Project 1: Martingale

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Experiment 1

1) *Estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning.*

Based on the plots, 100% probability. I hate to admit something involving chance and gambling could ever be 100% certain but I ran the experiment over 10 million episodes and the target amount of $80 was reached every, single, time. The graphs all reach $80. This is mind-bending because it seems, as a figure of speech so "improbable." When considering that 1000 bets will be place sequentially as a whole, there is no doubt what the outcome will be.

2) *What is the estimated expected value of our winnings after 1000 sequential bets? Explain your reasoning.*

Based solely on plots, we see all values reach our target score of $80. The expected value formula from Wikipedia article "Expected Value" for a finite number of bets instructs us that we take the sum of the products of all probabilities and the monetary values.

$$\mathrm{E}[X] = \sum_{i=1}^{k} x_i\, p_i = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k.$$

If we take the expected value formula for a single $1 bet on black, we will find that we expect to lose approximately 5 cents per $1 bet. This is because the chance of winning the dollar bet (landing on black) is 18/38 and the chance of losing a dollar (lands on red or zero(s)) is 20/38. This is illustrated with the formula:

$$1*(18/38) + (-1(20/38)) = (18/38) + -(20/38) = -2/38 = -0.0526 \text{ (approx.)}$$

After losing 5.26 cents on average over 1000 winnings, we would expect to have lost $52.60 on our $1000 investment, for a return of approximately $947.40. Because our target is much lower than this amount and our bankroll unlimited, we will quickly the hit $80.

3) In Experiment 1, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases? Explain why it does (or does not).

The standard deviation in the *mean* does converge on the target limit once reached but fluctuates wildly because the bets are growing rapidly due to doubling. For the median it diverges and linearly grows and ultimately flatlines into a stabilized value above and below the median value. The median represents a much more stable representation of the graph as it avoids the edges of episodes that are having bad luck during certain spins.

Experiment 2

1) *Estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning using the experiment. (not based on plots)*

Since the bankroll is limited here, we now have losers who go bust. In experiment two, 632 of the 1000 episodes were able to reach the $80 target (63.2%) of the time.

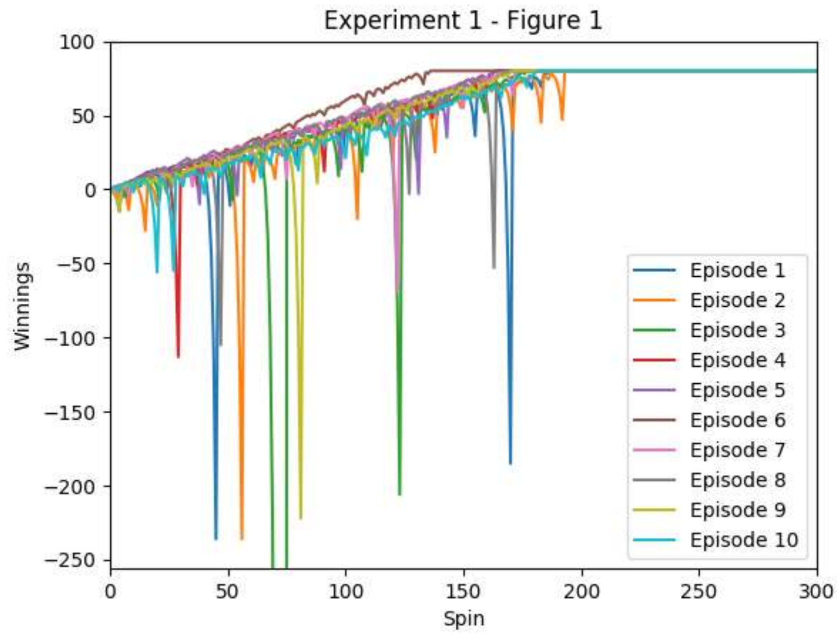
2) *What is the estimated expected value of our winnings after 1000 sequential bets? Explain your reasoning. (not based on plots)*

