The effect of reflection and polarizers on linear polarized light

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We study linearly polarized light in two simple experiments. The first involves reflecting linear polarized light from the surface of glass while observing the polarization angle of the reflected ray. The second examines how linear polarizers at relative angles affect the transmitted intensity of light. Both experiments produce results in agreement with theoretical predictions that arise from the electromagnetic theory of light.

I. INTRODUCTION

When light is incident to the surface of a dielectric medium some of the light is refracted while some is reflected. The refracted light entering the medium causes electrons to oscillate in the direction of the light's electric field, the electrons then re-emit this energy with maximum intensity in the direction of the reflected wave. The electron oscillators cannot emit energy parallel to the direction of their oscillation, so when the refracted ray and the reflected ray are 90° from one another the only component of the incidence ray reflected is that which is orthogonal the plane of incidence.

This is one way to make linearly polarized light. The angle at which this happens is called Brewster's angle θ_B and is a function of $\frac{n_t}{n_i}$ which is the index of refraction of the denser medium over the index of refraction of the less dense medium, the full equation is

$$\tan(\theta_B) = \frac{n_t}{n_i}. \tag{1}$$

In this experiment we will use Brewster's angle as a starting point to measure the effect changing the angle of incidence has on the polarization angle of the reflected light. We will do this in a simple setup with a laser, glass prism, and a linear polarizer.

Linear polarizers work by absorbing the component of light that is perpendicular to their transmission axis. This mean they can be used to find the polarization angle of linearly polarized light, and this is the method we use to measure the polarization angle of the reflected ray.

Light exiting the linear polarizer is necessarily polarized in the direction of the transmission axis. If this light is then passed through another linear polarizer the transmitted intensity will be a function of the angle between the polarizers. In the second experiment we will send light through two linear polarizers and using a power meter to record the change in intensity at several different angles.

II. THEORETICAL BACKGROUND

The electromagnetic theory of light describes light as a collection of waves with sinusoidally oscillating electric and magnetic fields that are orthogonal to one another. The waves are transverse in that the direction of propagation is perpendicular to the fields. The polarization of light is defined by the electric field vector as a function of time [1]. Linearly polarized light has an electric field vector that is constant with only its magnitude and sign varying in time [2]. Light of this type can be described by

$$\vec{E} = \left(\boldsymbol{i}E_{ox} + \boldsymbol{j}E_{oy}\right)\cos(kz - \omega t) \tag{2}$$

where z is the direction the wave propagates, k is the wave number, ω is the angular frequency of the wave and finally t is time.

When a wave of this kind hits an interface some of the field is reflected and some is refracted into the medium. The fraction reflected is given by the Fresnel Equations

$$r_{\perp} = \frac{n_i cos\theta_i - n_t cos\theta_t}{n_i cos\theta_i + n_t cos\theta_t} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \tag{3}$$

$$r_{||} = \frac{n_t cos\theta_i - n_i cos\theta_t}{n_t cos\theta_i + n_i cos\theta_t} = -\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \tag{4}$$

where $r_{\perp}$ is the fraction of the incident electric field perpendicular to the plane of incidence that is reflected, $r_{||}$ is the fraction parallel to the plane of incidence reflected, and θ_i and θ_t are the angles of incidence and transmission.

At Brewster's angle it is the case that $n_t cos\theta_B = n_i cos\theta_t$, therefore $r_{||} = 0$ and the reflected ray is made up of a fraction of the incident ray's perpendicular component. When the angle of incidence is not Brewster's angle the reflected light will have components in both the perpendicular and parallel direction and the angle of polarization α measured from the perpendicular axis will be

$$\alpha = tan^{-1}\frac{r_{||}}{r_{\perp}} \tag{5}$$

as shown in Figure 1.

When unpolarized light hits a linear polarizer half of the photons are absorbed and half are transmitted. This is because you can picture unpolarized light as a stream of randomly polarized photons that on average have at least half of their polarization on the axis parallel to the transmission axis and therefore a 50% chance of passing [3]. This accounts for the intensity change of unpolarized light through a linear polarizer of $I = \frac{I_0}{2}$ with I_0 being the incident intensity.

