Efficient and Compact Spreadsheet Formula Graphs

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***Abstract*—Spreadsheets are one of the most popular data analysis tools, wherein users can express computation as formulae alongside data. The ensuing dependencies are tracked as formula graphs. Efficiently querying and maintaining these formula graphs is critical for interactivity across multiple settings. Unfortunately, formula graphs are often large and complex such that querying and maintaining them is time-consuming, reducing interactivity. We propose TACO, a framework for efficiently compressing formula graphs, thereby reducing the time for querying and maintenance. The efficiency of TACO stems from a key spreadsheet property: tabular locality, which means that cells close to each other are likely to have similar formula structures. We leverage four such tabular locality-based patterns, and develop algorithms for compressing formula graphs using these patterns, directly querying the compressed graph without decompression, and incrementally maintaining the graph during updates. We integrate TACO into an open-source spreadsheet system and show that TACO can significantly reduce formula graph sizes. For querying formula graphs, the speedups of TACO over a baseline implemented in our framework and a commercial spreadsheet system are up to 34,972× and 632×, respectively.**

I. Introduction

Spreadsheets are widely used for data analysis, with a user-base of nearly 1 billion [1], [2]. They support a variety of applications, from planning and inventory tracking, to complex financial, medical, and scientific data analysis. Their popularity is attributable to an intuitive tabular layout and in-situ formula computation [3]. Users directly analyze their data by writing embedded formulae alongside data, akin to database views [4]. These formulae take the results of other formulae or raw data values as input, creating dependencies between the output of formulae and their inputs. These dependencies are internally represented as a *formula graph*. Formula graphs are critical to interactivity of spreadsheets for multiple purposes, including:

- **Formula recalculation:** When a spreadsheet cell is modified, the spreadsheet system needs to query the formula graph to find its dependents and calculate new formula results [5], [6]. Identifying dependents quickly is key to ensuring that users don't see stale or inconsistent results, and also helps spreadsheets return control early to users [7].
- **Formula dependency visualization:** Spreadsheet systems, such as Excel and LibreCalc, provide tools for tracing and visualizing the dependents and precedents of cells to help users check the accuracy of formulae and identify sources of errors [8]–[12]. For these applications, we need to query the formula graph to quickly return the dependents or precedents of cells to ensure interactivity.

Unfortunately, real-world spreadsheets often include large and complex formula graphs, where a cell update can have a

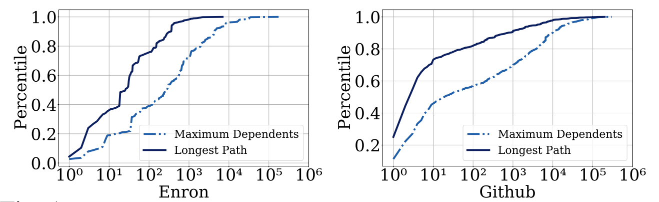

Fig. 1: CDFs for the maxinum number of dependents and the longest path in the Enron and Github datasets

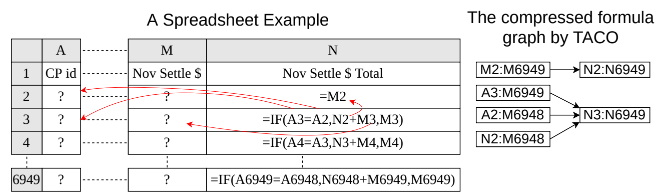

Fig. 2: A real spreadsheet with tabular locality [13]

large number of dependent cells. Traversing these graphs to find dependents can be time-consuming and lead to high response times. We analyze two real-world spreadsheet datasets, Enron [13] and Github (a dataset we crawled; details in Sec. VI-A). We compute, for each spreadsheet, the maximum number of dependents for a given cell, as well as the longest path in the formula graph. We plot the CDFs for these two quantities for the two datasets in Fig. 1. We see that the number of dependents of a single cell can be as high as 300K, while a path can be as long as 200K edges.

While identifying dependents interactively may seem to be an infeasible task, it turns out that many spreadsheet formulae follow certain predefined patterns. These patterns can be used to compress the formula graph and enable fast look-ups of dependents and precedents directly on the compressed graph. *Specifically, our key insight is that cells that are close to each other in the tabular spreadsheet layout often employ similar formula structures*, a property we refer to as **tabular locality**. Fig. 2 shows a column of formulae of a real-world spreadsheet that follows tabular locality. While the formulae in the column N look complicated at first glance, they follow the same pattern starting from N3: the IF formula in each row references the cell of the same row and the row above from column A (e.g., N3 references A3 and A2), the cell to the left (e.g., M3 for N3), and the cell above (e.g., N2 for N3). Tabular locality is prevalent in real-world spreadsheets mainly because users often do not write several distinct formulae by hand, but use convenient spreadsheet functionalities, such as copy-paste and autofill, to generate formulae automatically. Autofill, for instance, allows users to drag a cell to fill adjacent cells by repeating the pattern of the source cell. Users could also programmatically generate a large spreadsheet, which still likely respects tabular locality.

However, leveraging tabular locality to query formula

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graphs at interactive speed is challenging for several reasons. First, as we will show in the experiments (Sec. VI-B), formula references are complex: a formula could include multiple references to different rows or columns (e.g., N3 in Fig. 2 references 4 different cells). Disentangling these multiple references across cells, identifying common patterns across them and compressing these common patterns can be time-consuming. Therefore, the compression algorithm should balance between the quality of the compression (e.g., the number of common patterns detected) and the compression time. While it may be possible to track user actions (e.g., during autofill) to identify and compress formula dependencies directly, this approach does not apply when spreadsheets are shared via files (e.g., xlsx files), losing track of the actions that generated them. It also cannot compress formula dependencies generated programmatically [14], [15]. Furthermore, developing a compressed representation and algorithms for directly querying the compressed graphs to reduce look-up time is also an open challenge Finally, the formula graph needs to be maintained over time, which requires incrementally updating the compressed graph to avoid decompression overhead.

To address these challenges, we present *Tabular Locality-based Compression* or TACO. In TACO, we leverage four basic patterns that serve as building blocks for other more complicated patterns, and identify one extended pattern based on the analysis of real spreadsheets. We propose a generic framework that decomposes messy formulae and extracts their predefined patterns. This framework is also extensible to support new patterns. We prove that compressing a formula graph using predefined patterns is NP-HARD via a reduction from the *rectilinear picture compression* problem [16]. We develop a greedy algorithm to efficiently compress formula graphs while maintaining low compression overhead. Further, we design algorithms for finding dependents or precedents directly on the compressed graph, and for maintaining the graph incrementally, and analyze the complexity of each algorithm. Our experiments show that for querying formula graphs, the speedups of TACO over a baseline and a commercial spreadsheet system are up to **34,972**× and **632**×, respectively.

While there is a lot of work on graph compression [17], this work does not leverage tabular locality or take into account the spatial nature of spreadsheet ranges. In addition, most of this work does not support directly querying the compressed graph, so these compression algorithms will not be faster than an approach without compression in terms of finding dependents/precedents. Fan et al. [18] propose a method for directly executing reachability and pattern matching queries on a compressed graph, but do not leverage tabular locality nor supporting finding dependents/precedents. TACO is also different from columnar compression [19], [20] because TACO is designed to compress formula dependencies (i.e., edges) while columnar compression methods are used to compress columnar data. A recent paper proposes a specialized algorithm for compressing formula graphs [7]. However, this compression algorithm introduces false positives, which is not applicable to finding exact dependents for the formula dependency visualization. In addition, it has a high compression and maintenance time, which we will show in Sec. VI. Turns out Excel has a capability wherein it identifies identical formulae and stores duplicate formulae as pointers to the first formula [21]. But it does not consider compressing and querying the formula dependencies based on tabular locality.

II. BACKGROUND AND PROBLEM

In this section, we define spreadsheets and formula graphs. We also present the problem of compressing, querying, and maintaining formula graphs, the focus of this paper.

A. Background

Spreadsheets A spreadsheet consists of a set of *cells* organized in a tabular layout. Each cell is referenced using its column and row index. Columns are identified by letters A, $\cdots$, Z, AA, $\cdots$, and rows are identified by numbers 1, 2, $\cdots$. For simplicity, for a cell we also use integers (i, j) to represent its position, where i and j are column and row indices, respectively, both starting from 1. A *range*, akin to a 2D window, is a rectangular region of cells, identified by the top-left (called *head*) and bottom-right (called *tail*) cells. For example, the range A1:B2 contains cells A1, A2, B1, B2, with head and tail cells A1 and B2, respectively.

A cell contains a *formula* or a *pure value*. A pure value is a constant belonging to a fixed type while a formula is a mathematical expression that takes pure values and/or cell/range references as input. The result of an evaluated formula is an *evaluated value*. For example, the cells in column A in Fig. 2 include pure values while the cells in column N include formulae. In the rest of the paper, we use "value" to refer to either the pure or evaluated value of a cell. A cell that includes a formula is called a *formula cell*.

