Spherical solutions for stars

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General Relativity I Presentations December 14th, 2015



Spherical stars

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eneral Relativity I Presentation December 14th, 2015

2015-12-12

Introduction

Spherical stars
Introduction

Introduction

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Spherically symmetric coordinates

Spherical stars —Spherically symmetric coordinates

Spherically symmetric coordinates



General metric

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Metric on 2-sphere

Schutz (2009, p. 256)

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$$dl^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) = r^2d\Omega^2$$

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Spherical stars

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Spherical stars

___Two-sphere in flat spacetime

-Spherically symmetric coordinates

Two-sphere in flat spacetime $\frac{d^2-d^2+dr^2+r^2(dr^2+dr^2+dr^2+dr^2)}{dr^2-dr^2+dr^2+r^2(dr^2+dr^2+dr^2+dr^2)}$

Schutz (2009, p. 256)

Two-sphere in flat spacetime

General metric

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Metric on 2-sphere

$$dl^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2) \equiv r^2 d\Omega^2$$

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Spherical stars -Spherically symmetric coordinates

Two-sphere in flat spacetime

Two-sphere in flat spacetime $ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$ $dl^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2) \equiv r^2 d\Omega^2$

Schutz (2009, p. 256)

Schutz (2009, p. 256)

Metric on 2-sphere

$$dl^2 = f(r', t)d\Omega^2$$

Relation to r

$$f(r',t) \equiv r$$

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Schutz (2009, pp. 256–257)

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Spherically symmetric coordinates

☐Two-sphere in curved spacetime



Metric on 2-sphere

$$dl^2 = f(r', t)d\Omega^2$$

Relation to r

$$f(r',t) \equiv r^2$$

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Schutz (2009, pp. 256–257)

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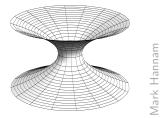
Spherical stars

-Spherically symmetric coordinates

__Two-sphere in curved spacetime



Meaning of r



- "curvature" or "area" coordinate
 - radius of curvature and area

Figure:

Surface with circular symmetry but no coordinate r = 0.

Schutz (2009, p. 257)

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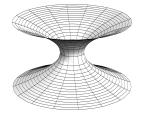
Spherical stars -Spherically symmetric coordinates

 \sqsubseteq Meaning of r



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Meaning of r



• "curvature" or "area" coordinate

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Figure:

Surface with circular symmetry but no coordinate r = 0.

Schutz (2009, p. 257)

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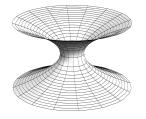
Spherical stars -Spherically symmetric coordinates

 \sqsubseteq Meaning of r



Mark

Meaning of r



• "curvature" or "area" coordinate

- radius of curvature and area
- *not* proper distance from center
- r = const, t = const
 - $A = 4\pi r^2$
 - $C=2\pi r$

Figure:

Surface with circular symmetry but no coordinate r = 0.

Schutz (2009, p. 257)

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Spherical stars -Spherically symmetric coordinates

 \sqsubseteq Meaning of r



General metric

$$ds^{2} = q_{00} dt^{2} + 2q_{0r} dr dt + q_{rr} dr^{2} + r^{2} d\Omega^{2}$$

 $g_{00}, g_{0r}, \text{ and } g_{rr} \text{ functions of } t \text{ and } r$

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-Spherically symmetric coordinates

Spherically symmetric spacetime

 $ds^2 = g_{00} dt^2 + 2g_{0r} dr dt + g_{rr} dr^2 + r^2 d\Omega^2$ g_{0r}, g_{0r} , and g_{rr} functions of t and r

Spherically symmetric spacetime

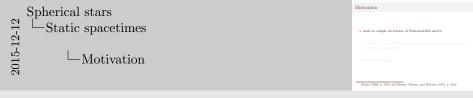
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Static spacetimes

Spherical stars
Static spacetimes

Static spacetimes

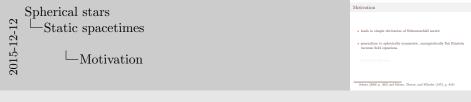
- generalizes to spherically symmetric, asymptotically flat Einstein
- a Dial-1- 07- 41- ----



• Birkhoff's theorem says that the Schwarzschild metric applies to point 2

Motivation

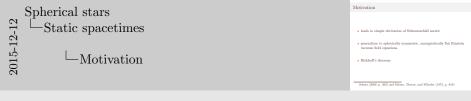
- leads to simple derivation of Schwarzschild metric
- generalizes to spherically symmetric, asymptotically flat Einstein vacuum field equations
- Dinlehoff's theorem



