Self Organised Criticality in Forest Fires

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1 Introduction

1.1 Self Organised Criticality

Self organised criticality is the spontaneous organisation of a system such that the system reaches a critical point where the state of the system changes. This is state change is analogous to other physical systems such as a ferroelectric material. This material has an internal structure where all magnetic dipoles are pointing in a certain direction in one portion of the material. This results in a net magnetic dipole in the material, and certain conditions such as temperature or voltage across a material can induce a change in state where the net magnetic dipole changes direction.

These types of systems require certain conditions to be met before they can undergo a state-transition. However, in a system exhibiting self-organised criticality, the system requires no deliberate tuning of conditions to undergo a state transition. The system will change its conditions spontaneously in such a way that it reaches a critical point, undergoes a state-transition, then begins to approach criticality again. In the case of the forest fire model, the system increases the density of the trees over time until the spontaneous appearance of a fire will spark a burn of the majority of the forest. Then the forest increases in density again, approaching criticality, and the cycle continues. The behaviour of a system exhibiting self organised criticality can be investigated by examining a forest fire model.

1.2 Power Law

As this system exhibits self organised criticality, the small events that make up the general behaviour of the system (in this case growing and burning down of trees), can combine to create a larger event (the burning down of the majority of the forest). The scale of these events and the probability of an event of a certain scale happening are related by a power law:

$$N(s) \approx s^{-\alpha} \tag{1}$$

where s is the magnitude of the event, α is a fixed exponent and $N(s)$ is the number of events. This relation is shown by Figure 1:

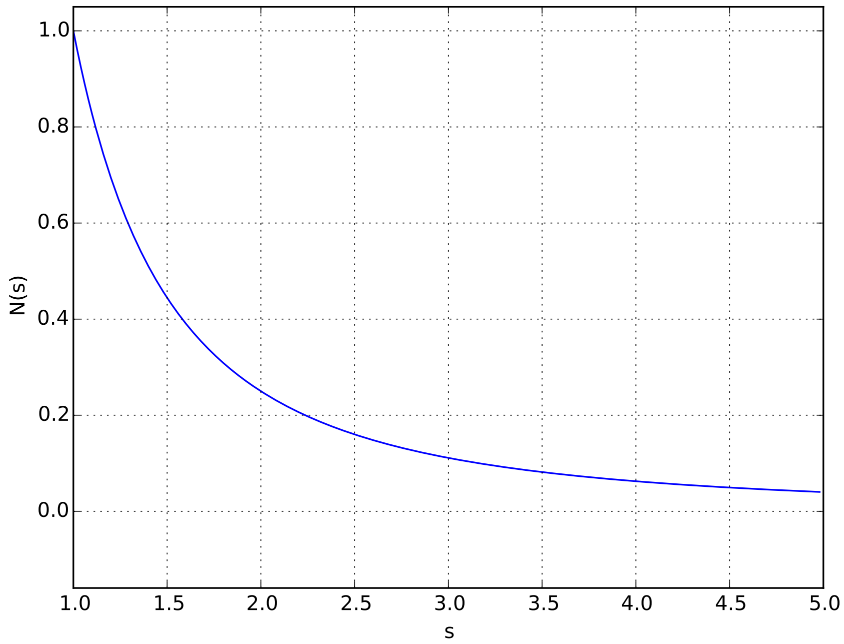

Figure 1: Graph of equation 1 between $s = 1$ and $s = 5$ with $\alpha = 2$. This shows the relation between the frequency of events and the magnitude of these events for self organised criticality.

1.3 Scale Invariance

Scale invariance means that a function keeps the same shape when the variable is scaled by any fixed amount. Consider a function $f(x)$. If $f(x)$ is scale invariant, the following equation is true:

$$f(Ax) = kf(x)$$

where A and k are constants.

Therefore, in the case of self organised criticality, scale invariance can be analytically shown:

$$\begin{aligned} N(s) &\approx s^{-\alpha} \\ N(As) &\approx (As)^{-\alpha} \\ &\approx A^{-\alpha} \times s^{-\alpha} \\ &\approx A^{-\alpha} \times N(s) \end{aligned}$$

where $A^{-\alpha}$ is a constant. This shows that the power law maintains its shape when scaled by any fixed factor.

1.4 Rules of Forest Fire Model

The forest fire model is a simple set of rules based on a 2-D grid of nodes. Each node can be one of three types:

1. Tree
2. Empty
3. Fire

In its fundamental form, there are four rules that the forest follows:

1. Empty nodes have a random chance p to grow a tree
2. Trees have a random chance f to spontaneously catch fire
3. Any tree adjacent to a fire will catch fire
4. Any fire will go out in the next time step

Generally in this set up, $f << p << 1$. The reasons for this are explained later. The forest is updated according to these four rules synchronously.