If we now consider starting with linear polarized light then all of the photons have the same polarization and probability of passing through that depends on one fixed angle θ measured from the transmission axis. The part of the incident intensity $I_0 = \frac{c\epsilon_o}{2}E^2$ that passes is then $I = \frac{c\epsilon_o}{2}E^2 cos^2(\theta)$ which can be rewritten as Malus' Law

$$I = I_0 cos^2(\theta). \tag{6}$$

We see the component parallel to the transmission axis ($\theta = 0°$) passes through unchanged and the component perpendicular ($\theta = 90°$) is completely absorbed. Malus' Law and Equation 5 is what we will test our experimental values against.

III. EXPERIMENT

PART I – Angle of Polarization

We set up a laser and a semicircular prism such that the beam was orthogonal to the planar end of the prism as shown in Figure 2. This allowed us to change the angle of incidence by rotating the prism, which was mounted to an adjustable stand. To carry out later calculations we first found the index of refraction of the prism using Equation 1. This was done by setting the laser to be horizontally polarized and rotating the prism until the reflected beam was at a minimum. Since the incident beam's electric field is parallel to the plane of incidence there is no perpendicular component to reflect at Brewster's angle.

The laser was then set up such that the beam was polarized at a $45°$ angle from the vertical so that the electric field components were equal in the perpendicular and parallel direction. Now at Brewster's angle there will be a reflected ray containing the perpendicular component of the incident ray as shown in Figure 3. The reflected ray is vertically polarized so to find Brewster's angle we sent it through a horizontal polarizer and adjusted the incident angle until the transmitted beam was at a minimum. We recorded the angle of the polarizer as $\beta(\theta_B)$.

Now that the angle of incidence is equal to Brewster's angle we could begin collecting data on how changing the angle of incidence changes the angle of polarization. This was done in iterative steps of changing the angle of incidence and then rotating the linear polarizer until the transmitted beam was at a minimum. We then recorded the new angle of incidence θ_i and the new angle of the polarizer $\beta(\theta_i)$ from the vertical. The angle of polarization is then calculated $\alpha(\theta_i) = \theta_i - \beta(\theta_i)$. We did this for several incident angles less than and greater than Brewster's angle. Our recorded data can be seen in Table 1 and a plot of our experimental values of $\alpha(\theta_i)$ vs. theoretically predicted values are shown to be a good fit in Figure 5.

PART II – Malus' Law

To measure the change in intensity we first sent the beam through a linear polarizer so that we could adjust the magnitude of the electric field vector being passed onto the second. This was done because our photometer was too sensitive to measure the full power of the laser. We aligned it such that the light leaving the first polarizer was polarized vertically. We recorded the power of the beam with the photometer before inserting the second polarizer, allowing us to then insert and adjust the second polarizer until the same intensity came through, assuring the polarizers were aligned. Figure 4 shows the setup with an additional lens added before the fiber optic cable to spread out the beam, because this made the reading less sensitive to movement by spreading out the beam.

The second polarizer was then rotated an angle θ from the vertical and the new intensity was recorded. This was done for several angles in two separate trials. The recorded data is shown in Table 2.

IV. ANALYSIS

We first calculated the index of refraction using Equation 1. Uncertainty in the measurement of $\theta_B,\ \delta_B = \pm 1.0° = 0.0174rad$ was propagated to n_t the following way:

$$\theta_B = 55.0° = 0.960rad$$

$$n(\theta) = tan(\theta) \qquad \rightarrow \qquad \frac{dn}{d\theta} = sec^2(\theta)$$

$$\delta_n = sec^2(\theta_B)\delta_B = sec^2(0.960rad) * 0.0174rad = \pm 0.0529 \tag{6}$$

which results in $n_t = 1.43 \pm 0.0529$.

With the index of refraction we can now calculate $r_\perp$ and $r_{||}$ using Equation 3 & 4 and use these to calculate the theoretical values of the polarization angles $\alpha(\theta_i)$ using Equation 5. Our experimental values of $\alpha(\theta_i)$ are calculated $\alpha(\theta_i) = \beta(\theta_i) - \beta(\theta_B)$ with uncertainty in both $\beta(\theta_i)$ and $\beta(\theta_B)$ of $\pm 1.0°$. Total uncertainty in $\alpha(\theta_i)$ is then

$$\delta_\alpha = \sqrt{(\pm 1.0°^2) + (\pm 1.0°)^2} = \pm 1.4°. \tag{7}$$

Figure 5 shows a plot of $\alpha(\theta_i)$ vs. θ_i with the experimental values plotted alongside the theoretical values.

