Interference effects with an amplitude splitting interferometer

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We study interference with a Michelson Interferometer in three different experiments. First we observe how counting fringes that pass when varying one path of the interferometer can be used to determine the wavelength of light. Next we use the same setup to calculate the difference between two slightly different wavelengths. Finally by observing white light interference we determine the coherence length of different white light sources.

I. INTRODUCTION

Amplitude splitting interferometers split an incident beam into two separate paths and later recombine the beams where interference effects may be observed. What happens to the split beams depends on the type of interferometer. In these experiments we use a Michelson Interferometer where a beam splitter splits an incident beam and each travels its own path to a mirror where it is reflected back the same path and recombines at the original beam splitter. Figure 1 shows a diagram of this process. The split path travel through a compensator which is an exact replica of the beam splitter but without the reflective coating. The compensator is needed to account for the reflected beam twice traveling the thickness of the beam splitter once split. The compensator also minimizes dispersion effects when working with broad wavelengths [1].

When the path lengths differ by even half multiples of the wavelength λ total constructive interference is observed and when the paths differ by $\frac{\lambda}{2}$ total destructive interference is observed.

When the paths have any difference in their length a viewer can see more than just interference occurring parallel to the optical path and several different path lengths can be seen alternating between constructive and destructive interference, giving the appearance of fringes shown in Figure 2. When the distances are equal only the zeroth order fringe is visible and this will be an important detail in Part III of the experiment.

Adjustment of the optical path length of one of the split beams affects the position of the fringes and this is how precise measurements can be made using the interferometer. Changing the optical path length can be done different ways, such as inserting a cell of different refractive index from the surrounding medium. We will adjust the optical path length using a micrometer attached to one of the mirrors for a net effect of changing the path length for one split beam equal to twice the displacement of the mirror.

II. THEORETICAL BACKGROUND

Interference is due to the superposition of electric fields. Consider two plane waves of equal amplitude described by

$$\boldsymbol{E}_1 = \boldsymbol{E}_0 e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}_1-\omega_1 t)} \qquad \text{and} \qquad \boldsymbol{E}_2 = \boldsymbol{E}_0 e^{i(\boldsymbol{k}_2\cdot\boldsymbol{r}_2-\omega_2 t)}$$

The superposition of the two waves is then

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0 e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}_1-\omega_1 t)} + \boldsymbol{E}_0 e^{i(\boldsymbol{k}_2\cdot\boldsymbol{r}_2-\omega_2 t)}$$

which interfere to give regions of high and low intensity described by

$$I(\boldsymbol{r},t) = \frac{n\varepsilon_0 c}{2}\boldsymbol{E}_0\boldsymbol{E}_0^*\left[e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}_1-\omega_1 t)} + e^{i(\boldsymbol{k}_2\cdot\boldsymbol{r}_2-\omega_2 t)}\right]\left[e^{-i(\boldsymbol{k}_1\cdot\boldsymbol{r}_1-\omega_1 t)} + e^{-i(\boldsymbol{k}_2\cdot\boldsymbol{r}_2-\omega_2 t)}\right]$$

$$I(\boldsymbol{r},t) = n\varepsilon_0 c\boldsymbol{E}_0\boldsymbol{E}_0^*[1 + \cos(\Delta\boldsymbol{k}\cdot\boldsymbol{r} - \Delta\omega t)] \quad (1)$$

where $\Delta\boldsymbol{k} = \boldsymbol{k}_2 - \boldsymbol{k_1},\ \Delta\omega = \omega_2 - \omega_1,\ c$ is the speed of light, ε_0 is the permittivity of free space, and n is the refractive index of the medium.

In the case of the Michelson Interferometer we have two split plane waves that differ only by delay τ. The result is $\Delta\boldsymbol{k} = 0$ and $\Delta\omega \rightarrow \Delta t = \tau$. Taking $n = 1$ and $I_0 = \frac{\varepsilon_0 c\boldsymbol{E}_0\boldsymbol{E}_0^*}{2}$ Equation 1 becomes

$$I(\boldsymbol{r},t) = 2I_0[1 + \cos(\omega\tau)]. \quad (2)$$

The time delay τ is the difference in time caused by the change in path length δ_d over the speed of the wave such that $\tau = \frac{\delta_d}{c}$. Figure 3 shows a graph of how the intensity at the detector changes with a change in τ [2]. Equation 2 shows a periodic relationship between the intensity and δ_d. Taking $\omega = 2\pi f$ with f the frequency of the light equal to $\frac{c}{\lambda}$ we see that max intensity, or a bright center fringe occurs when

$$\cos(\omega\tau) = 1 \ \rightarrow\ \omega\tau = N\pi = 2\pi f * N\frac{\delta_d}{c}$$

$$\lambda = \frac{2\delta_d}{N} \quad (3)$$

where N is an integer $0, \pm 1, \pm 2$ .... We will use Equation 3 to determine the wavelength of light by counting the number of fringes that pass when adjusting the optical path distance a known distance δ_d.