Formula graphs A formula graph is a directed acyclic graph (DAG) that stores the dependencies of each formula referencing other ranges as edges. Specifically, each formula is parsed to get the set of ranges the formula references, with a directed edge added from each referenced range to the formula cell. We call this directed edge a *dependency*. Given a directed edge $e = (prec, dep)$, we call *prec* the *direct precedent* of *dep* or the *precedent* of the edge e. Symmetrically, we call *dep* the *direct dependent* of *prec* or the *dependent* of e.

Fig. 3 shows a spreadsheet with four formulae along with its formula graph. The cells denoted as ? are pure values. B1 and B2 have the same formula SUM(A1:A3), so the direct dependents of A1:A3 include B1 and B2. C1 references B1 and B3, so we add two edges. Finally, C2 references B2:B3, which adds an edge with separate vertices although B2:B3 overlaps with the vertices B2 and B3.

The formula graph is used to quickly find the *dependents* or *precedents* of an input range. The dependents of an input range are the set of cells that are reachable from the input range in the formula graph. Symmetrically, the precedents are the set of cells that can reach the input range in this formula graph via a directed path. For example, the dependents of A1 are {B1, B2, C1, C2} in Fig. 3. Since a vertex in the graph can

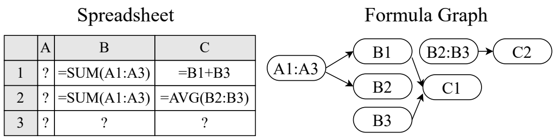


Fig. 3: An example spreadsheet and its formula graph

be a range, we can build an index (e.g., R-Tree [22]) over the vertices to quickly find the ranges that overlap with an input range. (e.g., find A1:A3 given a cell A1).

One application of the formula graph is to find the dependents when users update the spreadsheet to ensure that users do not see stale or inconsistent results. Specifically, when a cell is updated, the formulae of its dependents will be re-evaluated in sequence to refresh their values. A key prerequisite to updating the formulae is identifying which formulae require recomputation in the first place. If the update is to a formula cell, the formula graph will be modified. The formula graph is also useful for visualizing formula dependencies, which allows users to trace the dependents/precedents of a cell to check the accuracy of formulae or find the sources of errors [8], [9], [12], [14]. In both applications, the performance of querying the formula graph is critical to ensure interactivity.

B. Compressing, Querying, and Maintaining Formula Graphs

Formula graph problems Finding the dependents and precedents of an input range is often time-consuming due to the size or complexity of the graph. Therefore, we propose compressing formula graphs by leveraging tabular locality to significantly reduce graph size. Directly querying the compressed graph and incremental maintenance can decrease the time taken for finding dependents/precedents and maintaining the graph, respectively, while introducing the modest overhead of building the compressed graph.

Patterns in formula graphs We capture and distill tabular locality in a formula graph via *patterns*. For a set of edges A of arbitrary size, a pattern is a constant-size (i.e., $O(1)$) representation that can reconstruct A (with size $O(|A|)$). In addition, finding the direct dependents or precedents of an input range in a pattern should also be constant time. Consider Fig. 2 as an example. Each formula cell Ni starting from N3 follows the pattern that Ni depends on Ai, A(i-1), Mi, and N(i-1). By storing the relative positions between Ni and Ai, i.e., Ai is 13 columns left to Ni, and the valid range of Ni, i.e., N3:N6949, we can represent the edges Ai $\rightarrow$ Ni using constant-size information. We can also find dependents/precedents in this compressed edge in constant time. For example, for input range A3:A10, we can use the information of relative positions to find the dependents N3:N10 in constant time. We can represent and query other edges in a similar way. Therefore, leveraging patterns greatly reduces formula graph sizes and consequently reduces the time for finding dependents/precedents.

Compressed formula graph representation Given a formula graph $G'(E', V')$, we want to find a *lossless* compressed graph $G(E, V)$ that preserves the results of finding dependents/precedents. G is generated based on a partition of the edge set E' in G': $P = \{E'_1, E'_2, \cdots, E'_N\}$, where $E' = \cup_{i=1}^{N} E'_i$. Each E'_i must either contain a single edge (i.e., uncompressed), or be a set of edges that follow one of the predefined compression patterns, in which case these edges will be replaced with a compressed edge. We generate a compressed edge $e_i = (\mathit{prec}, \mathit{dep}, p, \mathit{meta})$ for each $E'_i = \{e^i_1, e^i_2, \cdots, e^i_{M_i}\}$, where M_i is the size of E'_i. The components $e_i.\mathit{prec}$ and $e_i.\mathit{dep}$ are the precedent and the dependent of e_i, computed as $e_i.\mathit{prec} = \bigoplus_{j=1}^{M_i} e^i_j.\mathit{prec}$ and $e_i.\mathit{dep} = \bigoplus_{j=1}^{M_i} e^i_j.\mathit{dep}$. $\bigoplus$ is the minimal bounding range of the input ranges. For example, $\bigoplus$ merges the ranges A1:A3 and A2:A5 into A1:A5. The component $e_i.p$ is the compression pattern (or NoComp if E'_i contains a single edge) while $e_i.\mathit{meta}$ encodes the underlying pattern information such that *meta* of an edge e_i in E is used to reconstruct the corresponding edges in E'_i. So E in G comprises the edges generated from the partition P and $|E| = N$. The compressed vertex set V is induced from E.

Problem statement Given a set of predefined pattern types, we want to build an equivalent compressed graph $G(E, V)$ of a formula graph $G'(E', V')$ such that the size of G and the time for finding the dependents/precedents of a cell are both significantly smaller compared to G', and the time for maintaining G with respect to updates is small.

To address this problem, we discuss the basic compression patterns we support (Sec. III). We then present the TACO framework that leverages the basic patterns to compress, query, and maintain a formula graph, and analyze the algorithmic complexity (Sec. IV). Finally, we discuss extending TACO to support new patterns beyond the basic ones (Sec. V).

III. Basic Patterns

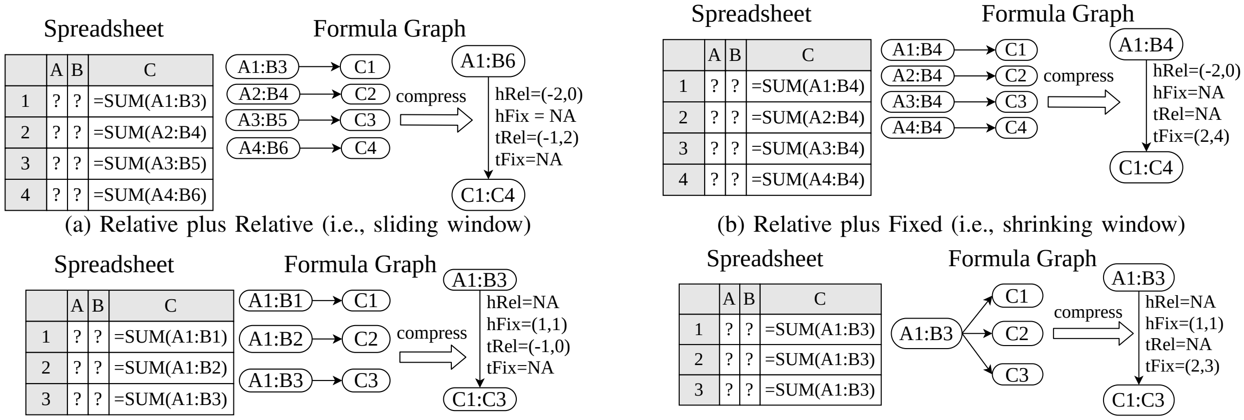
In this section, we define the basic compression patterns we consider and propose algorithms for using the pattern to build a compressed edge, finding dependents or precedents, and maintaining the edge.

A. Basic Patterns

The basic patterns consider tabular locality in adjacent formula cells and assume each formula cell references a single range. We can employ the basic patterns multiple times to compress dependencies when formula cells have multiple references, as discussed in Sec. IV-A. We focus on adjacent cells in a column for simplicity; the row-wise case can be derived symmetrically.

Let $A'(A' \subseteq E')$ be a set of dependencies in a column of adjacent formula cells. Our basic patterns capture various relationships between the precedent and dependent of each $e' \in A'$. Recall that $e'.\mathit{dep}$ is a formula cell and $e'.\mathit{prec}$ is a referenced range, represented by the positions of its head and tail cell. In a spreadsheet, there are two types of relationships between the formula cells (i.e., $e'.\mathit{dep}$) and the head/tail cells in the referenced range (i.e., $e'.\mathit{prec}$): *fixed* and *relative* [23].

The fixed relationship captures the scenario where each dependent $e'.\mathit{dep}$ references the same head or tail cell of the precedent $e'.\mathit{prec}$. For example, the formulae in a column



(a) Relative plus Relative (i.e., sliding window)

(b) Relative plus Fixed (i.e., shrinking window)

(c) Fixed plus Relative (i.e., expanding window)

(d) Fixed plus Fixed (i.e., fixed window)

Fig. 4: Examples of four basic patterns of tabular locality

all reference a common dollar conversion rate, stored in a fixed location, i.e., a cell. Here, we have a fixed relationship with both the head and the tail cell of the referenced range, which are identical. This is a case of an FF (or Fixed-Fixed) pattern. The referenced head and tail cell for fixed references are denoted *hFix* and *tFix*, respectively.