The intensity measurements from Table 2 can be plotted as $\frac{I}{I_o}$ vs. $cos^2\theta$. Plots for both trials are shown in Figure 6.

IV. CONCLUSION

These experiments show in practice these phenomena behave as theory would predict. Our experimental values of $\alpha(\theta_i)$ are a close fit to the theoretically predicted values. The measurements for angles of incidence greater than Brewster's angle are a slightly better fit than the angles that were less than. This could possibly be due to "resetting" the experiment in between collecting the two halves of data and one being slightly more aligned than the other. Figure 6 shows a linear relationship between $\frac{I}{I_o}$ and $cos^2\theta$ as predicted. The consistent difference in relative LUX between two trials may be due to the photometer heating up after extended use and becoming less sensitive.

References

[1] B.E.A Saleh, M.C. Teich. *Fundamentals of Photonics* John Wiley & Sons, Hoboken, NJ, 2007) 2nd ed.

[2] E. Hecht. *Optics* (Addison-Wesley, San Francisco, CA, 2002), 4th ed.

[3] R.P.Feynman, R. Leighton, M. Sands, *The Feynman Lectures on Physics (Vol. 1), (Addison-Wesley, Reading, MA, 1963-1965)*

$\beta(\theta_i)$ [$\pm 1.0°$]	θ_i [$\pm 1.0°$]	$\alpha(\theta_i)$ [$\pm 1.4°$]
122.0	80.0	-32.0
118.0	75.0	-28.0
110.0	70.0	-20.0
102.0	65.0	-12.0
96.0	60.0	-6.0
90.0	55.0*	0.0
83.0	50.0	7.0
76.0	45.0	14.0
68.0	40.0	22.0
63.0	35.0	27.0
58.0	30.0	32.0
54.0	25.0	36.0
51.0	20.0	39.0

Table 1 – Measured values for $\beta(\theta_i)$ & θ_i and calculated value of $\alpha(\theta_i)$ ** Brewster's angle*

θ [$\pm 1.0°$]	Trial 1 I [relative LUX]	Trial 2 I [relative LUX]	δ [$\pm$ relative LUX]
0	260.0	255	5
10.0	250.0	245	5
20.0	230.0	220	5
30.0	195.0	185	5
40.0	150.0	140	5
50.0	100.0	95	5
60.0	57.0	55.0	2.5
70.0	25.0	22.0	2.5
80.0	4.0	4.5	1.0

Table 2 – Measured values of angle between polarizers and relative LUX. The photometer used has different uncertainties that depend on selected range

