Smart Beta 2.0

June 2013

Noël Amenc
Professor of Finance, EDHEC Business School
Director, EDHEC-Risk Institute

Felix Goltz
Head of Applied Research, EDHEC-Risk Institute

Lionel Martellini
Professor of Finance, EDHEC Business School
Scientific Director, EDHEC-Risk Institute

EDHEC-RISK
Institute

Table of Contents

About the Authors

Noël Amenc is professor of finance at EDHEC Business School, director of EDHEC-Risk Institute and CEO of ERI Scientific Beta. He has conducted active research in the fields of quantitative equity management, portfolio performance analysis, and active asset allocation, resulting in numerous academic and practitioner articles and books. He is on the editorial board of the *Journal of Portfolio Management* and serves as associate editor of the *Journal of Alternative Investments* and the *Journal of Index Investing*. He is a member of the scientific board of the French financial market authority (AMF), the Monetary Authority of Singapore Finance Research Council and the Consultative Working Group of the European Securities and Markets Authority Financial Innovation Standing Committee. He co-heads EDHEC-Risk Institute's research on the regulation of investment management. He holds a master's in economics and a PhD in finance.

Felix Goltz is head of applied research at EDHEC-Risk Institute and research director at ERI Scientific Beta. He does research in empirical finance and asset allocation, with a focus on portfolio construction and indexing strategies. His work has appeared in various international academic and practitioner journals and handbooks. He obtained a PhD in finance from the University of Nice Sophia-Antipolis after studying economics and business administration at the University of Bayreuth and EDHEC Business School.

Lionel Martellini is professor of finance at EDHEC Business School, scientific director of EDHEC-Risk Institute and senior scientific advisor at ERI Scientific Beta. He has graduate degrees in economics, statistics, and mathematics, as well as a PhD in finance from the University of California at Berkeley. Lionel is a member of the editorial board of the *Journal of Portfolio Management* and the *Journal of Alternative Investments*. An expert in quantitative asset management and derivatives valuation, his work has been widely published in academic and practitioner journals and he has co-authored textbooks on alternative investment strategies and fixed-income securities.

Abstract

Recent years have seen increasing interest in new forms of indexation, referred to as Smart Beta strategies. Investors are attracted by the performance of these indices compared to traditional cap-weighted indices. However, by departing from cap-weighting, Smart Beta equity indices introduce new risk factors for investors, and no sufficient attention is presently given to the evaluation of these risks. In addition, the Smart Beta market appears to be inefficient today, due to restricted access to information, as well as lack of independent analysis.

This paper puts forth a new approach to Smart Beta Investment, called the Smart Beta 2.0 approach.

In fact, a first important step towards a better understanding of Smart Beta strategies is to conduct proper analysis of risk and performance of Smart Beta strategies rather than relying on demonstrations of outperformance typically conducted by the providers of the strategies.

Secondly, Smart Beta 2.0 allows investors to not only assess, but also to control the risk of their investment in Smart Beta equity indices. Rather than only proposing pre-packaged choices of alternative equity betas, the Smart Beta 2.0 approach allows investors to explore different Smart Beta index construction methods in order to construct a benchmark that corresponds to their own choice of risks. In particular, we discuss the following types of risk: i) exposure to systematic risk factors (which can be managed through stock selection decisions or factor constraints); ii) exposure to strategy specific risk (which can be managed by diversifying across strategies); and iii) relative performance risk with respect to traditional market cap-weighted benchmarks (which can be managed through tracking error control).

Introduction: Taking the Risks of Smart Beta Equity Indices Into Account

In recent years, in both the United States and Europe, there has been increasing talk of the pre-eminence of beta in asset management.

As such, in two studies published by EDHEC-Risk Institute analysing new index offerings and investor reactions in both Europe[1] and North America[2], these investors provide evidence of their increasing appetite for passive investment (about 90% of the respondents to the European Index Survey 2011 and to the North American Index Survey 2011 use indices as a reference for all or part of their investment in equities) and their interest in new forms of indexation referred to as advanced or Smart Beta (more than 40% of investors have already adopted alternative weighting schemes and over 50% see their current cap-weighted indices as problematic). Exhibit 1 below summarises some of the results taken from these two surveys.

From our viewpoint, this dual interest is part of an evolution in asset management that perhaps goes further than the growing momentum towards passive investment for cost reasons or doubts over active managers' capacity to justify their management fees by producing significant and consistent alpha.

The success of Smart Beta with institutional investors largely outstrips the initial framework that was established for it, namely that of replacing the natural passive investment reference represented by cap-weighted indices. For one thing, it is easy to observe that cap-weighted indices have no equivalent when it comes to representing market movements; for another, it is equally plain to see that they remain the simple reference understood by all investors and stakeholders in the investment industry. In the end, even the biggest critics of cap-weighted indices constantly refer to cap-weighted indices to evaluate the performance of their new indices.

In fact, the reason behind the new indices for the vast majority of investors, and doubtless their promoters, is probably the superiority of their performance compared to traditional cap-weighted indices. Everyone agrees that while cap-weighted indices are the best representation of the market, they do not necessarily constitute an efficient benchmark[3] that can be used as a reference for an informed investor's strategic allocation. In other words, they do not constitute a starting point (for active investment) or an end point (for passive investment) that offers, through its diversification, a fair reward for the risks taken by the investor. Alternative Beta, also known as Advanced Beta or Smart Beta is therefore a response from the market to a question that forms the basis of Modern Portfolio Theory since the work of the

Exhibit 1: Summary of the results on equity indices for EDHEC-Risk Institute Surveys in Europe and North America – The table summarises the responses of investment professionals to the questions relating to the use of indices in investment. The results shown here can be found in Exhibits 9, 12 and 15 of EDHEC-Risk North American Index Survey 2011 and in Table 2 and Exhibits 8 and 10 of EDHEC-Risk European Index Survey 2011.

Question	North America	Europe
Percentage of respondents who use indices in their equity investments	88.9%	91.4%
Percentage of respondents who are satisfied with their index investments in the area of equity	68.8%	67.1%
Percentage of respondents who see significant problems with cap-weighted equity indices	53.2%	67.7%
Percentage of respondents who have adopted any alternative weighting scheme in their equity investments	42.1%	45.2%

1 - Amenc, Goltz and Tang (2011).
2 - Amenc, Goltz, Tang and Vaidyanathan (2012).
3 - This question of the efficiency of the benchmark is moreover totally independent of the existence or otherwise of an efficient market. On this point, see "Inefficient Benchmarks in Efficient Markets" by Lionel Martellini, www.edhec-risk.com. Editorial, 4 October, 2012.

Introduction: Taking the Risks of Smart Beta Equity Indices Into Account

Nobel Prize winner Harry Markowitz: How is an optimally diversified portfolio constructed?

As with any technique or any model, implementation of these new forms of benchmarks is not risk-free. In order to justify why cap-weighted indices are no longer considered as good benchmarks, Smart Beta promoters raise their risks of concentration, and rightly so, but it is also necessary to grasp the risks to which investors are exposed when they adopt alternative benchmarks.

Talking about the superiority of Smart Beta equity indices over the long term is totally legitimate, but it is also perfectly legitimate to discuss the sources of this outperformance, the risks of the outperformance not being robust, or even the conditions of underperformance in the short or medium term.

This is one of the objectives of what EDHEC-Risk Institute calls the Smart Beta 2.0 approach. This new vision of Smart Beta investment, which over the past three years has been subject to a considerable research effort on the part of the Institute, ultimately aims to allow investors to control the risk of investment in Smart Beta equity indices so as to benefit fully from their performance.

Each Smart Beta solution contains risks, which can be filed in two categories: systematic risks and specific risks.

1.1 Systematic risks of Smart Beta strategies

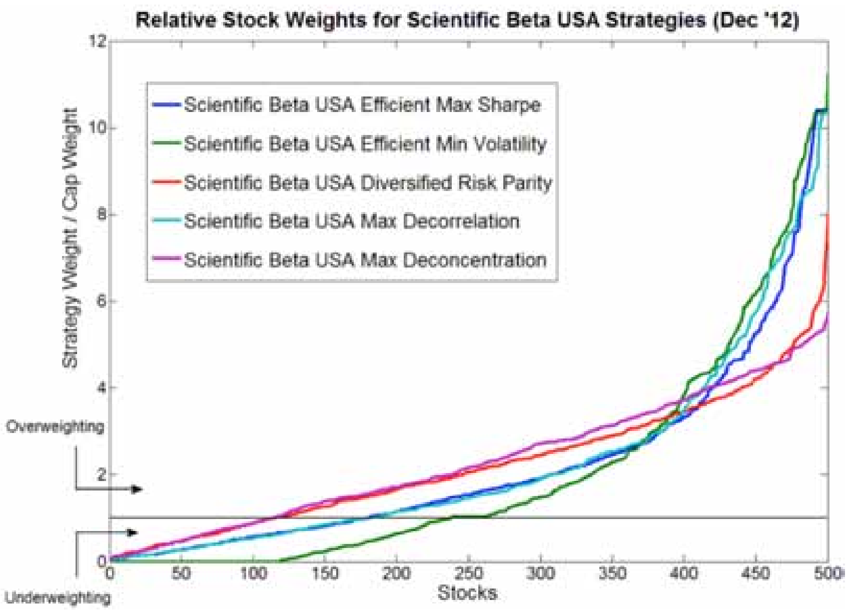
Systematic risks come from the fact that new indices or benchmarks can be more or less exposed to particular risk factors depending on the methodological choices guiding their construction; this exposure can be expressed either in absolute terms or more often in relative terms with respect to the cap-weighted index which is representative of the same universe of securities. For example, an index construction scheme that uses indicators of the firm's economic size leads more often than not to the index being exposed to style biases such as value or small cap biases compared to its cap-weighted reference index. In the same way, compared to the cap-weighted reference index, a scheme that favours low-volatility stocks will lead to overexposure to some sectors and of course to a very different exposure to the volatility factor. More globally, given that a cap-weighted index is typically concentrated in a very small effective number[4] of highly liquid stocks, any deconcentration of the benchmark will necessarily lead to an increase in the exposure to less liquid stocks. Exhibit 2 shows the weights of stocks relative to their market capitalisation weight in a range of Smart Beta indices. It is clear from this illustration that all strategies lead to significant increases of weights for some stocks relative to their market capitalisation weight.

In the area of systematic risk, much thought has been given in recent years to the construction quality of Smart Beta benchmarks.

The first generation of Smart Beta benchmarks are embedded solutions which do not distinguish the stock picking methodology from the weighting methodology. As such, they oblige the investor to be exposed to particular systematic risks which represent the very source of their performance.

As such, since they deconcentrate cap-weighted indices that are traditionally exposed to momentum and large growth

Exhibit 2: Weight profile of Smart Beta strategies – The figure plots the ratio of stock weight in the following Scientific Beta USA strategy indices – Efficient Max Sharpe, Efficient Min Volatility, Diversified Risk Parity, Max Decorrelation, and Max Deconcentration – to stock weight in the cap-weighted index based on same stock universe. The weights at the rebalancing date of 21 December 2012 are used for the analysis. Data on constituent weights and market cap weights was downloaded from www.scientificbeta.com.



4 - The effective number of stocks is defined as the reciprocal of the Herfindahl Index, a commonly used measure of portfolio concentration. The Herfindahl index is defined as the sum of squared weights across portfolio constituents.

risk, the first-generation Smart Beta indices are often exposed to value, small or mid-cap, and sometimes contrarian, biases. Moreover, specific tilts on risk factors that are not particularly related to deconcentration but to the objectives of the scheme itself, can amplify these biases. For example, fundamentally weighted indices have a value bias due to their use of accounting measures which are related to the ratios applied in the construction of value indices. Exhibit 3 reports factor exposures of a selection of popular Smart Beta indices from major index providers.