The relative relationship, on the other hand, captures the scenario where each dependent $e'.dep$ has the same relative position with respect to the head or tail cell of the precedent $e'.prec$. The relative position with respect to the head and tail cell is denoted *hRel* and *tRel*, respectively. One example of a relative position is each formula cell in a column references a cell to its left. Here there is a relative relationship to both the head and tail cell of the referenced range, which are identical. This is a case of an RR (or Relative-Relative) pattern. We use a pair (p, q) to represent the relative position; p denotes the relative column distance and q the relative row distance. Given two cells' positions u and v, we say u is *relative* to v by (p, q) if $v.i = u.i + p$ and $v.j = u.j + q$.

Combining the two types of relationships (fixed or relative) with the two cells that represent the precedent (head and tail), there are four basic patterns that capture the relationships between dependents and precedents for a column of dependencies. We additionally include a default pattern called NoComp for an uncompressed edge.

Relative plus Relative (RR) RR is the setting where each $e'.dep$ has the same relative positions to both head and tail cells of $e'.prec$. Fig. 4a shows an example. We see that each formula cell in column C is relative to the head cell of its referenced range by $(-2, 0)$ (i.e., to the left by two columns) and relative to the tail cell by $(-1, 2)$. The metadata *meta* is ($hRel = (-2, 0), hFix = \mathsf{NA}, tRel = (-1, 2), tFix = \mathsf{NA}$)). NA means that this pattern does not include this information (e.g., RR does not reference fixed head or tail cells). So the compressed edge in Fig. 4a is ($prec = \mathsf{A1{:}B6}, dep = \mathsf{C1{:}C4}, p = \mathsf{RR}, meta$), where *meta* is as defined above.

Relative plus Fixed (RF) RF is the setting where each $e'.dep$ has the same relative position to the head cell and references a fixed tail cell. Fig. 4b shows an example. Here, each formula in column C is relative to the head cell of its referenced range by $(-2, 0)$ and points to a fixed tail cell $\mathsf{B4} = (2, 4)$. The metadata *meta* equals ($hRel = (-2, 0), hFix = \mathsf{NA}, tRel = \mathsf{NA}, tFix = (2, 4)$).

Fixed plus Relative (FR) FR is the dual pattern of RF. So each $e'.dep$ points to the same head cell and has the same relative position for the tail cell. Fig. 4c shows that the metadata of the compressed edge is ($hRel = \mathsf{NA}, hFix = (1, 1), tRel = (-1, 0), tFix = \mathsf{NA}$).

Fixed plus Fixed (FF) FF is the setting where each $e'.dep$ references fixed head and tail cells, represented as *hFix* and *tFix*, respectively. Fig. 4d shows that each formula cell always points to (A1:B3) and its *meta* is ($hRel = \mathsf{NA}, hFix = (1, 1), tRel = \mathsf{NA}, tFix = (2, 3)$).

Applicability of basic patterns These basic patterns are common in real spreadsheets because they are building blocks for many spreadsheet applications. A concrete example that follows RR is in Fig. 2; RR is common in sliding-window-style computation. FR and RF are often employed for cumulative total computation. A real example involves a user sorting transactions by date and using a column of formulae to compute the year-to-date sales amount. FF is also widely used in real applications for referencing fixed ranges for point lookups, e.g., a fixed interest rate or monetary conversion rate in a cell, or range lookups. One example of a FF range lookup is a column of VLOOKUP formulae, each of which looks up a value in the same range.

In addition, spreadsheet systems provide intuitive tools to help spreadsheet users easily write formulae using these patterns [23]. For example, spreadsheet systems provide autofill tools for users to apply the pattern of one source cell to adjacent cells. The autofill varies the range references of the formula in the source cell based on the following rules: if a reference is pre-fixed with a dollar sign $, it is a fixed reference; otherwise it is a relative reference [24].

Discussion on leveraging the basic patterns One natural question is whether existing spreadsheets leverage the basic patterns for compression and querying. Excel does identify the same formulae and stores them efficiently [21], but does not leverage these patterns to accelerate traversing formula graphs, as is verified by our experiments in Sec. VI-E. While it is possible to track autofill expressions to compress formula dependencies, this approach does not apply to spreadsheets

Algorithm 1: Algorithms for the RR pattern

1 **Algorithm** `addDep` (e, e')
2 **if** $e.p ==$ NoComp **then**
3 **if** $rel(e) == rel(e')$ **then**
4 **return** $(e.prec \bigoplus e'.prec, e.dep \bigoplus e'.dep, \mathsf{RR}, rel(e'))$
5 **else if** $e.meta == rel(e')$ **then**
6 **return** $(e.prec \bigoplus e'.prec, e.dep \bigoplus e'.dep, \mathsf{RR}, meta)$
7 **end**
8 **return** NULL
9 **Procedure** `rel` (e)
10 $hRel \leftarrow e.prec.head - e.dep$
11 $tRel \leftarrow e.prec.tail - e.dep$
12 **return** $(hRel, tRel)$
13 **Algorithm** `findDep` (e, r)
14 $prec_t^{d_h} \leftarrow (e.prec.tail.i, r.head\ .j)$
15 $d_h \leftarrow prec_t^{d_h} - e.meta.tRel$
16 $prec_h^{d_t} \leftarrow (e.prec.head.i, r.tail\ .j)$
17 $d_t \leftarrow prec_h^{d_t} - e.meta.hRel$
18 **return** the intersection of (d_h, d_t) and $e.dep$
19 **Algorithm** `findPrec` (e, r)
20 $g_h \leftarrow r.head + e.meta.hRel$
21 $g_t \leftarrow r.tail + e.meta.tRel$
22 **return** (g_h, g_t);
23 **Algorithm** `removeDep` (e, s)
24 $newDepSet \leftarrow$ delete s from $e.dep$
25 **for** $newDep \in newDepSet$ **do**
26 $newPrec \leftarrow$ `findPrec` $(e, newDep)$
27 $p \leftarrow |newDep| == 1$? NoComp : RR
28 add $(newPrec, newDep, p, e.meta)$ to $retSet$
29 **end**
30 **return** $retSet$

generated programatically, and is coupled with a spreadsheet system. To the best of our knowledge, no spreadsheet systems compress and query formula dependencies via tabular locality.

Therefore, we develop general compression algorithms that apply to all spreadsheets. Our algorithms can certainly leverage user actions (e.g., autofill) or cues (e.g., dollar sign) for better compression, but do not rely on it. In addition, we also design novel algorithms for efficiently querying and incrementally maintaining these patterns.

B. Algorithms for the Basic Patterns

To integrate a pattern into TACO, TACO requires each pattern to implement four key functions, as shown in Fig. 5.

- `addDep`(e, e'): add a dependency e' to a compressed edge e, where $e'.dep$, the formula cell, is adjacent to $e.dep$;
- `findDep`(e, r): find the dependents of a range of cells r within a compressed edge e, where r is contained in $e.prec$;
- `findPrec`(e, s): find the precedents of a range of cells s within a compressed edge e, where s is contained in $e.dep$;
- `removeDep`(e, s): remove the dependencies for a range of formula cells s in an edge e, where s is contained in $e.dep$

The parameter assumptions are guaranteed by TACO framework, discussed in Sec. IV.

RR The four key functions for RR are shown in Algorithm 1. Consider `addDep`(e, e'), which adds a dependency e' to

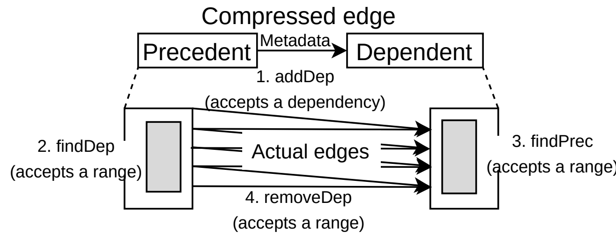


Fig. 5: An overview of the four key functions

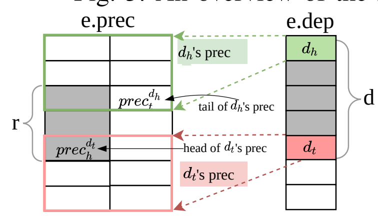


Fig. 6: An example of `findDep`(e, r) for RR

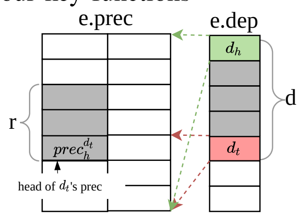


Fig. 7: An example of `findDep`(e, r) for RF

a compressed edge e. According to the definition of RR, the dependency e' can only be added to e if the relative positions of e' equal the relative positions $(hRel, tRel)$ in $e.meta$. We define $rel(e')$ as the procedure to compute the relative positions of $e'.dep$ with respect to the head and tail cell of $e'.prec$, respectively (i.e., line 9-12 in Algorithm 1). For example, if $e' = \mathsf{A5{:}B7} \rightarrow \mathsf{C5}$, the relative positions are $hRel = \mathsf{A5} - \mathsf{C5} = (-2, 0)$ and $tRel = \mathsf{B7} - \mathsf{C5} = (-1, 2)$. This dependency can be added to the compressed edge e in Fig. 4a because $e.meta.hRel$ and $e.meta.tRel$ equal $hRel$ and $tRel$, respectively. Therefore, we create a new compressed edge, where $prec = e.prec \bigoplus e'.prec$, $dep = e.dep \bigoplus e'.dep$, and $meta = e.meta$. If e is an uncompressed edge (a single dependency), we compare $rel(e')$ and $rel(e)$ to check whether the two edges can be compressed using the RR pattern.