When waves of different wavelengths, which is to say different frequencies interfere they form a localized pulse where the amplitudes constructively interfere for a given $\Delta\tau$ as shown in

Figure 4. The splitting sodium D lines are very close together and will periodically display coherence which a viewer will observe in the fringe pattern as crisp fringes turning to blurred fringes back to crisp. The δ_d over which this periodic cycle happens can be used to determine the difference in the two wavelengths. We then use Equation 3 with the number of counted fringes N and N+1 using the wavelength determined in the first experiment. Subtracting the two equations results in

$$\delta\lambda = \frac{\lambda^2}{2\delta_d} \tag{4}$$

where $\delta\lambda$ is the difference between the two sodium D lines.

The coherence length l_c is equal to the amount of delay τ_c necessary to destroy the coherence between waves multiplied by the speed of the wave. In the case of light $l_c = c\tau_c$ which is equal to $2\delta_d$ in our setup, if δ_d is the distance required to destroy coherence. This is the method we will use to determine the coherence length of a white light source.

III. EXPERIMENT

PART I – Wavelength of Sodium D Lines

We used a Michelson Interferometer set up as shown in Figure 5. Alignment of the interferometer is critical to performing any sensitive measurement. We placed a sodium D lamp behind a diffuser and adjusted the mirrors until the fringe pattern appeared as concentric circles, assuring the mirrors were aligned parallel. When adjusting the micrometer we verified the fringes disappeared or appeared from the center. At this point we recorded the reading of the micrometer and slowly adjusted the micrometer screw attached to one of the mirrors while counting the number of fringes that disappeared into the center. We repeated this process several times while counting 50 fringes, our recorded data is shown in Table 1 where d_1 and d_2 are our recorded starting and ending micrometer readings. Note that a change in micrometer reading is not equal to the change in path length at this point due to the gearing in the micrometer to mirror mechanism.

PART II – Splitting of the Sodium D Lines

With the experiment still set up as in Part I we can measure the splitting of the sodium D lines. Because the lines are close together coherence occurs periodically for longer changes in optical path distance compared to white light for example. This makes it easy to see the entire cycle where the lines become sharp, then blurry, and again sharp. We decided that it is easiest to tell when the fringes are most blurry rather than the sharpest. The path length was adjusted using the micrometer at a position where the fringes were most blurry, the micrometer reading was recorded as d_1. We turned the micrometer through the fringes coming into focus, passed their sharpest point, and then stopped when the fringes went maximally out of focus again. The micrometer reading was recorded as d_2. Total uncertainty in the measurement will include uncertainty in the exact position where the fringes were most blurry. Table 2 shows our results over two trials.

PART III – Coherence Length of White Light

With the sodium D lamp still in place we adjusted the path distance looking for the zeroth order fringe. This occurs when the path lengths are exactly equal and the concentric ring pattern turns into one blurred fringe. The coherence length for white light is very short due to many different frequencies having to align and the mirrors being aligned parallel is especially important for this measurement. Once we found the zeroth order fringe and centered it on the optical axis we recorded this position as a starting place for subsequent trials.

We then increased the path length until we saw white fringes begin to appear. Turning through the fringes we saw that they become very colorful over a short distance and then turn back to white and fade away over a longer distance. We recorded d_1 as the micrometer reading right before the bright fringes appeared and d_2 as the reading at which they disappear. Our results for different white light sources are show in Table 3.

IV. ANALYSIS

PART I - Wavelength of Sodium D Lines

Uncertainty in d_1 and d_2 is $\pm 0.005mm$ each. The uncertainty in Δd is calculated

$$\sigma_{\Delta d} = (0.005^2 + 0.005^2)^{1/2} = \pm 0.007\text{mm}. \tag{5}$$

Over four trials this becomes

$$\sigma_{\Delta d} = \frac{1}{4}\sqrt{4 * 0.007^2} = \pm 0.003mm \tag{6}$$

A change in the micrometer reading translates to

$$\delta_d = \frac{1}{5}(d_1 - d_2) \tag{7}$$

in our setup due to the gearing mechanism between the micrometer and mirror. We use Equation 7 to calculate

