Spherical solutions for stars

Daniel Wysocki

Rochester Institute of Technology

General Relativity I Presentations
December 14th, 2015

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Introduction

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Spherically symmetric coordinates

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Two-sphere in flat spacetime

General metric

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Metric on 2-sphere

$$dl^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2) \equiv r^2 d\Omega^2$$

Schutz (2009, p. 256)

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Relation to r

$$f(r', t) \equiv r^2$$

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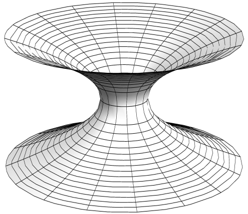
Meaning of r



Mark Hannam

Figure:
Surface with circular symmetry but no coordinate $r = 0$.

- “curvature” or “area” coordinate
  - radius of curvature and area
- *not* proper distance from center
- $r = \text{const}$, $t = \text{const}$
  - $A = 4\pi r^2$
  - $C = 2\pi r$

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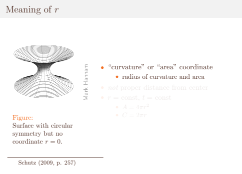

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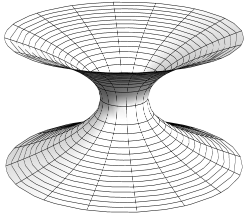

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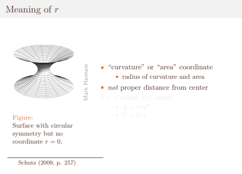
└Meaning of r



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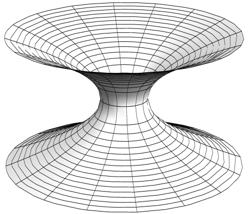

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g_{00}, g_{0r}, and g_{rr} functions of t and r

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Static spacetimes

Static spacetimes

Motivation

- leads to simple derivation of Schwarzschild metric
- generalizes to spherically symmetric, asymptotically flat Einstein vacuum field equations
- Birkhoff's theorem

Schutz (2009, p. 263) and Misner, Thorne, and Wheeler (1973, p. 843)

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Definition

A spacetime is static if we can find a time coordinate t for which

(i) the metric independent of t

$$g_{\alpha\beta,t} = 0$$

(ii) the geometry unchanged by time reversal

$$t \to -t$$

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  - e.g. rotating star

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Time reversal

$$\mathbf{\Lambda} : (t, x, y, z) \to (-t, x, y, z)$$

$$g_{\bar\alpha\bar\beta} = \Lambda^{\alpha}{}_{\bar\alpha}\Lambda^{\beta}{}_{\bar\beta}\, g_{\alpha\beta} = g_{\alpha\beta}$$

Transformation

$$\Lambda^{0}{}_{\bar 0} = x^{0}{}_{,\bar 0} = -x^{0}{}_{,0} = -1$$
$$\Lambda^{i}{}_{\bar j} = x^{i}{}_{,\bar j} = x^{i}{}_{,j} = \delta^{i}{}_{j}$$
$$\Lambda^{0}{}_{\bar i} = x^{0}{}_{,\bar i} = x^{0}{}_{,i} = 0$$

Metric

$$g_{\bar 0\bar 0} = (\Lambda^{0}{}_{\bar 0})^2 g_{00} = g_{00}$$
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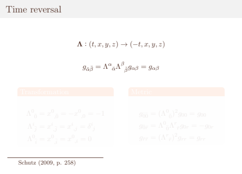

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The metric

Simplified metric

$$\mathrm{d}s^2 = g_{00}\,\mathrm{d}t^2 + g_{rr}\,\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

Replacement

$$g_{00} \to -e^{2\Phi}, \quad g_{rr} \to e^{2\Lambda}, \quad \text{provided } g_{00} < 0 < g_{rr}$$

Static spherically symmetric metric

$$\mathrm{d}s^2 = -e^{2\Phi}\,\mathrm{d}t^2 + e^{2\Lambda}\,\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

$$\lim_{r\to\infty} \Phi(r) = \lim_{r\to\infty} \Lambda(r) = 0$$

Schutz (2009, pp. 258–259)