The results show that all the Smart Beta indices analysed exhibit significant exposures to equity risk factors other than the market factor.[5] The FTSE RAFI, FTSE EDHEC-Risk Efficient and the S&P 500 equal weighted indices show large and significant exposure to the small cap factor. The MSCI USA Minimum Volatility index, on the other hand, has a significantly negative small cap exposure suggesting exposure to large caps. The FTSE RAFI U.S. 1000 Index has a significant exposure to the value factor. All strategies exhibit a negative exposure to the momentum factor. It is well known that cap-weighting has a built in momentum feature by the nature of its construction and any strategy that deviates from cap weighting, and hence avoiding this feature, is expected to show a negative exposure to the momentum factor.[6]

Another type of bias is the low-volatility bias of the minimum-volatility weighting method, which often favours the low-volatility argument over the decorrelation one in reaching the low volatility objective. The biases of low volatility portfolios towards the least volatile stocks have been well documented in the literature. For example, Clarke, de Silva and Thorley (2011) write that the long-only Global Minimum Variance (GMV) "portfolio averages about 120 long securities, i.e., about 12% of the 1000-security investable set". Likewise, DeMiguel, Garlappi, Nogales and Uppal (2009) argue that "Short sale-constrained minimum-variance portfolios […] tend to assign a weight different from zero to only a few of the assets". This concentration leads to biases towards low risk (beta)

Exhibit 3: Factor Exposures of Commercial Smart Beta Equity Strategies – The table shows the risk factor exposures of FTSE RAFI U.S. 1000 Index, FTSE EDHEC Risk Efficient U.S. Index, MSCI USA Minimum Volatility Index, and S&P 500 Equal Weight Index using the Carhart 4 factor model (Carhart, 1997). The Market factor is the daily return of cap-weighted index of all stocks that constitute the Scientific Beta USA universe. SMB factor is the daily return series of a portfolio (cap-weighted) that is long the top 30% of stocks (small market-cap stocks) and short the bottom 30% of stocks (large market-cap stocks) sorted on market capitalisation. HML factor is the daily return series of a portfolio (cap -weighted) that is long the top 30% of stocks (value stocks) and short the bottom 30% of stocks (growth stocks) sorted on book-to-market value. MOM factor is the daily return series of a portfolio (cap-weighted) that is long the top 30% of stocks (winner stocks) and short the bottom 30% of stocks (loser stocks) sorted on past returns in descending order. . The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate in US Dollars. Risk-free rate is the return of 3 months US Treasury Bill. The following regression is run for each index over the period of analysis.

$$R_P = \alpha + \beta_{Market}.(R_{CW} - R_{rf}) + \beta_{Size}.SMB + \beta_{Value}.HML + \beta_{Momentum}.MOM \qquad (1)$$

Daily total return data for all indices has been obtained from Datastream and factor returns are obtained from www.scientificbeta.com. Period of analysis is from 23 December 2002 to 31 December 2012 and betas significant at the 1% confidence level are highlighted in bold. Reported alphas are geometrically averaged and are annualised.

	FTSE RAFI U.S. 1000 Index	FTSE EDHEC Risk Efficient U.S. Index	MSCI USA Minimum Volatility Index	S&P 500 Equal Weight Index
Annualised Alpha	2.3%	**3.3%**	2.9%	**2.7%**
Market Beta	**97.9%**	**93.1%**	**79.9%**	**102.4%**
Size (SMB) Beta	**14.8%**	**39.9%**	**-5.8%**	**40.5%**
Value (HML) Beta	**15.8%**	0.4%	4.7%	1.5%
Momentum (MOM) Beta	**-11.6%**	**-5.6%**	**-0.7%**	**-8.1%**
Adjusted R-square	98.5%	98.7%	95.1%	98.9%

5 - The fact that the R-squared in such a regression is often close to 1 is sometimes seen as evidence that all weighting schemes are very similar and simply consist of factor exposures to standard equity risk factors. This argument is however misleading as a high R-squared does not contradict that the source of outperformance is improved diversification. In fact, if these index strategies were able to outperform with an R-squared of 1, this would mean that this outperformance would be entirely due to a value-added of such strategies. Technically speaking, the outperformance of such strategies may be due to cancelation of the negative alpha of cap-weighted indices which is due to their poor diversification.
6 - For more discussion, please refer to Amenc, Goltz, Martellini and Ye (2011).

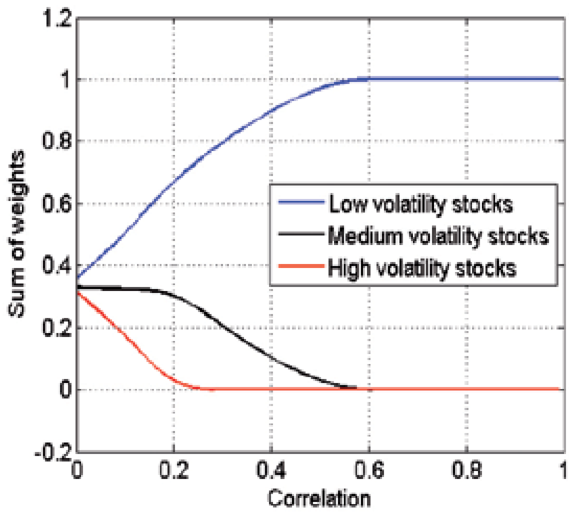
stocks and utility stocks (Chan, Karceski and Lakonishok, 1999).

These style biases, and more globally these risk factors, had been ignored in the promotion of the performance of first-generation Smart Beta indices. The foundation paper on fundamental indexation (Arnott, Hsu and Moore, 2005), which highlights the outperformance of this new form of beta in comparison with cap-weighted indices, does not contain any measure of the exposure of fundamentally-weighted indices to different style factors. The first documentation on these risk factors, and notably the value bias that explains a large part of the outperformance of fundamental indices was produced by others than the promoters of these indices and was only published in 2007/2008.[7] Likewise, promoters of equal-weighted indices have documented outperformance without correcting performance for risk factor exposures such as small cap and liquidity risk[8] and promoters of Equal Risk Contribution (ERC) strategies, while pointing out that fundamental equity indexation suffers from value exposure, did not report any results on value and small cap exposures of the ERC strategy in their foundation paper (Demey, Maillard and Roncalli, 2010).

We can also note that certain weighting schemes that are supposed to, for example, provide well-diversified proxies for efficient frontier portfolios can lead to considerable concentration in a small number of stocks and/or exhibit very pronounced sector or style biases.

For example, focusing on volatility minimisation leads to selecting the least volatile stocks irrespective of their other properties. In particular, it has been widely recognised that low volatility stocks display, more often than not, severe sector biases such as biases towards utility stocks (see Chan, Karceski and Lakonishok, 1999). Exhibit 4 shows an illustration of the concentration of minimum volatility portfolios in low volatility stocks, taken from Amenc, Goltz and Stoyanov (2011). They create minimum volatility portfolios based on a simulated universe of 100 stocks and then sort the stocks by volatility into three groups of equal size. Exhibit 4 figure shows the weights of the groups

Exhibit 4: Concentration of Minimum Volatility portfolios in low volatility stocks – The universe contains 100 stocks, the volatilities of which are equally spaced in the corresponding intervals. Correlations between stock returns are assumed constant. The Global Minimum Variance (GMV) portfolios are long-only. For additional details, see Amenc, Goltz and Stoyanov (2011). These results are based on equally spaced volatilities between 16% and 18%.



7 - Perold (2007) argues that "Fundamental indexing is a strategy of active security selection through investing in value stocks". Jun and Malkiel (2008) assess the performance of the FTSE RAFI index and find that the alpha of this index is zero after adjusting for market, value and small cap exposure.
8 - For example, Dash and Zheng (2010) analyse the performance of the S&P 500 equal-weighted index. While they provide evidence on style exposure to small cap and value, they do not provide any formal attribution of performance to these exposures.

as a function of correlation in the stock universe. The results show that Global Minimum Variance (GMV) portfolios get severely concentrated in the low volatility group of stocks as correlation across stocks increases.

Ultimately, it is often only by imposing specific sector constraints associated with minimal and maximal weights per stock that indices based on minimum volatility approaches avoid the problems posed by their natural over-concentration. The question then arises of the influence of these constraints, which are necessarily defined ex-post on the historically simulated performance of these indices that were created very recently.

1.2 Identifying Specific Risks of Smart Beta Investing

The second type of risk to which investors are exposed when they use a benchmark is the risk that is specific to the construction of that benchmark. Whatever the weighting scheme envisaged, it relies on modelling assumptions and on parameter estimation, which obviously always leads to a risk of a lack of out-of-sample robustness. Any investor who strays from a weighting scheme such as capitalisation weighting, for which the assumptions that determine the construction are largely open to criticism and not proven, and whose outputs are hardly compatible with the definition of a well-diversified portfolio, will probably take a well-rewarded risk, in the sense that there is a strong probability of doing better in the long term.[9] However, by moving away from the consensus, from the default option constituted by the cap-weighted indices, this investor will be questioned on the relevance of the new model chosen and on the robustness of the past performance that will probably underpin their choice to a large degree. In this sense, like in the area of systematic risk, every informed Smart Beta investor will have to be clear-sighted and carry out sound due diligence to evaluate the specific risks rather than rely only on an assessment of the past performance of the index.

We believe that the specific risk dimension should be better taken into account in the choices that investors make in the area of Smart Beta. Too often investors stop at performances that are composed over fairly short periods, or are the fruit of simulated track records. There is no reason to criticise this situation in itself, because firstly the track records are often limited by the availability of data, and secondly, since they were created recently, Smart Beta indices cannot exhibit live performance over the long term. Nonetheless, this weakness in the statistics should logically lead investors to analyse the robustness conditions of the performance displayed. In the area of specific risk, two competing effects, namely *parameter estimation risk* and *optimality risk*, should be taken into account.

In this document, we will describe these two dimensions of specific risk for an equity portfolio construction strategy according to the following decomposition, and we refer the reader to Martellini, Milhau and Tarelli (2013) for more details.

$$\textit{Total specific risk} = \textit{parameter estimation risk} + \textit{optimality risk} \qquad (1)$$

9 - It is often argued that cap-weighting can be justified by Sharpe's (1964) Capital Asset Pricing Model (CAPM). It should be recognised that not only the many assumptions underlying the CAPM are highly unrealistic (e.g., the presence of homogenous expectations and the absence on non tradable assets to name just a few), but also that the CAPM predicts that the true market portfolio, as opposed to any given cap-weighted equity index, is an efficient portfolio. In fact, it is internally inconsistent for the unobservable (Roll's, 1977, critique) cap-weighted true market portfolio to be efficient and for a cap-weighted equity portfolio extracted from the whole investment universe to be also efficient. This is because the design of an efficient equity portfolio taken in isolation from the rest of the investment universe ignores the correlation of selected stocks with the rest of the investment universe, while these correlations are taken into account in the design of an efficient portfolio for the whole investment universe. In other words, if it was the true asset pricing model the CAPM would predict that a cap-weighted equity portfolio cannot be an efficient portfolio since it instead predicts that the true market portfolio is an efficient portfolio. See also "Cap-Weigted Portfolios" section in the Appendix.

Parameter estimation risk: Risk of errors in parameter estimates

Parameter estimation risk relates to the risk of an imperfect estimation of the required parameters. Due to the presence of estimation errors on expected return, volatility and correlation parameters, portfolios that rely on Maximum Sharpe Ratio (MSR) optimisation based on sample estimates typically perform poorly out of sample (DeMiguel, Garlappi, and Uppal, 2009). Particularly critical is the presence of errors in expected return parameter estimates, given that such estimates are more noisy compared to risk estimates due to the slow convergence of sample-based expected return estimators (Merton, 1980), and optimisation procedures are more sensitive to errors in expected return parameters versus errors in risk parameters (e.g., Chopra and Ziemba, 1993).[10]

In this context, one first natural approach to addressing the concern over sensitivity to errors in parameter estimates consists of *improving parameter estimates* typically by imposing some structure to the statistical problem so as to alleviate the reliance on pure sample-based information. It is in this area that the research in financial econometrics has led to the most progress, whether it involves reducing the dimensionality of the set of parameters to be estimated (robust estimation of the variance-covariance matrices) or having less sample-dependent estimators to take account of the dynamics of their variation (GARCH model for example). In particular, expected returns and risk parameters can be inferred from an asset pricing model such as Sharpe's (1964) CAPM or Fama and French (1993) three factor model. In this context, one needs to estimate the sensitivity to each asset with respect to the systematic factors, as well as the expected return and volatility of the factors, which typically involves (for parsimonious factor models and large portfolios) a dramatic reduction in the number of parameters to estimate, and consequently an improvement in the accuracy of each parameter estimate. The key trade-off, however, is between model risk, namely the risk of using the wrong asset pricing model, e.g., using a single-factor model while the true data generating process originates from a multi-factor model, and sample risk involved in purely relying on sample-based information with no prior on the prevailing asset pricing model.

Hence we conclude that, in general, parameter estimation risk can be further decomposed as follows:

$$\textit{Parameter estimation risk} = \textit{parameter sample risk} + \textit{parameter model risk} \tag{2}$$

Here again, we feel it is important to stress that parameter estimation risk, with the notable exception of Equal-Weighting, does exist in the construction of Smart Beta benchmarks. On the other hand, the rhetoric from the promoters of fundamental or qualitative approaches, according to whom parameter estimation risk would arise only when some kind of optimisation is performed, does not seem to us to correspond to the reality of the risks. Indeed, any portfolio construction technique that uses parameters is confronted with the risk of estimating the parameters. The fact that these parameters are averaged accounting values as in the case of fundamental equity indexation strategies, which gives them less variability, does not in any way solve the problem of the out-of-sample robustness of the estimation of the parameters. On the contrary, the highly backward-looking aspect of parameter estimations based both on accounting values, and especially their average, often

10 - In order to avoid the pitfalls of estimating expected returns directly from past realised returns, Amenc, Goltz, Martellini and Retkowsky (2011) estimate expected returns indirectly by assuming a positive relation between expected returns and downside risk of stock deciles.

leads to parameter values that are highly sample-dependent. If we refer to the economic size of the banking sector, using an average of bank sizes between 2004 and 2008 as a proxy for the economic size in 2009 does not necessarily give particularly relevant values. Ultimately, by denying the estimation risks of non-quantitative schemes, the promoters of *ad-hoc* approaches that are referred to as qualitative do not position themselves well to manage these risks properly or to allow investors to analyse them. Indeed, the performance of fundamentals-based strategies are very sensitive to strategy specification as can be seen from Exhibit 5, which shows the maximum calendar year difference between different forms of fundamental indices, where fundamental weighted portfolios are constructed using different variants for each of the three specification choices – the choice of a fundamental weighting variable, that of a stock selection methodology and that of the rebalancing timing (while using the default choices for the other two specification choices[11]).

