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RISK MEASUREMENT WHEN SHARES ARE SUBJECT TO INFREQUENT TRADING

Comment

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When securities are thinly traded OLS techniques yield biased beta estimates. Procedures for calculating consistent estimates are proposed by Scholes and Williams (1977) and by Dimson (1979). This comment examines both procedures and concludes that the Dimson procedure is incorrect and cannot generally be expected to yield consistent beta estimates. However, a variant of this procedure can yield results which are identical to Scholes and Williams' and is, therefore, correct.

Both Dimson (1979) and Scholes and Williams (1977) have developed simple but elegant methods for obtaining estimators of beta in the presence of thin trading. While Scholes and Williams have shown their method to yield a consistent estimator, unfortunately, Dimson's estimator is not specified correctly. The purpose of this comment is to demonstrate that Dimson's estimator is not consistent with that of Scholes and Williams and to provide a corrected version of the former that is.

Dimson's estimator is obtained by first regressing the observed security return against leading, synchronous and lagged values of the appropriate market index to obtain a set of slope coefficients, β_{j+k},

$$R_{jt}=\alpha_j+\sum_{k=-m}^{m}\beta_{j+k}R_{It+k}+\mu_{jt}, \tag{1}$$

where R_{jt} is the security return, R_{It+k} is the market index return with the appropriate lead and lag, α_j is a constant and μ_{jt}, is a 'well-behaved' random variable. Dimson's estimator is then obtained by summing the slope

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coefficients,

$$\text{(DIM)}\quad \operatorname{plim}\hat{\beta}_j = \sum_{k=-m}^{+m} \beta_{j+k}. \tag{2}$$

The values of m are chosen using information about the degree of thinness of the security and/or the index.

Scholes and Williams have shown that, for securities that do not miss an observation, i.e., m in eq. (1) between -1 and $+1$, the following gives a consistent estimate of beta:

$$\operatorname{plim}\hat{\beta}_j = (\beta_j^{-1} + \beta_j^0 + \beta_j^{+1})/(1+2\rho_1), \tag{3}$$

where

- β_j^{-1} is the parameter estimate obtained from the simple regression of R_t against I_{t-1},
- β_j^0 is obtained from the synchronous simple regression,
- β_j^{+1} is obtained from the simple regression of R_t against I_{t+1},
- ρ_1 is the first order serial correlation coefficient for the index.

When securities have single missed observations this technique can be extended to compute a consistent estimate of beta using two period returns instead of a single period. If the trading process is stationary over two periods and a geometric index is used, then two period returns can be expressed as simple aggregates of single period returns and the Scholes–Williams estimator becomes

$$\operatorname{plim}\hat{\beta}_j = ({}_2\beta_j^{-1} + {}_2\beta_j^0 + {}_2\beta_j^{+1})/(1+2{}_2\rho_I), \tag{4}$$

where ${}_2\beta_j^{-1}$, ${}_2\beta_j^0$, ${}_2\beta_j^{+1}$ are the OLS estimators corresponding to eq. (3) but based on two period returns. The two period betas and the serial correlation coefficient are determined as follows:[1]

$$\begin{aligned} {}_2\beta_j^{+1} &= \frac{\operatorname{cov}({}_2R_{jt}, {}_2R_{It+1})}{\operatorname{var}({}_2R_{It+1})} \\ &= \frac{\operatorname{cov}(R_{jt}+R_{jt-1}, R_{It+2}+R_{It+1})}{\operatorname{var}(R_{It+2}+R_{It+1})}, \\ &= \frac{2\operatorname{cov}(R_{jt}, R_{It+2}) + \operatorname{cov}(R_{jt}, R_{It+1})}{2\operatorname{var}(R_{It}) + 2\operatorname{cov}(R_{It}, R_{It-1})}, \end{aligned} \tag{5}$$

[1]It is assumed that only covariances with lags of two or less are non-zero. By assumption no returns with greater lags have any synchronous portions.

$$
\begin{aligned}
{}_2\beta_j^0 &= \frac{\operatorname{cov}({}_2R_{jt}, {}_2R_{It})}{\operatorname{var}({}_2R_{It})} \\
&= \frac{2\operatorname{cov}(R_{jt}, R_{It}) + \operatorname{cov}(R_{jt}, R_{It-1}) + \operatorname{cov}(R_{jt}, R_{It+1})}{2\operatorname{var}(R_{It}) + 2\operatorname{cov}(R_{It}, R_{It-1}).}, \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
{}_2\beta_j^{-1} &= \frac{\operatorname{cov}({}_2R_{jt}, {}_2R_{It-1})}{\operatorname{var}({}_2R_{It-1})} \\
&= \frac{2\operatorname{cov}(R_{jt}, R_{It-2}) + \operatorname{cov}(R_{jt}, R_{It-1})}{2\operatorname{var}(R_{It}) + 2\operatorname{cov}(R_{It-1})}, \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
{}_2\rho_1 &= \frac{\operatorname{cov}({}_2R_{It}, {}_2R_{It-1})}{\operatorname{var}({}_2R_{It-1})} \\
&= \frac{\operatorname{cov}(R_{It}, R_{It-1}) + 2\operatorname{cov}(R_{It}, R_{It-2})}{2\operatorname{var}(R_{It}) + 2\operatorname{cov}(R_{It}, R_{It-1})}, \qquad (8)
\end{aligned}
$$

$$
1 + 2{}_2\rho_1 = \frac{\operatorname{var}(R_{It}) + 2\operatorname{cov}(R_{It}, R_{It-1}) + 2\operatorname{cov}(R_{It}, R_{It-2})}{\operatorname{var}(R_{It}) + \operatorname{cov}(R_{It}, R_{It-1})}, \qquad (9)
$$

where ${}_2R_{jt}$ and ${}_2R_{It}$ denote two period returns. Dividing the numerator and denominator of eqs. (5), (6), and (7) by $\operatorname{var}(R_{It})$ and summing leads to

$$
{}_2\beta_j^{+1} + {}_2\beta_j^0 + {}_2\beta_j^{-1} = (\beta_j^{+2} + \beta_j^{+1} + \beta_j^0 + \beta_j^{-1} + \beta_j^{-2})/(1 + \rho_1). \qquad (10)
$$

