	$7 \times 6/36 = 1.17$
8	5/36	$8 \times 5/36 = 1.11$
9	4/36	$9 \times 4/36 = 1.00$
10	3/36	$10 \times 3/36 = 0.83$
11	2/36	$11 \times 2/36 = 0.61$
12	1/36	$12 \times 1/36 = 0.33$
		Total 7.00

The expected value, or the mathematical expectation, of rolling two dice is exactly 7, confirming our calculation of 77/11. Now we can see why a roll of 7 plays such a critical role in the game of craps.

Bernoulli recognizes that such calculations are fine for games of chance but insists that everyday life is quite a different matter. Even when the probabilities are known (an oversimplification that later mathematicians would reject), rational decision-makers will try to maximize expected *utility*—usefulness or satisfaction—rather than expected value. Expected utility is calculated by the same method as that used to calculate expected value but with utility serving as the weighting factor.[11]

For example, Antoine Arnauld, the reputed author of the Port-Royal *Logic*, accused people frightened by thunderstorms of overestimating the small probability of being struck by lightning. He was wrong. It was he who was ignoring something. The facts are the same for everyone, and even people who are terrified at the first rumble of thunder are fully aware that it is highly unlikely that lightning will strike precisely where they are standing. Bernoulli saw the situation more clearly: people with a phobia about being struck by lightning place such a heavy weight on the consequences of that outcome that they tremble even though they know that the odds on being hit are tiny.

Gut rules the measurement. Ask passengers in an airplane during turbulent flying conditions whether each of them has an equal degree of anxiety. Most people know full well that flying in an airplane is far safer than driving in an automobile, but some passengers will keep the flight attendants busy while others will snooze happily regardless of the weather.

And that's a good thing. If everyone valued every risk in precisely the same way, many risky opportunities would be passed up. Venturesome people place high utility on the small probability of huge gains and low utility on the larger probability of loss. Others place little utility on the probability of gain because their paramount goal is to preserve their capital. Where one sees sunshine, the other sees a thunderstorm. Without the venturesome, the world would turn a lot more slowly. Think of what life would be like if everyone were phobic about lightning, flying in airplanes, or investing in start-up companies. We are indeed fortunate that human beings differ in their appetite for risk.

Once Bernoulli has established his basic thesis that people ascribe different values to risk, he introduces a pivotal idea: "[The] *utility resulting from any small increase in wealth will be inversely proportionate to the quantity of goods previously possessed.*" Then he observes, "Considering the nature of man, it seems to me that the foregoing hypothesis is apt to be valid for many people to whom this sort of comparison can be applied."

The hypothesis that utility is inversely related to the quantity of goods previously possessed is one of the great intellectual leaps in the history of ideas. In less than one full printed page, Bernoulli converts the process of calculating probabilities into a procedure for introducing subjective considerations into decisions that have uncertain outcomes.

The brilliance of Bernoulli's formulation lies in his recognition that, while the role of facts is to provide a single answer to expected value (the facts are the same for everyone), the subjective process will produce as many answers as there are human beings involved. But he goes even further than that: he suggests a systematic approach for determining how much each individual desires more over less: the desire is inversely proportionate to the quantity of goods possessed.

For the first time in history Bernoulli is applying measurement to something that *cannot be counted*. He has acted as go-between in the wedding of intuition and measurement. Cardano, Pascal, and Fermat provided a method for figuring the risks in each throw of the dice, but Bernoulli introduces us to the risk-*taker*—the player who chooses how much to bet or whether to bet at all. While probability theory sets up the choices, Bernoulli defines the *motivations* of the person who does the choosing. This is an entirely new area of study and body of theory. Bernoulli laid the intellectual groundwork for much of what was to follow, not just in economics, but in theories about how people make decisions and choices in every aspect of life.

Bernoulli offers in his paper a number of interesting applications to illustrate his theory. The most tantalizing, and the most famous, of them has come to be known as the Petersburg Paradox, which was originally suggested to him by his "most honorable cousin the celebrated Nicolaus Bernoulli"—the dilatory editor of *The Art of Conjecture*.