Next, consider `findDep`(e, r), which finds the dependents (denoted as a range d) of a range of cells r that are contained in $e.prec$. To determine d, we need to find its head cell d_h and tail cell d_t. Since the precedent of each cell in d forms a sliding window on $e.prec$ as shown in Fig. 6, the intuition for computing d_h is that the top row of r must intersect with the bottom row of d_h's precedent. Similarly, the bottom row of r must intersect with the top row of d_t's precedent. So we "back calculate" d_h and d_t based on r. Specifically, we use the following invariant to compute d_h,

$$d_h + tRel = prec_t^{d_h}$$

where $prec_t^{d_h}$ is d_h's precedent's tail cell and $tRel$ is the relative position of d_h with respect to $prec_t^{d_h}$ as shown in Fig. 6. Since $tRel$ is known, the remaining task is to compute $prec_t^{d_h}$. We know that $prec_t^{d_h}$ is in the bottom row of d_h's precedent since it is a tail cell and that the bottom row of d_h's precedent intersects the top row of r. So $prec_t^{d_h}$ is in the top row of r and its row index is the row index of r's head cell (i.e., $r.head.j$). Since $prec_t^{d_h}$ is a tail cell, it is in the right-most column of $e.prec$. So its column index is $e.prec.tail.i$.

Finding the tail cell d_t adopts a dual procedure. Based on the invariant $d_t + hRel = prec_h^{d_t}$, we need to find d_t's precedent's head cell $prec_h^{d_t}$. As shown in Fig. 6, $prec_h^{d_t}$ should be in the last row of r and in the left-most column of $e.prec$. Therefore, we have $prec_h^{d_t}.i = e.prec.head.i$ and $prec_h^{d_t}.j = r.tail.j$. We note this procedure can output a range d that is beyond $e.dep$. In this case, we take the intersection between d and $e.dep$ to return a valid range.

The third function, $\texttt{findPrec}(e, s)$ finds the precedents (denoted as a range g) for a range of cells s contained in $e.dep$. By the definition of RR, the precedents of the cells in s form "sliding windows" on $e.prec$ as we move from $s.head$ to $s.tail$; g is simply the union of the precedents of all cells in s. So $g.head$ is the head cell of $s.head$'s precedent and $g.tail$ is the tail cell of $s.tail$'s precedent. We have $g.head = s.head + hRel$ and $g.tail = s.tail + tRel$.

Finally, consider $\texttt{removeDep}(e, s)$, which removes the dependencies for a range of formula cells s in $e.dep$. We first subtract s from $e.dep$, after which we are left with a range or a union of two ranges. For example, if we remove C2 from C1:C4, the remainder is composed of two ranges: C1 and C3:C4. For each range *newDep* in the remaining dependents, we generate its corresponding precedent *newPrec* using $\texttt{findPrec}(e, newDep)$.

RF, FR, FF We now discuss the key functions for RF. The function $\texttt{addDep}(e, e')$ for the RF pattern has similar logic to the one for RR and is different only in the compression condition. By definition of RF, we first compute the relative position between $e'.dep$ and the head cell of $e'.prec$ (denoted *hRel*) and check whether *hRel* and $e.meta.hRel$ are the same. If so, we additionally check whether the tail cell of $e'.prec$ is the same as $e.meta.tFix$.

$\texttt{findDep}(e, r)$ finds the range of dependents d for a range of cells r contained in $e.prec$. Similar to RR, we need to find d's head cell d_h and tail cell d_t. To compute d_h, we use the intuition shown in Fig. 7: the precedent of $e.dep.head$ equals $e.prec$ and the precedent of each cell in $e.dep$ shrinks when we move from $e.dep$'s head to its tail cell. That is, $e.dep.head$ references the entire range of $e.prec$ and is the dependent of any r contained in $e.prec$. So $e.dep.head$ equals d_h.

To compute d_t, we use the observation that the precedent of each cell in $e.dep$ shrinks as we move from d_h to d_t such that the bottom row of r should intersect with the top row of the precedent of d_t. Therefore, to compute d_t, we leverage the invariant $d_t + hRel = prec_h^{d_t}$, where $prec_h^{d_t}$ is the head cell of d_t's precedent. Since *hRel* is known, we need to compute $prec_h^{d_t}$. As Fig. 7 shows, since $prec_h^{d_t}$ is in the bottom row of r, its row index is $r.tail.j$ and since $prec_h^{d_t}$ is a head cell, its column index is $e.prec.head.i$.

Consider $\texttt{findPrec}(e, s)$, which finds the precedents (denoted as a range g) of s contained in $e.dep$. Our observation is that the precedent of $s.head$ contains all of the precedents of other cells in s since the precedent of each cell in $e.dep$ is shrinking as we move from $s.head$ to $s.tail$. Therefore, g is the precedent of $s.head$, and is computed as $g.head = s.head + hRel$ and $g.tail = tFix$. The function $\texttt{removeDep}(e, s)$ for RF follows the same logic as RR. We first remove s from $e.dep$ to return one or two ranges. For each returned range *newDep*, we generate their corresponding precedent *newPrec* using $\texttt{findPrec}(e, newDep)$ of RF.

FR is a dual pattern of RF, so its algorithms can be easily derived from the algorithms above. FF's algorithms can also be derived from FR and RF. We omit these algorithms due to space limits.

Algorithmic complexity All of the four algorithms for the basic patterns are $O(1)$, independent of the number of edges compressed.

IV. TACO Framework

In this section, we present a compression algorithm for efficiently extracting patterns from spreadsheets (Sec. IV-A). In addition, we design an algorithm to quickly find dependents and precedents in a compressed formula graph (Sec. IV-B), and introduce an algorithm to incrementally maintain the compressed graph in response to updates (Sec. IV-C). We analyze the complexity of each algorithm. Finally, we compare the complexity of TACO against an approach that does not compress the formula graph (Sec. IV-D). We assume the formula graphs in both approaches are implemented via an adjacency list and we build an R-Tree index on the vertices to quickly find the overlapping vertices for an input range. The complexity of operations on an R-Tree varies based on the design choices. In our analysis, we assume the complexity for searching, inserting, and deleting one range is $O(N)$, $O(\log N)$, and $O(\log N)$, respectively, where N is the number of ranges stored in the R-Tree.

A. Compressing a Formula Graph

We formalize the problem of minimizing the number of edges of the compressed formula graph based on the predefined basic patterns and present our compression algorithm.

1) Problem formalization: Using the definition of the compressed graph G, the problem of minimizing the number of edges in G is equivalent to the problem of finding a partition $P = \{E'_1, E'_2, \cdots, E'_N\}$ of the uncompressed edge set E' such that N is minimum, and each E'_i is compressed by a single pattern or only includes one uncompressed edge. The optimization problem, which we call Compressed Edge Minimization (or CEM for short), is defined as follows:

$$\begin{array}{ll} \underset{P=\{E'_1,\cdots,E'_N\}}{\text{minimize}} & N \\ \text{where} & E'_i \text{ is compressed by a pattern or is} \\ & \text{an uncompressed edge}, \forall i \in 1 \ldots N \\ & \cup_i E'_i = E' \end{array}$$

We show CEM is NP-Hard even when we only consider FF.

Theorem 1 (CEM Hardness)**.** *Compressing the graph G' into G while minimizing the number of edges of G, even when restricted to the FF pattern, is* NP-Hard.

Proof. (Sketch) We reduce the *rectilinear picture compression* (RPC for short) problem, which is known to be NP-Hard [16], to CEM. The input to RPC problem is a $m \times n$ matrix of 0's and 1's, with the goal to find the minimal number of rectangles that precisely cover the 1's. We reduce RPC to

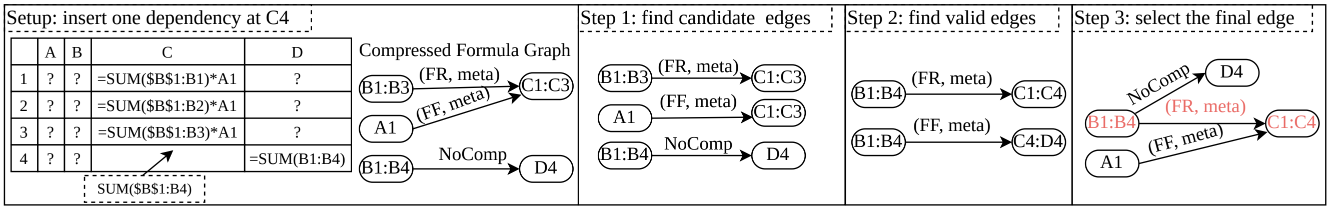


Fig. 8: An example of compressing one dependency inserted at C4

Algorithm 2: Compressing a dependency e' into $G(E, V)$

```
Algorithm addDep(G(E,V), e')
    isCompressed ← false
    pSet ← pre-defined patterns
    eSet ← find all e ∈ E whose e.dep is adjacent to
            e'.dep on column or row axis
    for candE ∈ eSet do
        edgePairs ← genCompEdges(candE, e', pSet)
        edgePairSet.add(edgePairs)
    end
    if edgePairSet is not empty then
        edgePair ← sort edgePairSet by heuristics and
                    take the first
        maintain G using edgePair
        isCompressed ← true
        break
    end
    if isCompressed is false then
        insert e' into G
Procedure genCompEdges(candE, e', pSet)
    if candE.p == NoComp then
        for p ∈ pSet do
            pair ← (p.addDep(candE, e'), candE)
            edgePairs.addIfValid(pair)
        end
    else
        pair ← (candE.p.addDep(candE, e'), candE)
        edgePairs.addIfValid(pair)
    end
    return edgePairs
```