$$\delta_d = 0.016 \pm 0.003mm$$

which allows us to calculate the wavelength using Equation 4 with N=50 to be $\lambda = 640nm$. We estimate the amount of uncertainty in N due to miscounting to be ± 1 fringe. Total uncertainty in λ is calculated

$$\frac{\sigma_\lambda^2}{\lambda^2} = \frac{\sigma_{\delta_d}^2}{{\delta_d}^2} + \frac{\sigma_N^2}{N^2}$$

$$\sigma_\lambda = \pm 100nm.$$

To give the final wavelength of

$$\lambda = 640 \pm 100nm.$$

The large uncertainty is due to the uncertainty in the micrometer reading being close to the difference of our measured values which we talk more about in the conclusion. The known values of 589.0nm and 589.6nm for the two lines fall well within the uncertainty.

PART II – Splitting of the Sodium D Lines

Change in optical path distance is calculated as in Part 1 to be $\delta_d = 0.28 \pm .003mm$. We then calculate the difference between the two sodium D lines using Equation 4 with our experimental value of $\lambda = 640nm$ to be $\delta\lambda = 0.73nm$. Uncertainty in $\delta\lambda$ is propagated by first calculating the uncertainty in λ^2 to be

$$\frac{\sigma_{\lambda^2}^2}{{\lambda^2}^2} = \frac{\sigma_\lambda^2}{\lambda^2} + \frac{\sigma_\lambda^2}{\lambda^2} \qquad \sigma_{\lambda^2} = \pm 90500nm^2$$

and then calculating the uncertainty in $\delta\lambda$ to be

$$\frac{\sigma_{\delta\lambda}^2}{\delta\lambda^2} = \frac{\sigma_{\lambda^2}^2}{{\lambda^2}^2} + \frac{\sigma_{\delta_d}^2}{{\delta_d}^2} \qquad \sigma_{\delta\lambda} = \pm 0.16nm$$

giving the final result

$$\delta\lambda = 0.73 \pm 0.16nm.$$

The known value of 0.58nm falls just inside the uncertainty. The difference is due to our wavelength being much higher than the known value, and the error propagating through to the value of the line spacing. If instead we used the known wavelength of sodium D of $\lambda = 590nm$ we would get a value for $\delta\lambda$ closer to 0.62nm.

PART III – Coherence Length of White Light

The fringes appeared very sharply when observing the incandescent, flash light, and LED making it was easier to tell when the fringes began and ended. Any additional uncertainty in starting an ending position would be much less than the uncertainty in the micrometer reading. The fluorescent light however showed fringes that appeared more slowly and lasted for a longer time before slowing fading out again, so we will include a larger uncertainty of $\pm 0.1mm$ which is 8.3% of our measured value for Δd.

The coherence length of the sources are calculated using Δd from Table 3 along with Equation 7 to find the mirror displacement, multiplied by two to obtain total path distance. Total uncertainty for δ_d is calculated in the form of Equation 5 with the addiontal uncertainty added for the fluorescent light, the results are shown in Table 4.

IV. CONCLUSION

The interferometer allowed us to make three measurements by observing the effects of interference that would otherwise be very difficult to obtain. Our measured values for the wavelength and splitting of the sodium D lines are within uncertainty of the known values and we were able to estimate the coherence length of different white light sources. Large uncertainties arise due to the measurement uncertainty of the micrometer being close to differences in length we set out to measure. Performing more trials to lower uncertainty as we did in Equation 6 would help make the uncertainties tighter. A micrometer that can measure another order of precision would have helped with some of the rounding that lead to our wavelength being much larger than the known value.

References

[1] E. Hecht. *Optics* (Addison-Wesley, San Francisco, CA, 2002), 4th ed.

[2] J. Peatross, M. Ware. *Physics of Light and Optics* (J. Peatross, M. Ware. Brigham Young University 2015).

Tables

d_1 [$\pm$.005mm]	d_2 [$\pm$.005mm]	Δd [$\pm$.007mm]
8.27	8.35	0.08
8.27	8.35	0.08
5.00	5.07	0.07
5.00	5.08	0.08

Table 1 – Recorded change in micrometer during experiment part 1

d_1 [$\pm$.005mm]	d_2 [$\pm$.005mm]	Δd [$\pm$.007mm]
4.00	5.37	1.37
5.37	6.83	1.46

Table 2 – Recorded change in micrometer during experiment part 2

	d_1 [$\pm$.005mm]	d_2 [$\pm$.005mm]	Δd [$\pm$.007mm]
Incandescent	11.85	11.87	0.02
Flash Light (white)	11.83	11.85	0.02
Overhead Fluorescent	11.80	13.00	1.20
LED	11.81	11.85	0.04