Nicolaus proposes a game to be played between Peter and Paul, in which Peter tosses a coin and continues to toss it until it comes up heads. Peter will pay Paul one ducat if heads comes up on the first toss, two ducats if heads comes up on the second toss, four ducats on the third, and so on. With each additional throw the number of ducats Peter must pay Paul is doubled.* How much should someone pay Paul—who stands to rake in a sizable sum of money—for the privilege of taking his place in this game?

*With the assistance of Richard Sylla and Leora Klapper, the best information I have been able to obtain about the value of ducats in the early 18th century is that one ducat could have purchased the equivalent of about $40 in today's money. Baumol and Baumol, Appendix, provides an approximate confirmation of this estimate. See also McKuster, 1978, and Warren and Pearson, 1993.

The paradox arises because, according to Bernoulli, "The accepted method of calculation [expected value] does, indeed, value Paul's prospects at infinity [but] no one would be willing to purchase [those prospects] at a moderately high price . . . . [A]ny fairly reasonable man would sell his chance, with great pleasure, for twenty ducats."*

Bernoulli undertakes an extended mathematical analysis of the problem, based on his assumption that increases in wealth are inversely related to initial wealth. According to that assumption, the prize Paul might win on the two-hundredth throw would have only an infinitesimal amount of additional utility over what he would receive on the one-hundredth throw; even by the 51st throw, the number of ducats won would already have exceeded 1,000,000,000,000,000. (Measured in dollars, the total national debt of the U.S. government today is only four followed by twelve zeroes.)

Whether it be in ducats or dollars, the evaluation of Paul's expectation has long attracted the attention of leading scholars in mathematics, philosophy, and economics. An English history of mathematics by Isaac Todhunter, published in 1865, makes numerous references to the Petersburg Paradox and discusses some of the solutions that various mathematicians had proposed during the intervening years.[12] Meanwhile, Bernoulli's paper remained in its original Latin until a German translation appeared in 1896. Even more sophisticated, complex mathematical treatments of the Paradox appeared after John Maynard Keynes made a brief reference to it in his *Treatise on Probability*, published in 1921. But it was not until 1954—216 years after its original publication—that the paper by Bernoulli finally appeared in an English translation.

The Petersburg Paradox is more than an academic exercise in the exponents and roots of tossing coins. Consider a great growth company whose prospects are so brilliant that they seem to extend into infinity. Even under the absurd assumption that we could make an accurate forecast of a company's earnings into infinity—we are lucky if we can

*Bernoulli's solution to the paradox has been criticized because he fails to consider a game in which the prize would rise at a faster rate than the rate Nicolaus had specified. Nevertheless, unless there is a point where the player has zero interest in any additional wealth, the paradox will ultimately come into play no matter what the rate is.

make an accurate forecast of next quarter's earnings—what is a share of stock in that company worth? An infinite amount?*

There have been moments when real, live, hands-on professional investors have entertained dreams as wild as that—moments when the laws of probability are forgotten. In the late 1960s and early 1970s, major institutional portfolio managers became so enamored with the idea of growth in general, and with the so-called "Nifty-Fifty" growth stocks in particular, that they were willing to pay any price at all for the privilege of owning shares in companies like Xerox, Coca-Cola, IBM, and Polaroid. These investment managers defined the risk in the Nifty-Fifty, *not as the risk of overpaying, but as the risk of* not owning *them: the* growth prospects seemed so certain that the future level of earnings and dividends would, in God's good time, always justify whatever price they paid. They considered the risk of paying too much to be minuscule compared with the risk of buying shares, even at a low price, in companies like Union Carbide or General Motors, whose fortunes were uncertain because of their exposure to business cycles and competition.

This view reached such an extreme point that investors ended up by placing the same total market value on small companies like International Flavors and Fragrances, with sales of only $138 million, as they placed on a less glamorous business like U.S. Steel, with sales of $5 billion. In December 1972, Polaroid was selling for 96 times its 1972 earnings, McDonald's was selling for 80 times, and IFF was selling for 73 times; the Standard & Poor's Index of 500 stocks was selling at an average of 19 times. The dividend yields on the Nifty-Fifty averaged less than half the average yield on the 500 stocks in the S&P Index.