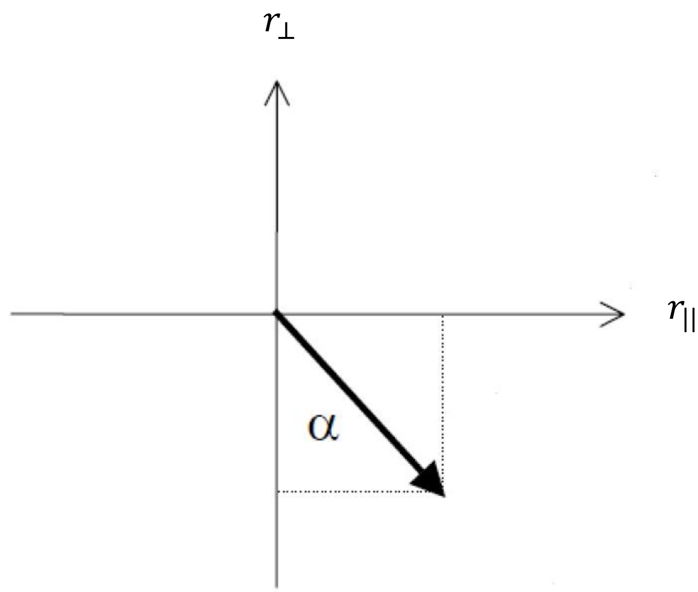

Figure 1 - Relationship between α, $r_\perp$, and $r_{||}$

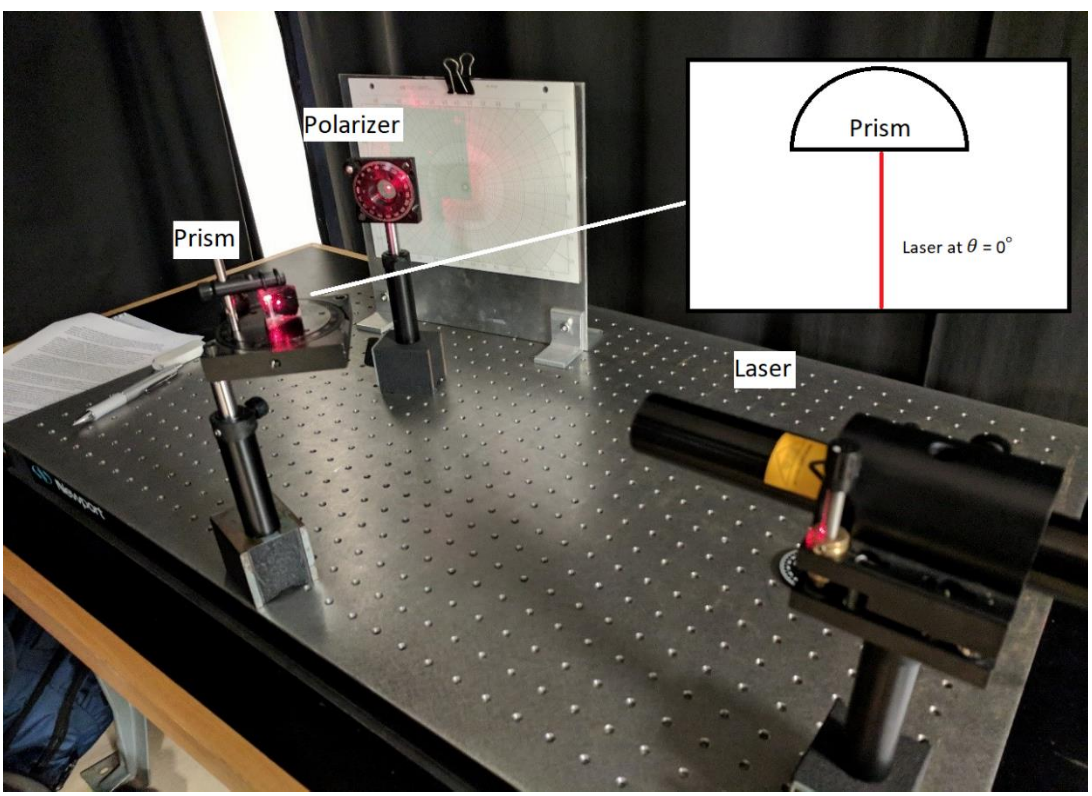

Figure 2 - Setup for measuring θ_i and $\beta(\theta_i)$

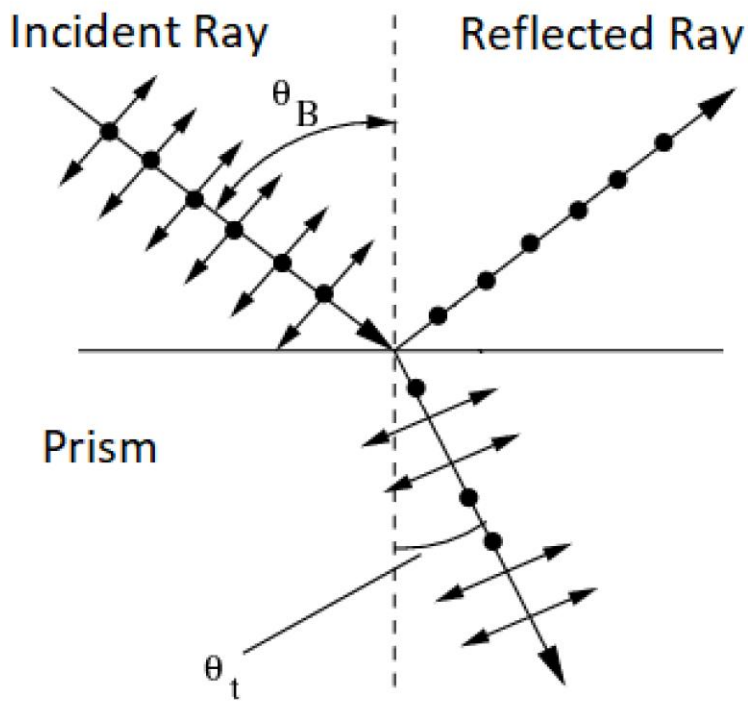

Figure 3 - At Brewster's angle only the perpendicular component of the incident ray is reflected