Dividing numerator and denominator of eq. (9) by $\operatorname{var}(R_{It})$ gives

$$
1 + 2{}_2\rho_I = (1 + 2\rho_1 + 2\rho_2)/(1 + \rho_1). \qquad (11)
$$

Dividing eq. (10) by eq. (11) gives the extended Scholes and Williams estimator,

$$
\operatorname{plim} \hat{\beta} = (\beta^{-2} + \beta^{-1} + \beta^0 + \beta^{+1} + \beta^{+2})/(1 + 2\rho_1 + 2\rho_2). \qquad (12)
$$

In this equation ρ_2 is the second order serial correlation coefficient and the -2 and $+2$ superscripts imply a lead and lag of 2, respectively. Eq. (12) makes better use of the available information than eq. (3) when a security skips price observations because all the data can be used in the process. The procedure can be generalized to securities that skip two or more consecutive price observations.

Dimson's approach can now be related to Scholes and Williams' procedure. The Dimson regression that should be used when securities skip single price observations is

$$R_{jt} = \alpha_j + \beta_{j+2}R_{It+2} + \beta_{j+1}R_{It+1}\beta_{j0}R_{It} + \beta_{j-1}R_{It-1} + \beta_{j-2}R_{It-2} + \mu_{jt}. \tag{13}$$

Minimizing the sum of the squared error term in eq. (13) leads to the following set of five equations:[2]

$$\beta_{j+2} + \rho_1\beta_{j+1} + \rho_2\beta_{j0} = \beta_j^{+2}, \tag{14}$$

$$\rho_1\beta_{j+2} + \beta_{j+1} + \rho_1\beta_{j0} + \rho_2\beta_{j-1} = \beta_j^{+1}, \tag{15}$$

$$\rho_2\beta_{j+2} + \rho_1\beta_{j+1} + \beta_{j0} + \rho_1\beta_{j-1} + \rho_2\beta_{j-2} = \beta_j^{0}, \tag{16}$$

$$\rho_2\beta_{j+1} + \rho_1\beta_{j0} + \beta_{j-1} + \rho_1\beta_{j-2} = \beta_j^{-1}, \tag{17}$$

$$\rho_2\beta_{j0} + \rho_1\beta_{j-1} + \beta_{j-2} = \beta_j^{-2}. \tag{18}$$

The coefficients on the left-hand side of each of these equations are those derived from the Dimson multiple regression whereas those on the right-hand side are the Scholes and Williams coefficients from simple regressions.

Summing the right-hand side of these equations and dividing by $(1+2\rho_1+2\rho_2)$ gives the Scholes and Williams $\operatorname{plim}\hat{\beta}_j$ estimator as in eq. (12). Summing the left-hand side of the equations and again dividing by $(1+2\rho_1+2\rho_2)$ gives

$$\begin{aligned}\operatorname{plim}\hat{\beta}_j = {} & \frac{(1+\rho_1+\rho_2)}{(1+2\rho_1+2\rho_2)}\beta_{j+2} + \frac{(1+2\rho_1+\rho_2)}{(1+2\rho_1+2\rho_2)}\beta_{j+1} + \beta_{j0} \\ & + \frac{(1+2\rho_1+\rho_2)}{(1+2\rho_1+2\rho_2)}\beta_{j-1} + \frac{(1+\rho_1+\rho_2)}{(1+2\rho_1+2\rho_2)}\beta_{j-2}.\end{aligned} \tag{19}$$

Eq. (19) demonstrates that Dimson's estimator [eq. (2)] is inconsistent with Scholes and Williams' and, therefore, incorrect.[3] A weighted rather than an

[2]This derivation assumes that ρ_1 and ρ_2 are non-zero and all other serial correlation coefficients are zero.

[3]In an earlier version of this paper (available on request) we showed, somewhat tediously, the fallacies in Dimson's argument that led him to claim that his estimator was consistent. Our interest in this problem was sparked by some tests of the comparative ability of several different techniques for removing thin trading bias. In these empirical tests the Dimson estimator proved generally inferior to Scholes and Williams' and was frequently outperformed by simple OLS. See Fowler et al. (1980).

unweighted sum of the slope coefficients is required to obtain a consistent beta estimate.[4] Eq. (19) does, however, present a corrected procedure for Dimson's estimator that is quite operational since the coefficients must be weighted by functions of the observable serial correlation coefficients for the index. Even with this modification Dimson's procedure may still be more economical than Scholes and Williams'. In the example with two leads and two lags, the former only requires one multiple regression per security and two for the index whereas the latter requires five simple regressions for each security and two for the index. The issue then is whether one multiple regression is more expensive than several single regressions.

[4]Eq. (19) gives no indication of the direction of the bias as the non-synchronous β_k can be either positive or negative. In aggregate, however, as Dimson's simulations show, the biases may cancel out. Dimson's procedure is clearly consistent when a fat index is used because $\rho_1=\rho_2=0$.

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