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1. In Experiment 1, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning.
   The probability of winning $80 after 1000 bets is very close to 1. In my simulations, the average number of spins before winning $80 is about 170 (and STD at spin 200 is nearly 0 in all simulations). This can also be seen by simply glancing at Figure 1.

   The code used to compute this probability is the following:

   ```
   wins = 0
   for s in simulations:
       if s[-1] == 80:
           wins += 1

   probability = wins/1000.0
   ```

   From an analytical perspective, losing N straight spins is not a problem because the spin N + 1 recovers all the eariler loses and wins you $1. Not only that, the probability of losing N straight times is remarkably low even when N is a low as 10: P^N = (20/38)^10 = 0.1631%. The probability is not exactly 1 because, although highly slim, there is always a chance of being 'unlucky' in 1000 spins.

   This would let people to believe this is a winning strategy. However, this only happens because there is unlimited bankroll.

2. In Experiment 1, what is the estimated expected value of our winnings after 1000 sequential bets? Explain your reasoning. Go here to learn about expected value: https://en.wikipedia.org/wiki/Expected_value

   The expected value of the winnings after 1000 bets is very close to $80. This make sense because, as stated in question 1, the probability is close to 1 on winning $80 after 1000 bets (EV = P * V = ~1 * $80 = ~$80). To estimate the expected value, I simply calculated the average of all resulting winnings with the following code:

   ```
   for s in simulations:
       total += s[-1]
   EV = total/len(simulations)
   ```

3. In Experiment 1, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases? Explain why it does (or does not).

   First, I would like to answer this question in terms of the 'mean'. The 'mean' does *tend* to converge. This is because as more spins happen, a greater number of simulations reach the $80 limit. Having said that, even when dealing with the 'mean', it is not guaranteed to reach a local maximum and then never see that maximum again.

   Similar reasoning for can be applied to standard deviation. Does the standard deviation reach a maximum value then stabilize? My answer would be no. The likelihood/probability of STD converging is higher as the number of spins increase because more simulations would have reached the $80 bucket. However, that is not a guaranteed scenerio.

   To explain my reasoning, I'll use a corner case scenario. Let say that 999 of the simulations have reached the $80 bucket. There can still be one simulation with very huge loses. Let's say that one single simulation is losing $1T in the spin number 170 (the mean where the $80 gain is reached). If you do standard deviation of values [999 x $80, -$1T x 1] it could lead to a new maximum STD.

   It goes without saying that once all simulations reach the $80 bucket, STD 'stabilizes' to 0.

   I wanted to point out this is a great question by the way!!! It made me think in ways I don't normally think about. My first reaction/response to this question was an obvious 'yes' but the more I thought about it, the more I changed my mind.

4. In Experiment 2, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning using the experiment. (not based on plots)

   The probability of winning $80 after 1000 spins was estimated to be 0.652. That is, on average you will win about 652 out of 1000 simulations. Running more simulations (> 1000) would produce a more accurate estimate but 1000 seems to be good enough. To gather this data, I used the same code as described in question 1.

   This 'winning' probability is significant lower compared to when we had unlimited bankroll. On all simulations, as soon as the bankroll limit is reached -- we run out of money -- and simulation effective ends. This can happen surprisingly pretty quickly. After 8 straight losses, you cannot keep doubling and you are left with $1.

Winnings	Bet	Outcome
1	1	Loss
3	2	Loss

7	4	Loss
15	8	Loss
31	16	Loss
63	32	Loss
127	64	Loss
255	128	Loss
511	256	Can't bet

5. In Experiment 2, what is the estimated expected value of our winnings after 1000 sequential bets? Explain your reasoning. (not based on plots)

   The winnings expected value is estimated to be -$36.729. I estimated this through the MC simulation, in a similar manner described in question 2. Running more simulations (eg 1M) would produce a more accurate EV estimate.

   Although the probability of winning $80 is relatively high (0.652), the expected value says that on the long-run on average we can expected to lose ~$37 after 1000 spins (over many bet days).