2 Theory

2.1 Analysis of Scale Invariance

If a large number of independently random events are combined, the magnitude of the resulting event is not expected to be scale invariant. Consider two random variables A(x) and B(x). Another function, C is related to these random variables by this equation:

$$C(x) = A(x) + B(x) \tag{2}$$

For a given value of x, A(x) = a and C(x) = c. Equation 2 states that B(x) must there be equal to $c - a$. As A(x) and B(x) are independent variables, the probability of C(x) = c is given by this equation:

$$P(C(x) = x) = \sum_{a=-\infty}^{\infty} P(A(x) = a) \times P(B(x) = c - a) \tag{3}$$

Therefore, the sum of these two random variables results in a distribution that is the result of a multiplication of the distributions of the random variables. This resulting probability distribution is a different shaped function to the random variables being summed, so this process is not scale invariant. This argument could be extended to large numbers of independent random variables, meaning that one does not expect scale invariance when summing multiple random independent variables. The forest fire model exhibiting self organising criticality is interesting as the algorithm is effectively combining the effects of multiple random variables however the process is scale invariant. This may be because each random variable is not independent as the values of the nodes surrounding a node can affect the value of that node (e.g. a tree with a neighbouring fire will deterministically turn into a fire itself).

2.2 Criticality

In this study a critical regime is defined as a system that exhibits critical behaviour. This means that a clear transition can be measured when a system reaches a certain condition. In the case of the forest fire model, the density of the trees can be seen to increase up to $\approx 50\%$. At this point, the system becomes critical and the next spontaneous burning of a tree results in a large scale burning of the forest. The criticality of this forest fire model depends primarily on the two probability factors f and p. The process of choosing suitable values for these parameters is explained in the next section.

3 Method

3.1 Choosing Parameters

In order to choose the correct parameters for the forest fire model, a combination of trial and error while observing the behaviour of the forest and parameters found in previous studies was used.

In literature [1], a value of $p \ll T(s)$ (the time for a cluster of trees to burn down) is suitable for critical behaviour. A ratio of $f/p \to 0$ is recommended for critical behaviour.

In order to test these recommendations, forest models were created with various values of p and f, with the behaviour of the forest being recorded. Figure 2 shows the graph of number of trees in a forest as a function of time with different values of p.

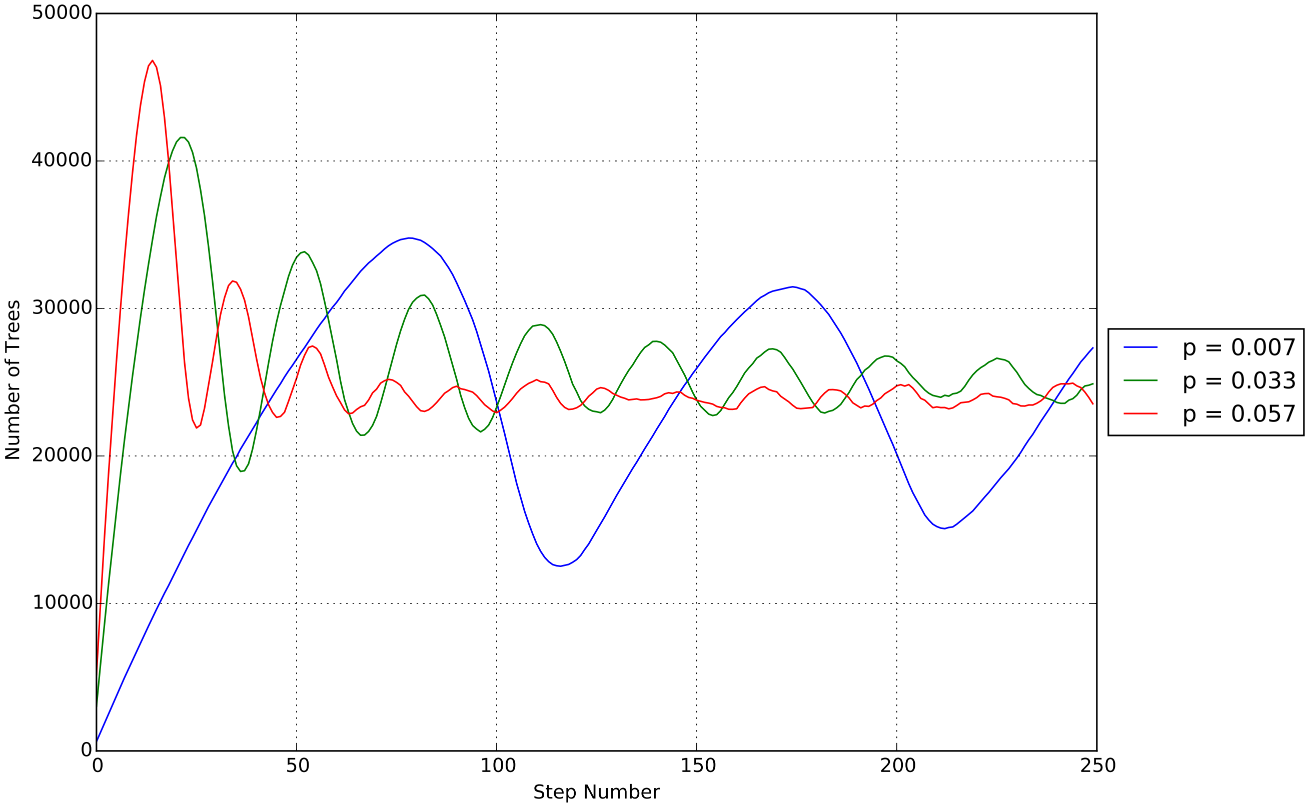

Figure 2: Graph of number of trees per time step for different values of p

This graph shows that as the value of p increases, the behaviour of the forest quickly stops exhibiting self-organised criticality. This occurs with certain values of p and f because the system reaches an equilibrium state where the rate of trees being burnt down is approximately the same as the rate of trees growing. After some trial and error, a value of p of 0.01 and a value of f of 0.0001 are determined to be suitable to produce criticality. Figure 3 shows the critical behaviour of a 100 by 100 grid forest with $p = 0.01$ and $f = 0.0001$:

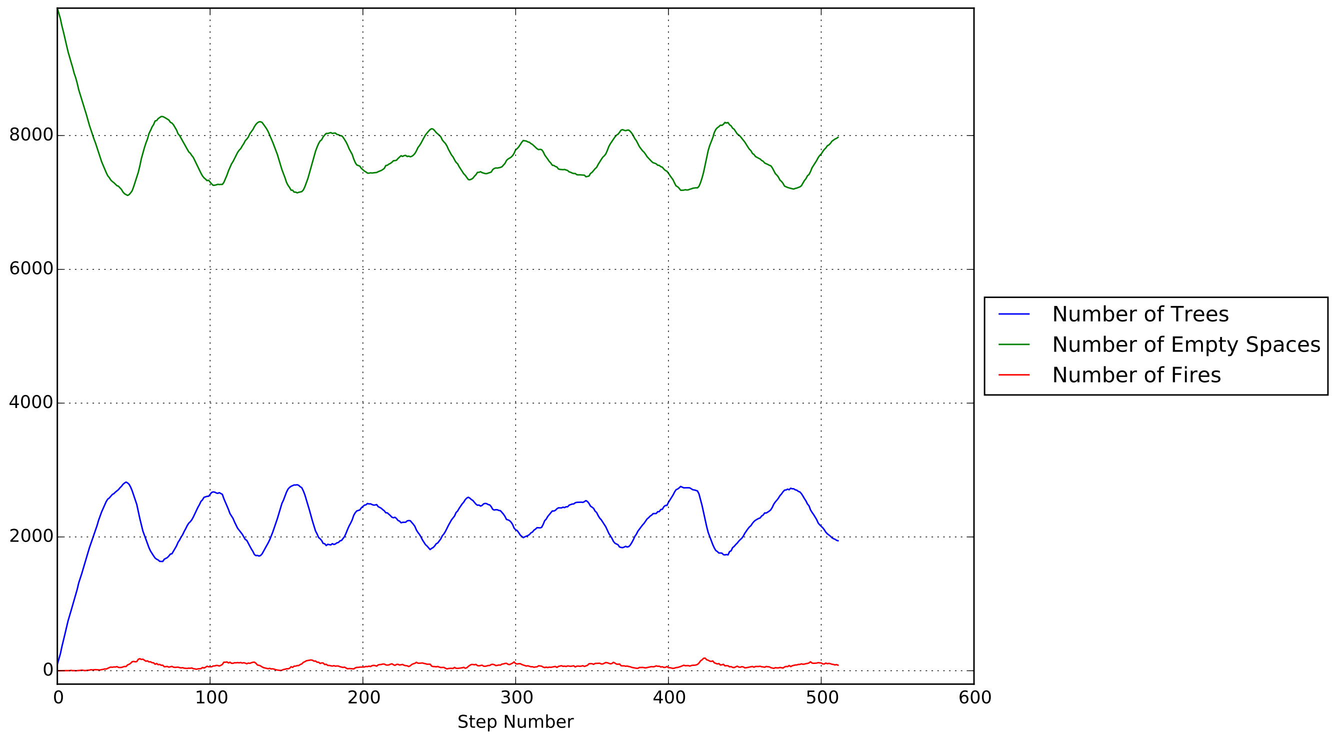

Figure 3: Graph showing number of trees, empty spaces and fires for 500 steps with $p = 0.01$ and $f = 0.0001$.

This graph shows that as the density of the trees automatically increases to a certain value as the trees grow, the density reaches such a point that a spontaneous fire will burn down a significant portion of the trees. This critical behaviour shows that the default values of $p = 0.01$ and $f = 0.0001$ are satisfactory for investigating the Forest Fire model.

3.2 Quantifying the Behaviour of the Model

3.2.1 Console Display Method

There are multiple methods for quantifying the behaviour of the model. The simplest method is to simply observe the forest as the algorithm progresses. This can be done in two ways, firstly by printing a grid of coloured characters to console and updating the grid of characters once per step of the algorithm. Figure 4 shows the output from this method:

Figure 4: A 40*40 forest outputted with the console method

This method allows for step by step analysis of the progress of the algorithm as the whole progression is printed to console so any point in the process can be viewed and analysed. This method allows the magnitude of a forest fire to be measured by counting the number of trees burnt from a single spontaneous forest fire. This is useful when investigating the power law relation in self-organised criticality as the frequency of forest fires of various magnitudes can be recorded. This method also has an advantage in that the forest can be displayed in an intuitive graphical way with a less resource intensive method than using Gnuplot, the second method. However, there is a limit to the size of forest that can be displayed with the console method as the characters have a set size so this method is disabled by default.