CEM by mapping the input matrix to a spreadsheet range R: for each value at column i and row j in the matrix, if the value is 1, we place a formula at (i, j) in R; otherwise, we place a pure value at (i, j) in R. We assume all of the formulae reference the same fixed range outside R such that we only need to use FF to compress R. There are no other formulae in this spreadsheet. The problem is thereby reduced to finding the minimal number of ranges that precisely cover the formulae. Since the references of each range of the formulae can be compressed into a single edge using FF, this problem is equivalent to minimizing the number of edges of the compressed graph G for R. $\square$

CEM is also trivially NP-COMPLETE since verifying that a partition using FF is correct is in PTIME. We tested the algorithm that enumerates all possible partitions and found it cannot finish within 30 mins for a spreadsheet with 96 edges because the number of possible partitions is a Bell number [25]. To reduce the compression overhead, we propose a greedy compression algorithm.

2) Greedy compression algorithm: Our algorithm compresses a list of dependencies between formula cells and their referenced ranges by repeatedly inserting each dependency into the compressed graph and determining the partitions as well as the corresponding compression patterns. Observing that pre-defined patterns compress the dependencies in adjacent formula cells, we use this constraint to quickly find the candidate edges that one inserted dependency can be compressed into. If there are multiple candidate edges, we leverage several heuristics based on our analysis of real-world spreadsheets to decide the edge that can best reduce the graph size (e.g., by leveraging the dollar sign cues in the formula expression if available).

Algorithm 2 shows our approach for compressing one dependency $e' = (prec, dep)$ into a compressed formula graph G. We use the example in Fig. 8 to explain compressing e' into G. The setup of Fig. 8 (the left pane) shows each formula cell in column C referencing two ranges. The references to column B follow FR and the references to column A follow FF. In addition, we have an uncompressed edge of D4 referencing B1:B4. Our example assumes that a formula SUM($B$1:B4) is inserted at C4 (i.e., $e' = (\text{B1:B4}, \text{C4})$).

Find candidate edges: The first step is to quickly find candidate edges that the dependency e' can be compressed into. Specifically, an edge e is a candidate if $e.dep$ is adjacent to $e'.dep$ along the row or column axis. Step 1 in Fig. 8 shows that all three edges meet this condition because $e'.dep =$ C4 is adjacent to both C1:C3 and D4. To find these edges, we first shift $e'.dep$ by one cell in all four directions (i.e., up/down/left/right) and use the index on the vertices (e.g., an R-Tree [22]) to quickly find ranges that overlap with the shifted $e'.dep$. Then, for each overlapping range (e.g., D4), we find its precedent (e.g., B1:B4) and add this edge (e.g., B1:B4 $\rightarrow$ D4) into the candidate edge set.

Find valid candidates: Next, we check whether e' can be compressed into each candidate edge using addDep(e, e') from Sec. III to find the valid compressed edges. (i.e., genCompEdges in Algorithm 2). We consider two cases. First, if the candidate edge *candE* is not compressed, we check whether e' and *candE* can be compressed into a new edge *newEdge* using the predefined patterns. If so, we store *newEdge* as a valid candidate edge (i.e., addIfValid in Algorithm 2). If, instead, *candE* is a compressed edge, we check whether e' can be compressed into *candE* and if so, we generate a valid edge. Step 2 in Fig. 8 shows two valid compressed edges because the edge B1:B4 $\rightarrow$ C4 can be compressed into B1:B3 $\rightarrow$ C1:C3 or B1:B4 $\rightarrow$ D4.

Select the final edge: The final step is to select the final

Algorithm 3: Find dependents of a column/row of cells r in $G(E, V)$

```
initiate queue as a queue containing only r
initiate result as an empty set and an R-Tree for it
while queue is not empty do
    precToVisit ← remove the first element in queue
    precs ← find vertices that overlap with
            precToVisit via the R-Tree on V
    for prec ∈ precs do
        edges ← {e : e ∈ E and e.prec = prec}
        for e ∈ edges do
            dep ← e.p.findDep(e, precToVisit)
            newDepSet ← Find the subset of dep not
                contained in result via the R-Tree on result
            for newDep ∈ newDepSet do
                add newDep to result and its R-Tree
                add newDep to queue
            end
        end
    end
end
return result
```

edge from the valid ones. The selection is based on the following heuristics, in order. First, we prioritize column-wise compression over row-wise compression. If this heuristic does not return a single edge, we further compare the priority of each remaining edge's pattern. If one pattern p_a is a special case of another pattern p_b, then we choose p_a over p_b because we expect the special pattern p_a to be more efficient. In Sec. V, we will describe one such special pattern of RR. Otherwise, we leverage the dollar sign ($) information, if available, sometimes specified as part of the formula strings. For example, for the formula string SUM($B$1:B4) at C4, we will prioritize compressing its dependency B1:B4 $\rightarrow$ C4 using FR over other patterns because the head cell $B$1 in SUM($B$1:B4) has dollar sign annotations, but its tail cell does not, which indicates that SUM($B$1:B4) follows the FR pattern if it is generated via autofill. For our example in Fig. 8, we choose the compressed edge (B1:B4 $\rightarrow$ C1:C4) over (B1:B4 $\rightarrow$ C4:D4) because the former one uses column-wise compression. Finally, we delete the old edge and insert the newly compressed edge.

Algorithmic complexity Our analysis assumes the inserted dependencies have no duplicates, so its size is $|E'|$, the number of uncompressed edges. For each inserted dependency, we leverage the R-Tree to find the candidate edges, taking $O(|V|)$ operations, where $|V|$ is the number of vertices in the compressed formula graph G. The number of the candidate edges is $O(|E|)$. For these candidate edges, it takes $O(|E|)$ operations to find the valid compressed edges and the final edge that the input dependency is compressed into. In addition, we need to maintain the R-Tree by removing the old and inserting the new vertices, which takes $O(\log |V|)$ operations. In total, the complexity of inserting $|E'|$ dependencies is $O(|E'| \times (|V| + |E| + \log |V|))$ = $O(|E'| \times |E|)$ since each vertex is connected to at least one edge.

B. Querying a Formula Graph

We now discuss finding the dependents or precedents of a column/row of cells r in G using the key functions from Sec. III. Since finding dependents is the dual problem of finding precedents, we focus on the former. We apply Breadth-First-Search (BFS), but with three major differences. First, when we find the direct dependents of r, we need to consider all of the vertices in G that overlap with r. Second, since an edge e in G can be a compressed edge, finding the direct dependents of r in e may not be the full $e.dep$, but a subset instead. So we need to find the real dependents within $e.dep$. Third, for a "real" dependent, which will in turn serve as a precedent for subsequent searches, we need to add the subset of this dependent that has not yet been visited during BFS. We illustrate the three modifications below.

Algorithm 3 shows the modified BFS algorithm: it takes a column or row of cells r as input and returns the set of ranges that depend on r. We explain this algorithm using the compressed graph in Step 3 of Fig. 8. Our example involves finding the dependents of B2. Our algorithm uses a queue to store the ranges to be visited in the future and a set *result* to store the ranges that depend on r and have been visited. An additional R-Tree is also built for *result*. For each range *precToVisit* in this queue, we find its direct dependents. As mentioned earlier, we need to consider the ranges that overlap with *precToVisit* (i.e., B1:B4 for B2). Next, we find the direct dependents of each overlapping range and the corresponding edges (i.e., B1:B4 $\rightarrow$ C1:C4 and B1:B4 $\rightarrow$ D4). Since some of these edges can be compressed, for a compressed edge e we need to find the real direct dependent within $e.dep$ for *precToVisit*, which is done by the key function $\texttt{findDep}(e, \textit{precToVisit})$. For the example of the input B2, we return C2:C4 for the edge B1:B4 $\rightarrow$ C1:C4 since C1 does not depend on B2. Finally, we find the subset of the real dependent that has not yet been visited via the R-Tree on the *result* set, and add the subset to the queue, the set *result*, and the R-Tree on *result*. We repeat the process until the queue is empty. Continuing our example, for the dependent C2:C4, if we have visited C2:C3, which is stored in *result*, we will only store C4 in the queue and *result*.