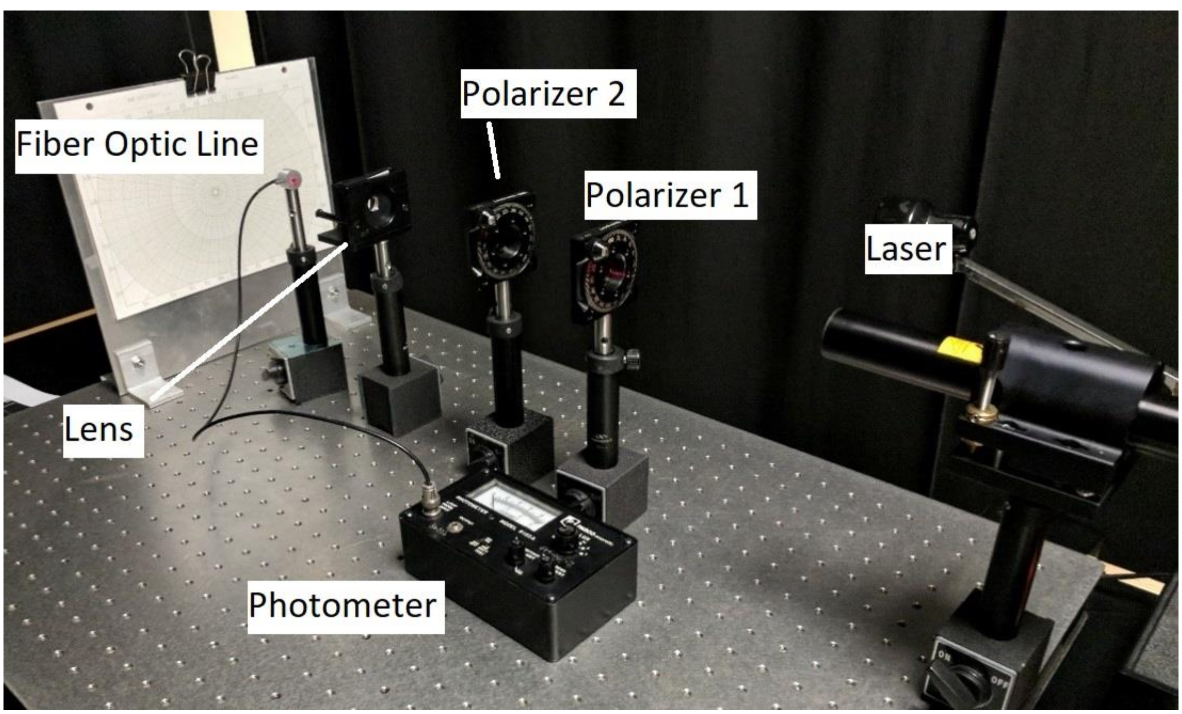

Figure 4 - Setup for intensity measurements

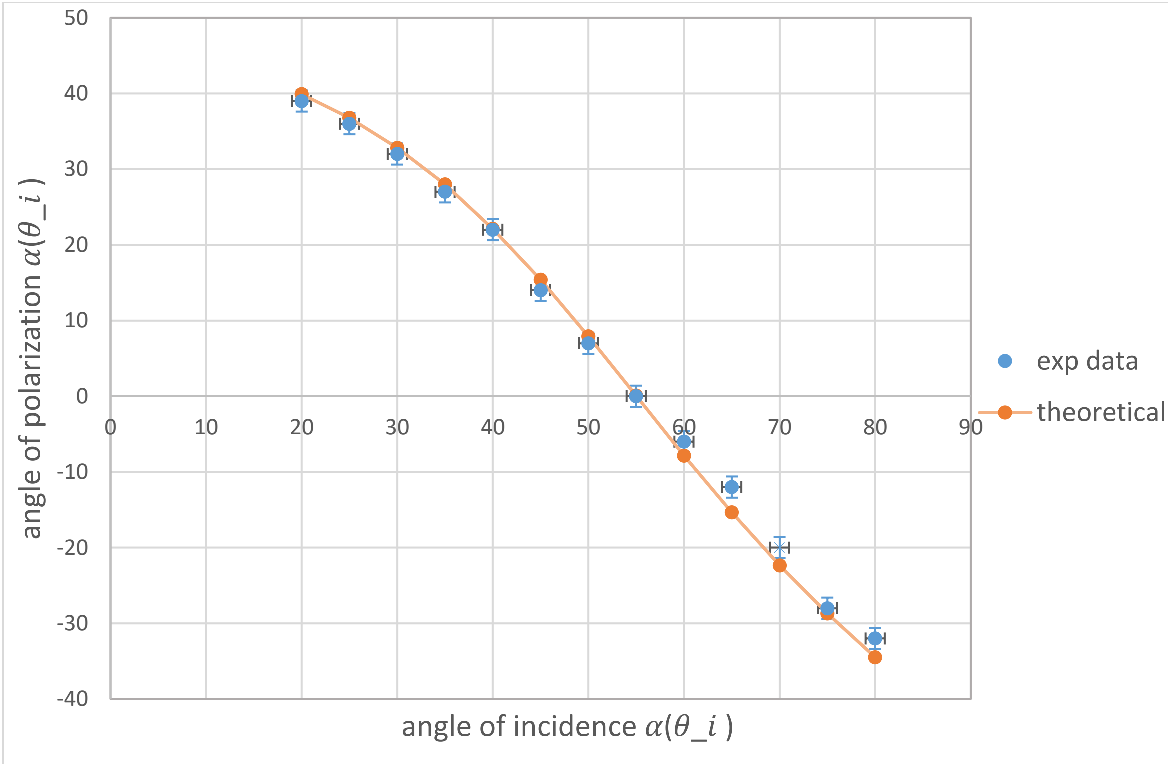