3.2.2 Gnuplot Display Method

The second method is to use the Gnuplot program to display a scatter plot of the trees and fire in such a way that the forest can be shown graphically, updating once per second. The output from this method is shown in Figure 5:

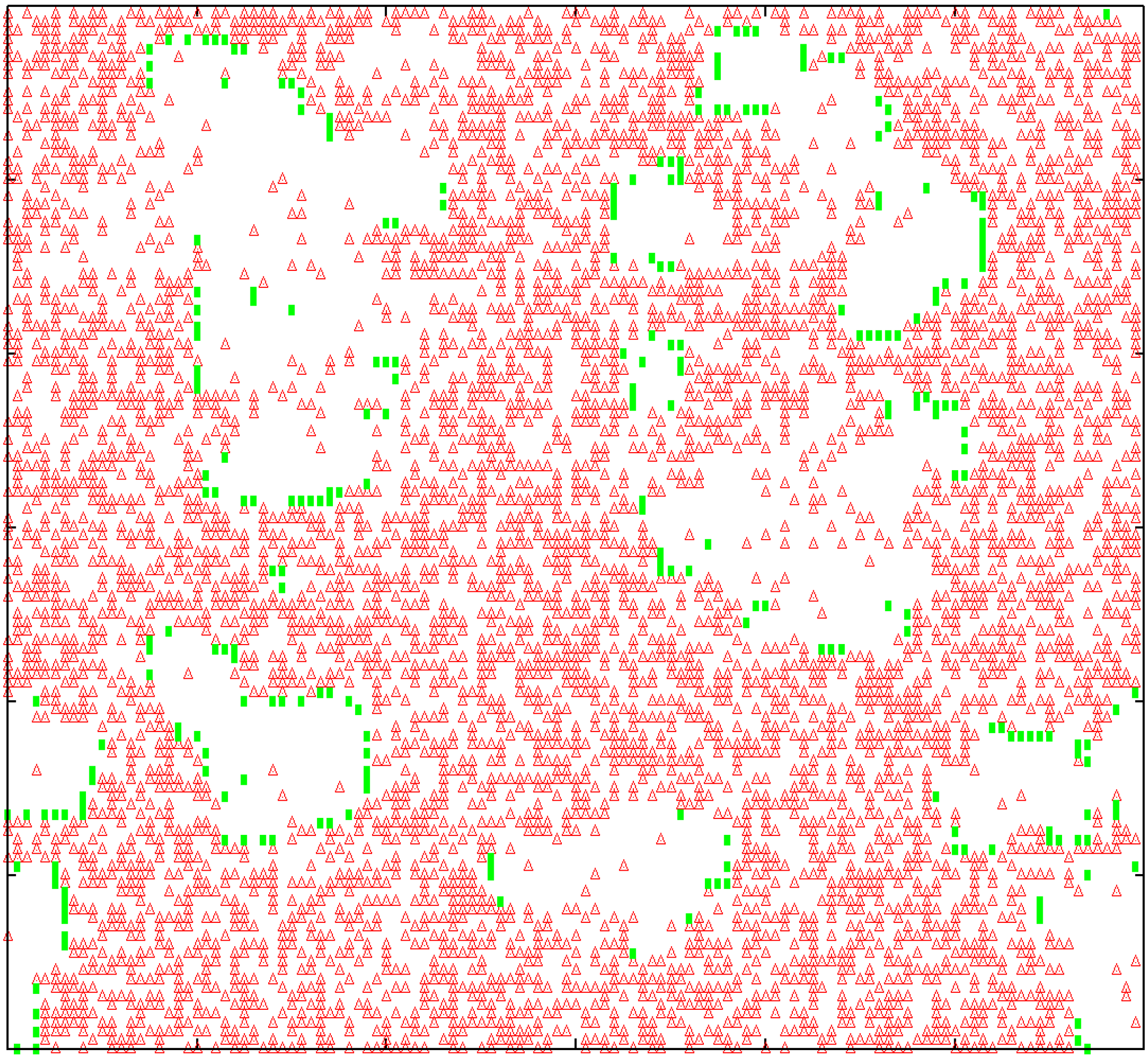

Figure 5: A 120*120 forest outputted with the gnuplot method.

This method is advantageous as the density of the forest and the progression of the fire is much easier to see. This graph shows the state of the forest just before the majority of the trees burn down, The density of this forest is high enough that the existing fires shown will continue to progress outwards until the forest is mostly burned down.

3.2.3 Quantitative Output

The final method of quantifying the behaviour of the forest is a count of the number of trees, empty spaces and fires in the forest. This method is useful as it allows the overall condition of the forest to be summarised by three values. These values can be recorded to a text file to be used with graphing software (This study uses graphs produced by the matplotlib module in Python). These graphs can be used to show the critical behaviour of the forest as the fluctuation in number of trees corresponds to the transition in state of the system when it reaches criticality.

3.3 Verification of Results

In the forest fire model, the results are verified by checking the outputs to see if the behaviour of the forest is what is expected given the starting conditions. For example, the forest starts with either 0 trees or a random distribution of trees in a certain percentage of places, so the quantitative output can be used to check that the percentage of trees is correct. Once a fire spontaneously appears, the algorithm is meant to progress in a deterministic way through the trees. This can be verified using the console display method or the gnuplot display method. The criticality of the system can be verified by checking that the population of trees fluctuates in a predictable manner.