The proof of this particular pudding was surely in the eating, and a bitter mouthful it was. The dazzling prospect of earnings rising up to the sky turned out to be worth a lot less than an infinite amount. By 1976, the price of IFF had fallen 40% but the price of U.S. Steel had more than doubled. Figuring dividends plus price change, the S&P 500 had surpassed its previous peak by the end of 1976, but the Nifty-Fifty did not surpass their 1972 bull-market peak until July 1980. Even worse, an equally weighted portfolio of the Nifty-Fifty lagged the performance of the S&P 500 from 1976 to 1990.

*A theoretical exploration into this question appears in Durand, 1959, which anticipated *the events described in the paragraphs* immediately following.

But where is infinity in the world of investing? Jeremy Siegel, a professor at the Wharton School of Business at the University of Pennsylvania, has calculated the performance of the Nifty-Fifty in detail from the end of 1970 to the end of 1993.[13] The equally weighted portfolio of fifty stocks, even if purchased at its December 1972 peak, would have realized a total return by the end of 1993 that was less than one percentage point below the return on the S&P Index. If the same stocks had been bought just two years earlier, in December 1970, the portfolio would have outperformed the S&P by a percentage point per year. The negative gap between cost and market value at the bottom of the 1974 debacle would also have been smaller.

For truly patient individuals who felt most comfortable owning familiar, high-quality companies, most of whose products they encountered in their daily round of shopping, an investment in the Nifty-Fifty would have provided ample utility. The utility of the portfolio would have been much smaller to a less patient investor who had no taste for a fifty-stock portfolio in which five stocks actually lost money over twenty-one years, twenty earned less than could have been earned by rolling over ninety-day Treasury bills, and only eleven outperformed the S&P 500. But, as Bernoulli himself might have put it in a more informal moment, you pays your money and you takes your choice.

Bernoulli introduced another novel idea that economists today consider a driving force in economic growth—human capital. This idea emerged from his definition of wealth as "anything that can contribute to the adequate satisfaction of any sort of want . . . . There is then nobody who can be said to possess nothing at all in this sense unless he starves to death."

What form does most people's wealth take? Bernoulli says that tangible assets and financial claims are less valuable than productive capacity, including even the beggar's talent. He suggests that a man who can earn 10 ducats a year by begging will probably reject an offer of 50 ducats to refrain from begging: after spending the 50 ducats, he would have no way of supporting himself. There must, however, be some amount that he would accept in return for a promise never to beg again. If that amount were, for instance, 100 ducats, "we might say that [the beggar] is possessed of wealth worth one hundred."

Today, we view the idea of human capital—the sum of education, natural talent, training, and experience that comprise the wellspring of future earnings flows—as fundamental to the understanding of major shifts in the global economy. Human capital plays the same role for an employee as plant and equipment play for the employer. Despite the enormous accretions of tangible wealth since 1738, human capital is still by far the largest income-producing asset for the great majority of people. Why else would so many breadwinners spend their hard-earned money on life-insurance premiums?

For Bernoulli, games of chance and abstract problems were merely tools with which to fashion his primary case around the desire for wealth and opportunity. His emphasis was on decision-making rather than on the mathematical intricacies of probability theory. He announces at the outset that his aim is to establish "rules [that] would be set up whereby anyone could estimate his prospects from any risky undertaking in light of one's specific financial circumstances." These words are the grist for the mill of every contemporary financial economist, business manager, and investor. Risk is no longer something to be faced; risk has become a set of opportunities open to choice.

Bernoulli's notion of utility—and his suggestion that the satisfaction derived from a specified increase in wealth would be inversely related to the quantity of goods previously possessed—were sufficiently robust to have a lasting influence on the work of the major thinkers who followed. Utility provided the underpinnings for the Law of Supply and Demand, a striking innovation of Victorian economists that marked the jumping-off point for understanding how markets behave and how buyers and sellers reach agreement on price. Utility was such a powerful concept that over the next two hundred years it formed the foundation for the dominant paradigm that explained human decision-making and theories of choice in areas far beyond financial matters. The theory of games—the innovative twentieth century approach to decision-making in war, politics, and business management—makes utility an integral part of its entire system.

