Quantum Entanglement and the Theta Tau Puzzle

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The phenomena of quantum entanglement is conceptually introduced along with an example of Bell's inequality suggesting the universe is fundamentally nonlocal in nature. Quantum entanglement, nonlocality, and Bell's inequality is also discussed in relation to the "Theta Tau Puzzle".

I. INTRODUCTION

By the mid 1930's Einstein, Podolsky, and Rosen could see that quantum mechanics and relativity were not compatible and had published a paper that would become known as "EPR" or the "EPR Paradox". The argument considers two particles that have interacted with one another. Quantum mechanics says until measured these two particles are still in a single quantum state with one another even if they are separated by an arbitrarily large distance. EPR claims that once the particles have been separated such that the two systems could not possibly still interact with one another they are safe to be considered in their own quantum states. EPR shows in general these states could be eigenfunctions of noncommuting operators [1]. This however would allow two physical quantities with noncommuting operators to be measured with certainty which is a violation of the uncertainty principle and quantum mechanics in general. The alternative seemed to be if the states did not separate then when one particle was measured it would need to signal the other particle with information of its state faster than the speed of light, which is a violation of locality and relativity in general.

Schrodinger over the next year published papers that used the word “entanglement” for the first time. As a response to EPR he also stated that it is not the case that once two systems interact with one another that it is possible for them to separate to their own states until a measurement is made, calling this not only an important feature but “the characteristic trait of quantum mechanics” [2].

I. Quantum Entanglement

To understand what is meant by entanglement we can consider an electron positron pair formed by the decay of some other particle. The particles are moving opposite directions in the state

$$\psi = \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2). \tag{1}$$

This means that when one of the particles is measured there is a 50% chance of measuring spin-up and a 50% chance of measuring spin-down. However once this first measurement is made the spin of the other particle can be known with 100% certainty even if they have separated (with no other interactions) for an arbitrarily large distance, as long as spin measurements are made along the same axis. This doesn’t seem too bizarre at first but remember that it does not make sense to say one particle had “really” been spin-up and the other was “really” spin-down the whole time but just unknown to observers. Quantum mechanics says they are in a state of 50% probability of spin-up or spin-down until one is measured. The second particle must somehow know the result of the first measurement.

EPR suggested that since there is no way to signal faster than the speed of light that quantum mechanics is incomplete and the particles must contain a “hidden variable” that the current theory doesn’t predict. It wasn’t until 1964 that Bell thought of an experiment to test the hidden variable theory [3]. Bell allows the two spin measurements to randomly be made in any direction perpendicular to the direction of particle motion. We will show an example where only three are allowed.

Consider the electron positron pair from before. If the particles are moving parallel to the z-axis we could imagine three possible detector locations spaced evenly apart in the x-y plane. The detectors will each randomly be placed at 0°, 120°, or 240° with respect to the x direction, one such configuration is shown in Figure 1.