The annual return difference between using 'earnings' and 'dividends' to construct a fundamental equity indexation strategy can be as high as 10.8% in a given year. Moreover, note that in 2009, the March rebalanced portfolio outperformed the September rebalanced portfolio by 11.1%.

Optimality risk: Risk of ignoring parameter estimates

A second approach to the challenge posed by sensitivity of portfolio optimization procedures to errors in parameter estimates consists of *ignoring parameter estimates* by using an objective different from Sharpe ratio maximisation that requires fewer if any parameter estimates.[12] For example, one may decide to use a cap-weighted portfolio or an equally-weighted portfolio, which requires no information about the risk and return characteristics of the portfolio constituents. In the same spirit, weighting schemes based on fundamental accounting information also do not rely on risk and return parameter estimates even though they require, as explained below, other parameter estimates. Other strategies such as Global Minimum Variance, Risk Parity or Maximum Diversification Ratio strategies, to name a few, solely rely on risk parameter estimates, thus avoiding the risk of using highly noisy estimates for expected returns.

In order for the methodologies proposed by the promoters of indices to be robust, i.e., to allow long-term outperformance over cap-weighted indices, the selection or weighting model must not be dictated by an in-sample choice but correspond to realistic assumptions. Within the framework of Modern Portfolio Theory, the realism or relevance of the assumptions that underlie

Exhibit 5: Maximum calendar year performance difference under different strategy specifications – The table shows the returns of best and worst performing variants for each specification of the fundamental weighting scheme on the universe of top 1000 US stocks. Variants for weighting variable selection are Book Value, Cash Flow, Dividends, Earnings, Net Sales, and an equally-weighted composite of these 5 fundamentals. Variants for stock selection are the top 1,000 stocks by fundamentals size and the top 1,000 stocks by market-cap. Variants for rebalancing are June, September, December, and March annual rebalancing dates. Portfolios are formed using fundamental data and monthly returns from the period January 1982 to December 2010 and are rebalanced annually. Returns and fundamental data are obtained from Datastream and Worldscope. All returns reported are geometric averaged and are annualised.

Specification	Best Performing Fundamental Equity Strategy		Worst Performing Fundamental Equity Strategy		Maximum return difference	Year
	Variant	Annual return	Variant	Annual return		
Variable selection	Earnings	-12.2%	Dividends	-23.0%	10.8%	1999
Selection effect	Fundamental selection	4.6%	Market Cap selection	2.3%	2.3%	2003
Rebalancing	Annual in March	11.3%	Annual in September	0.2%	11.1%	2009

11 - The default choices of the specifications are composite weighting using the five variables, using the same fundamentals for stock selection, and rebalancing in June.
12 - A related effort to alleviate the concern over estimation risk consists in introducing portfolio weight constraints (see Jagannathan and Ma, 2003, for hard constraints, DeMiguel, Garlappi, Nogales and Uppal, 2009, for soft (norm) constraints).

the model are often appreciated through the concept of optimality. In that case, it involves understanding how a particular portfolio diversification weighting scheme is situated in comparison with the optimal portfolio constituted by the Maximum Sharpe Ratio (MSR) portfolio and we wish here to stress the importance for investors of paying attention to the assumption of optimality of the weighting model proposed. In other words, giving up on (some) parameter estimates, as opposed to trying and improving parameter estimates, implies an efficiency cost related to the use of a portfolio that is *a priori* suboptimal, since it only coincides with the Maximum Sharpe Ratio portfolio under what are sometimes heroic assumptions. This is what we call optimality risk. For example, a Global Minimum Variance (GMV) portfolio will coincide with a Maximum Sharpe Ratio portfolio (MSR) if expected returns happen to be identical for all stocks, which is hardly a reasonable assumption. In Exhibit 6, we

Exhibit 6: Overview and Specific Risk of Popular Equity Weighting Schemes – The table indicates, for a range of smart beta strategies, the weighting scheme used, the required parameters, and the relevant foundation paper. The column "Optimality conditions" indicates under which conditions each diversification strategy would result in the Maximum Sharpe Ratio portfolio of Modern Portfolio Theory. N is the number of stocks, μ_i is the expected return on stock i, σ_i is the volatility for stock i, ρ_{ij} is the correlation between stocks i and j, μ is the (Nx1) vector of expected return, 1 is the (Nx1) vector of ones, σ is the (Nx1) vector of volatilities, Ω is the (NxN) correlation matrix and Σ is the (NxN) covariance matrix. Please refer to the Appendix for a brief presentation of each strategy.

Strategy	Weighting scheme	Required parameter	Foundation paper	Optimality conditions
Market Cap Weights (CW)	w_{CW}	Observable market cap information	Sharpe (1964)	CAPM assumptions + no other assets*
Diversity Weights (DW)	$w_{DW} = \frac{w_{CW}^{p}}{\mathbf{1}' w_{CW}^{p}}$ **	Observable market cap information	Fernholz and Shay (1982)	Unclear
Fundamental Weights (FW)	$w_{FW} = \frac{\mathbf{s}}{\mathbf{1}'\mathbf{s}}$ ***	Unobservable accounting information	Arnott, Hsu and Moore (2005)	Unclear
Max Deconcentration (MD) / Equal Weights (EW)	$w_{EW} = \frac{1}{N}\mathbf{1}$	None	DeMiguel, Garlappi and Uppal (2009)	$\mu_i = \mu$ $\sigma_i = \sigma$ $\rho_{ij} = \rho$
Risk Parity (RP) also known as Equal Risk Contribution (ERC)****	$w_{RP} = \frac{\beta^{-1}}{\mathbf{1}'\beta^{-1}}$	σ_i, ρ_{ij}	Maillard *et al.* (2010)	$\lambda_i = \lambda$ $\rho_{ij} = \rho$
Diversified Risk Parity (DRP)	$w_{DRP} = \frac{\sigma^{-1}}{\mathbf{1}'\sigma^{-1}}$	σ_i	Maillard *et al.* (2010)	$\lambda_i = \lambda$ $\rho_{ij} = \rho$
Maximum Diversification Ratio (MDR)	$w_{MD} = \frac{\Sigma^{-1}\sigma}{\mathbf{1}'\Sigma^{-1}\sigma}$	σ_i, ρ_{ij}	Choueifaty and Coignard (2008)	$\lambda_i = \lambda$
Global Minimum Variance (GMV)	$w_{GMV} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}$	σ_i, ρ_{ij}	MPT (many papers following Markowitz, 1952)	$\mu_i = \mu$
Max Decorrelation (MDC)	$w_{MDC} = \frac{\Omega^{-1}\mathbf{1}}{\mathbf{1}'\Omega^{-1}\mathbf{1}}$	ρ_{ij}	Christoffersen *et al.* (2010)	$\mu_i = \mu$ $\sigma_i = \sigma$
Diversified Minimum Variance (DMV)	$w_{DMV} = \frac{\sigma^{-2}}{\mathbf{1}'\sigma^{-2}}$	σ_i	N/A	$\mu_i = \mu$ $\rho_{ij} = 0$
Maximum Sharpe Ratio (MSR)	$w_{MSR} = \frac{\Sigma^{-1}\mu}{\mathbf{1}'\Sigma^{-1}\mu}$	μ_i, σ_i, ρ_{ij}	MPT (many papers following Tobin, 1958)	Optimal by construction

**CAPM assumptions imply that the true market portfolio CW is an efficient portfolio. For a given CW equity index to be efficient, the CAPM assumptions are therefore not enough; one also needs to assume that the equity index is the true market portfolio, that is one needs to assume away the existence of any asset other than the constituents of the stock index under consideration.*
***$0 \leq p \leq 1$. If p = 1, DW is similar to CW and if p = 0, DW is similar to EW.*
****Here $s=(s_1,\ldots, s_N)$ is a vector containing for each stock some fundamental accounting measure of company size.*
*****Here the beta $\beta=(\beta_1,\ldots, \beta_N)$ is the vector of betas with respect to the RP portfolio, hence portfolio weights for the RP portfolio appear on both sides of the equation, which needs to be solved numerically (no analytical solution).*

provide a list of popular equity weighting schemes, and discuss the conditions on the true risk and return parameter values under which each of these schemes can be regarded as a Maximum Sharpe Ratio portfolio.

On the other hand, ignoring parameter estimates might intuitively be a reasonable approach in the presence of an overwhelming amount of estimation risk even for investors using improved parameter estimates. For example, DeMiguel, Garlappi and Uppal (2009) argue that mean-variance optimisation procedures do not consistently outperform, from an out-of-sample Sharpe ratio perspective, naive equally-weighted portfolio strategies.[13] Similarly, GMV portfolios typically outperform MSR portfolios based on sample-based parameter estimates from an out-of-sample risk-adjusted perspective[14]. However, the risk remains for the investor to select the model or investment strategy with a substantial efficiency cost.

In this area, we also wish to stress that it is not because a methodology claims to be a common sense approach and aims not to be "quantitative" that optimality risk and parameter estimation risk do not exist. On the contrary, the further removed one is from the academic mainstream when proposing *ad-hoc* models based on intuition, the greater the risk that the model chosen was selected for its performance over the back-test period. As such, fundamental weighting schemes could outperform in the back-tests proposed simply because the methodological choices, and notably the appreciation of the economic size of the firms, led to the avoidance of large-cap stocks of small economic size[15], such as the Internet stocks. Reasoning in that way in 2003, publication date of the first articles justifying fundamental indexation (Wood and Evans, 2003), was probably easier than in 1999! In the same way, constructing a value bias through the design of economic size measurement parameters will enable good historical performances to be shown over the long term, since it has been shown that value stocks, because they are exposed to particular risks (see e.g. Petkova and Zhang, 2005), outperform growth stocks, which are more strongly represented in cap-weighted indices. Naturally, this empirical evidence gives no indication of the out-of-sample robustness of the fundamental strategy[16]. In order to assess the out-of-sample robustness of a fundamentals-based strategy, we look at the role that the burst of the internet bubble had on fundamentals-based portfolio construction approaches.

Exhibit 7 (taken from Amenc, Goltz, Ye, 2012) shows results for a fundamental weighted portfolio in the global equity universe. The results show that the inclusion of dotcom crisis pushes up the outperformance of the strategy by approximately 2.5% compared to the outperformance that would have been if the crisis was excluded. In other words, the strategy benefited heavily during the dotcom crisis period, which only seems natural as fundamentals-based weighting leads to an underweighting of technology stocks at the start of that period. Including this period in a simulated track record thus tends to increase its simulated outperformance compared to the case where one excludes this episode from the track record.

13 - Naturally, an Equal-Weighted (EW) portfolio can have numerous attributes and can benefit for example from a very positive rebalancing effect (see Plyakha, Uppal and Vilkov, 2012) in comparison with a buy-and-hold strategy such as cap-weighted, but one could argue that this benefit of fixed-mix strategies documented by Perold and Sharpe (1995) is not limited to an Equal-Weighting approach.

14 - It has also been shown (for example Martellini and Ziemann, 2010) that attempting to take account of the higher moments of the distribution to propose portfolios minimising extreme risks rapidly encountered such a large number of parameters that ultimately the ambition of the optimisation was often compromised by the low out-of-sample robustness of the optimisation.

15 - We refer to economic size as measured by accounting variables such as earnings or sales.

16 - A related issue is to assess the conditions of optimality of a fundamental weighting scheme. Indeed, if one wants to rely on an assessment which is independent of looking at past track records, one may consider under which conditions a fundamental weighting scheme would be optimal in the sense of Modern Portfolio Theory, that is under which assumptions it would provide the Maximum Sharpe Ratio. Melas and Kang (2010) show that this would be the case if risk parameters (correlations and volatilities) are identical for all stocks and expected return of each stock is identical to the accounting measure used for the fundamental weighting. In this sense, this weighting scheme, like any weighting scheme, takes on a model risk, i.e. the risk that this assumption needed for optimality could be wrong.

Exhibit 7: Impact of dotcom crisis period on the track record of a developed global fundamentals-based portfolio – The table shows the excess returns of fundamentals-based portfolio for global developed markets over the cap-weighted benchmark for full period and the period excluding internet bubble. The fundamentals-based portfolio is constructed on the basis of book value, dividend, cash flow, sales and earnings and is rebalanced yearly at the end of June based on accounting variables for the past year. The benchmark used is the portfolio of the largest 1,000-stocks by market cap held in proportion to their market cap, and is reconstituted annually at the end of June. The analysis is based on monthly return data from March 2000 to June 2011. All returns reported are geometrically averaged and are annualised. The dotcom crisis period is defined as the drawdown period of the NASDAQ 100 index from March 2000 to September 2002. The returns and fundamental data are obtained from Worldscope and Datastream.