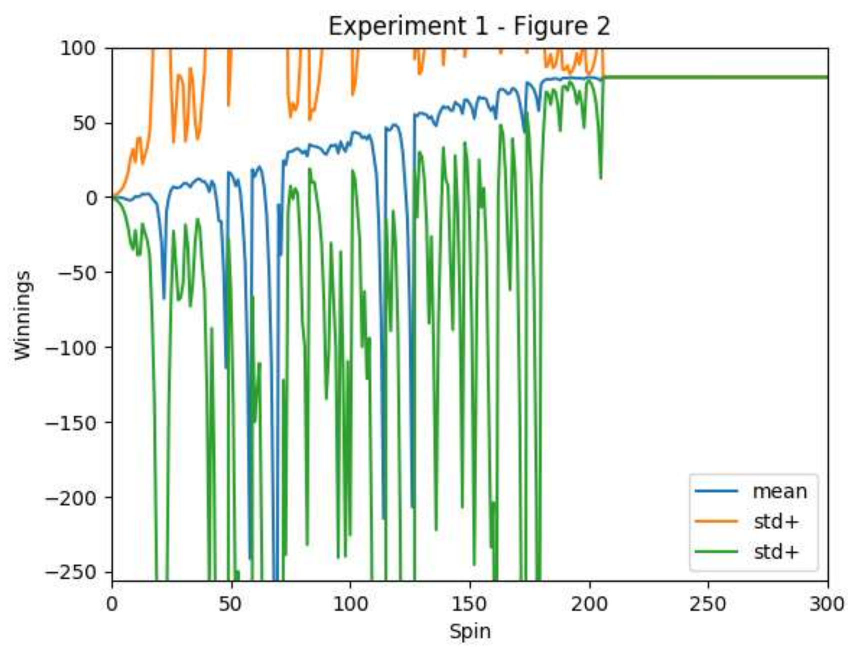
The expected value formula as provided in question 2 of Experiment 1 provided the mathematical basis for this answer, essentially we lose 5.26 cents for each dollar bet spent on each spin. Over 1000 spins this would be a $52.60 loss on our $1000 investment. In actuality, the average earnings extracted from the experiment was negative $44 dollars - not too far off the theoretical EV loss.

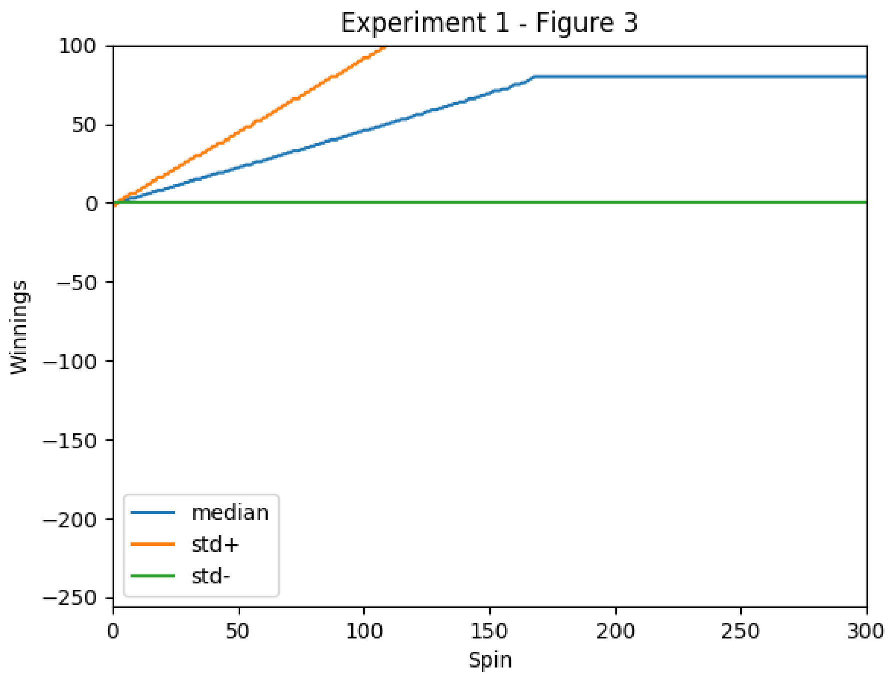
3) *Does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases? Explain why it does (or does not).*