Utility has had an equally profound influence on psychology and philosophy, for Bernoulli set the standard for defining human rationality. For example, people for whom the utility of wealth *rises* as they grow richer are considered by most psychologists—and moralists—as neurotic; greed was not part of Bernoulli's vision, nor is it included in most modern definitions of rationality.

Utility theory requires that a rational person be able to measure utility under all circumstances and to make choices and decisions accordingly—a tall order given the uncertainties we face in the course of a lifetime. The chore is difficult enough even when, as Bernoulli assumed, the facts are the same for everyone. On many occasions the facts are not the same for everyone. Different people have different information; each of us tends to color the information we have in our own fashion. Even the most rational among us will often disagree about what the facts mean.

Modern as Bernoulli may appear, he was very much a man of his times. His concept of human rationality fitted neatly into the intellectual environment of the Enlightenment. This was a time when writers, artists, composers, and political philosophers embraced the classical ideas of order and form and insisted that through the accumulation of knowledge mankind could penetrate the mysteries of life. In 1738, when Bernoulli's paper appeared, Alexander Pope was at the height of his career, studding his poems with classical allusions, warning that "A little learning is a dangerous thing," and proclaiming that "The proper study of mankind is man." Denis Diderot was soon to start work on a 28-volume encyclopedia, and Samuel Johnson was about to fashion the first dictionary of the English language. Voltaire's unromantic viewpoints on society occupied center stage in intellectual circles. By 1750, Haydn had defined the classical form of the symphony and sonata.

The Enlightenment's optimistic philosophy of human capabilities would show up in the Declaration of Independence and would help shape the Constitution of the newly formed United States of America. Carried to its violent extreme, the Enlightenment inspired the citizens of France to lop off the head of Louis XVI and to enthrone Reason on the altar of Notre Dame.

Bernoulli's boldest innovation was the notion that each of us—even the most rational—has a unique set of values and will respond accordingly, but his genius was in recognizing that he had to go further than that. When he formalizes his thesis by asserting that utility is inversely proportionate to the quantity of goods possessed, he opens up a fascinating insight into human behavior and the way we arrive at decisions and choices in the face of risk.

According to Bernoulli, our decisions have a predictable and systematic structure. In a rational world, we would all rather be rich than poor, but the intensity of the desire to become richer is tempered by how rich we already are. Many years ago, one of my investment counsel clients shook his finger at me during our first meeting and warned me: "Remember this, young man, you don't have to make me rich. I am rich already!"

The logical consequence of Bernoulli's insight leads to a new and powerful intuition about taking risk. If the satisfaction to be derived from each successive increase in wealth is smaller than the satisfaction derived from the previous increase in wealth, then the *dis*utility caused by a loss will always exceed the positive utility provided by a gain of equal size. That was my client's message to me.

Think of your wealth as a pile of bricks, with large bricks at the foundation and with the bricks growing smaller and smaller as the height increases. Any brick you remove from the top of the pile will be larger than the next brick you might add to it. The hurt that results from losing a brick is greater than the pleasure that results from gaining a brick.

Bernoulli provides this example: two men, each worth 100 ducats, decide to play a fair game, like tossing coins, in which there is a 50-50 chance of winning or losing, with no house take or any other deduction from the stakes. Each man bets 50 ducats on the throw, which means that each has an equal chance of ending up worth 150 ducats or of ending up worth only 50 ducats.

Would a rational person play such a game? The mathematical expectation of each man's wealth after the game has been played with this 50-50 set of alternatives is precisely 100 ducats (150 + 50 divided by 2), which is just what each player started with. The expected value for each is the same as if they had not decided to play the game in the first place.