• Birkhoff's theorem says that the Schwarzschild metric applies to point 2

• generalizes to spherically symmetric, asymptotically flat Einstein vacuum field equations

• Birkhoff's theorem



• Birkhoff's theorem says that the Schwarzschild metric applies to point 2

Definition

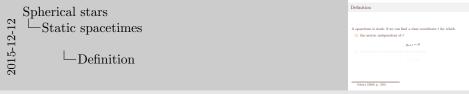
A spacetime is static if we can find a time coordinate t for which

(i) the metric independent of t

$$g_{\alpha\beta,t} = 0$$

(ii) the geometry unchanged by time reversal

$$t \rightarrow -t$$



- ullet a spacetime which only satisfies the first condition is stationary
 - e.g. rotating star

Definition

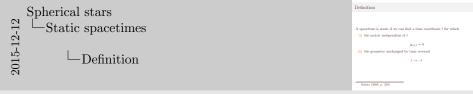
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Time reversal

$$\Lambda: (t, x, y, z) \rightarrow (-t, x, y, z)$$

$$g_{\bar{\alpha}\bar{\beta}} = \Lambda^{\alpha}_{\ \bar{\alpha}} \Lambda^{\beta}_{\ \bar{\beta}} g_{\alpha\beta} = g_{\alpha\beta}$$

$$\Lambda^{0}_{\ \overline{0}} = x^{0}_{\ ,\overline{0}} = -x^{0}_{\ ,0} = -1$$

$$\Lambda^{i}_{\ \overline{j}} = x^{i}_{\ ,\overline{j}} = x^{i}_{\ ,j} = \delta^{i}_{\ j}$$

$$\Lambda^{0}_{\ \overline{i}} = x^{0}_{\ ,\overline{i}} = x^{0}_{\ ,i} = 0$$

$$g_{\bar{0}\bar{0}} = (\Lambda^0_{\bar{0}})^2 g_{00} = g_{00}$$

$$g_{\bar{0}\bar{r}} = \Lambda^0_{\bar{0}} \Lambda^r_{\bar{r}} g_{0r} = -g_{0r}$$

$$g_{\bar{r}\bar{r}} = (\Lambda^r_{\bar{r}})^2 g_{rr} = g_{rr}$$

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-Static spacetimes

__Time reversal



•
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Schutz (2009, p. 258)

Time reversal

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Transformation

$$\begin{split} & \Lambda^0_{\ \bar{0}} = x^0_{\ ,\bar{0}} = -x^0_{\ ,0} = -1 \\ & \Lambda^i_{\ \bar{j}} = x^i_{\ ,\bar{j}} = x^i_{\ ,j} = \delta^i_{\ j} \\ & \Lambda^0_{\ \bar{i}} = x^0_{\ \bar{i}} = x^0_{\ ,i} = 0 \end{split}$$

Metric

$$g_{\bar{0}\bar{0}} = (\Lambda^0_{\bar{0}})^2 g_{00} = g_{00}$$

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Spherical stars
—Static spacetimes
—Time reversal



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Spherical stars

Static spacetimes

__Time reversal



Time reversal

$$\bullet \ g_{\bar{0}\bar{r}} = -g_{0r} = 0$$



Simplified metric

$$ds^{2} = g_{00} dt^{2} + g_{rr} dr^{2} + r^{2} d\Omega^{2}$$

Replacement

$$g_{00} \to -e^{2\Phi}$$
, $g_{rr} \to e^{2\Lambda}$, provided $g_{00} < 0 < g_{rr}$

Static spherically symmetric metric

$$ds^2 - e^{2\Phi} dt^2 + e^{2\Lambda} dr^2 + r^2 d\Omega^2$$

$$\lim_{r \to \infty} \Phi(r) = \lim_{r \to \infty} \Lambda(r) = 0$$

Schutz (2009, pp. 258–259)



Spherical stars
—Static spacetimes



└─The metric

- constraint $g_{00} < 0 < g_{rr}$ holds for stars but not black holes
- ullet limits at infinity tell us that spacetime is asymptotically flat

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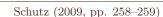
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Static spacetimes —The metric

