

Genetic Differential Equations

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December 1, 2014

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Differential Equations

Definition

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Demos

- a differential equation is any equation which relates a function with its derivatives

Definition

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- a differential equation is any equation which relates a function with its derivatives
- in other words, it is an equation which relates some quantity with its rates of change

Ordinary Differential Equations

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- an ordinary differential equation is the easiest to deal with, as it only has derivatives with respect to a single variable

Ordinary Differential Equations

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- an ordinary differential equation is the easiest to deal with, as it only has derivatives with respect to a single variable

$$\frac{dy}{dx} = y(x)$$

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$$\frac{dy}{dx} = y(x)$$

$$\frac{dy}{dx} = x \cdot y(x)$$

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- an ordinary differential equation is the easiest to deal with, as it only has derivatives with respect to a single variable

$$\frac{dy}{dx} = y(x)$$

$$\frac{dy}{dx} = x \cdot y(x)$$

$$\frac{d^2y}{dx^2} = x^2 y(x)$$

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$$\frac{dy}{dx} = y(x)$$

$$\frac{dy}{dx} = x \cdot y(x)$$

$$\frac{d^2y}{dx^2} = x^2 y(x)$$

$$y''(x) + 2y'(x) + y(x) = 0$$

Real World Examples of ODE's

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Population equation:

$$\frac{dp}{dt} = rp$$

Real World Examples of ODE's

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Population equation:

$$\frac{dp}{dt} = rp$$

Pendulum equation:

$$\frac{d^2\theta}{dt^2} + \frac{g}{\ell} \sin \theta = 0$$

Real World Examples of ODE's

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Population equation:

$$\frac{dp}{dt} = rp$$

Pendulum equation:

$$\frac{d^2\theta}{dt^2} + \frac{g}{\ell} \sin \theta = 0$$

Schrödinger wave equation in 1D:

$$\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + (E - \frac{1}{2}kx^2)\psi = 0$$

Exact Solutions to Differential Equations

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- the simplest differential equations can have their function solved for exactly

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$$\frac{dy}{dx} = x \cdot y(x)$$

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- the simplest differential equations can have their function solved for exactly

$$\frac{dy}{dx} = x \cdot y(x)$$
$$\implies \frac{1}{y} dy = x dx$$

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- the simplest differential equations can have their function solved for exactly

$$\begin{aligned}\frac{dy}{dx} &= x \cdot y(x) \\ \implies \frac{1}{y} dy &= x dx \\ \implies \int \frac{1}{y} dy &= \int x dx\end{aligned}$$

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- the simplest differential equations can have their function solved for exactly

$$\begin{aligned}\frac{dy}{dx} &= x \cdot y(x) \\ \implies \frac{1}{y} dy &= x dx \\ \implies \int \frac{1}{y} dy &= \int x dx \\ \implies \ln y &= \frac{1}{2} x^2 + C\end{aligned}$$

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- the simplest differential equations can have their function solved for exactly

$$\begin{aligned}\frac{dy}{dx} &= x \cdot y(x) \\ \implies \frac{1}{y} dy &= x dx \\ \implies \int \frac{1}{y} dy &= \int x dx \\ \implies \ln y &= \frac{1}{2} x^2 + C \\ \implies y(x) &= C e^{x^2/2}\end{aligned}$$

Numerical Solutions to Differential Equations

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- most differential equations cannot be solved exactly

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- most differential equations cannot be solved exactly
- numerical solutions seek to find the value of the function $y(x)$ at specific values of x

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- most differential equations cannot be solved exactly
- numerical solutions seek to find the value of the function $y(x)$ at specific values of x
- need some initial value $y(x_0) = y_0$, as otherwise there are infinitely many solutions

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- most differential equations cannot be solved exactly
- numerical solutions seek to find the value of the function $y(x)$ at specific values of x
- need some initial value $y(x_0) = y_0$, as otherwise there are infinitely many solutions
- this is the approach used for the genetic algorithm

Genetic Algorithm

- find values of $y(x)$ at evenly spaced values of x which best satisfy

$$\frac{dy}{dx} = f(x, y); \quad y(x_0) = y_0; \quad x \in \{x_0, x_0 + h, \dots, x_0 + (N - 1)h\}$$

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■ $f(x, y)$

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- $f(x, y)$
- y_0, x_0, x_N

- $f(x, y)$
- y_0, x_0, x_N
- N

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Demos

- list of y_i values

Initial Solutions

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- list of y_i values
- y_0 is exactly equal to the provided y_0 (never changes)