	Annualised Excess Returns of fundamentally weighted strategy	Impact of including dotcom crisis period on annualised returns
Full period (03/ 2000 to 06/2011)	4.3%	
Full period excluding dotcom crisis (March 2000 to September 2002)	1.8%	+2.5%

A somewhat similar concern over insufficient attention to robustness issues exists with low volatility strategies. Promoters of low-volatility strategies often refer to a particular condition of optimality which would be that low-volatility stocks actually have higher returns than high-volatility stocks. This demonstration is based on results obtained with particular methodological choices[17]. Ultimately, the robustness of this low-volatility anomaly is questionable and the fragility of the assumption of the negative relationship between returns and risks should be understood as a model risk which the investor could decide to take, but which should be documented. It is here again curious that the promoters of the low-volatility anomaly who are so prone to referring to a scientific article of Ang *et al.* which appeared in 2006 in the Journal of Finance forget to mention other articles in leading academic finance journals that called the results of this article into question, notably by Bali and Cakici in 2008 in the Journal of Financial & Quantitative Analysis, Fu in 2009 in the Journal of Financial Economics and Huang *et al.* 2010 in the Review of Financial Studies.

17 - Ang, Hodrick, Xing, and Zhang (2006, 2009) find that high idiosyncratic volatility stocks have low returns. However, the outperformance of low volatility stocks is subject to a few caveats. The negative risk-return relation is not robust to changes in data frequency, portfolio formation period, to the screening out of illiquid stocks (Bali and Cakici, 2008), or to adjusting for short-term return reversals (Huang *et al.*, 2010) and considering risk measures obtained over longer horizons even inverses the relation between volatility and returns (see e.g. Fu, 2009).

2. Controlling the Risks of Smart Beta Investing: The Smart Beta 2.0 Approach

The second generation of Smart Beta clearly addresses the problem of measuring and controlling the risks of these new forms of indices. Even though the majority of Smart Beta indices have a strong probability of outperforming cap-weighted indices over the long term because of the high level of concentration of the latter, it should be noted that through their exposure to sources of risk that are different from those of cap-weighted indices, these new benchmarks can sometimes significantly underperform market indices for a considerable length of time. Exhibit 8 shows the magnitude and duration of the worst underperformance for a range of Smart Beta indices.

The results show that Smart Beta strategies can encounter severe relative drawdowns relative to the cap-weighted reference index which are at least in the order of 10%. Moreover, as the time under water statistics suggest, underperformance can last for extended time periods.

2.1 Choosing and controlling exposure to systematic risk factors

Paying attention to the systematic risks of Smart Beta is today not only a genuine opportunity to create added value, but is also a condition for its durability. While Smart Beta can play an important role in institutional investors' allocations, we think that this can only be at the price of implementing a genuine risk management process. It would seem fairly contradictory for investors to accept the idea that what drives their global asset allocation is more the integration of the risks of asset classes or categories rather than the names of the latter (as evidenced by increasing interest in modern approaches which look at risks rather than asset categories when defining an allocation, as for example in the Risk Parity and more generally in risk allocation approaches) and that they forget to look into the risks of the benchmarks used to invest in these asset categories.

The first approach to take into account the systematic risks of investing in Smart Beta

Exhibit 8: Relative risk of various alternative beta strategies – The table summarises the maximum relative drawdown numbers for selected commercial Smart Beta indices and 9 Scientific Beta USA flagship Indices with respect to the S&P 500 Index. Maximum relative drawdown is the maximum drawdown of the long-short index whose return is given by the fractional change in the ratio of strategy index to the benchmark index. Daily total return data for the period 23 December 2002 to 31 December 2012 has been used for the analysis as this is the earliest date since data for all indices are available. Returns data has been downloaded from Datastream and from www.scientificbeta.com.

		Maximum Relative Drawdown	Time Under Water (business days)
Indices from Traditional Index Providers	FTSE RAFI U.S. 1000 Index	12.71%	439
	FTSE EDHEC Risk Efficient U.S. Index	8.72%	46
	MSCI USA Minimum Volatility Index	12.82%	371
	S&P 500 Equal Weight Index	13.72%	453
Scientific Beta USA Flagship Indices	High Liquidity Max Decorrelation	14.27%	104
	High Liquidity Max Decorrelation	14.27%	104
	High Liquidity Max Deconcentration	15.53%	110
	High Liquidity Efficient Max Sharpe	10.14%	103
	High Liquidity Efficient Min Volatility	6.17%	507
	High Liquidity Diversified Risk Parity	8.81%	108
	Low Volatility Max Deconcentration	7.75%	222
	Value Max Deconcentration	14.44%	44
	Growth Max Deconcentration	17.84%	109
	High Dividend Yield Max Deconcentration	11.55%	122

that we feel is very compatible with the idea that an index must remain a simple construction is the disentangling of the two ingredients that form the basis of any Smart Beta index construction scheme: the stock selection and weighting phases.

A very clear separation of the selection and weighting phases enables investors to choose the risks to which they do or do not wish to be exposed. This choice of risk is expressed firstly by a very specific and controlled definition of the investment universe. An investor wishing to avail of a better diversified benchmark than a cap-weighted index but disinclined to take on liquidity risk can decide to apply this scheme solely to a very liquid selection of stocks. In the same way, an investor who does not want the diversification of his benchmark to lead him to favour stocks with a value bias can absolutely decide that the diversification method chosen will only be applied to growth, or at least not strictly value, stocks, etc.

In an article published recently in the *Journal of Portfolio Management* (Amenc, Goltz and Lodh, 2012) we have been able to show that the distinction between the selection and weighting phases (which can be made for most Smart Beta construction methods) could add value both in terms of performance and in controlling the investment risks. Exhibits 9 and 10 reproduce some of the results in the article.

The results show that the three optimised strategies[18] included in the analysis all have significant implicit small-cap exposure relative to the S&P 500 index when no stock selection is made.

Exhibit 9: Size exposure of diversification strategies based on different size-based stock selections – The table shows the excess (over S&P 500) risk factor exposures of the Global Minimum Volatility, Maximum Sharpe Ratio, and Maximum Decorrelation portfolios based on broad S&P 500 stock universe and three size based stock selections. The stock selection is done at each rebalancing. Weekly return data from 5 July 1963 to 31 December 2010 obtained from CRSP is used for the analysis. We run the following regressions to identify factor exposures

$$R_P - R_{CW} = \alpha + \beta_M . R_{CW} \qquad (2)$$

$$Res = \beta_S . R_S \qquad (3)$$

R_P is the time series of test portfolio returns, R_{CW} is the S&P 500 time series returns, β_M is the market beta, β_S is the size (big-small) beta, R_S is the size factor which is the return of a portfolio (cap-weighted) long in 1/5th largest cap stocks and short in 1/5th smallest cap stocks that constitute the NYSE, AMEX and NASDAQ universe, and Res is the residual time series from equation 3 regression. This two-step process is used for each risk factor and for each test portfolio. The bold values indicate that the beta for the size factor tilt is significant at 1% confidence level. All averages reported are geometric averages and all statistics are annualised.

	Global Minimum Volatility (GMV)				Maximum Sharpe Ratio (MSR)				Maximum Decorrelation (MDC)			
Universe	All stocks	Small size universe	Medium size universe	Large size universe	All stocks	Small size universe	Medium size universe	Large size universe	All stocks	Small size universe	Medium size universe	Large size universe
Market exposure of excess returns over CW	-26.20%	-25.41%	-28.03%	-23.92%	-21.92%	-23.69%	-23.94%	-20.09%	-8.60%	-7.93%	-10.57%	-6.59%
Size (Big - Small) exposure of excess returns over CW	-19.00%	-43.75%	-19.32%	1.83%	-21.13%	-46.28%	-21.40%	0.29%	-37.07%	-65.59%	-27.26%	-3.15%

18 - GMV (Global Minimum Variance) portfolios aim to minimise portfolio volatility. GMV optimization is performed in the presence of norm constraints (DeMiguel, Garlappi, Nogales and Uppal, 2009) with a lower bound of N/3 on the effective number where N is the total number of stocks in the relevant universe. MSR (Maximum Sharpe Ratio) optimization aims to maximise the Sharpe Ratio of the portfolio. The downside risk of stocks are used as a proxy for their expected returns (Amenc, Goltz, Martellini and Retkowsky, 2011) and the covariance matrix is obtained using principal component analysis. MDC (Maximum Decorrelation) optimization is an approach which exploits low correlations across stocks to reduce portfolio risk rather than concentrating in low volatility stocks, which is a limit often underlined with GMV approaches. The MDC approach aims to minimise portfolio volatility under the assumption that volatility across all stocks is identical (Christoffersen *et al.*, 2010), hence focusing on exploiting differences in correlations rather than on exploiting differences in volatility across stocks.

However, when applying the weighting schemes only on the largest cap stocks in the universe, none of the weighting schemes leads to significant pronounced small-cap exposure. Amenc, Goltz and Lodh (2012) show that qualitatively similar results hold when selecting stocks based on dividend yield or low and high volatility characteristics. For these different stock characteristics, it is possible to reduce or cancel implicit factor tilts of a weighting scheme through an appropriate stock selection decision. An additional question is whether improvements in risk/return properties of the weighting scheme relative to the broad cap-weighted index still hold after having corrected for the factor tilts. Exhibit 10 below shows results from the article which addresses this question for the case of size-based selection.

Overall, the results suggest that when reducing the small cap exposure of these weighting schemes through an appropriate stock selection, each weighting scheme still improves its respective diversification objective relative to the standard cap-weighted index. For example, Minimum Volatility (GMV) portfolios created for a stock selection of the largest stocks achieve a considerable reduction in volatility compared to the broad cap-weighted portfolio. Applying a stock selection on a Maximum Sharpe Ratio (MSR) portfolio shows qualitatively similar results meaning that increase in the Sharpe Ratio over the S&P 500 index is observed in all size selected portfolios. The Maximum Decorrelation (MDC) portfolios on mid and large cap stock selection show higher GLR measures[19] than broad MDC portfolio, suggesting that they are less well diversified by this criterion, but their GLR measures are still considerably lower than that of the broad cap-weighted index, suggesting improved diversification. Overall, the results show that - even after controlling for specific risk factor through stock selection – the risk-return objectives of a diversification scheme can still be met and important improvements over the cap-weighted reference index are possible.

Next we test if the specific risk of liquidity can be controlled using stock selection approach without compromising the performance of the Smart Beta strategy. In other words we analyse if diversification benefits can be exploited in a high liquid stock universe. For this, we construct four

Exhibit 10: Attainment of objective of diversification strategies with size based stock selection – The table compares the attainment of investment objective of the three optimised portfolios: Global Minimum Volatility, Maximum Sharpe Ratio, and Maximum Decorrelation portfolios (see the Appendix for a description of these terms) each based on broad S&P 500 stock universe and three size-based stock selections, which each includes one third of stocks in the universe (respectively the smallest, medium and largest cap stocks in the S&P 500 universe). Weekly return data from 2 January 1959 to 31 December 2010 obtained from CRSP is used for the analysis. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate in US Dollars. All averages reported are geometric averages and all statistics are annualised.

Panel 1	Global Minimum Volatility (GMV)			
Universe	All	Small	Medium	Large
Annual Volatility	12.40%	13.67%	12.67%	12.59%
% Reduction relative to S&P 500	19.8%	11.6%	18.0%	18.6%
Panel 2	Maximum Sharpe Ratio (MSR)			
Universe	All	Small	Medium	Large
Sharpe Ratio	0.51	0.65	0.51	0.35
% Increase relative to S&P 500	85.6%	139.9%	85.9%	30.2%
Panel 3	Maximum Decorrelation (MDC)			
Universe	All	Small	Medium	Large
GLR Measure	0.139	0.134	0.167	0.208
% Reduction relative to S&P 500	43.1%	45.3%	31.9%	15.0%

19 - The GLR measure (Goetzmann, Li and Rouwenhorst, 2005) is the ratio of the portfolio variance to the weighted variance of its constituents and a low GLR measure indicates that correlations have been well exploited and in this sense the portfolio is well diversified.

Exhibit 11: Diversification strategies based on high liquidity stock selection – The table shows the risk and return statistics of the Scientific Beta USA strategy indices based on all stocks (in Panel 1) and their high liquidity counterparts (in Panel 2). High Liquidity portfolios are constructed using the top 50% of stocks in the parent universe in terms of liquidity. The analysis is based on daily total return data for the period 21 June 2002 to 31 December 2012 downloaded from www.scientificbeta.com. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate in US Dollars. All averages reported are geometric averages and all statistics are annualised.

Panel 1	Efficient Max Sharpe	Efficient MinVolatility	Max Decorrelation	Max Deconcentration
Ann Returns	7.79%	8.23%	7.60%	8.09%
Ann Volatility	20.49%	18.32%	21.39%	22.71%
Sharpe Ratio	0.30	0.36	0.28	0.28

Panel 2	Efficient Max Sharpe	Efficient MinVolatility	Max Decorrelation	Max Deconcentration
Ann Returns	7.86%	7.88%	7.29%	7.90%
Ann Volatility	21.52%	19.20%	22.60%	24.01%
Sharpe Ratio	0.29	0.32	0.25	0.26

Exhibit 12: Controlling sector exposures of minimum volatility indices – The table shows the risk and return statistics of Scientific Beta USA Min Volatility Index, sector neutral Scientific Beta USA Min Volatility Index and the S&P 500 Index. The analysis is based on daily total return data for the period 21 June 2002 to 31 December 2012 downloaded from Bloomberg and from www.scientificbeta.com. All averages reported are geometric averages and all statistics are annualised.