Figure 5 - angle of polarization vs. angle of incidence

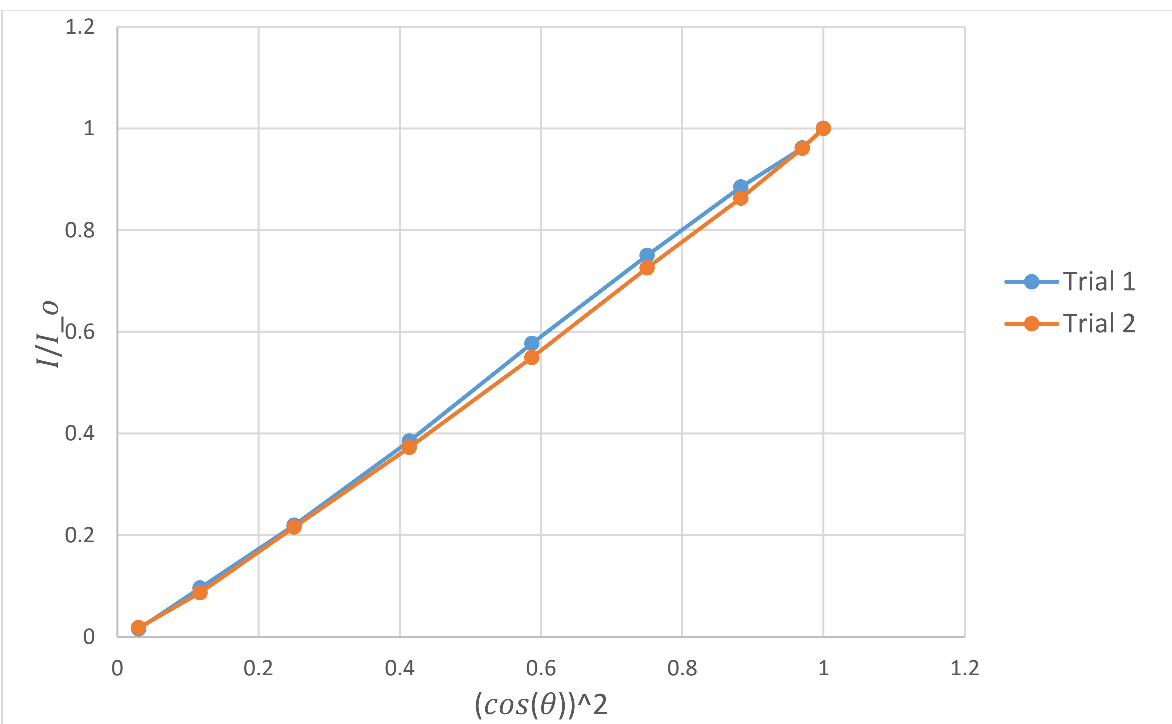

Figure 5 - Plot of Intensity ratio vs. relative angle of polarizers