   Intuitively, the EV looks correct. To confirm though, we can solve the following:

   EV = P(winning) * $80 + P(losing)*$-256 + P(neither)*V

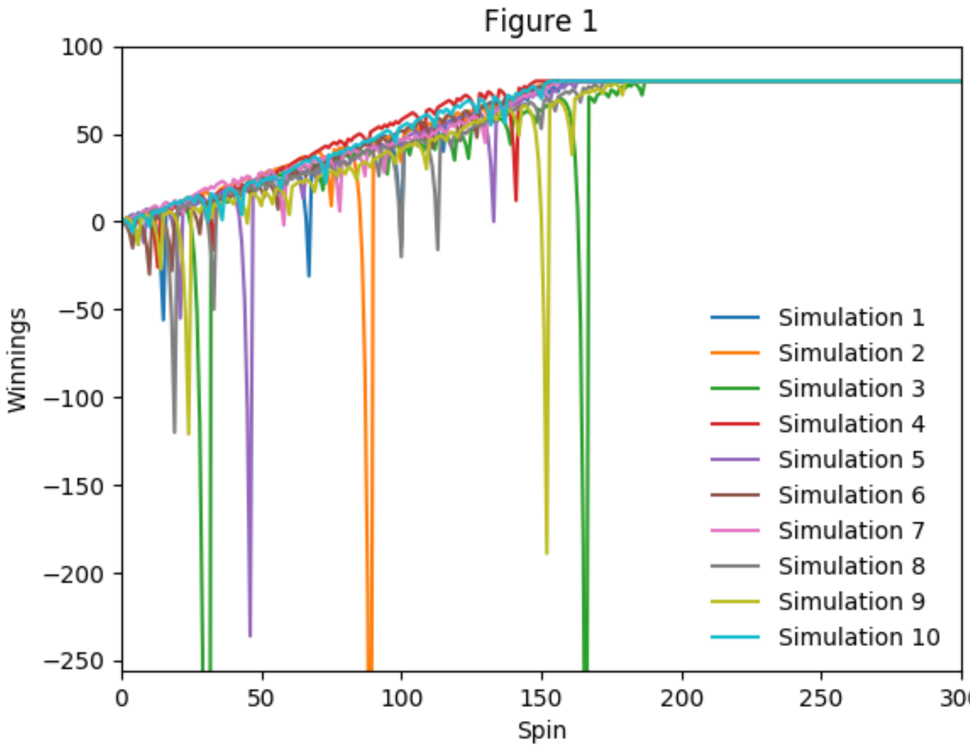
   The last term can be dropped because on average the probability of not winning AND not losing is really low. EV = 0.652*$80 + 0.348 * -$256 = ~-36.928

6. In Experiment 2, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases? Explain why it does (or does not).

   The standard deviation does indeed stabilize in experiment 2. One of the reasons is that the winnings are hard capped (+80 to -256), this prevents outliers from distorting or deviating to far away from the mean. Also, as more spins happen the probability of winning or losing increases and values become more stable.

   One can think of this as having three buckets: A[$80 winners], B[$-256 losers], C[Random]. The more spins, the bigger A and B buckets get. On the other hand, the 'noise' bucket (bucket C) decreases in size.