	Scientific Beta USA Efficient Min Volatility Index	Sector Neutral Scientific Beta USA Efficient Min Volatility Index	S&P 500 Index (cap weighted benchmark)
Ann Returns	8.23%	8.02%	5.66%
Ann Volatility	18.32%	19.24%	21.81%
% Reduction in Vol compared to the S&P 500 Index	16.0%	11.8%	-
Sharpe Ratio	0.36	0.33	0.18

Smart Beta strategies on the top 50% of stocks in the USA universe in terms of liquidity and compare their performance with the strategies based on the full USA universe. The results are reported in Exhibit 11.

The results show that high liquidity stock selection does not have a large effect on the performance of the strategies. The annualised returns tend to be decreased slightly. The Sharpe ratios also tend to be decreased but not by a big amount. These results suggest that by a simple stock selection approach of selecting the most liquid stocks, one can avoid the liquidity problems of Smart Beta strategies while maintaining most of the potential for improved risk/reward properties.

Naturally, in addition to, or as a substitution for this stock selection, the construction of Smart Beta indices can use a second approach to take into account systematic risks linked to investing in Smart Beta. This approach consists of the well-known constrained optimisation techniques that allow the maximal or minimal exposures to specific risk factors to be determined.

In this way, nothing prevents investors from having Smart Beta indices that have, for example, controlled sector exposures that reduce the difference with respect to the cap-weighted reference index. Exhibit 12 shows a comparison of performance statistics for the S&P 500 index with Scientific Beta USA Minimum Volatility portfolios with and without sector neutrality constraints to examine the effect of additional constraints on strategy's ability to achieve its portfolio level objective.

The results show that imposing sector neutrality constraints does not affect the performance of Scientific Beta USA Min Volatility strategy by a large amount. The unconstrained portfolio achieves 16.0% volatility reduction over cap-weighted benchmark while the sector constrained portfolio still achieves 11.8% reduction.

2.2 Measuring and managing the specific risk of Smart Beta investing

The evaluation and especially the management of specific risks have not given rise to any real application that is appropriate for Smart Beta indices. However, here again, like for the estimation of systematic risk factors, turning to the analysis framework of Modern Portfolio Theory provides a relevant conceptual structure for replying to this question.

Modern Portfolio Theory has a very straightforward prescription, namely that every investor should optimally seek to allocate to risky assets so as to achieve the highest possible Sharpe ratio.

Implementing this objective, however, is a complex task because of the presence of estimation risk for the required expected returns and covariance parameters. As recalled above, the costs of estimation error may entirely offset the benefits of optimal portfolio diversification (see DeMiguel, Garlappi and Uppal, 2009, for evidence of the domination of naively-diversified portfolios over scientifically-diversified portfolios from an out-of-sample Sharpe ratio perspective).

In this context, the choice in risk and return parameter estimation for efficient diversification is between "trying", which has a cost related to *parameter estimation risk*, i.e., the risk of a substantial difference between the estimated parameter value and the true parameter value, or "giving up", which also has an *optimality risk*, related to the risk that the heuristic benchmark (such as Global Minimum Variance (GMV) or Equal-Weighted (EW)) can be very far from the optimal MSR benchmark. The trade-off occurs because using objectives that involve fewer parameters leads to a smaller amount of parameter risk, but a higher amount of optimality risk (see Section 1.2), since one is using fewer dimensions for optimisation. In this sense, it can perfectly happen that a "good" proxy (i.e., a proxy based on parameters with little estimation risk) for a "bad" target (i.e., a target *a priori* far from the true MSR based on true population values) eventually dominates a "bad" proxy (i.e., a proxy based on parameters plagued with substantial estimation risk) for a "good" target (i.e., a target *a priori* close to the true MSR based on true population values).

Hence, different portfolios are intuitively expected to incur more estimation risk or more optimality risk. For example, investing in Equal-Weighted (EW) benchmarks involves no estimation risk, since no parameter estimates are required, but arguably a large amount of optimality risk, since these benchmarks are not expected to be good proxies for the corresponding true MSR portfolios, unless all constituents have the same expected return, the same volatility and the same correlations. In other words, holding EW portfolios, which are not subject to estimation risk, involves an opportunity cost related to the fact that their Sharpe ratio may be dramatically inferior to the Sharpe ratio of the true MSR. On the other hand, investing in GMV or ERC (Equal Risk Contribution) benchmark involves more estimation risk compared to EW benchmarks, because covariance parameter estimates are needed, and possibly less optimality risk if it turns out that these heuristic benchmarks are closer to the optimal MSR benchmarks than is the EW benchmark. Finally, investing in MSR benchmarks involves even more estimation risk, since (possibly very noisy) expected return parameters are used in addition to covariance parameters; on the other hand, it does not involve any optimality risk since the target portfolio would coincide with the true optimal portfolio in the absence of estimation risk.

In this context, it is useful to first propose an empirical analysis of optimality risk taken in isolation, i.e., in the absence of any estimation risk. To conduct this analysis, we consider a large number of possible equity universes, defined in terms of many different possible reasonable true population values for risk and return parameters, and measure the difference for these parameter values (in terms of *ex-ante* Sharpe ratios, i.e., based on true parameter values) between the true MSR portfolios and various heuristic portfolios, as well as various combinations of these portfolios (see Martellini, Milhau and Tarelli, 2013, for more details). We then analyse the distribution of this distance across all possible sets of parameter values so as to generate an absolute assessment of optimality risk for various heuristic portfolios, as well as a relative assessment of optimality risk amongst competing heuristic portfolios. For example, this analysis allows us to answer questions such as what is the probability (across all tested parameter values) that the GMV portfolio is closer than the EW portfolio to the (true) MSR portfolio, hence allowing us to provide a quantitative comparison of the optimality risk involved in EW versus GMV (or any other heuristic) benchmark.

In a second step, estimation risk is introduced so as to help measure the distance of various heuristic benchmarks using *imperfect* estimates with respect to the true MSR portfolio. This analysis allows us to analyse the interaction between estimation risk and optimality risk, and allows us to answer questions such as the following: Given realistic estimation errors in the covariance matrix and expected returns, what are the chances that an imperfectly estimated MSR, which suffers only from estimation risk (estimated MSR different from true MSR) will be closer to the true MSR portfolio in terms of *ex-ante* Sharpe ratios compared to a GMV portfolio (for example) which is subject to optimality risk (because the true GMV portfolio is different from the true MSR portfolio), but to a lower amount of estimation risk (there is less difference between the estimated GMV and the true GMV than between the estimated MSR and the true MSR, since the GMV does not require any expected return parameters)?

Overall, our analysis allows us to provide a detailed empirical assessment of total specific risk of Smart Beta benchmarks (in terms of differences in *ex-ante* Sharpe ratios) between a given benchmark and the true MSR portfolio, by decomposing this specific risk as indicated in equation (1), which we rewrite as follows:

Total distance (in terms of ex-ante Sharpe ratio based on true parameter values) of a given benchmark with respect to the true MSR portfolio = distance of the given target benchmark with respect to the true MSR portfolio assuming away estimation risk (optimality risk in the absence of estimation risk) + distance between the imperfectly estimated target and the true target (estimation risk).

This analysis can also be used to manage specific risks of Smart Beta benchmarks. In particular, one may seek to have a strategic exposure to various Smart Beta benchmarks so as to diversify away these risks. For example, Exhibit 13 below shows the average Sharpe ratio across 2,226 different sets of reasonable parameter values for the S&P 500 universe.[20] This exhibit shows that assuming that true covariance and expected returns parameters are known, an exceedingly large value of 13.34 is generated for the average Sharpe ratio of the Maximum Sharpe Ratio portfolio. This value by far exceeds the value obtained for GMV, EW and cap-weighted (CW) portfolios, thus underlining the opportunity costs involved in optimality risk for such portfolios. On the other hand, when a realistic estimate of estimation error is introduced for covariance and expected return parameters (assuming that such estimates are generated by the use of a parsimonious factor model), the average Sharpe ratio of the scientifically-diversified portfolios is substantially reduced, much more substantially so for the MSR portfolio which suffers from the presence of estimation errors in both covariance and expected return parameters, compared to the MSR portfolio, which only suffers from estimation errors in covariance parameters.

Interestingly, we see that the GMV dominates the MSR portfolio after estimation risk is taken into account, as also does the EW portfolio albeit by a lower margin. Moreover, a mixture of GMV and EW portfolios generates the highest average Sharpe ratio. These results, which can be extended to other possible combinations of Smart Beta indices, are consistent with theoretical results by Kan and Zhou (2007), who show that a portfolio that combines the sample-based MSR and GMV portfolios dominates the sample-based MSR alone in the presence of parameter uncertainty. These results suggest that the presence of estimation risk completely alters the standard prescriptions of the fund separation theorem, and also suggest that the benefits of diversifying away the specific risks of Smart Beta benchmarks can be substantial. More generally, beyond a static diversification approach, one may also implement an improved dynamic diversification approach based on making the allocation to various Smart Beta benchmarks conditional upon market conditions such as average correlation levels, volatility levels, etc.

2.3 Managing tracking error risk

If the goal of investment in Smart Beta is to outperform cap-weighted market indices, we should note that this goal is exactly the same as that of a benchmarked active manager. In this approach, it seems logical to take account of the tracking error of

Exhibit 13: Sharpe ratios for selected weighting schemes in the presence of estimation errors in expected excess returns and covariances - Results taken from Martellini, Milhau and Tarelli (2013). The table shows statistics on the ex-ante Sharpe ratio of different portfolios. These results have been obtained by simulating ("true") population parameters and estimation errors. The first column contains results when expected excess returns and the covariance matrix are perfectly estimated (no estimation risk) in particular the average annualised Sharpe Ratio. The average is taken across different sets of "true" parameters. The 2nd and 3rd columns contain results when we simulate estimation errors for risk and return parameters. We calculate the mean and standard deviation of the distribution of Sharpe ratios that we obtain across our simulations for each set of "true" parameters. The 2nd and 3rd columns show the average of these statistics across all sets of "true" parameters.

Portfolio strategy	Average Sharpe ratio with no estimation risk	Average Sharpe ratio with estimation risk	St. dev. of Sharpe ratio with estimation risk
MSR (Maximum Sharpe Ratio)	13.3377	0.5587	0.6114
GMV (Global Minimum Variance)	2.4904	0.8859	0.5743
EW (Equal-Weighted)	0.6048	0.6048	0.0000
CW (Cap-Weighted)	0.4972	0.4972	0.0000
50% GMV + 50% EW	1.0773	0.9443	0.3003

20 - To generate realistic parameter values, we use the following approach. We elect to perform the analysis on weekly data over a 2-year rolling window, where we choose the rolling step to be equal to one week. For each position of the time-window considered, we take into account all stocks belonging to the universe, without introducing a survivorship bias. We then calculate a robust estimator. We use an implicit multi-factor approach, where principal component analysis is used to extract the factors and where random matrix theory is used to select the number of relevant factors. As far as the expected excess returns are concerned, we use expected returns implied by the Fama-French three-factor model.

these new benchmarks with regard to cap-weighted indices, not only to establish an equivalent comparison between them, but also to control the risk of these new benchmarks underperforming the market indices. We feel that integrating the risk of underperforming cap-weighted indices is all the more important in that, even if the Chief Investment Officer (CIO) has made the choice of a new benchmark, there is no guarantee that the fund's stakeholders will agree with this choice. The popularity of cap-weighted indices and the fact that they represent the average of the market mean as far as we are concerned that they will remain the ultimate reference for equity investing for many years. As proof, even though all the promoters of Smart Beta indices base their sales proposals on the absolute necessity of no longer using cap-weighted indices, which they rightfully consider to be poor-quality investment references, they systematically use these same cap-weighted indices to benchmark the performance of their Smart Beta indices. Ultimately, CIOs who replace a cap-weighted index with a Smart Beta index take considerable reputation risk. They will often be recommending to their trustees and subscribers that they pay more for a benchmark for which the CIO will be entirely responsible. While in active investment management, the CIO can always relieve the manager whose mandate is underperforming the benchmark of his duties, in the case of the choice of benchmark only the CIO will be to blame.

We can of course argue that any management of a relative risk budget with respect to an inefficient benchmark such as a cap-weighted index can penalise the potential outperformance generated by a Smart Beta index over the long term, but who really believes that short-term performance constraints do not exist in asset management, private wealth management or institutional investment management? It would be a shame if the good idea of tracking a smart benchmark were called into question due to short-term underperformance. In our view, the level of relative risk budget with respect to cap-weighted indices should be proportional on the one hand to the knowledge that the stakeholders have of the Smart Beta concept, and on the other, to the confidence that the CIO has in his Smart Beta choices.

In this context, controlling the tracking error and the choice of stocks in the indices is ultimately an essential tool for managing the systematic risks of these new indices. In an article published in the Spring 2012 issue of the *Journal of Portfolio Management* (Amenc, Goltz, Lodh and Martellini, 2012), we were able to show that using a suitably designed relative risk control approach, it is possible to obtain a reliable control of both the average tracking error and the extreme tracking error of Smart Beta strategies. This approach involves an alignment of the risks of the Smart Beta benchmark with those of the cap-weighted index. In addition to achieving a very effective control of the tracking error, such an approach maintains overall significant potential for outperformance of the Smart Beta investment.