3.4 Optimisation

The model has multiple configuration options which can limit the processor demands of the program. The time taken for each step increases as the grid size of the forest increases, so the default grid size is 120*120. However, larger and smaller grid sizes can be selected by the user. The grid size has a large impact because it increases the processing time of each step, so any time increase is multiplied by the number of steps. The number of steps is directly proportional to the processing time so the number of steps is by default 100 steps, but this can also be changed by the user.

The console display method, Gnuplot display method and quantitative output can all be turned on or off by the user. The gnuplot display method is more resource intensive so if speed is required, a small grid used with the console display method will proceed faster. The quantitative output also has a small impact on performance.

4 Results

4.1 General Behaviour of the Model

Figure 6 shows the cyclic nature of the self organised criticality in the forest fire model:

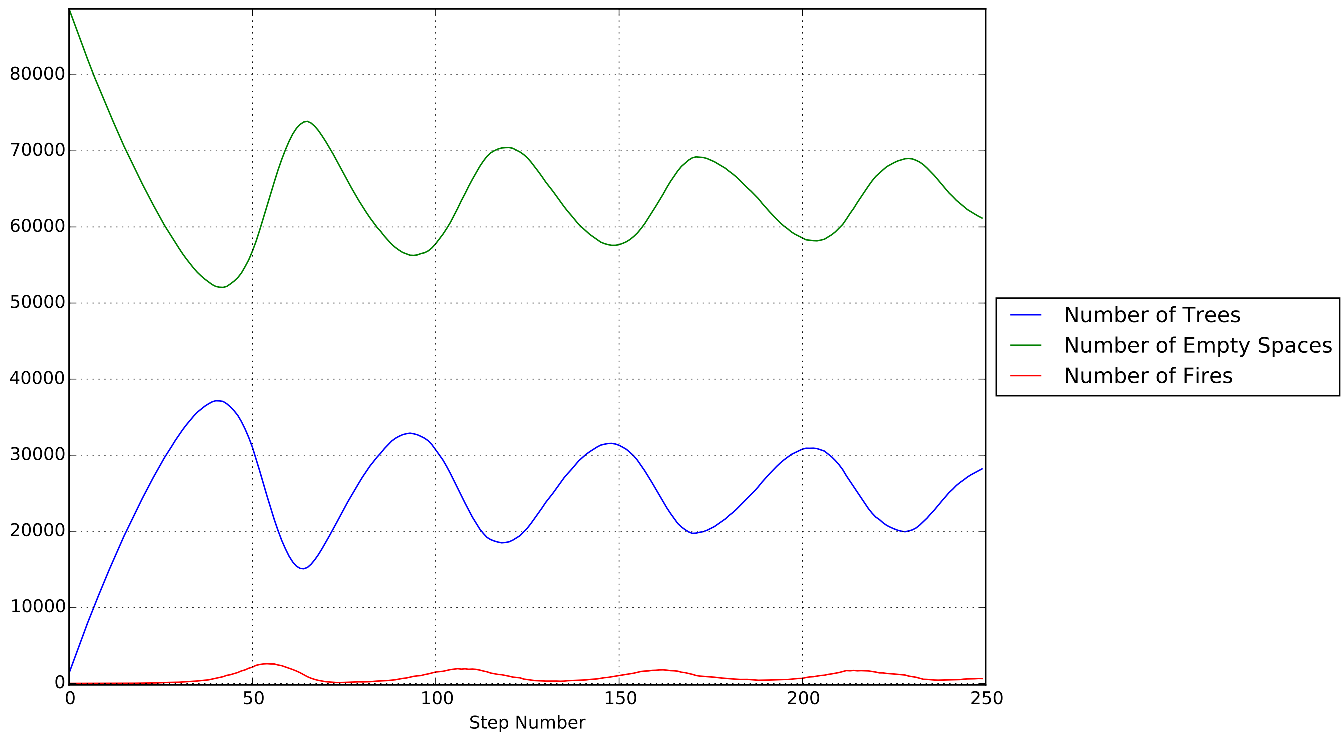

Figure 6: The number of trees, empty spaces and fires in 300*300 forest over 250 time steps with p = 0.001

This forest begins completely empty. Trees begin to grow, increasing the density until the spontaneous fire can travel across the width of the forest. This self-organising criticality is cyclic, with a frequency of ≈ 50 between one peak and another. The number of trees appears to mirror the number of empty spaces as when a tree grows it replaces an empty space and when a tree burns it turns into an empty space. The number of fires appears to be one quarter wavelength out of phase with the other waves. This occurs because the fires are at their greatest density when the trees are decreasing in number at their greatest rate.