7. Include figures 1 through 5.



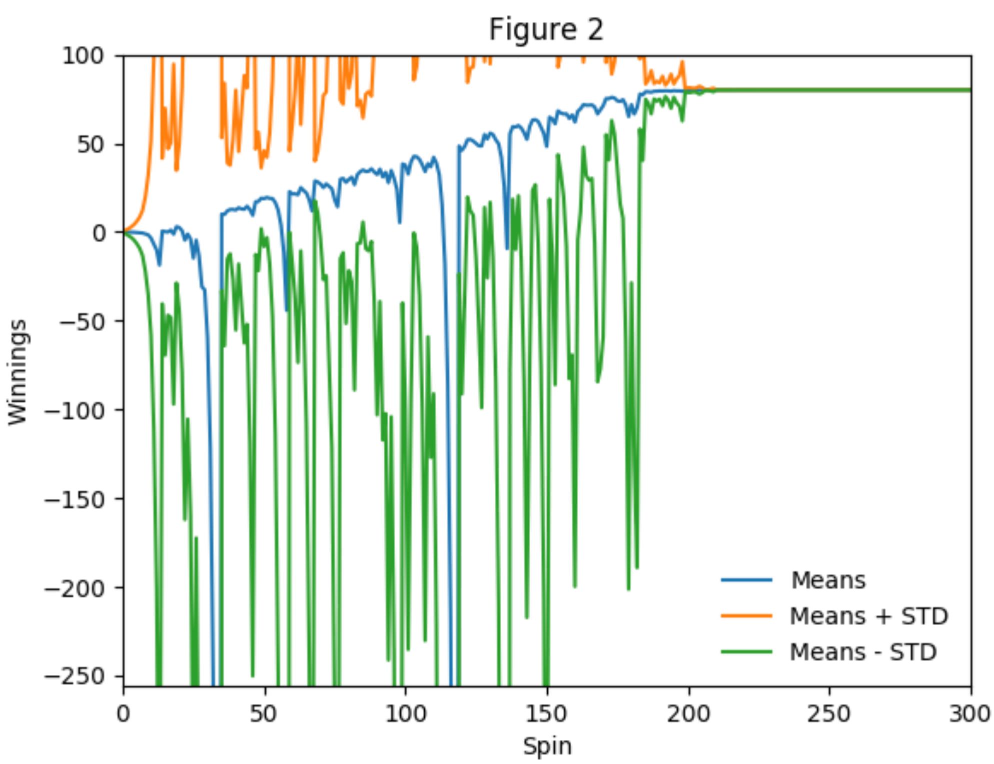


Figure 3

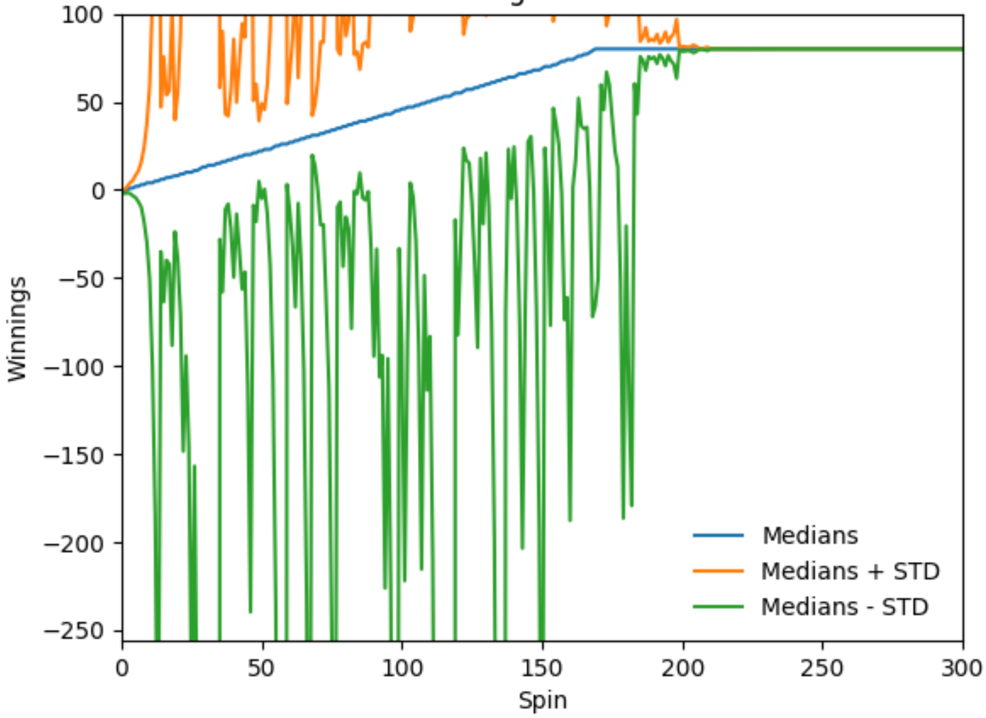
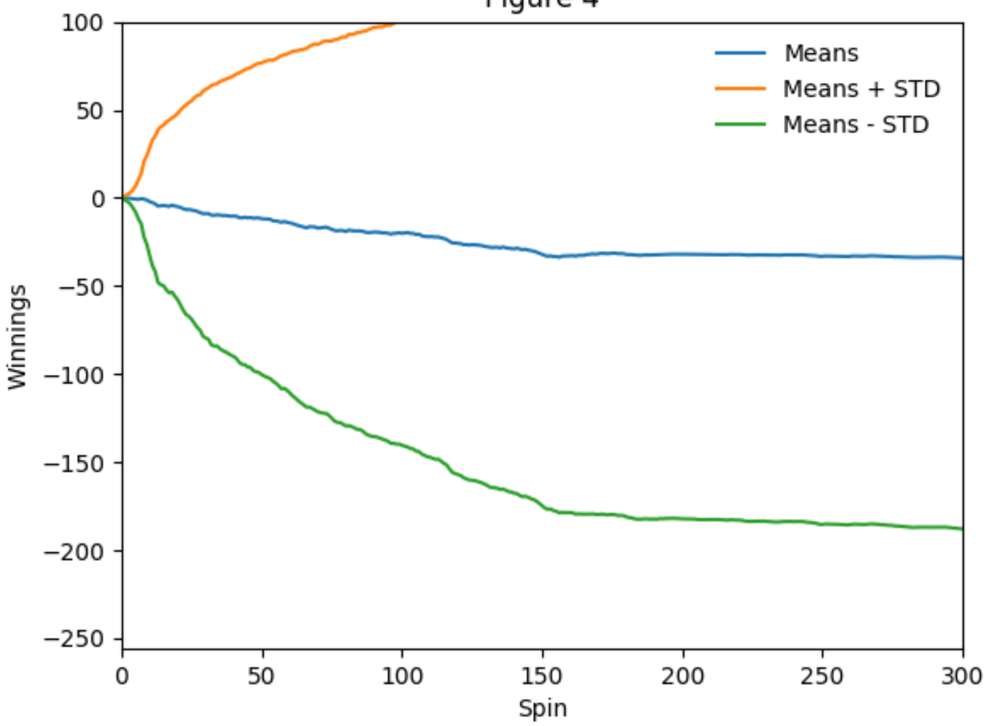