The results in Exhibit 14 show the effect of imposing tracking error constraints on Scientific Beta USA strategy indices. The 3% tracking error target is not exceeded substantially by any of the strategies even for the extreme observations of tracking error, as can be seen from the extreme (95%) tracking error observations. Despite the risk control, the relative risk controlled versions are still able to outperform their cap-weighted benchmarks, albeit by a

Exhibit 14: Relative risk of Scientific Beta USA strategy indices – The table summarises, in panel 1 the relative risk and return statistics of four Scientific Beta USA strategy indices without any relative risk control, and in panel 2 the relative risk and return statistics of the same strategies with 3% tracking error control. 95% tracking error is computed using a rolling window of 1 year length and 1 week step size over the analysis period. The benchmark used is Scientific Beta USA cap-weighted index and the analysis is based on daily total return data for the period 21 June 2002 to 31 December 2012. All averages reported are geometric averages and all statistics are annualised. All returns data has been downloaded from www.scientificbeta.com.

Panel 1	Smart Beta *without* Relative Risk Control: Scientific Beta USA Indices			
	Efficient Max Sharpe	Efficient Min Volatility	Max Decorrelation	Max Deconcentration
Excess Returns over CW	1.72%	2.16%	1.53%	2.02%
Tracking Error	3.39%	4.60%	3.57%	3.62%
95% Tracking Error	5.28%	8.01%	5.58%	6.36%
Panel 2	**Smart Beta *with* Relative Risk Control: Scientific Beta USA Indices (3% Tracking Error)**			
	Efficient Max Sharpe	Efficient Min Volatility	Max Decorrelation	Max Deconcentration
Excess Returns over CW	0.68%	0.71%	0.99%	0.90%
Tracking Error	1.83%	2.10%	2.03%	1.86%
95% Tracking Error	3.01%	4.30%	3.55%	2.83%

smaller margin. Investors thus face a clear trade-off between taking on relative risk and generating outperformance. However, in the relative risk controlled indices both overall tracking error and extreme tracking error figures are brought down substantially without eroding all of the potential for over-performance.

Conclusion: Smart Beta 2.0 – New Ethics in the Relationship with Investors?

Ultimately, we observe that investing in Smart Beta springs from the same logic and the same approach as choosing a manager. Investment in Smart Beta presupposes measurement of the systematic risk factors and integration of the factors, not only in absolute terms to evaluate the real risk-adjusted performance created by better diversification of the benchmark, but also in relative terms to limit the tracking error risk and therefore the risk of underperformance in comparison with the cap-weighted index. These statistical analyses should then be completed by thorough due diligence on the specific risk represented, not by the manager here, but by the diversification model and the implementation rules and methods.

In our opinion, this similarity in the investment processes raises a major question. What is the minimal level of information which the Smart Beta investor should possess in order to evaluate genuine performance and risks? In the area of Smart Beta indices, one is forced to conclude that the situation is currently inadequate.

Access to data on the performance, composition and risk of Smart Beta indices is restricted or costly. As far as the model and the implementation rules are concerned, the desire to protect proprietary know-how and the profusion of *ad-hoc* schemes whose conditions of optimality and robustness are highly inexplicit makes the analysis of the specific risks of Smart Beta difficult, not to say impossible.

Moreover, while research traditionally helps to increase the knowledge of market participants, in the area of Smart Beta, and despite the abundance of publications, we consider that this research is not currently contributing to an improvement in the efficiency of the Smart Beta market.

Due on the one hand to the restriction in access to information and on the other to the sales and marketing stakes represented by the validation in a scientific publication of the relevance and the robustness of the model proposed by its promoters, research in the area of Smart Beta is polluted by conflicts of interest and the difficulties in distinguishing between what is scientific evidence and what stems from the author's desire to promote a particular weighting scheme. Even worse, most of the articles that are positioned as objective comparisons of different forms of Smart Beta are written by index promoters or asset managers who sell one strategy, or a very small number of strategies. Unsurprisingly, these comparison articles[21] more often than not conclude on the objective superiority of the solution(s) marketed by the authors.

Here too, Smart Beta market practices are no different from those of active managers who like to choose the periods of comparison with their peers.

That is why, for the past two years,[22] EDHEC-Risk Institute has been demanding that information enabling all investors to analyse the risks and performances of Smart Beta strategies be available at a cost that is not prohibitive and above all without any particular restrictions on usage. This is not to deny the economic value of the data and information but to consider that an index that is sold as a reference for the market

21 - It seems to us to be an anomalous situation that comparison of competing strategies in the area of smart beta more often than not is conducted by promoters of a particular strategy, who compare themselves to their competitors' strategies, or even to their own replication of their competitors' strategies. In fact, articles published by providers of a given smart beta strategies often contain confusing statements about their competitors' strategies. For example, in an article published by promoters of fundamentals-based equity indexation in the *Financial Analysts Journal* (Chow, Hsu, Kalesnik and Little, 2011a), the authors, who highlight the importance of implementation rules to evaluate alternative equity indices, omitted to include the turnover management rules integrated by the competitors so as to then show that the turnover they calculated themselves for these strategies was higher than that of their own index. For more details on this issue, see Amenc, Goltz and Martellini (2011) and Chow, Hsu, Kalesnik and Little (2011b). In the same way, a recent article published in the *Journal of Indexes Europe* (Blitz, 2013), by promoters of low-volatility strategies, asserts that Efficient Maximum Sharpe Ratio indices are exposed to higher volatility than cap-weighted indices over the long term, even though the article cited by the author to justify his argument (Clarke, de Silva & Thorley, 2013,) does not specifically refer to Efficient MSR strategies. Moreover, this statement is inconsistent with an article published in the *Journal of Investment Management* (see Amenc, Goltz, Martellini and Retkowsky, 2011) analysing the performances and risks of this strategy which finds that the volatility of Efficient MSR is lower than that of a cap-weighted index containing the same stocks.

22 - Amenc, Ducoulombier, Goltz, and Tang (2012), Amenc and Ducoulombier (2012a), Amenc and Ducoulombier (2012b), Amenc and Ducoulombier (2012c), Amenc and Ducoulombier (2013).

should be able to be genuinely analysed and criticised by all market participants.

The Smart Beta 2.0 approach, which aims to allow investors to invest in these advanced forms of benchmarks with full knowledge while controlling the risks of their choice, can only be conceived with an efficient market for indices that are supposed to represent references for the implementation of Smart Beta strategies.

That is why we consider that market information on the rules and historical compositions that underlie the performances of the track records used to promote these indices is indispensable.[23]

In the same way, since Smart Beta 2.0 allows the risks to which investors wish to be exposed via a Smart Beta benchmark to be controlled, it is time to stop continually equating Smart Beta strategies with a predetermined set of risk factors.

The growing of Smart Beta strategies offers interesting opportunities for investors, as such strategies recognise the importance of equity portfolio construction (or "beta") as a determinant for risk and return of portfolios over the long run which has been widely documented in the literature. Given that some in the industry for a long time have tended to neglect the importance of betas due to an exclusive focus on alpha, increasing the attention to the choice of betas is a promising development. However, rather than accepting pre-packaged choices of alternative equity betas, investors should be able to explore different Smart Beta index construction methods in order to construct a benchmark that corresponds to their own choice of risks. This can be done by the means of a Benchmark Builder which allows you to choose flexibly among a wide range of options for each of the key steps in the benchmark construction process, rather than relying on a pre-packaged bundle of choices proposed by commercial indices, by selecting the different characteristics (regional universe, stock selection weighting, and risk control schemes) among the 2,442 smart beta indices available on the platform. For the sake of transparency about the risks they are taking, they also need to be able to analyse risk and performance of Smart Beta strategies openly rather than depend on the sole analysis published by the providers of particular strategies. It is against the backdrop of these requirements from investors that we argue for the need for a second generation of Smart Beta approaches.

23 - It is on that basis that EDHEC-Risk Institute will set out a series of initiatives from the second quarter of 2013 to create the conditions for an efficient market of smart beta indices.

In what follows, we provide a brief description of the Smart Beta indices shown in Exhibit 6 in the main text of this paper, with a focus on outlining the implicit or explicit assumptions that would legitimise the use of each given weighting scheme.

Cap-Weighted (CW) Portfolios

The Capital Asset Pricing Model (CAPM) introduced by Sharpe (1964) predicts that the cap-weighted market portfolio of all assets provides the highest Sharpe ratio. In practice, "holding the market" is virtually impossible, but its approximate implementation in terms of some market-capitalisation weighted equity indices has become the standard practice for most investors and asset managers. Capitalisation weighting in equity index construction has, however, been subject to severe criticism. Early papers by Haugen and Baker (1991) or Grinold (1992) provide empirical evidence that market-cap weighted indices provide an inefficient risk-return trade-off. From the theoretical standpoint, the poor risk-adjusted performance of such indices should not come as a surprise given that the risk-return efficiency of market cap weighting schemes can only be warranted at the cost of heroic assumptions. An extensive literature (see Goltz and Le Sourd, 2011, for a review) has shown that the theoretical prediction of an efficient market portfolio breaks down when some of the highly unrealistic assumptions of the CAPM are not met. In particular, financial theory does not predict that the market portfolio is efficient if investors have different time horizons, if they derive wealth from non-traded assets such as housing, social security benefits or human capital, if short sales are constrained or if frictions in the form of taxes exist. Besides, even if the CAPM was the true asset pricing model, which it is not, holding a market cap weighted equity index would be a rather poor proxy for holding the market portfolio, which, in principle, is a combination of all assets, traded or non-traded, financial or non-financial, including, for example, human capital. In the end, there is no good theoretical reason why a cap-weighted scheme should lead to an efficient portfolio. See also footnote nine for related argumentation.

Diversity-Weighted (DW) Portfolios

In what is perhaps the first historical approach to argue in favour of a deviation from cap-weighted indices, Fernholz, Garvey and Hannon (1998) propose to use *stock market diversity*, a measure of the distribution of capital in an equity market first introduced in Fernholz (1999), as a weighting factor. While there various possible measures of diversity exist, Fernholz (1999) focuses on the following one:

$$D_p(w)=\left(\sum_{i=1}^{n} w_i^p\right)^{1/p}$$

It can easily be seen that D_p reaches its minimum value equal to *1* if there is a stock *i* such that $w_i = 1$ and D_p reaches its maximum value equal to $n^{(1-p)/p}$ if $w_i = 1/n$ for all *n*. Fernholz *et al.* (1998) propose a focus on diversity-weighted benchmarks that use market cap weighting as a basis. To do so, he suggests defining a Diversity-Weighted Portfolio scheme as:

$$\boldsymbol{w}_{DW}=\frac{\boldsymbol{w}_{CW}^{p}}{\mathbf{1}'\boldsymbol{w}_{CW}^{p}},\ 0 \leq p \leq 1$$

As an example, one obtains for *p=1/2*:

$$w_{DW,i} = \frac{\sqrt{w_{CW,i}}}{\sum_{i=1}^{n}\sqrt{w_{CW,i}}}$$

More generally, one recovers the market cap index for p=1, and the equally-weighted portfolio for p=0: more generally, one can use p as a way to control the tracking error with respect to the market cap weighting scheme. To emphasise the benefits of DW benchmarks, Fernholz (1999) shows that the return of a diversity weighting scheme relative to the market cap weighting schem can be decomposed according to the following equation:

$$d\log\left(Z_w(t)/Z_\mu(t)\right) = d\log D_p(w)(t) + d\Theta(t)$$

with:

$$d\Theta(t) = (1-p)\left(\sum_{i=1}^{n} w_i(t)\sigma_i^2 - \sum_{i,j=1}^{n} w_i(t)w_j(t)\sigma_{ij}\right)dt.$$

In words, it says that the relative return of the diversity-weighted index can be decomposed into a "variation in diversity" term (arguably rather small over the long-term) and a (positive) "diversification potential" term (also called "kinetic differential").

Fundamentally-Weighted (FW) Portfolios

Fundamental equity indices (see Wood and Evans, 2003, Arnott, Hsu and Moore, 2005, or Siegel, Schwartz and Siracusano, 2007) maintain key characteristics of standard cap-weighted indices in order to facilitate their adoption as substitutes for the latter. Such indices simply replace the market cap by a different measure of firm size based on variables such as profits, book-value, and revenue. The idea behind such ad-hoc weighting schemes is not to optimise the risk/reward trade-off but to have measures of firm size more reliable than market capitalisation. For this characteristics-based index to be efficient in a Sharpe ratio maximization sense, one would need to assume that the volatilities and pair-wise correlations are identical across stocks and expected returns are proportional to the accounting measures that are used to attribute the weights. Clearly these assumptions are simple but unrealistic, and there are no theoretical foundations for the choice of which accounting parameters should be used for weighting. In other words, there is an inherent parameter selection risk which is the risk of specification of the strategy that works only in-sample. Schwartz and Siracusano (2007) propose that earnings are relevant weighting criteria. On the other hand, Arnott, Hsu and Moore (2005) use sales, cash flow, book value and dividends while some index providers prefer revenues to sales.