Table 3 – Recorded change in micrometer during experiment part 3

	Coherence Length = 2* δ_d [μm]
Incandescent	8.0 ± 7
Flash Light (white)	8.0 ± 7
Overhead Fluorescent	240 ± 100
LED	16 ± 7

Table 4 – Coherence length calculated values

Figures

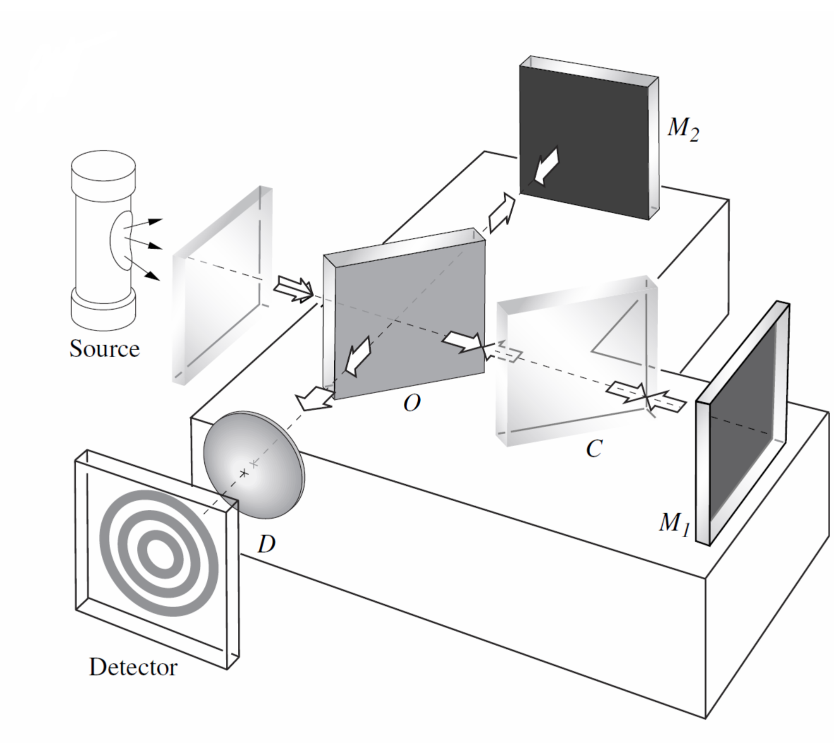
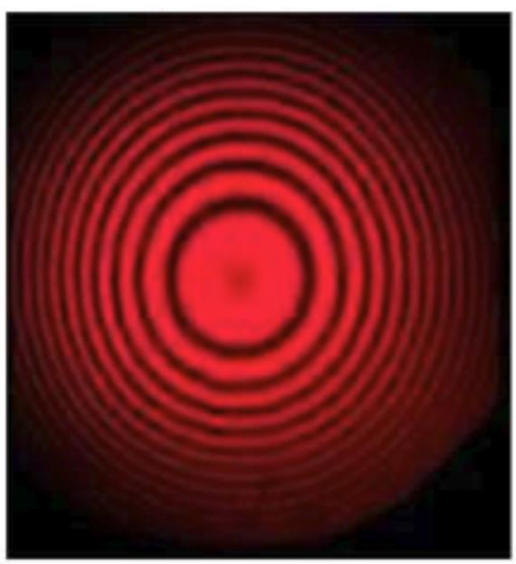


Figure 1 –Diagram of Michelson Interferometer [1]



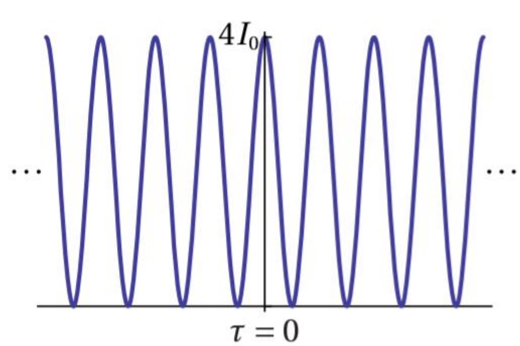


Figure 2 – Interference fringes observed at interferometer detector

Figure 3 – Interference as a function of delay time [2]