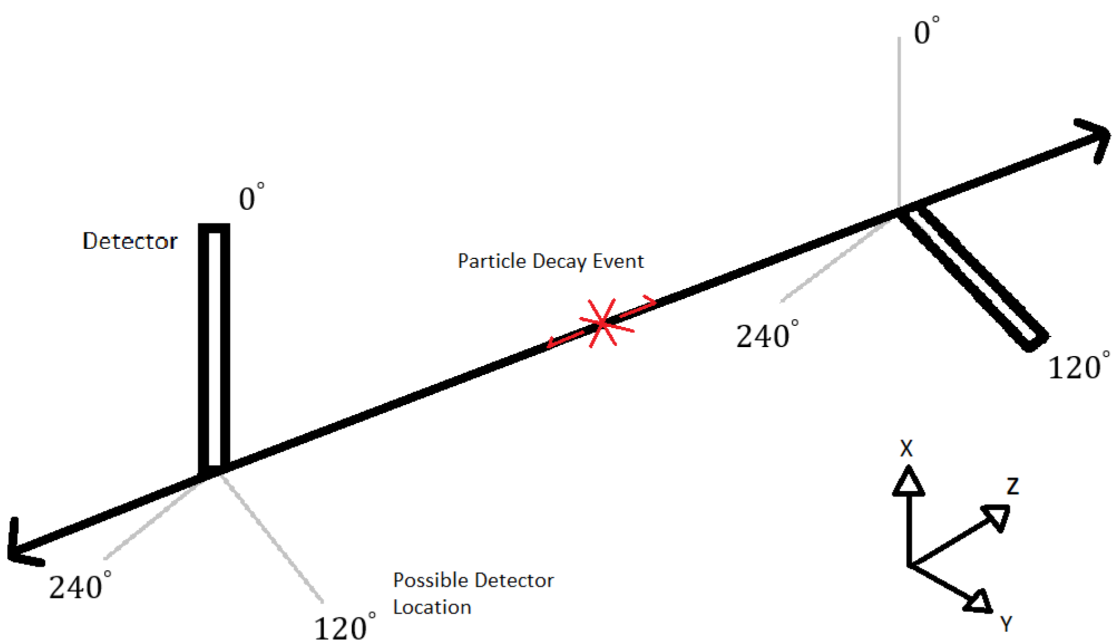

Figure 1 - Detectors located at 0° for one particle and 120° for the other with respect to x-axis. Particle motion is parallel to $\pm$z.

We can now imagine what possible spin arrangements could arrive at the detectors and count how many times we measure opposite spins. The only requirement is that the particles always have equal and opposite spin when measured along the same axis. We can find the case of maximum opposite spin agreement by imaging the hidden variable contains enough information that the particles always have opposite spin no matter what direction the two are measured at. The worst agreement would be when the chosen spin direction is as disordered as possible without breaking the rule that the particles must have equal and opposite spin along the same axis. Table 1 shows a possible configuration which gives the minimum number of opposite spin agreements which we see is 5/9. What this means is that for the hidden variable theory to be plausible the experiment would have to measure opposite spin with a probability 5/9 or greater [4].

Detector 1 direction	Detector 2 direction	Spin direction particle 1	Spin direction particle 2	Opposite spin?
0°	0°	↑	↓	yes
0°	120°	↑	↑	no
0°	240°	↑	↓	yes
120°	0°	↓	↓	no
120°	120°	↓	↑	yes
120°	240°	↓	↓	no
240°	0°	↑	↓	yes
240°	120°	↑	↑	no
240°	240°	↑	↓	yes

Table 2 – A detector at position 1 measures spin in direction 1 while detector 2 measures spin in direction 2. Note that all measurements of particle 1 at 0° are opposite of all measurements of particle 2 at 0° which is true for the other angles as well. This is the limiting feature of the hidden variable theory.

Bell generalizes this idea by defining an average value of the product of the spins (in units of $\frac{\hbar}{2}$) $P(\boldsymbol{a}, \boldsymbol{b})$ where $\boldsymbol{a}$ and $\boldsymbol{b}$ are vectors in the direction of the detectors. The final result known as the *Bell inequality* is

$$|P(\boldsymbol{a}, \boldsymbol{b}) - P(\boldsymbol{a}, \boldsymbol{c})| \leq 1 + P(\boldsymbol{b}, \boldsymbol{c}) \tag{2}$$

where **c** is an additional arbitrary direction vector [3] [5]. This in general is incompatible with the quantum mechanic prediction that the probability average should be equal to

$$P(\boldsymbol{a}, \boldsymbol{b}) = -\boldsymbol{ab}\cos(\theta) \ \text{[4].} \tag{3}$$

We can see this is the case for the configuration in Figure 1 by adding $\boldsymbol{c}$ with direction