Bernoulli's theory of utility reveals an asymmetry that explains why an even-Steven game like this is an unattractive proposition. The 50 ducats that the losing player would drop have greater utility than the 50 ducats that the winner would pocket. Just as with the pile of bricks, losing 50 ducats hurts the loser more than gaining 50 ducats pleases the winner.* In a mathematical sense a zero-sum game is a loser's game when it is valued in terms of utility. The best decision for both is to refuse to play this game.

Bernoulli uses his example to warn gamblers that they will suffer a loss of utility even in a fair game. This depressing result, he points out, is:

> Nature's admonition to avoid the dice altogether . . . . [E]veryone who bets any part of his fortune, however small, on a mathematically fair game of chance acts irrationally . . . . [T]he imprudence of a gambler will be the greater the larger part of his fortune which he exposes to a game of chance.

Most of us would agree with Bernoulli that a fair game is a loser's game in utility terms. We are what psychologists and economists call "risk-averse" or "risk averters." The expression has a precise meaning with profound implications.

Imagine that you were given a choice between a gift of $25 for certain or an opportunity to play a game in which you stood a 50% chance of winning $50 and a 50% chance of winning nothing. The gamble has a mathematical expectation of $25—the same amount as the gift—but that expectation is uncertain. Risk-averse people would choose the gift over the gamble. Different people, however, are risk-averse in different degrees.

You can test your own degree of risk aversion by determining your "certainty equivalent." How high would the mathematical expectation of the game have to go before you would prefer the gamble to the gift? Thirty dollars from a 50% chance of winning $60 and a 50% chance of winning nothing? Then the $30 expectation from the gamble would be the equivalent of the $25 for certain. But perhaps you would take the gamble for an expectation of only $26. You might even discover that at heart you are a *risk-seeker,* willing to play the game even when the mathematical expectation of the payoff is less than the certain return of $25. That would be the case, for example, in a game where the payoff differs from 50-50 so that you would win $40 if you toss tails and zero if you toss heads, for an expected value of only $20. But most of us

*This is an oversimplification. The utility of any absolute loss depends on the wealth of the loser. Here the implicit assumption is that the two players have equal wealth.

would prefer a game in which the expected value is something in excess of the $50 in the example. The popularity of lottery games provides an interesting exception to this statement, because the state's skim off the top is so large that most lotteries are egregiously unfair to the players.

A significant principle is at work here. Suppose your stockbroker recommends a mutual fund that invests in a cross section of the smallest stocks listed on the market. Over the past 69 years, the smallest 20% of the stock market has provided an income of capital appreciation plus dividend that has averaged 18% a year. That is a generous rate of return. But volatility in this sector has also been high: two-thirds of the returns have fallen between –23% and +59%; negative returns over twelve-month periods have occurred in almost one out of every three years and have averaged 20%. Thus, the outlook for any given year has been extremely uncertain, regardless of the high average rewards generated by these stocks over the long run.

As an alternative, suppose a different broker recommends a fund that buys and holds the 500 stocks that comprise the Standard & Poor's Composite Index. The average annual return on these stocks over the past 69 years has been about 13%, but two-thirds of the annual returns have fallen within the narrower range of –11% and +36%; negative returns have averaged 13%. Assuming the future will look approximately like the past, but also assuming that you do not have 70 years to find out how well you did, is the higher average expected return on the small-stock fund sufficient to justify its much greater volatility of returns? Which mutual fund would you buy?

Daniel Bernoulli transformed the stage on which the risk-taking drama is played out. His description of how human beings employ both measurement and gut in making decisions when outcomes are uncertain was an impressive achievement. As he himself boasts in his paper, "Since all our propositions harmonize perfectly with experience, it would be wrong to neglect them as abstractions resting upon precarious hypotheses."

A powerful attack some two hundred years later ultimately revealed that Bernoulli's propositions fell short of harmonizing perfectly with experience, in large part because his hypotheses about human rationality were more precarious than he as a man of the Enlightenment might want to believe. Until that attack was launched, however, the concept of utility flourished in the philosophical debate over rationality that prevailed for nearly two hundred years after Bernoulli's paper was published. Bernoulli could hardly have imagined how long his concept of utility would survive—thanks largely to later writers who came upon it on their own, unaware of his pioneering work.