Algorithmic complexity To analyze the complexity of Algorithm 3, we consider two cases: 1) Algorithm 3 accesses each edge in G at most once; 2) otherwise. For the first case, each vertex in G will serve as the precedent at most once when we find direct dependents (i.e., the inner part of the first **for** loop in Algorithm 3). So it will only repeat $O(|V|)$ times. To find a precedent (i.e., *prec* in the first **for** loop in Algorithm 3), we need to search the R-Tree, taking $O(|V|)$. To find the real direct dependents for a precedent using findDep, we need to spend $O(dep\_num)$ operations, where dep_num is the number of direct dependents of a precedent and findDep takes a constant time. For each real direct dependent, we need to additionally find the subset that is not contained in *result* using the R-Tree on *result*, taking $O(\text{size of } \textit{result})$. Since each range in *result* will be

	TACO	NoComp
Building	$O(\lvert E'\rvert \times \lvert E\rvert)$	$O(\lvert E'\rvert \times \log \lvert V'\rvert)$
Querying	Case 1: $O(\lvert V\rvert^2 + \lvert E\rvert \times \lvert V\rvert)$ Case 2: $O(\lvert V'\rvert^2 \times \lvert E\rvert)$	$O(\lvert V'\rvert^2 + \lvert E'\rvert)$
Maintaining	$O(\lvert E\rvert \log \lvert V\rvert)$	$O(\lvert E'\rvert \log \lvert V'\rvert)$

TABLE I: Complexity comparison between TACO and NoComp

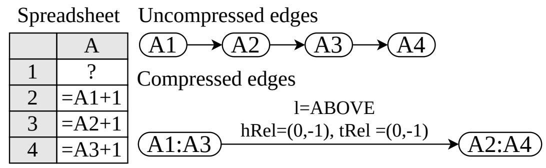


Fig. 9: One example of RR-Chain

a precedent, the size *result* is $O(|V|)$. In addition, the total cost for maintaining the R-Tree on *result* is $O(|V| \times \log |V|)$ since the size of *result* is $O(|V|)$. To sum up, the complexity here is $O(|V|) \times (O(|V| + O(dep\_num) \times O(|V|)) + O(|V| \times \log |V|) = O(|V|^2 + |E| \times |V|)$.

For the second case, the first **while** loop in Algorithm 3 runs $O|V'|$ times, where $|V'|$ is the number of vertices in the uncompressed graph. This is because the size of *result* is $O(|V'|)$, so the total number of ranges inserted into the queue is also $O(|V'|)$. For each range *precToVisit* in the queue, we check $O(|E|)$ edges in G. For each edge, we find the real dependent using $O(1)$ and take $O(|V'|)$ operations to find the subset of the real dependent that is not contained in *result*. The total cost for maintaining the R-Tree on *result* is $O(|V'| \times \log |V'|)$. So the complexity is $O(|V'|) \times (O(|V|) + O(|E| \times |V'|)) + O(|V'| \times \log |V'|$ = $O(|V'|^2 \times |E|)$.

C. Maintaining a Formula Graph

We now discuss maintaining the formula graph when users insert, clear, or update formula cells. We process inserts using Algorithm 2. Since an update can be modeled as a clearing operation plus an insert, we focus on clearing formula cells.

The idea of clearing a column/row of formula cells s is to delete s from the edges whose dependents (i.e., formula cells) overlap with s using removeDep(e, s) from Sec. III. First, we find the relevant edges *relEdges* whose dependents overlap with s. Second, for each $e \in$ *relEdges*, we generate new edges *newESet* after clearing s for e, which is done by removeDep(e, s). Finally, we maintain the graph by deleting the old edge *relEdges* and inserting the new edges *newESet*.

Algorithmic complexity For this algorithm, the size of the relevant edges whose dependents overlap with s is $O(|E|)$ and searching the R-tree takes $O|V|)$, so the cost for finding relevant edges is $O(|E|)$. From each relevant edge, clearing s and maintaining the graph is $O(1)$ while maintaining the R-tree takes $O(\log |V|)$ time. In total, the complexity for removing s is $O(|E| \log |V|)$.

D. Comparison with a No Compression Approach

We now compare the complexity of TACO with an approach that does not compress the formula graph, called NoComp. Table I summarizes the results.

NoComp builds the uncompressed formula graph G' by inserting a list of dependencies into G'. For each dependency, we need to insert its precedent and dependent into an R-Tree, taking $O(\log |V'|)$ operations, and insert the dependency into the adjacency list, taking $O(1)$. In total, the complexity of inserting $|E'|$ dependencies is $O(|E'| \times \log |V'|)$. TACO can be more expensive than NoComp for building the formula graph because it needs to search the R-Tree and find the edge that an inserted dependency can be compressed into.

Next, we analyze the complexity of finding dependents of an input range r via a modified BFS. During BFS, when we find the direct dependents of an input range r, we need to consider all of the vertices in G' that overlap with r (i.e., via an R-Tree search). Similar to conventional BFS, it recursively finds dependents starting from the input range r. Each vertex in G' may serve as a precedent when we find direct dependents of a precedent (i.e., $O(|V'|)$ times), while the cost for finding one precedent via the R-Tree is $O(|V'|)$. Finding direct dependents of one precedent is $O(dep\_num)$, where dep_num is the number of the direct dependents. The overall complexity for finding dependents is $O(|V|') \times (O(|V'|) + O(dep\_num)) = O(|V'|^2 + |E'|)$.

We see thatTACO is more efficient than NoComp if the query algorithm of TACO accesses each edge in the compressed graph G at most once (i.e., Case 1 in Table I). For Case 2, TACO can be potentially more expensive than NoComp in theory. To understand the performance of Case 2, we analyze real spreadsheets to find when this case will happen and become the performance bottleneck, and adopt an extended pattern to reduce the cost for this case in Sec. V. In practice, we find the average number of accesses for an edge during BFS is relatively low. For the tests for finding dependents in Sec. VI, the average number of edge accesses during BFS is no larger than 3 for 95% of the cases. In addition, our experiments in Sec. VI show that TACO is much more efficient than NoComp on real spreadsheets.

Finally, clearing a column/row of formula cells s requires searching the R-tree to find relevant edges (i.e., $O(|E'|)$). Then, we will delete each relevant edge and update the R-tree (i.e., $O(\log |V'|)$). In total, the complexity is $O(|E'| \log |V'|)$. As shown in Table I, TACO is more efficient here.

V. Supporting a New Pattern: RR-Chain

As shown in Sec. IV, our algorithm for finding dependents or precedents may be slow if an edge is repeatedly accessed multiple times. By examining real spreadsheets, we find one pattern that leads to these cases and becomes a performance bottleneck for TACO. In this section, we discuss supporting this pattern to further accelerate TACO. Our discussion focuses on a column of cells and finding dependents as before; the other cases can be derived symmetrically.

Consider a column of formula cells that form a chain of dependencies, where each formula cell references its adjacent formula cell above or below. We will compress these dependencies using RR because each formula cell has the same relative position with respect to its referenced range. Consider

	Enron		Github	
	Vertices	Edges	Vertices	Edges
NoComp	18.6M	23.7M	165.8M	179.8M
TACO-InRow	7.7M (41.2%)	12.5M (52.8%)	55.2M (33.3%)	55.2M (30.7%)
TACO-Full	1.2M (6.3%)	1.2M (5.0%)	4.2M (2.5%)	3.5M (1.9%)

TABLE II: Graph sizes after TACO compression (lower is better)

		Max	75th per.	Median	Mean
Enron	TACO-InRow	142,396	18,196	12,489	18,876
	TACO-Full	700,155	37,286	18,380	37,963
Github	TACO-InRow	1,693,698	42,728	19,704	45,303
	TACO-Full	3,139,011	75,553	31,608	78,633

TABLE III: The num. of edges reduced by TACO (higher is better)

		Min	25th per.	Median	Mean
Enron	TACO-InRow	0.0042%	6.32%	39.81%	42.27%
	TACO-Full	0.0042%	0.47%	1.93%	7.37%
Github	TACO-InRow	0.0005%	0.10%	17.45%	36.48%
	TACO-Full	0.0005%	0.03%	0.19%	3.44%

TABLE IV: Remaining edges after compression (lower is better)

Pattern	Enron Total	Enron Max	Github Total	Github Max
RR	17,412,246	525,026	141,876,182	2,094,936
RF	1,880	1,413	13,361	9,999
FR	150,845	13,815	178,609	39,008
FF	3,844,351	174,948	24,784,621	1,043,702
RR-Chain	566,348	24,596	5,867,728	399,996

TABLE V: Num. of edges reduced by each pattern (higher is better)

the example in Fig. 9, where each formula cell starting from A2 increments the value of the above formula cell by one. To find the dependents of A1, we first find its overlapping vertex A1:A3 and then compute its real direct dependent: A2. Afterwards, the compressed edge is repeatedly accessed until we reach the end of this chain, which introduces high searching overhead.

To solve this problem, we introduce a new pattern RR-Chain as a special case of RR. RR-Chain's *meta* additionally includes a variable l to indicate the direction of a formula cell referencing its adjacent cell. For example, l is ABOVE in Fig. 9 because each formula cell references its adjacent cell above. Our discussion focuses on l = ABOVE; the case for l = BELOW can be easily derived. To compress a dependency e' into e for RR-Chain, we first check the condition of RR, and then further check whether $e'.prec$ is above $e'.dep$ and if they are adjacent. To find dependents of a range r, we return a range d between $r.head$'s direct dependent and the tail cell of $e.dep$. Consider finding dependents of A2 in Fig. 9. We return the range between A3 (i.e., A2's direct dependent) and the tail cell of $e.dep$ (i.e., A4). Finally, clearing formula cells in $e.dep$ follows the same logic as RR and is omitted.