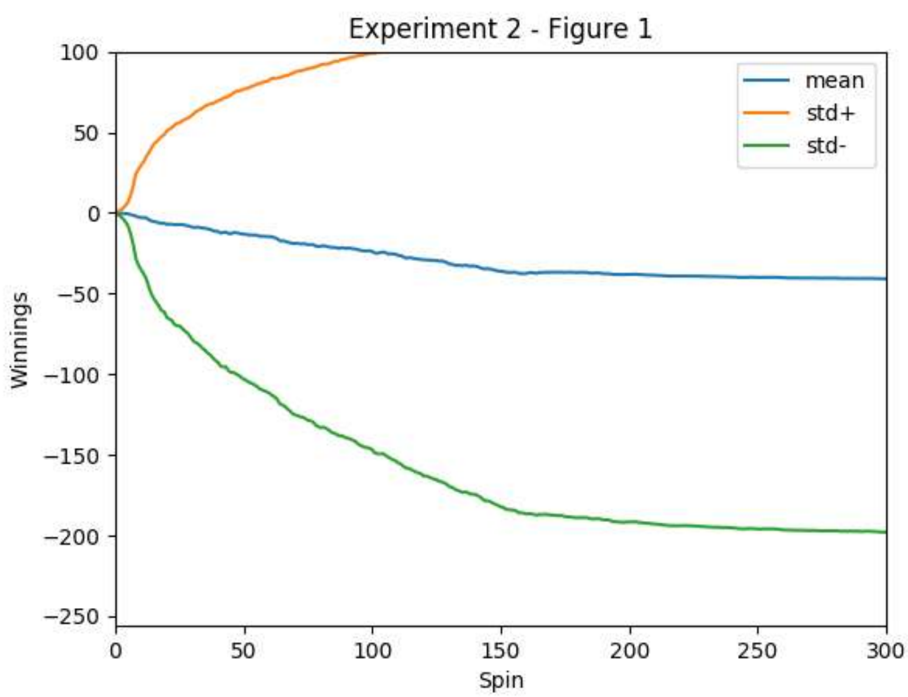
Standard deviation appears to eventually reach a maximum value due to two factors: First the target values are sometimes reached by lucky players, and secondly losers start to go bust on their bankroll. Initially the behavior is one of wild divergence as losing spins force the players to double their bets and dip deeper into their bankrolls. As we have more players lose, we cease to see their effects on these graphs and likewise when players reach their target scores the do not weigh on the results as much.

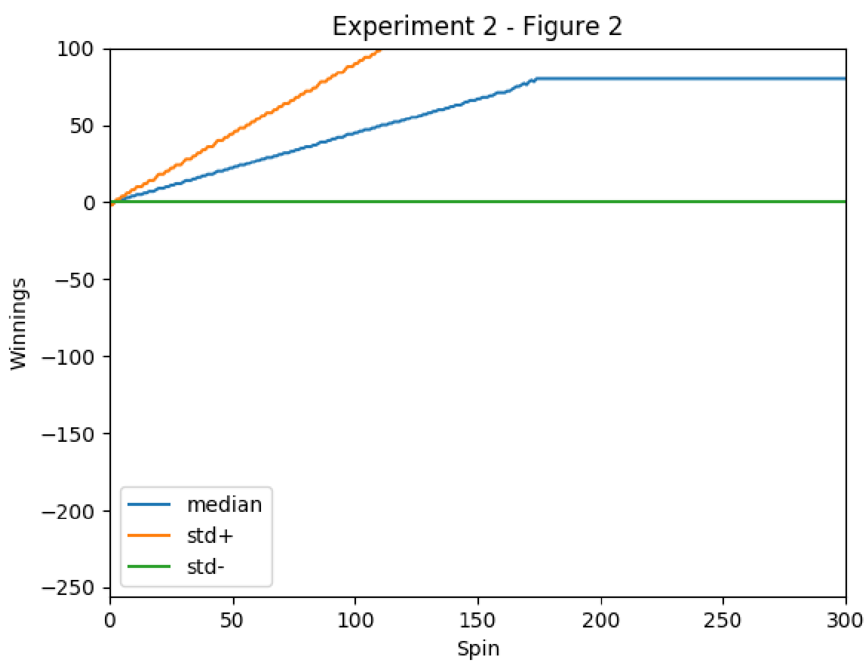
Graphs


Experiment 1 - Figure 1
Winnings
Spin
Episode 1
Episode 2
Episode 3
Episode 4
Episode 5
Episode 6
Episode 7
Episode 8
Episode 9
Episode 10


Experiment 1 - Figure 2
Winnings
Spin
mean
std+
std+


Experiment 1 - Figure 3
Winnings
Spin
median
std+
std-
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300


Experiment 2 - Figure 1
Winnings
Spin
mean
std+
std-
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300



References

1. Expected Value. (n.d.). In Wikipedia. Retrieve 5/20/2019, from https://en.wikipedia.org/wiki/Expected_value