$60°$ between **a** and **b** and using equation 2 and 3 to get

$$|-\cos(120) - (-\cos(60))| \leq 1 + (-\cos(60)) \quad \rightarrow \quad 1 \leq 1/2 \tag{4}$$

which is a contradiction. What quantum mechanics predicts for our three possible detector location experiment is that when the detectors do happen to align parallel the spins are always opposite, however when the detectors do not align then the quantum mechanic prediction is used. This results in a opposite spin agreement probability of

$$\frac{1}{3}(1) + \frac{2}{3}cos^2(60) = \frac{1}{2} \tag{5}$$

because once the first particle is detected the second particle suddenly has a well defined spin and there is a 1/3 chance of the second particle being measured along the same direction and a 2(1/3) chance of the particle making a $60°$ angle to the detector. Experiments like this have shown to be in good agreement to quantum mechanics [6][7]. This suggests that the universe is fundamentally nonlocal nature and rules out a local hidden variable theory.

III. Theta Tau Puzzle

By the mid 1900's the strong and weak nuclear forces were being investigated and the field of particle physics was beginning to grow rapidly. Experiments had shown that nature oddly had provided two particles that were identical in mass (within experimental error) but decayed into two different sets of particles. The tau (τ) meson decayed into three pions (π) and the theta (θ) meson decayed into two pions as shown in Figure 2 [8].

The reason people thought they could not be the same particle was due to *parity* conservation which describes the left-right symmetry in systems. It was proposed by Block, Feynman, and Gell-Man at a conference in 1956 that perhaps parity is not conserved in the weak force and experiments soon proved this to be the case [9] [10].

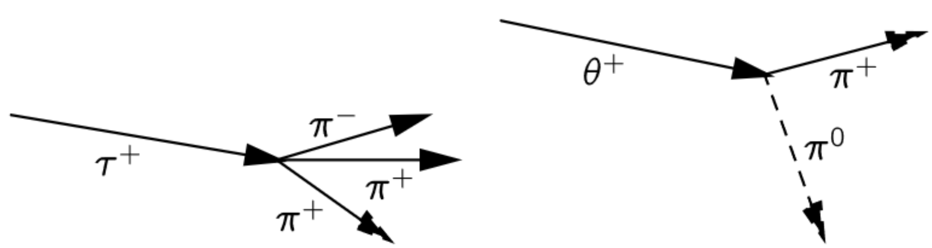

Figure 2 - The decay of the tau and theta meson [8]

The role quantum entanglement plays is that a Bell inequality experiment can be used to determine the degree to which parity is violated and this in itself can be used to test localism similar to the spin example. The τ and θ meson are now known as the kaon (K). The kaon is an entangled state of two possibilities, one of which decays like the tau and the other like the theta [11]. These two states can be thought of as the spin states in the previous example, but since parity is not conserved the particles are not forced to have a 50% chance of being one or the other and instead have a CP violation parameter that determines the degree of parity violation [12].

The way locality can be tested is to create a stream of decays into neutral kaon anti-kaon particles. When the kaon decays it must choose either the state that decays into three pions or the state that decays into two. If for example the kaon decays into two pions at time t, we know that at time t the anti-kaon cannot decay into two pions, it must choose the other state and decay into three pions. The different decay times of the particles now play the role of the different angles in our previous spin example and a Bell inequality modified for the specific case of kaons can be constructed and used in locality experiments [11]. The results again agree with the quantum mechanic predictions [12].

IV. Conclusion

Entanglement is at the center of a lot of the groundbreaking physics that took place in the last hundred years that has drastically changed the way we view the world. Locality and parity conservation are two examples of what physicist in the early 1900s did not see a reason to question but turned into some of the biggest questions of the century. Quantum entanglement is now enabling new ideas and technologies creating whole new fields of study like quantum teleportation, quantum cryptography, and quantum computing/information, while still playing its role in CP violation which is one of the great unsolved mysteries that large teams of physicist at particle accelerators are working to better understand.

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