Spherical stars



- constraint $g_{00} < 0 < g_{rr}$ holds for stars but not black holes
- limits at infinity tell us that spacetime is asymptotically flat

$$G_{\alpha\beta} = R^{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R$$

Einstein tensor components

$$G_{tt} = \frac{1}{r^2} e^{2\Phi} \frac{d}{dr} [r(1 - e^{-2\Lambda})],$$

$$G_{rr} = -\frac{1}{r^2} e^{2\Lambda} (1 - e^{-2\Lambda}) + \frac{2}{r} \Phi'$$

$$G_{\theta\theta} = r^2 e^{-2\Lambda} [\Phi'' + (\Phi')^2 + \Phi'/r - \Phi'\Lambda' - \Lambda'/r],$$

$$G_{\phi\phi} = \sin^2 \theta G_{\theta\theta}$$

Schutz (2009, pp. 165, 260)



Spherical stars

Static spacetimes

LEinstein Tensor

ein tensor

Einstein Tensor

Schutz (2009, pp. 165, 260)

Sinstein tensor $G_{\alpha\beta} = R^{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R$

 $G_{12} = \frac{1}{r^2} e^{2\phi} \frac{i\sigma}{dr} [r(1 - e^{-2\Lambda})],$ $G_{12} = -\frac{1}{r^2} e^{2\Lambda} (1 - e^{-2\Lambda}) + \frac{2}{r} \Phi'$ $G_{32} = r^2 e^{-2\Lambda} [\Phi'' + (\Phi')^2 + \Phi'/r - \Phi'/\Lambda' - \Lambda'/r]$ $G_{32} = \sin^2\theta G_{32}$

• $x' \equiv dx/dr$

Einstein Tensor

General Einstein tensor

$$G_{\alpha\beta} = R^{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R$$

Einstein tensor components

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Spherical stars

Static spacetimes

Einstein Tensor

record Function to some $G_{n,j} = R^{-1} - \frac{1}{2} \mu_{ij} M$ which is the order components $G_{n} = \frac{1}{12} 2^{2n} \frac{d}{dr} [i(1-e^{-i\phi})_{i}],$ $G_{n} = -\frac{1}{12} 2^{2n} \frac{d}{dr} [i(1-e^{-i\phi})_{i}],$ $G_{m} = -\frac{1}{2} e^{2n} [1-e^{-i\phi}]_{i} - e^{i\phi} N - e^{i\phi} N - A/r],$ $G_{m} = e^{2n} M G_{m,j}$ where $G_{m} = e^{2n} M G_{m,j}$ and $G_{m,j} = e^{2n} M G_{m,j}$ Solution (2.5).

Einstein Tensor

• $x' \equiv dx/dr$

Schutz (2009, pp. 165, 260)

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Spherical stars
Static perfect fluid

Static perfect fluid

Static perfect fluid



$$\vec{U} \cdot \vec{U} = -1$$
 (conservation law)

Solving for U^0

$$a_{00}(U^0)^2 = -1 \implies U^0 = (-a_{00})^{-1/2} = e^{-\Phi}$$

Solving for U

$$U_0 = a_{00}U^0 = -e^{\frac{a_0}{2}}$$

Spherical stars
—Static perfect fluid

 \sqsubseteq Four-velocity



• "conservation law" is the conservation of four-momentum

$$g_{00}U^0U^0 = -1 \implies (U^0)^2 = (-g_{00})^{-1}$$

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Schutz (2009, p. 260)

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Spherical stars
—Static perfect fluid

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Schutz (2009, p. 260)

Constraints

$$U^i = 0 \text{ (static)}$$
 $\vec{U} \cdot \vec{U} = -1 \text{ (conservation law)}$

Solving for U^0

$$q_{00}(U^0)^2 = -1 \implies U^0 = (-q_{00})^{-1/2} = e^{-\Phi}$$

Solving for U_0

$$U_0 = q_{00}U^0 = -e^{\Phi}$$

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Spherical stars

-Static perfect fluid

 $\chi_{00}(U^{0})^{2} = -1 \implies U^{0} = (-g_{00})^{-1/2} = e^{-\Phi}$ Schutz (2009, p. 200)

Four-velocity

Four-velocity

• "conservation law" is the conservation of four-momentum

$$g_{00}U^0U^0 = -1 \implies (U^0)^2 = (-g_{00})^{-1}$$

 $\implies U^0 = (-g_{00})^{-1/2}$
 $\implies U^0 = (e^{2\Phi})^{-1/2} = e^{-\Phi}$

Schutz (2009, p. 260)

$$T_{\alpha\beta} = (\rho + p)U_{\alpha}U_{\beta} + pg_{\alpha\beta}$$

Components of $T_{\alpha\beta}$

$$T_{00} = (\rho + p)e^{2\Phi} + p(-e^{2\Phi}) = \rho e^{2\Phi}$$

$$T_{\alpha\beta} = 0 \text{ for } \alpha \neq \beta; \quad T_{ii} = pg_{ii}$$

$$T_{max} = ne^{2\Lambda}; \quad T_{na} = nr^2; \quad T_{na} = nr^2 \sin^2 \theta = T_{na} \sin^2 \theta$$