4.2 Scale Invariance

Figure 7 shows the scale invariance of the self organised criticality in the forest fire model:

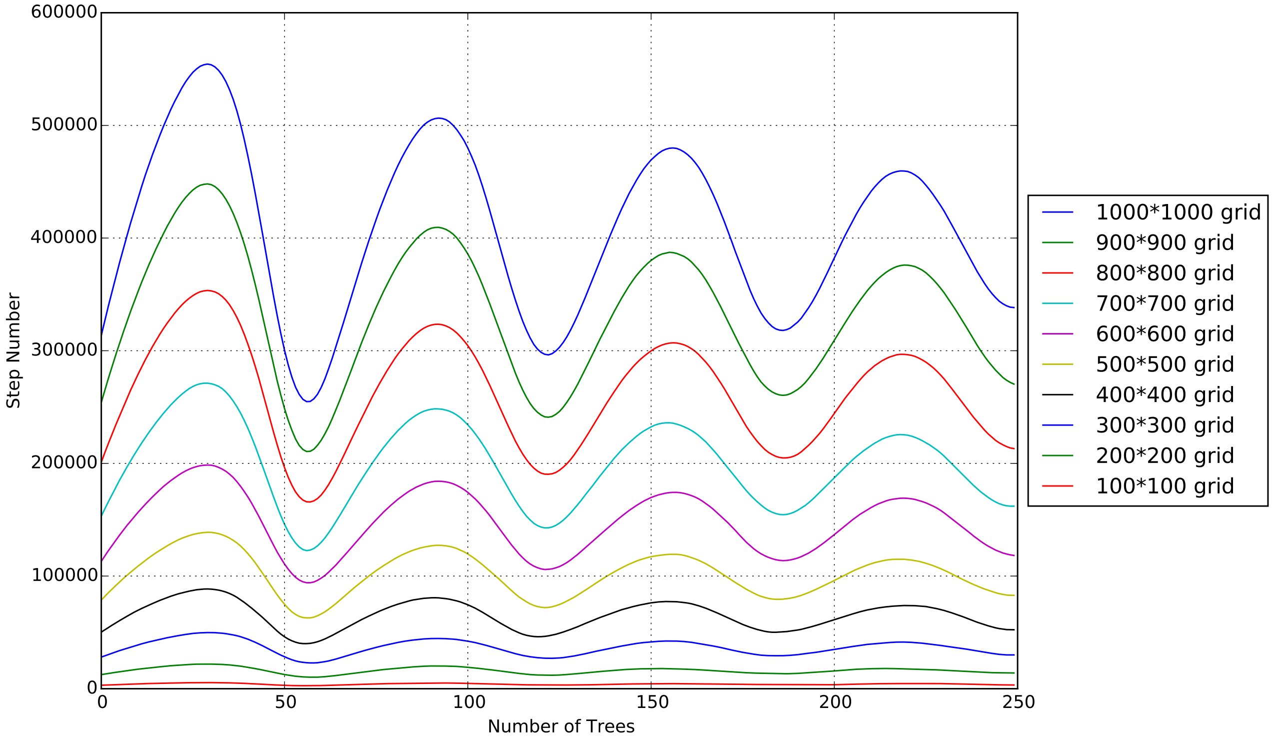

Figure 7: The number of trees over 250 iterations of the algorithms with grids of various sizes.

This graph shows that the critical behaviour of the forest model behaves independent of scale, with a 500*500 grid acting in exactly the same way as a 1000*1000 grid forest. The peaks and troughs of each wave occur after the same number of iterations, with the wavelength being consistent across the sizes at ≈ 70.

4.3 Power Law

Figure 8 shows the relation between the magnitude of the event and the frequency of the event occurring in the self organised criticality in the forest fire model:

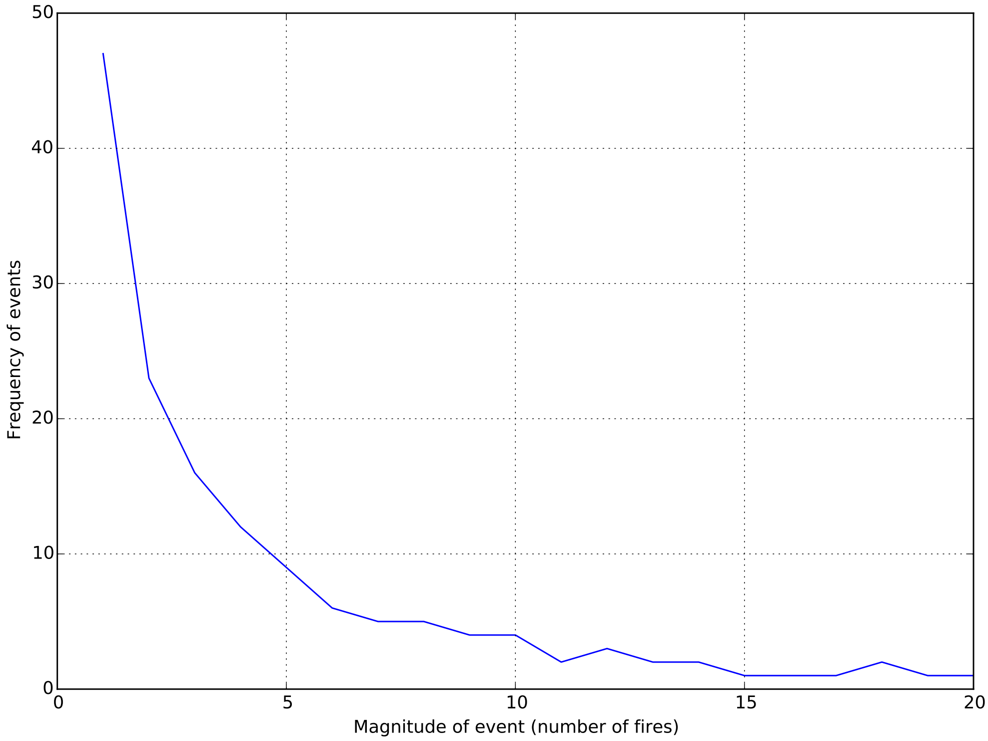

Figure 8: The frequency of an event vs the magnitude of the event

In this case the magnitude of the event is measured by the number of trees burnt down by a spontaneously occurring fire. This graph shows that the probability of an event occurring decreases with a negative exponent with the magnitude of the event. The relationship shown here is similar to other systems of the same universality class such as the sand-pile model. This result is predicted by equation 1.