VI. Experiments

Our experiments address the following research questions:

- How much do TACO's predefined patterns reduce formula graph sizes for real-world spreadsheets? (Sec. VI-B)
- How much time does TACO take to build, query, and maintain a formula graph compared to NoComp and an approach specialized for formula graph compression? (Sec. VI-C and Sec. VI-D)
- How much faster does TACO query a formula graph compared to a commercial spreadsheet system? (Sec. VI-E)

A. Prototype, Benchmark, and Configurations

Prototype TACO is implemented as a Java library, which can be used by third-party tools to analyze formula dependencies or integrated into spreadsheet systems to accelerate formula computation. TACO takes an xls or xlsx file as input, leverages the POI library [15] to parse it, and builds a compressed formula graph for the parsed dependencies. The compressed formula graph is implemented using an adjacency list. We build an R-Tree [22] on the vertices of the formula graph to quickly find vertices that overlap with a given range. TACO provides interfaces of finding dependents or precedents of a range, and adding or deleting a dependency. TACO is integrated into DataSpread [26]–[28], an open-source spreadsheet system. DataSpread returns control to users after it has identified all of the dependents of an update and hides them; so finding dependents of an update is the bottleneck for returning control to users. In DataSpread, a formula graph is used to find the dependents of an update and TACO acts as a drop-in replacement for this formula graph. TACO can also be integrated into other spreadsheet systems, such as LibreCalc or MS Excel, because these systems similarly trace dependencies across formulae and raw values [5], [6], [29]. Beyond the integration with existing spreadsheet systems, TACO can be adopted by third-party tools to visualize formula dependencies [12] and help users quickly identify the sources of errors.

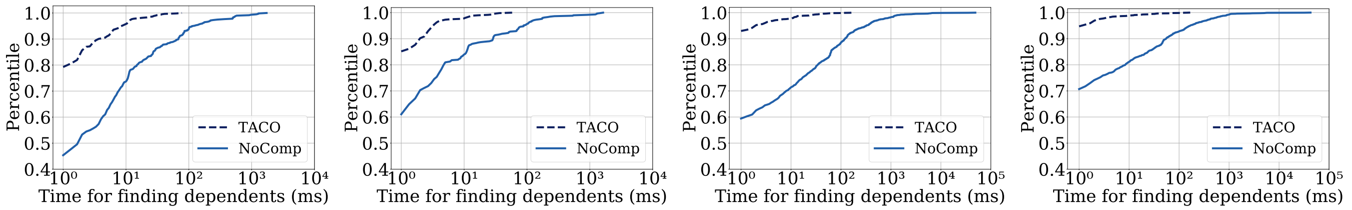
Benchmark Our tests are based on two real-world spreadsheet datasets. The first one is the Enron dataset [13] with 17K xls files. We focus on the large spreadsheets (i.e., with no less than 10K dependencies) that do not cause exceptions (e.g., those requiring passwords), and are left with 593 xls files. Since the Enron dataset includes only xls files, we further crawl 7.8K xlsx files from Github that are larger than 10 KB[1]. We focus on large spreadsheets and skip the erroneous ones, and get 2,238 xlsx files. In total, we test 2,831 files.

Configurations Unless otherwise specified, the experiments are run on a t2.2xlarge instance from AWS EC2, which has 32 GB memory and 8 vCPUs, and uses Ubuntu 22.04 as the OS. We use a single thread, and run each test three times and report the average number. We configure the POI library to load spreadsheets by columns.

B. Compressed Formula Graph Sizes

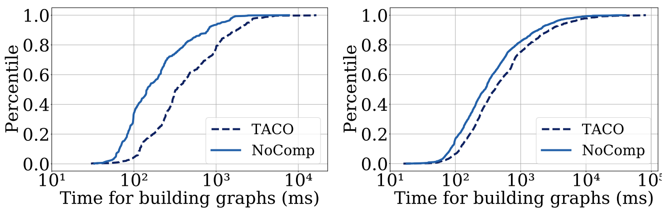
We first test the effectiveness of TACO in reducing the graph sizes. We test two variants: TACO-InRow and TACO-Full. TACO-InRow only compresses adjacent column formulae that reference ranges in the same row, using RR to perform the compression. This approach captures the pattern of derived columns, where a subset of columns are computed using the remaining, which is common in data science and feature engineering (e.g., storing normalized versions of values in

[1]Xlsx files, unlike xls files, support larger spreadsheets (e.g., the row limits for xlsx and xls files are 1M and 66K, respectively.)



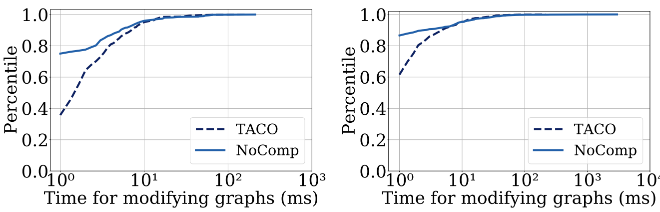
(a) Maximum Dependents (Enron)
(b) Longest Path (Enron)
(c) Maximum Dependents (Github)
(d) Longest Path (Github)

Fig. 10: CDFs for the time for finding dependents



(a) Enron
(b) Github

Fig. 11: CDFs for the time for building formula graphs



(a) Enron
(b) Github

Fig. 12: CDFs for the time for modifying formula graphs

a column as a new column, or extracting substrings of an existing column and storing them as a new column). TACO-Full considers any formulae and adopts all predefined patterns.

Overall effectiveness of reducing graph sizes We first report the total number of vertices and edges of the compressed and uncompressed formula graphs across all the files in Enron and Github, respectively, in Table II. The uncompressed graphs are built using NoComp, as discussed in Sec. IV-D. Both TACO-InRow and TACO-Full significantly reduce the total number of vertices and edges compared to NoComp. For example, TACO-Full reduces the number of edges in Github from 179.8M to 3.5M. In addition, TACO-Full has much smaller graph sizes than TACO-InRow (e.g., 3.5M vs. 55.2M edges for Github), which shows that many formulae reference different rows and TACO-Full can efficiently compress these complex cases that TACO-InRow does not consider.

To further understand the effectiveness of TACO, we compute two additional metrics for each spreadsheet's uncompressed formula graph $G'(E', V')$ and compressed formula graph $G(E, V)$: the number of edges reduced by the TACO (i.e., $|E'| - |E|$) and the fraction of the number of compressed edges compared to the uncompressed ones (i.e., $\frac{|E|}{|E'|}$). Table III reports the max, 75th percentile, median, and mean value of the number of reduced edges across all files for both datasets. Table IV reports the min, 25th percentile, median, and mean value of the edge fraction after compression.

Table III shows that TACO-Full can reduce the number of edges by **up to 700K and 3.1M in a single spreadsheet** for Enron and Github, respectively. The average edge reduction by TACO-Full is 38K and 79K for the two datasets. Table IV shows that the average edge fractions after compression by TACO-Full are **as low as 7.4% and 3.4%** for Enron and Github, respectively. These results show that TACO can effectively reduce formula graph sizes of real spreadsheets.

Effectiveness of TACO patterns Next, we evaluate the effectiveness of each TACO pattern in reducing the number of edges. Recall that a partition of edges E'_i in the original uncompressed graph $G'(E', V')$ corresponds to one compressed edge e_i in $G(E, V)$ in TACO. So the number of reduced edges of a pattern p in G is computed as: $\sum_{e_i \in E}[e_i.meta.p = p](|E'_i| - 1)$, where $[e_i.meta.p = p]$ considers the compressed edges for the pattern p, E'_i is the set of edges that are compressed into e_i, and $|E'_i| - 1$ is the number of reduced edges by e_i. We compute the above metric for each pattern, and report the total and maximum number of reduced edges across the tested spreadsheets.

The results in Table V show that RR and FF compress the most edges. The number of edges reduced by RR is more than 17.4M and 141.9M for Enron and Github, respectively. FF reduces around 3.8M and 24.8M edges in total for the two respective datasets. Other patterns also reduce a significant number of edges in some spreadsheets. For example, in the Github dataset RR-Chain reduces the number of edges up to around 400K for a single spreadsheet. FR and RF, while not as common, can reduce up to around 39K and 10K edges for a single spreadsheet, respectively. These results show that TACO's patterns are prevalent in real spreadsheets and can significantly reduce graph sizes.

C. Comparison with NoComp

We now compare the performance of TACO and NoComp, including the time for finding dependents, building formula graphs, and modifying formula graphs. We focus on finding dependents because finding precedents is the dual problem.