Figure 4



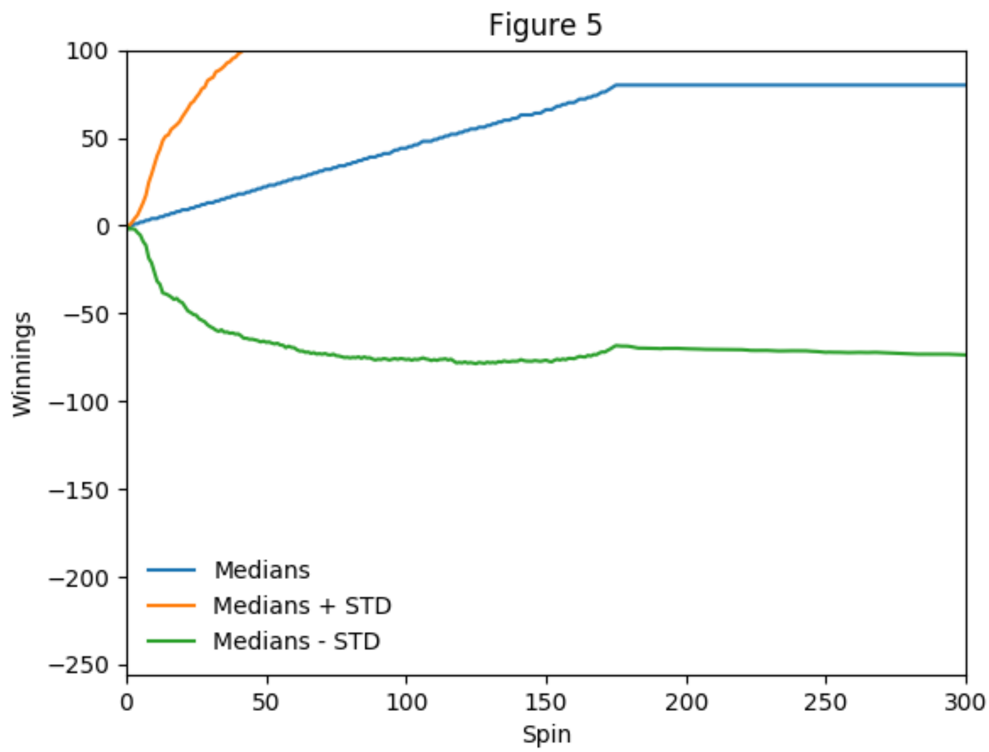


Figure 5
100
50
0
−50
−100
−150
−200
−250
Winnings
0
50
100
150
200
250
300
Spin
Medians
Medians + STD
Medians - STD