4.4 Variations to the algorithm

Initially when investigating the forest fire model, an algorithm similar to the one used here was developed, except fires had a burn time of 3 turns rather than 1. This caused some interesting effects, as the wavefront of the fire was much easier to see. The fire burned a larger proportion of the forest, perhaps because there was less space behind the wavefront of the fire to allow for new trees to grow. Weather conditions such as wind direction or rain to limit to propagation of the fire could be implemented to increase the real world usefulness of this forest fire model. However, this model may not exhibit self organised criticality.

Other factors can be used to change the propagation of the fire. For instance, increasing the number of burning neighbours required to set a tree alight causes the fire to only propagate in denser areas of forest. Another factor that can be changed is whether a neighbouring fire is defined as being either above, below or next to a tree or whether diagonal neighbours are also counted. The following graphs show the difference in behaviour between the four-neighbour method and the 8-neighbour method:

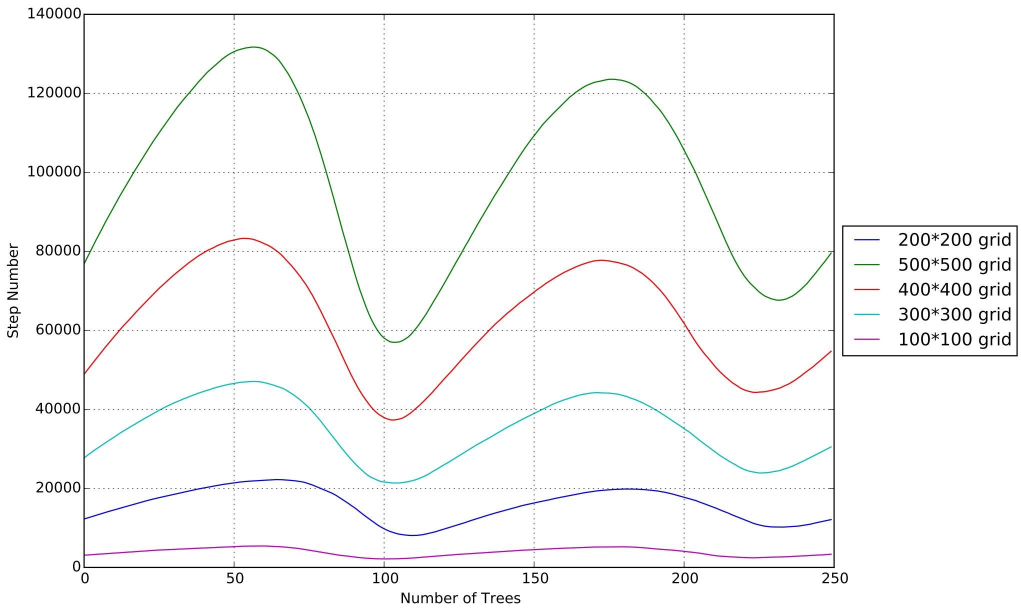

Figure 9: The number of trees over 250 iterations of various sized grids at constant p and f values with the 4 neighbour method

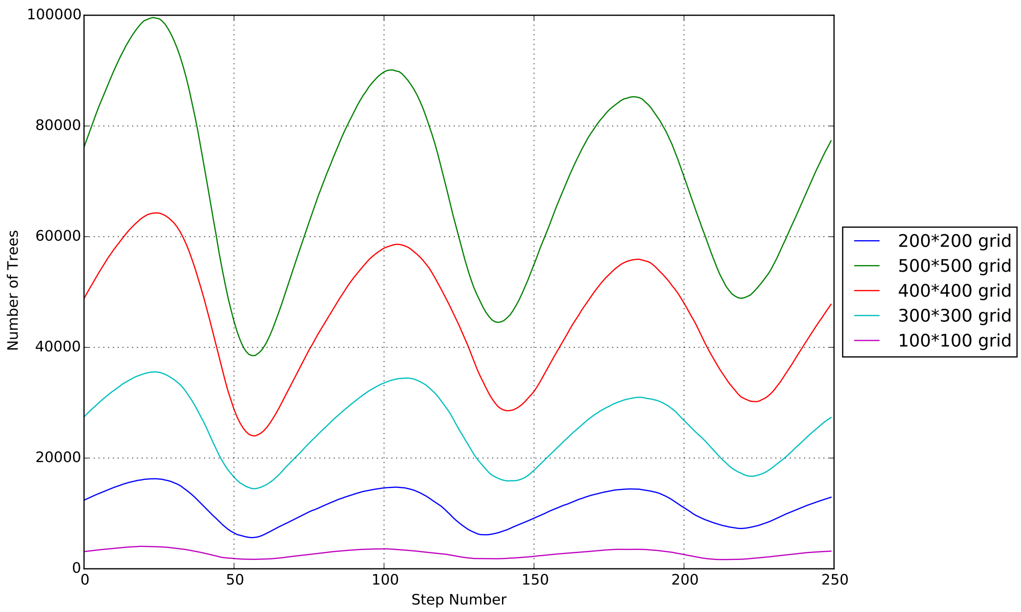

Figure 10: The number of trees over 250 iterations of various sized grids at constant p and f values with the 8 neighbour method