Initial Solutions

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- list of y_i values
- y_0 is exactly equal to the provided y_0 (never changes)
- y_{i+1} is constrained by $f(x_i, y_i)$

Initial Solutions

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- list of y_i values
- y_0 is exactly equal to the provided y_0 (never changes)
- y_{i+1} is constrained by $f(x_i, y_i)$
 - if $f(x_i, y_i) > 0$, then $hf(x_i, y_i) > y_{i+1} - y_i > 0$

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 - if $f(x_i, y_i) > 0$, then $hf(x_i, y_i) > y_{i+1} - y_i > 0$
 - if $f(x_i, y_i) < 0$, then $hf(x_i, y_i) < y_{i+1} - y_i < 0$

- list of y_i values
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 - if $f(x_i, y_i) < 0$, then $hf(x_i, y_i) < y_{i+1} - y_i < 0$
 - if $f(x_i, y_i) = 0$, then $y_{i+1} = y_i$

- select an index $0 < i < N$

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- add a number ϵ to all values y_i, \dots, y_{N-1}

- select an index $0 < i < N$
- add a number ϵ to all values y_i, \dots, y_{N-1}
- ϵ is selected from a gaussian distribution centered on 0 with some spread σ

- select an index $1 < i < N - 1$

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- take y_0, \dots, y_i from mother

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- take y_0, \dots, y_i from mother
- take y_{i+1}, \dots, y_{N-1} from father

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- take y_{i+1}, \dots, y_{N-1} from father
- shift the y values of the father so $y_{i+1} - y_i$ in the child is the same as it was in the mother

- select an index $1 < i < N - 1$
- take y_0, \dots, y_i from mother
- take y_{i+1}, \dots, y_{N-1} from father
- shift the y values of the father so $y_{i+1} - y_i$ in the child is the same as it was in the mother
- this has the effect of inheriting the values of $\frac{dy}{dx}$, instead of just $y(x)$

- approximate value of the derivative is taken by the Euler method

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$$\frac{dy_i}{dx_i} \approx \frac{y_{i+1} - y_i}{h}$$

- approximate value of the derivative is taken by the Euler method



$$\frac{dy_i}{dx_i} \approx \frac{y_{i+1} - y_i}{h}$$

- the difference between this and the provided function $f(x_i, y_i)$ is taken

- approximate value of the derivative is taken by the Euler method



$$\frac{dy_i}{dx_i} \approx \frac{y_{i+1} - y_i}{h}$$

- the difference between this and the provided function $f(x_i, y_i)$ is taken
- the negation of L_1 -norm of this difference is the fitness function

- approximate value of the derivative is taken by the Euler method

■

$$\frac{dy_i}{dx_i} \approx \frac{y_{i+1} - y_i}{h}$$

- the difference between this and the provided function $f(x_i, y_i)$ is taken
- the negation of L_1 -norm of this difference is the fitness function
- the less negative a solution's fitness is, the better

Demos

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■ population size: 100

- population size: 100
- sample size: 8

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- proportion mutate: 0.50

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- proportion crossover: 0.60

- population size: 100
- sample size: 8
- proportion mutate: 0.50
- proportion copy: 0.40
- proportion crossover: 0.60
- σ : 0.1

- population size: 100
- sample size: 8
- proportion mutate: 0.50
- proportion copy: 0.40
- proportion crossover: 0.60
- σ : 0.1
- iterations: 350

Demo 1

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- $f(x, y) = xy; y(0) = 1; x \in \{0, 0.02, 0.04, \dots, 0.98\}$

Demo 1

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Demos

- $f(x, y) = xy; \quad y(0) = 1; \quad x \in \{0, 0.02, 0.04, \dots, 0.98\}$
- $y(x) = e^{x^2/2}$

Demo 2

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Genetic
Algorithm

Demos

- $f(x, y) = xy; y(0) = 1; x \in \{0, 0.02, 0.04, \dots, 0.98\}$

Demo 2

Genetic
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Equations

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Algorithm

Demos

- $f(x, y) = xy; y(0) = 1; x \in \{0, 0.02, 0.04, \dots 0.98\}$
- $y(x) = 2 \cot^{-1} \left(e^{-x^2/2} \cot \left(\frac{1}{2} \right) \right)$

Mateescu, G. D., 2006

Howard, D & Kolibal, J, 2005