Max Deconcentration (MD) and Equally-Weighted (EW) Portfolios

The most straightforward, and also most naïve approach, towards the construction of better diversified portfolios consists in attributing the same weight to each of their constituents (this is sometimes called the *1/N rule).* For example, an equal-weighted version of the S&P 500 would use the same 500 constituents as the cap-weighted version of the index but it attributes an equal weight of 0.2% to each constituent. These *equal-weighted* benchmarks are attractive because they avoid the concentration and trendfollowing of cap-weighted indices and typically lead to higher Sharpe ratios compared to their cap-weighted counterparts (DeMiguel, Garlappi and Uppal, 2009, Platen and Rendek, 2010). However, for investors to adopt an equal weighting scheme is an extreme stance since it implies that they would accept to give up on any form of fundamental or statistical analysis to gather useful insights about the stocks' characteristics. In fact, the equal-weighted portfolio would have the highest possible Sharpe ratio if and only if pairwise correlations, volatilities and expected returns were identical for all stocks.

Equal-weighting is a simple way of "de-concentrating" a portfolio and allows us to benefit from systematic rebalancing back to fixed weights. Depending on the universe and on whether additional implementation rules are used, the rebalancing feature of equal-weighting can be associated with relatively high turnover and liquidity problems. Maximum Deconcentration can be perceived as a generalisation of a simple equal weighting scheme, the aim being to maximise the effective number of stocks defined as the inverse of the Herfindahl index (see footnote four):

$$ENC = \left(\sum_{i=1}^{N} w_i^2 \right)^{-1}$$

Maximum Deconcentration minimises the distance of weights from the equal weights subject to constraints on turnover and liquidity. In addition, investors may have the option to add constraints on tracking error, sector weight deviations or country weight deviations with respect to the cap-weighted reference index.

In the absence of any constraints, Maximum Deconcentration coincides with the equal-weighting (also known as the "1/N" weighting scheme), which owes its popularity mainly to its robustness and it has been shown to deliver attractive performance despite highly unrealistic conditions of optimality, even when compared to sophisticated portfolio optimisation strategies (De Miguel, Garlappi and Uppal, 2009).

Risk Parity (RP, also known as Equal Risk Contribution or ERC) and Diversified Risk Parity (DRP) Portfolios

The starting point in this approach consists of recognising that contribution to risk is not proportional to dollar contribution. To see this, we use the following decomposition for the portfolio volatility:

$$\sigma_p = \sqrt{\sum_i w_i^2 \sigma_i^2 + 2 \sum_{i<j} w_i w_j \sigma_{ij}}$$

$$\frac{\partial \sigma_p}{\partial w_i} = \frac{1}{\sigma_p} \left(w_i \sigma_i^2 + \sum_{i \neq j} w_j \sigma_{ij} \right)$$

$$\sum_i w_i \frac{\partial \sigma_p}{\partial w_i} = \frac{1}{\sigma_p} \sum_i w_i \left(w_i \sigma_i^2 + \sum_{i \neq j} w_j \sigma_{ij} \right) = \sigma_p$$

where w_i is the portfolio weight and σ_p the portfolio volatility. Hence, we define the contribution to risk as:

$$p_i = w_i \frac{\partial \sigma_p}{\partial w_i} = \frac{w_i}{\sigma_p} \left(w_i \sigma_i^2 + \sum_{i \neq j} w_j \sigma_{ij} \right)$$

To correct for these imbalances, and to generate portfolios that are better diversified in the sense of exhibiting a more balanced contribution to risk by the constituents of the portfolio, Qian (2005) and Maillard, Roncalli and Teiletche (2010) suggest to form so-called *equal risk portfolios* by choosing the portfolio weight w_i so that for all i, j:

$$p_i = p_j \Leftrightarrow w_i \frac{\partial \sigma_p}{\partial w_i} = w_j \frac{\partial \sigma_p}{\partial w_j}$$

(see Qian, 2005, or Maillard, Roncalli and Teiletche, 2010, for more details). It should be noted that no analytical solution is available to this program, which therefore needs to be solved numerically. However, in a recent paper Clarke *et al.* (2013) provide a semi-analytical solution. The RP portfolio weights can be written as:

$$\mathbf{w}_{RP} = \frac{\boldsymbol{\beta}^{-1}}{\mathbf{1}'\boldsymbol{\beta}^{-1}}$$

Here the beta β=(β_1,..., β_N) is the vector of betas with respect to the RP portfolio, hence portfolio weights for the Risk Parity portfolio appear on both sides of the equation, which needs to be solved numerically.

In an attempt to rationalise equal-risk contribution portfolios (also known as *Risk Parity* portfolios), Maillard, Roncalli and Teiletche (2010) show that risk-parity portfolios would be optimal Maximum Sharpe Ratio (MSR) portfolios if all Sharpe ratios are identical for all stocks, and if correlations are identical for all pairs of stocks, obviously a very restrictive assumption.

The Diversified Risk Parity (DRP) benchmark is a specific case of the Risk Parity benchmark where one makes the explicit assumption that all pairwise correlation coefficients are identical. In this case, the portfolio weights are proportional to the inverse of the volatility (Maillard, Roncalli and Teiletche, 2010):

$$\mathbf{w}_{\mathrm{DRP}} = \frac{\boldsymbol{\sigma}^{-1}}{\mathbf{1}'\boldsymbol{\sigma}^{-1}}$$

where **1** is a vector of ones and σ is the vector of volatilities.

Maximum Diversification Ratio (MDR) Portfolios

Since the focus should be on improving efficiency with respect to cap-weighted portfolios, a different approach consists in introducing a measure of portfolio diversification and trying to find the portfolio that is the most diversified under this criterion. Such a diversification measure, known as the diversification index (DI), can be defined in terms of distance between portfolio volatility and individual components' volatility:

$$DI = \left(\frac{\sum_i w_i \sigma_i}{\sqrt{\sum_{i,j} w_i w_j \sigma_{ij}}} \right)$$

where w_i is the portfolio weight, and σ_i the volatility of stock *i*, and σ_{ij} the covariance between stocks *i* and *j*. This diversification index has been used by Choueifaty and Coignard (2008) in a portfolio optimisation context to generate a *Maximum Diversification Ratio* (MDR) benchmark, with weights given by:

$$\boldsymbol{w}_{MDR} = \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{\sigma}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\sigma}}$$

where **1** is a vector of ones, σ is the vector of volatilities and Σ is the covariance matrix. While achieving an optimal risk-reward ratio is not the explicit focus in this approach, it is straightforward to see that Maximum Diversification portfolios would actually coincide with Maximum Sharpe Ratio portfolios if all Sharpe ratios were identical for all stocks.

Global Minimum Variance (GMV), Maximum Decorrelation (MDC) and Diversified Minimum Variance (DMV) Portfolios

Minimum variance portfolios are remarkable portfolios that achieve the lowest possible portfolio volatility. This means that the only optimisation inputs required are correlations and volatilities. The fact that the minimum volatility portfolio relies only on risk parameters is an appealing feature, especially when the estimation risk inherent in expected returns is substantial.

Formally, *Global Minimum Variance* (GMV) portfolios are defined by:

$$\boldsymbol{w}_{GMV} = \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}$$

where **1** is a vector of ones and Σ is the covariance matrix. The true minimum volatility portfolio lies on the efficient frontier and coincides with the Maximum Sharpe Ratio portfolio if and only if expected returns are identical across all stocks.

One serious concern is that minimum variance portfolios are typically heavily concentrated in the assets with the lowest volatility. The high concentration in GMV portfolios is a widely recognised issue. Clarke, de Silva and Thorley (2011) note that their long-only minimum variance "portfolio averages about 120 long securities, i.e., about 12% of the 1000-security investable set". Likewise, DeMiguel, Garlappi, Nogales and Uppal (2009) note that "shortsale-constrained minimum-variance portfolios [...] tend to assign a weight different from zero to only a few of the assets". In equity portfolio construction, such concentration in low volatility stocks leads to a pronounced sector bias towards utility stocks. Chan, Karceski and Lakonishok (1999 Table 4) report that a typical minimum variance portfolio invests 47% in the utility sector while the corresponding weight in the market-cap-weighted portfolio is 9% and the corresponding weight in the equally-weighted portfolio is 15%. Researchers have recognised this limitation of minimum volatility portfolio construction, and have proposed various ways to remedy the concentration of optimised portfolios in low volatility stocks. The most straightforward solution to any concentration problem is to impose weight constraints. Imposing lower and/or upper bounds on weights provides quite rigid constraints which leave reduced room for optimisation, but can help to obtain more reasonable portfolios. Recently, more flexible weight constraints have been proposed by DeMiguel, Garlappi, Nogales and Uppal

(2009), who use so-called "norm constraints". Such constraints put limits on the overall amount of concentration in the portfolio (e.g. on the sum of squares of portfolio weights) rather than limiting the weight of each stock in the portfolio, thus leaving more room for the optimiser while avoiding concentration overall.

An alternative to the use of weight constraints is to avoid making a difference between stocks based on their volatilities. Christoffersen *et al.* (2010) minimise volatility with the assumption that volatilities are identical across stocks. Hence the only differences across stocks that the optimiser then takes into account are differences in correlations. The minimum volatility portfolio under this assumption will thus be unaffected by a concentration in low volatility stocks, at the cost of an implicit assumption that all volatilities are equal. This portfolio, known as the *Maximum Decorrelation* (MDC) portfolio has weights given by:

$$w_{\mathrm{MDC}} = \frac{\mathbf{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\mathbf{\Omega}^{-1}\mathbf{1}}$$

where **1** is a vector of ones and Ω is the correlation matrix.

Another simplification of the GMV portfolio consists in assuming that all pairwise correlations are zero, in which case we obtain the so-called *Diversified Minimum Variance* (DMV) portfolio, where the weights are inversely proportional to each stock variance:

$$w_{\mathrm{DMV}} = \frac{\boldsymbol{\sigma}^{-2}}{\mathbf{1}'\boldsymbol{\sigma}^{-2}}$$

where **1** is a vector of ones and σ is the vector of volatilities.

Maximum Sharpe Ratio (MSR) Portfolios

In line with Modern Portfolio Theory, the Maximum Sharpe Ratio (MSR) portfolio is an implementable proxy for the tangency portfolio. As in any mean-variance optimisation, the estimation of input parameters is a central ingredient in the construction of the methodology. In contrast to minimum volatility strategies which only estimate risk parameters (volatilities and correlations), the Maximum Sharpe Ratio strategy attempts to estimate both risk parameters and expected returns. The Maximum Sharpe Ratio portfolio weights are given by:

$$w_{MSR} = \frac{\mathbf{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{1}'\mathbf{\Sigma}^{-1}\boldsymbol{\mu}}$$

where μ is the (Nx1) vector of expected return, **1** is the (Nx1) vector of ones, and Σ is the (NxN) covariance matrix.

As direct estimation of expected returns is well known to lead to large estimation error, Amenc, Goltz, Martellini and Retkowsky (2011) propose to use an indirect estimation of these parameters by assuming that they are positively related to a stock's downside risk. To ensure parsimony and robustness, they sort stocks by their total downside risk (in particular a stock's semi-deviation, which incorporates higher moments) and distinguish stocks based on their riskiness only across groups rather than on a stock by stock basis. Based on this assumption, one can conduct an explicit Sharpe Ratio maximisation.

On the one conceptual front, this approach is intuitively appealing since it suggests focusing on the rational objective for risk-averse investors, namely maximising the benefits to expect from investing in risky assets by generating the highest possible performance per unit of risk. On the pragmatic front, this approach also effectively penalises low risk stocks since it is based upon the assumption that expected returns are proportional to risk. Hence in a formal Maximum Sharpe Ratio portfolio optimisation procedure, this penalty on the expected returns side counterbalances the attractiveness of low-risk stocks from a risk perspective.

References

- Amenc, N. and F. Ducoulombier. 2013. Public Comment on IOSCO Financial Benchmarks Consultation Report CR01/13 (February).

- Amenc, N. and F. Ducoulombier. 2012a. EDHEC-Risk Institute Comments on ESMA Consultation Paper. ESMA/2012/44 (March).

- Amenc, N. and F. Ducoulombier. 2012b. Guidelines on ETFs and Other UCITS Issues – Response to ESMA ETF Guidelines of July 25, 2012.

- Amenc, N. and F. Ducoulombier. 2012c. Comments from EDHEC-Risk Institute on the IOSCO Consultation Report CR05/12 Concerning the Principles for the Regulation of Exchange Traded Funds.

- Amenc, N., F. Ducoulombier, F. Goltz and L. Tang. 2012. What are the Risks of European ETFs? EDHEC-Risk Institute Position Paper (January).

- Amenc N., F. Goltz and A. Lodh. 2012. Choose Your Betas: Benchmarking Alternative Equity Index Strategies. *Journal of Portfolio Management* 39 (1): 88-111.

- Amenc N., F. Goltz, A. Lodh and L. Martellini. 2012. Diversifying the Diversifiers and Tracking the Tracking Error: Outperforming Cap-Weighted Indices with Limited Risk of Underperformance. *Journal of Portfolio Management* 38 (3): 72–88.