These graphs shows that the number of neighbours considered when updating the algorithm has a significant effect on the self organised criticality of the fire. The 4-neighbour method (not including diagonal neighbours) has a much longer wavelength for the fluctuation in numbers of trees than the 8-neighbour method. Perhaps this is because the fire can more easily traverse a sparse area of trees when it can travel diagonally as well as up, down, left or right.

The starting distribution of trees also affects how the forest changes in subsequent iterations. The user can set a percentage of spots to be empty spaces or trees when the forest is first created. If the forest is completely empty to begin with, the forest tends to grow to a larger density before burning down than if the forest begins with an existing distribution of trees. This could happen because the system is oscillating around an equilibrium point where the rate of burning trees is the same as the rate of growth. If the system is initiated with a larger displacement from the equilibrium (with 0% trees rather than 30% trees), the amplitude of oscillation around the equilibrium point is likely to be larger. Figure 11 shows the effect of starting with an empty forest. The initial rise in density of trees is very pronounced, then starts to decrease in amplitude.

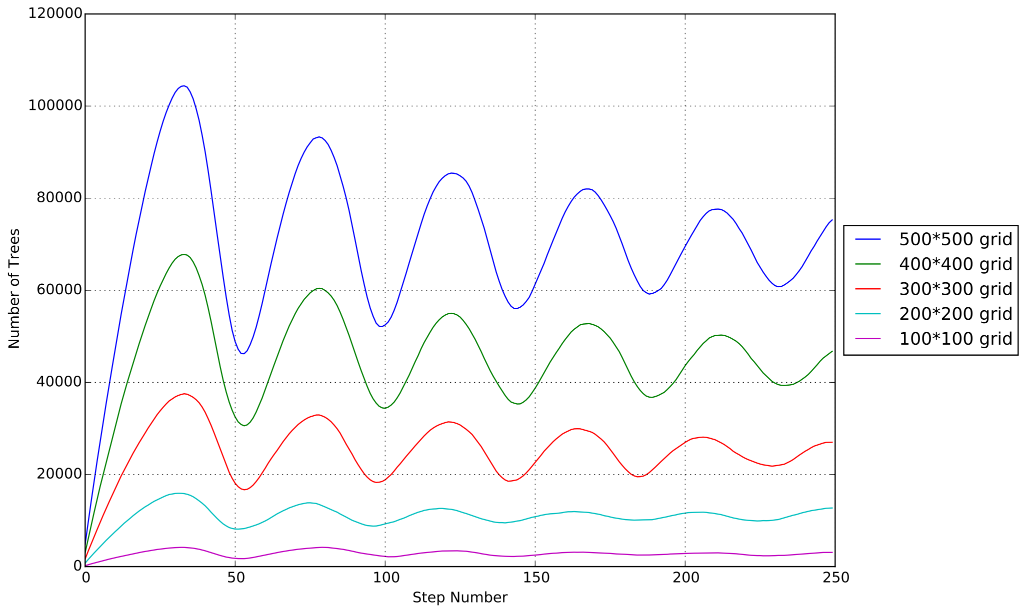

Figure 11: The number of trees over 250 iterations of various sized grids at constant p and f values starting with an empty forest

5 Conclusion

Having observed the behaviour of the forest fire model over multiple configurations, I conclude that the model does express self organised criticality when the starting conditions are correctly chosen. The correct starting configurations include a forest size of 50*50 or greater, with a periodic boundary. This allows for the forest to act as if it were effectively infinite, meaning that there is no special behaviour to observe at the boundaries. A burn time of one time step allows the fire to quickly move through the forest at a maximum speed of a square with its size increasing by 2 after each iteration.

The value of p and f chosen are also important, larger values of p causing the amplitude of the oscillation of number of trees to quickly decay into an equilibrium state. This behaviour is not self-organised criticality, so a value of p below 0.02 is recommended. The value of f should be significantly smaller than p because this gives the forest a chance to increase the density of the trees to such a point where a fire can burn down a large portion of the forest. If f is not much smaller than p, the forest will have lots of small fires and self organised criticality cannot be conserved.

The behaviour of this model has been shown both analytically and empirically to be scale invariant. This means that given similar probabilities and starting conditions, a forest of any size will exhibit self organised criticality. The frequency of the forest reaching a critical point is also independent of the size of the forest.

The power law associated with self organised criticality has been empirically observed, with larger magnitude events having a much lower probability than smaller magnitude events. The relative frequency of these events follows a negative exponential which is expected given the power law in equation 1.

References

[1] B. Drossel and F. Schwabl, Self-Organized Critical Forest-Fire Model, Published 14/09/92, Accessed 20/12/15, The American Physical Society