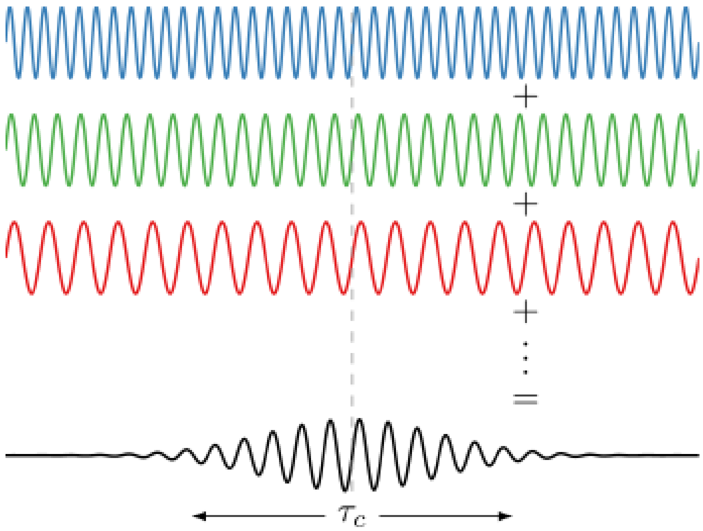


Figure 4 – Coherence of three different frequencies forming a localized pulse

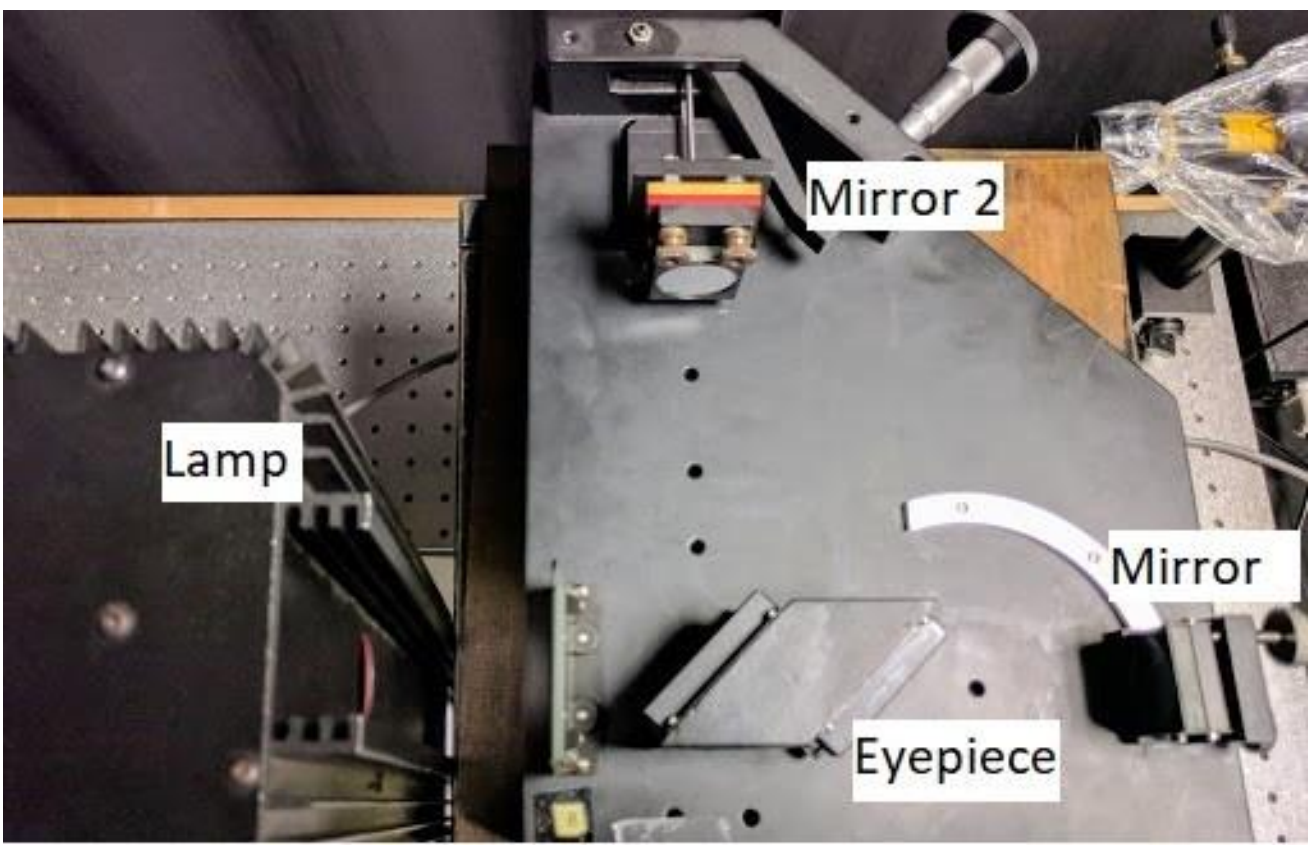


Figure 5 – Michelson Interferometer used for this experiment