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Spherical stars

Static perfect fluid

_Stress-energy tensor

Stress-energy tensor $T_{ab} = (p+p)U_aU_b + p_{ba}$ $T_{ab} = (p+p)U_aU_b + p_{ba}$

Schutz (2009, p. 200)

- $T_{\alpha\beta} = 0$ because
 - the cross terms make $g_{\alpha\beta} = 0$ - T_{0i} makes one of the U's $U_i = 0$
- likewise, $T_{ii} = pg_{ii}$ because $U_iU_i = 0$

$T_{\alpha\beta}$ for perfect fluid

$$T_{\alpha\beta} = (\rho + p)U_{\alpha}U_{\beta} + pg_{\alpha\beta}$$

Components of $T_{\alpha\beta}$

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$$T_{rr} = pe^{2\Lambda}; \quad T_{\theta\theta} = pr^2; \quad T_{\phi\phi} = pr^2 \sin^2 \theta = T_{\theta\theta} \sin^2 \theta$$

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Spherical stars Static perfect fluid

_Stress-energy tensor

Stress-energy tensor $T_{\alpha\beta} = (\rho + p)U_{\alpha}U_{\beta} + pg_{\alpha\beta}$ $T_{rr} = pe^{2\Lambda}; \quad T_{\theta\theta} = pr^2; \quad T_{\phi\phi} = pr^2 \sin^2\theta = T_{\theta\theta} \sin^2\theta$ Schutz (2009, p. 200)

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Local thermodynamic equilibrium

$$p = p(\rho, S) \approx p(\rho)$$

- pressure related to energy density and specific entropy
- we often deal with negligibly small entropies

Spherical stars Static perfect fluid -Equation of state



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Conservation laws

$$T^{\alpha\beta}_{\ \ ;\beta}=0$$

$$(\rho + p)\frac{\mathrm{d}\Phi}{\mathrm{d}r} = -\frac{\mathrm{d}}{\mathrm{d}r}$$

Schutz (2009, pp. 175, 261)

Spherical stars -Static perfect fluid

Equations of motion



Conservation laws

$$T^{\alpha\beta}_{\ \ ;\beta} = 0$$

• symmetries make only non-trivial solution $\alpha = r$ TODO: prove

$$(\rho + p)\frac{\mathrm{d}\Phi}{\mathrm{d}r} = -\frac{\mathrm{d}}{\mathrm{d}r}$$

Equations of motion

Spherical stars -Static perfect fluid

Equations of motion



Schutz (2009, pp. 175, 261)

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Equation of motion

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-Static perfect fluid

Equations of motion

Schutz (2009, pp. 175, 261)

Equations of motion • symmetries make only non-trivial solution $\alpha = r$ $(\rho + p)\frac{d\Phi}{dr} = -\frac{d\rho}{dr}$

Schutz (2009, pp. 175, 261)

Equation of motion (continued)

TODO

Show 10.31



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—Static perfect fluid

TODO Show 10.31

Equation of motion (continued)

Equation of motion (continued)

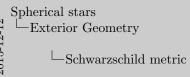
Exterior Geometry

Exterior Geometry



Schwarzschild metric

TODO



Schwarzschild metric

TODO

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Birkhoff and Langer (1923)

Spherical stars Exterior Geometry

□Birkhoff's Theorem

Birkhoff's Theorem

Let the geometry of a given region of spacetime

Let the geometry of a given region of spacetime:

- be spherically symmetric

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Spherical stars

Exterior Geometry

Birkhoff's Theorem

Birkhoff and Langer (1923)

Birkhoff's Theorem

Let the geometry of a given region of spacetime

Let the geometry of a given region of spacetime:

- be spherically symmetric
- 2 be a solution to the Einstein field equations in vacuum.

Then that geometry is necessarily a piece of the Schwarzschild geometry.

(Proof given in Misner, Thorne, and Wheeler (1973, pp. 843–844))

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Spherical stars

Exterior Geometry

└─Birkhoff's Theorem

Birkhoff's Theorem

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Birkhoff and Langer (1923)

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Birkhoff and Langer (1923)

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—Exterior Geometry

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-Birkhoff's Theorem

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(Proof given in Misner, Thorne, and Wheeler (1973, pp. 843–844)

Birkhoff and Langer (1923)

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