Finding dependents For each spreadsheet we test two cases: finding dependents for the cell that has the maximum number of dependents (denoted as the Maximum Dependents case) and the cell that has the longest path (denoted as the Longest Path case) in the uncompressed graph. Fig. 10 reports the CDFs for the time of finding dependents for the two cases in the two datasets. We see that TACO has much smaller execution time for finding dependents than NoComp. For Enron and Github datasets, TACO's maximum execution time for finding dependents is 78ms and 167ms, respectively, while NoComp's

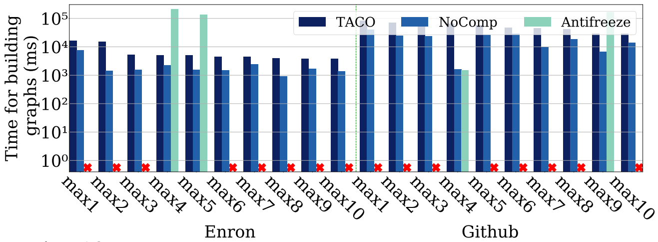


Fig. 13: Performance comparison with Antifreeze (building graphs)

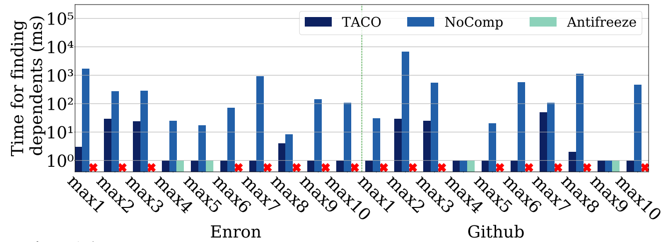


Fig. 14: Performance comparison with Antifreeze (finding dependents)

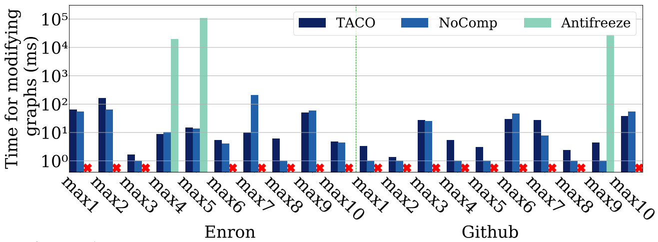


Fig. 15: Performance comparison with Antifreeze (modifying graphs)

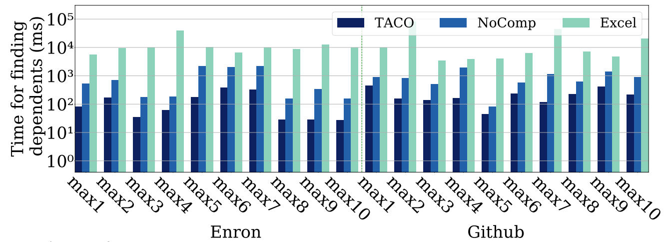


Fig. 16: Performance comparison with Excel on finding dependents

maximum execution time is 1,730ms and 48,889ms, respectively. Across all of the tested spreadsheets, **the speedup of TACO over NoComp is up to 34,972×**.

Building and modifying formula graphs We also test the time for building and modifying formula graphs for the two datasets. To modify a formula graph of a spreadsheet, we remove the content of a column of 1K cells starting from the cell that has the most dependents.

Fig. 11 reports the CDFs for the time of building formula graphs in two datasets. We see TACO takes more time to build the formula graphs compared to NoComp due to the compression overhead. For Enron, the longest time for building a formula graph for TACO and NoComp is 16,626ms and 7,704ms, respectively. For Github, this number for TACO and NoComp is 82,567ms and 40,103ms, respectively. We believe this overhead is acceptable because building formula graphs only happens once when we load the spreadsheet, and it can be executed in the background asynchronously and will not be on the critical path of users interacting with the system. Fig. 12 reports the CDFs for the time taken to modify formula graphs. We see that for the easy cases composing the first 90% with less than 10ms, TACO takes more time to modify formula graphs than NoComp. For the harder cases, TACO takes less time than NoComp, which is consistent with our complexity analysis. For example, for Github, the 99th percentiles for TACO and NoComp are 33ms and 41ms, respectively.

D. Comparison with Antifreeze

We now compare TACO with an approach that also involves compressing formula graphs, Antifreeze [7][2]. Antifreeze builds an uncompressed formula graph for the input dependencies, pre-computes the dependents for each cell, compresses the dependents for each cell via bounding ranges, and stores each cell along with the compressed dependents in a look-up table. If formula cells are changed, it modifies the uncompressed graph and builds the look-up table from scratch. The number of bounding ranges is set to 20, as in the original paper [7].

We find Antifreeze takes a long time for building the look-up table and it is too time-consuming to test all of the spreadsheets. Therefore, we choose the top 10 spreadsheets for which TACO has the longest time for building formula graphs from each dataset. Note that we have tried the spreadsheets where TACO has the longest time for finding dependents, but Antifreeze cannot finish for any of them. We rename top 10 spreadsheets to max_i, where i represents the order. If the time for building a formula graph is higher than 300s, this test is regarded as *did not finish* and the corresponding experimental numbers are marked as **X** in the experiment figures.

Fig. 13-15 report the time for finding dependents of the cell that has the maximum number of dependents, and the time for building and modifying a formula graph. We see that Antifreeze only finishes for 4 out of the 20 spreadsheets due to the overhead of building compressed formula graphs. For the spreadsheets where Antifreeze can finish the tests, TACO has the same execution time for finding dependents as Antifreeze, and has much smaller execution time for building and modifying a formula graph than Antifreeze.

E. Comparison with Excel

We now compare TACO's performance of finding dependents with Excel. For Excel, we test the VBA API for finding the dependents of a cell [14]. We use top 10 spreadsheet files for which TACO spends the most time for finding dependents in each dataset from Sec. VI-C. We rename the 10 spreadsheets to max_i, where i represents the order. In each spreadsheet, we test the time for finding dependents of the cell that has the maximum number of dependents. These experiments are done on a laptop that has one Intel Core i5 CPU with 4 physical cores and 8 GB of memory, and uses Windows 10 as the OS. The results in Fig. 16 show that TACO is much faster than Excel in all cases. The longest time for finding dependents for TACO and Excel is 442ms and 79,761ms, respectively. The speedup of TACO over Excel is up to 632× (i.e., max4 from Enron). It is surprised that Excel takes longer time for finding

[2]Note that graph compression is one of the Antifreeze paper's contributions; its main focus is on the asynchronous execution model, metric, and interface.

dependents than NoComp in all cases. One possible reason is that Excel compresses formula graphs to reduce memory consumption, which introduces the overhead of decompression when the formula graphs are used for finding dependents.

VII. Related Work

TACO is related to formula computation, graph compression, column-oriented databases, and scalable spreadsheets.

Formula computation in spreadsheets There has been some work on improving the interactivity of spreadsheets during updates. MS Excel [5] and other spreadsheet systems [6], [29], [30] track dependents of formula cells such that they can quickly identify the cells impacted by an update and recalculate them. DataSpread approaches this problem using asynchronous execution [31]–[33]. It uses the formula graph to identify the impacted formula cells and mark them dirty, return control to users immediately, and calculate the dirty cells asynchronously [7]. Unlike TACO, none of these approaches leverage tabular locality to compress formula graphs, thereby improving query interactivity. In addition, TACO approach is orthogonal to the execution models and can be integrated into an existing spreadsheet system to improve interactivity.

Graph compression Graph compression has been studied in many scenarios, such as in the Web [34] and social networks [35]. A recent survey [17] shows that different graph compression methods are designed for different goals, including understanding the structure of a graph [36], reducing graph sizes with bounded errors [37], or accelerating queries on graphs [38]. None of these papers leverage tabular locality and consider the spatial nature of formula graphs. In addition, most of them do not support directly querying the compressed graph. Fan et al. [18] support directly executing reachability and pattern matching queries on a compressed graph, but do not leverage tabular locality and support finding dependents/precedents. While a recent paper proposes a compressed graph for spreadsheets [7], we show that building such a compressed graph is time-consuming, making it not appropriate for large spreadsheets.

Column-oriented databases Column-oriented databases [19], [20] employ lightweight compression for data in each column and execute queries directly on the compressed data without decompression [39]. TACO is instead designed to compress dependencies (i.e., edges) while column-oriented compression methods are used to compress the columnar data. In addition, column-oriented databases do not consider decomposing complex patterns in a column of formulae into predefined patterns as TACO proposes.

Spreadsheets at scale Many prior papers focus on supporting large-scale data analysis on spreadsheets [26], [27], [40]–[45]. DataSpread [26], [27] adopts databases as back-end storage such that users can use spreadsheet interfaces to analyze large datasets. ABC [41] provides a spreadsheet interface and uses approximate query processing techniques, such as online aggregation [46], to quickly return approximate results to users. Mondrian [45] maps spreadsheets to visual images to detect different regions in spreadsheets and extract layout templates, but does not consider formula dependency compression. TACO is different from these papers because it approaches the scalability problem using compression techniques. TACO can be used together with the prior work to further improve usability or interactivity of spreadsheets.

VIII. Conclusion

We presented TACO—a framework that efficiently compresses formula graphs in spreadsheets to improve interactivity. TACO exploits tabular locality, wherein cells close to each other have formulae with similar structures, and represents tabular locality via four basic and one extended patterns. As part of TACO, we introduce algorithms for building the compressed formula graph based on predefined patterns, querying this graph without decompression, and incremental maintenance. Our experiments show that TACO can quickly find the dependents of spreadsheet cells to significantly reduce the time of returning control to users while achieving fast graph maintenance at the same time.

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