Bayesian linear regression using estimation maximization

In a linear regression model, we have to maximize the evidence function with respect to the real parameters, alpha and beta. Where alpha represents the slope of the line and beta represents the intercept of the line. We use the technique of estimation maximization to perform this task.

E-Step of Estimation Maximization Algorithm:

We first need to calculate the log likelihood function, considering all the four cases of missing variables, i.e. both x and y are present, x is present y is missing, x is missing y is present, both are missing. The expectation can be calculated as following:

$$
\begin{aligned}
Q(\theta|\theta^{(r)}) = \sum_{i=1}^{n_1} l(\theta) + \sum_{i=n_1+1}^{n_2} \int l(\theta) p\left(y_{i,mis}|X_i, r_i, s_i, \theta^{(r)}\right) dy_{i,mis} \\
+ \sum_{i=n_2+1}^{n_3} \int l(\theta) p\left(x_{2i,mis}|X_{i,obs}, y_i, r_i, s_i, \theta^{(r)}\right) dx_{2i,mis} \\
+ \sum_{i=n_3+1}^{n} \sum_{y_i=1}^{\infty} \int l(\theta) p\left(y_{i,mis}, x_{2i,mis}|X_{i,obs}, r_i, s_i, \theta^{(r)}\right) dy_{i,mis} dx_{2i,mis}
\end{aligned}
$$

The above expression can be simplified and then the Metropolis - Hastings algorithm can be utilized to calculate the above given conditional expectation.

M step of estimation maximization algorithm

Now, we need to find the value of theta, which maximizes the above calculated conditional expectation (posterior function).