

# **Angular momentum at every scale, an exercise in circular reasoning**

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An exercise in writing journal articles in L<sup>A</sup>T<sub>E</sub>X.

## ABSTRACT

Since the works of Newton et al., it has been believed that a correlation exists between an object’s mass and angular momentum. Taking a sample of objects on varying scales, and using some circular reasoning, we find the relation  $\log L = -0.6306 + 1.5810 \log m$ , with an  $R^2$  of 0.993. Using this relation, we determine the mass of globular cluster 47 Tucanae to be  $3.25 \times 10^{56} \text{ kg m}^2 \text{ d}^{-1}$ , answering a century-old question.

*Subject headings:* angular momentum, extrapolation, circular reasoning, science

## 1. Data

Our data were obtained from Wikipedia et al., and are shown in Table 1. Just the masses and rotational angular momenta are shown in Table 2. We were able to obtain the mass ( $m$ ), effective radius ( $r$ ), and rotation period ( $P$ ) for each object (with the exception of 47 Tuc and the Milky Way). To determine angular momentum ( $L$ ), we assumed each object to be a sphere, and used

$$L = I\omega = \left(\frac{2}{5}mr^2\right)\left(\frac{2\pi}{P}\right) = \frac{4\pi}{5}\frac{mr^2}{P}. \quad (1)$$

For the Milky Way, we cannot assume it to be spherical, and so we took an estimate of the angular momentum from Wikipedia et al., which is based on a more complex model.

## 2. Analysis

We fit a linear model to the mass and angular momenta in log-space, finding a relation of

$$\log L = -0.6306 + 1.5810 \log m, \quad (2)$$

with an  $R^2 = 0.993$ , and  $F$ -statistic of 722.1. If we return from log-space, we get a relationship of

$$L = 0.234m^{1.5810}. \quad (3)$$

A log-log plot of the angular momenta as a function of mass are given in Figure 1.

Using this relation, we determine the angular momentum of 47 Tucanae to be  $3.25 \times 10^{56} \text{ kg m}^2 \text{ d}^{-1}$ .

### 3. Conclusions

We find that log–log plots are the “bread and butter” of astronomical publications. However, we hypothesize that one’s academic career cannot survive on bread and butter alone.

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	$m(\text{kg})$	$r(\text{m})$	$P(\text{d})$	$L(\text{Js})$
951 Gaspra	$2.5 \times 10^{16}$	6100	0.293	...
Ceres	$9.393 \times 10^{20}$	473000	0.37809	...
Earth	$5.972\,37 \times 10^{24}$	$6.3781 \times 10^6$	0.99727	...
Jupiter	$1.8986 \times 10^{27}$	$7.1492 \times 10^7$	0.41007	...
Sun	$1.988\,55 \times 10^{30}$	$6.9634 \times 10^8$	25.38	...
Betelgeuse	$2.7 \times 10^{31}$	$7.5 \times 10^{11}$	$4.32 \times 10^8$	...
47 Tuc	$1.39 \times 10^{36}$	$5.7 \times 10^{17}$	...	...
Milky Way	$2.3 \times 10^{42}$	...	...	1e+67

Table 1: Data obtained from Wikipedia et al.

	$m(\text{kg})$	$r(\text{m})$
951 Gaspra	$2.5 \times 10^{16}$	$9.235\,45 \times 10^{19}$
Ceres	$9.393 \times 10^{20}$	$1.6168 \times 10^{28}$
Earth	$5.972\,37 \times 10^{24}$	$7.086\,68 \times 10^{33}$
Jupiter	$1.8986 \times 10^{27}$	$6.883\,61 \times 10^{38}$
Sun	$1.988\,55 \times 10^{30}$	$1.105\,13 \times 10^{42}$
Betelgeuse	$2.7 \times 10^{31}$	$1.022\,65 \times 10^{42}$
47 Tuc	$1.39 \times 10^{36}$	...
Milky Way	$2.3 \times 10^{42}$	$1 \times 10^{67}$

Table 2: Mass and angular momenta calculated from Wikipedia et al. data.

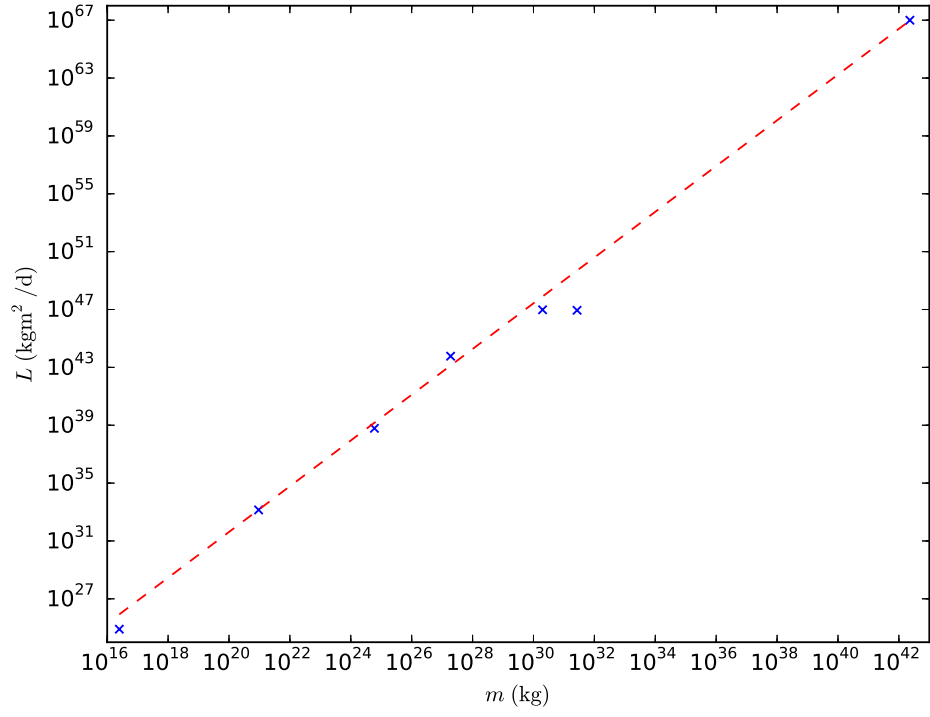


Fig. 1.— log–log plot of angular momentum as a function of mass.