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- limits at infinity tell us that spacetime is *asymptotically flat*

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Einstein Tensor

General Einstein tensor

$$G_{\alpha\beta} = R^{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$$

Einstein tensor components

$$G_{tt} = \frac{1}{r^2} e^{2\Phi} \frac{\mathrm{d}}{\mathrm{d}r} [r(1 - e^{-2\Lambda})],$$

$$G_{rr} = -\frac{1}{r^2} e^{2\Lambda} (1 - e^{-2\Lambda}) + \frac{2}{r} \Phi'$$

$$G_{\theta\theta} = r^2 e^{-2\Lambda} [\Phi'' + (\Phi')^2 + \Phi'/r - \Phi'\Lambda' - \Lambda'/r],$$

$$G_{\phi\phi} = \sin^2 \theta G_{\theta\theta}$$

Schutz (2009, pp. 165, 260)

- $x' \equiv \mathrm{d}x/\mathrm{d}r$

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Static perfect fluid

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Four-velocity

Constraints

$$U^i = 0 \text{ (static)} \qquad \vec{U} \cdot \vec{U} = -1 \text{ (conservation law)}$$

Solving for U^0

$$g_{00} (U^0)^2 = -1 \implies U^0 = (-g_{00})^{-1/2} = e^{-\Phi}$$

Solving for U_0

$$U_0 = g_{00} U^0 = -e^{\Phi}$$

Schutz (2009, p. 260)

- “conservation law” is the conservation of four-momentum

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Stress–energy tensor

$T_{\alpha\beta}$ for perfect fluid

$$T_{\alpha\beta} = (\rho + p) U_\alpha U_\beta + p g_{\alpha\beta}$$

Components of $T_{\alpha\beta}$

$$T_{00} = (\rho + p) e^{2\Phi} + p(-e^{2\Phi}) = \rho e^{2\Phi}$$

$$T_{\alpha\beta} = 0 \text{ for } \alpha \neq \beta; \quad T_{ii} = p g_{ii}$$

$$T_{rr} = p e^{2\Lambda}; \quad T_{\theta\theta} = p r^2; \quad T_{\phi\phi} = p r^2 \sin^2\theta = T_{\theta\theta} \sin^2\theta$$

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Equation of state

Local thermodynamic equilibrium

$$p = p(\rho, S) \approx p(\rho)$$

- pressure related to energy density and specific entropy
- we often deal with negligibly small entropies

Schutz (2009, p. 261)

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Equations of motion

Conservation laws

$$T^{\alpha\beta}{}_{;\beta} = 0$$

- symmetries make only non-trivial solution $\alpha = r$ **TODO: prove**

Equation of motion

$$(\rho + p)\frac{\mathrm{d}\Phi}{\mathrm{d}r} = -\frac{\mathrm{d}p}{\mathrm{d}r}$$

Schutz (2009, pp. 175, 261)

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$$(\rho + p)\frac{\mathrm{d}\Phi}{\mathrm{d}r} = -\frac{\mathrm{d}\rho}{\mathrm{d}r}$$

Schutz (2009, pp. 175, 261)

Equations of motion

Conservation laws

$T^{\alpha\beta}{}_{;\beta} = 0$

- symmetries make only non-trivial solution $\alpha = r$
  TODO: prove

Equation of motion

$(\rho + p)\frac{\mathrm{d}\Phi}{\mathrm{d}r} = -\frac{\mathrm{d}\rho}{\mathrm{d}r}$

Schutz (2009, pp. 175, 261)

Equation of motion (continued)

TODO
Show 10.31

Equation of motion (continued)

TODO
Show 10.31

Exterior Geometry

Exterior Geometry

Schwarzschild metric

TODO

Schwarzschild metric

TODO

Birkhoff's Theorem

Let the geometry of a given region of spacetime:

1. be spherically symmetric
2. be a solution to the Einstein field equations in vacuum.

Then that geometry is necessarily a piece of the Schwarzschild geometry.

(Proof given in Misner, Thorne, and Wheeler (1973, pp. 843–844))

Birkhoff and Langer (1923)

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Spherical stars
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