- Amenc, N., F. Goltz and L. Martellini. 2011. A Survey of Alternative Equity Index Strategies: A Comment. Letters to the Editor. *Financial Analysts Journal* 67(6): 14-16.

- Amenc, N., F. Goltz, L. Martellini and P. Retkowsky. 2011. Efficient Indexation. *Journal of Investment Management* 9 (4): 1-23.

- Amenc, N., F. Goltz, L. Martellini and S. Ye. 2011. Improved Beta? A Comparison of Index-Weighting Schemes. EDHEC-Risk Institute Publication (September).

- Amenc, N., F. Goltz and S. Stoyanov. 2011. A Post-crisis Perspective on Diversification for Risk Management. EDHEC-Risk Publication (May).

- Amenc, N., F. Goltz and L. Tang. 2011. EDHEC-Risk European Index Survey 2011. EDHEC-Risk Institute Publication (October).

- Amenc, N., F. Goltz, L. Tang and V. Vaidyanathan. 2012. EDHEC-Risk North American Index Survey 2011. EDHEC-Risk Institute Publication (April).

- Amenc, N., F. Goltz and S. Ye. 2012. Seeing through the Smoke Screen of Fundamental Indexers: What are the Issues with Alternative Equity Index Strategies? EDHEC-Risk Institute Publication (June).

- Ang, A., R. Hodrick, Y. Xing and X. Zhang. 2009. High Idiosyncratic Volatility and Low Returns: International and Further U.S. Evidence. *Journal of Financial Economics* 91(1): 1-23.

- Ang, A., R. Hodrick, Y. Xing and X. Zhang. 2006. The Cross-section of Volatility and Expected Returns. *Journal of Finance* 61(1): 259–299.

- Arnott, R., J. Hsu and P. Moore. 2005. Fundamental Indexation. *Financial Analysts Journal* 61(2): 83-99.

- Bali, T. and N. Cakici. 2008. Idiosyncratic Volatility and the Cross-section of Expected Returns? *Journal of Financial and Quantitative Analysis* 43(1): 29-58.

• Blitz, D. 2013. How Smart is 'Smart Beta'? *Journal of Indexes Europe* (March/April).

• Carhart, M. 1997. On the Persistence in Mutual Fund Performance. *Journal of Finance* 52(1): 57-82.

• Chan, L., J. Karceski and J. Lakonishok. 1999. On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model. *Review of Financial Studies* 12(5): 937-974.

• Chopra, V. and W. Ziemba. 1993. The Effect of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice. *Journal of Portfolio Management* 19(2): 6-11.

• Choueifaty, Y. and Y. Coignard. 2008. Toward Maximum Diversification. *Journal of Portfolio Management* 35(1): 40-51.

• Chow, T., J. Hsu, V. Kalesnik and B. Little. 2011a. A Survey of Alternative Equity Index Strategies. *Financial Analysts Journa*l 67(5): 37-57.

• Chow, T., J. Hsu, V. Kalesnik and B. Little. 2011b. A Survey of Alternative Equity Index Strategies. Author Response. Letters to the Editor. *Financial Analysts Journal* 67(6): 16-20.

• Christoffersen, P., V. Errunza, K. Jacobs and J. Xisong. 2010. Is the Potential for International Diversification Disappearing? The Rotman School. Working Paper. (October).

• Clarke, R., H. de Silva and S. Thorley. 2013. Risk Parity, Maximum Diversification, and Minimum Variance: An Analytic Perspective. *Journal of Portfolio Management* (Forthcoming).

• Clarke, R., H. de Silva and S. Thorley. 2011. Minimum Variance Portfolio Composition. *Journal of Portfolio Management* 37(2): 31-45.

• Dash S. and L. Zheng. 2010. Equal Weight Indexing, Seven Years Later. Standard and Poor's. Available at: http://us.spindices.com/documents/research/EqualWeightIndexing_7YearsLater.pdf.

• De Miguel, V., L. Garlappi and R. Uppal. 2009. Optimal versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy? Review of Financial Studies 22(5): 1915-1953.

• DeMiguel, V., L. Garlappi, F. Nogales and R. Uppal. 2009. A Generalized Approach to Portfolio Optimization: Improving Performance by Constraining Portfolio Norms. Management Science 55(5): 798-812.

• Demey, P., S. Maillard and T. Roncalli. 2010. Risk-Based Indexation. Lyxor. White Paper: http://www.lyxor.com/fileadmin/_fileup/lyxor_wp/document/16/docs/all.pdf.

• Fama, E. and K. French. 1993. Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics* 33(1): 3-56.

• Fernholz, R. 1999. On the Diversity of Equity Markets. *Journal of Mathematical Economics* 31(3): 393-417.

• Fernholz, R., R. Garvy and J. Hannon. 1998. Diversity-Weighted Indexing. *Journal of Portfolio Management* 24(2): 74-82.

• Fernholz, R. and B. Shay. 1982. Stochastic Portfolio Theory and Stock Market Equilibrium. *Journal of Finance* 37(2): 615-624.

• Fu, F. 2009. Idiosyncratic Risk and the Cross-section of Expected Stock Returns. *Journal of Financial Economics* 91(1): 24-37.

• Goetzmann, W.N., L. Li and K.G. Rouwenhorst. 2005. Long-Term Global Market Correlations. *Journal of Business* 78(1): 1-38.

• Goltz, F. And V. Le Sourd. 2011. Does Finance Theory Make the Case for Capitalization-Weighted Indexing? *Journal of Index Investing* 2(2): 59-75.

• Grinold, R. C. 1992. Are Benchmark Portfolios Efficient? *Journal of Portfolio Management* 19(1): 34-40.

• Haugen, R. A. and N. L. Baker. 1991. The Efficient Market Inefficiency of Capitalization-Weighted Stock Portfolios. *Journal of Portfolio Management* 17(1): 35-40.

• Huang, W., Q. Liu, S.G. Rhee and L. Zhang. 2010. Return Reversals, Idiosyncratic Risk, and Expected Returns. *Review of Financial Studies* 23(1): 147-168.

• Jagannathan, R. and T. Ma. 2003. Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps. *Journal of Finance* 58(4): 1651-1684.

• Jun, D. and B. Malkiel. 2008. New Paradigms in Stock Market Indexing. *European Financial Management* 14(1): 118–126.

• Kan, R. and G. Zhou. 2007. Optimal Portfolio Choice with Parameter Uncertainty. *Journal of Financial and Quantitative Analysis* 42(3): 621-656.

• Maillard, S., T. Roncalli and J. Teiletche. 2010. The Properties of Equally Weighted Risk Contribution Portfolios. *Journal of Portfolio Management* 36(4): 60-70.

• Markowitz, H. 1952. Portfolio Selection. *Journal of Finance* 7(1): 77-91.

• Martellini, L. and V. Ziemann. 2010. Improved Estimates of Higher-Order Comoments and Implications for Portfolio Selection. *Review of Financial Studies* 23(4): 1467-1502.

• Martellini, L., V. Milhau and A. Tarelli. 2013. To Try or not to Try – An Ex-ante Efficiency Analysis of Heuristic and Scientific Equity Portfolio Construction Strategies. EDHEC-Risk Institute Publication.

• Melas, D. and X. Kang. 2010. Applications of Systematic Indexes in the Investment Process. *Journal of Indexes* (Sept/Oct).

• Merton, R. 1980. On Estimating the Expected Return on the Market: An Exploratory Investigation. *Journal of Financial Economics* 8(4): 323-361.

• Perold, A. 2007. Fundamentally Flawed Indexing. *Financial Analysts Journal* 63(6): 31-37.

• Perold, A. and W. Sharpe. 1995. Dynamic Strategies for Asset Allocation. *Financial Analysts Journal* 51(1): 149-160.

• Petkova, R. and L. Zhang. 2005. Is Value Riskier than Growth? *Journal of Financial Economics* 78: 187-202.

• Platen E. and R. Rendek. 2010. Approximating the Numéraire Portfolio by Naive Diversification. Working paper.

• Plyakha, Y., R. Uppal and G. Vilkov. 2012. Why Does an Equal-Weighted Portfolio Outperform Value- and Price-Weighted Portfolios? SSRN Working Paper.

• Qian, E. 2005. Risk Parity Portfolios: Efficient Portfolios through True Diversification. Working paper, Panagora Asset Management (September).

References

- Roll, R. 1977. A Critique of the Asset Pricing Theory's Tests. Part I: On Past and Potential Testability of the Theory. *Journal of Financial Economics* 4(2): 129-176.
- Schwartz, J. D and L. Siracusano. 2007. WisdomTree Earnings-Weighted Indexes – The Market at a Reasonable Price. WisdomTree White Papers.
- Sharpe, W. 1964. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance* 19(3): 425-442.
- Siegel, J. J., J. D. Schwartz and L. Siracusano. 2007. The Unique Risk and Return Characteristics of Dividend-Weighted Stock Indexes. WisdomTree White Papers.
- Tobin, J. 1958. Liquidity Preference as Behavior towards Risk. *Review of Economic Studies* 25(2): 65-85.
- Wood, P. and C. Evans. 2003. Fundamental Profit Based Equity Indexation. *Journal of Indexes*, 2nd Quarter Edition.

EDHEC-Risk Institute Position Papers and Publications (2010-2013)

2013 Publications

- Blanc-Brude, F. and O.R.H. Ismail. Who is afraid of construction risk? (March)
- Lixia, L., L. Martellini, and S. Stoyanov. The relevance of country- and sector-specific model-free volatility indicators (March).
- Calamia, A., L. Deville, and F. Riva. Liquidity in european equity ETFs: What really matters? (March).
- Deguest, R., L. Martellini, and V. Milhau.. The benefits of sovereign, municipal and corporate inflation-linked bonds in long-term investment decisions (February).
- Deguest, R., L. Martellini, and V. Milhau. Hedging versus insurance: Long-horizon investing with short-term constraints (February).
- Amenc, N., F. Goltz, N. Gonzalez, N. Shah, E. Shirbini and N. Tessaromatis. The EDHEC European ETF survey 2012 (February).
- Padmanaban, N., M. Mukai, L . Tang, and V. Le Sourd. Assessing the quality of Asian stock market indices (February).
- Goltz, F., V. Le Sourd, M. Mukai, and F. Rachidy. Reactions to "A review of corporate bond indices: Construction principles, return heterogeneity, and fluctuations in risk exposures" (January).
- Joenväärä, J., and R. Kosowski. An analysis of the convergence between mainstream and alternative asset management (January).
- Cocquemas, F. Towards better consideration of pension liabilities in European Union countries (January).
- Blanc-Brude, F. Towards efficient benchmarks for infrastructure equity investments (January).

2012 Publications

- Arias, L., P. Foulquier and A. Le Maistre. Les impacts de Solvabilité II sur la gestion obligataire (December).
- Arias, L., P. Foulquier and A. Le Maistre. The Impact of Solvency II on Bond Management (December).
- Amenc, N., and F. Ducoulombier. Proposals for better management of non-financial risks within the european fund management industry (December).
- Cocquemas, F. Improving Risk Management in DC and Hybrid Pension Plans (November).
- Amenc, N., F. Cocquemas, L. Martellini, and S. Sender. Response to the european commission white paper "An agenda for adequate, safe and sustainable pensions" (October).
- La gestion indicielle dans l'immobilier et l'indice EDHEC IEIF Immobilier d'Entreprise France (September).
- Real estate indexing and the EDHEC IEIF commercial property (France) index (September).
- Goltz, F., S. Stoyanov. The risks of volatility ETNs: A recent incident and underlying issues (September).
- Almeida, C., and R. Garcia. Robust assessment of hedge fund performance through nonparametric discounting (June).
- Amenc, N., F. Goltz, V. Milhau, and M. Mukai. Reactions to the EDHEC study "Optimal design of corporate market debt programmes in the presence of interest-rate and inflation risks" (May).
- Goltz, F., L. Martellini, and S. Stoyanov. EDHEC-Risk equity volatility index: Methodology (May).

- Amenc, N., F. Goltz, M. Masayoshi, P. Narasimhan and L. Tang. EDHEC-Risk Asian index survey 2011 (May).
- Guobuzaite, R., and L. Martellini. The benefits of volatility derivatives in equity portfolio management (April).
- Amenc, N., F. Goltz, L. Tang, and V. Vaidyanathan. EDHEC-Risk North American index survey 2011 (March).
- Amenc, N., F. Cocquemas, R. Deguest, P. Foulquier, L. Martellini, and S. Sender. Introducing the EDHEC-Risk Solvency II Benchmarks – maximising the benefits of equity investments for insurance companies facing Solvency II constraints - Summary - (March).
- Schoeffler, P. Optimal market estimates of French office property performance (March).
- Le Sourd, V. Performance of socially responsible investment funds against an efficient SRI Index: The impact of benchmark choice when evaluating active managers – an update (March).
- Martellini, L., V. Milhau, and A.Tarelli. Dynamic investment strategies for corporate pension funds in the presence of sponsor risk (March).
- Goltz, F., and L. Tang. The EDHEC European ETF survey 2011 (March).
- Sender, S. Shifting towards hybrid pension systems: A European perspective (March).
- Blanc-Brude, F. Pension fund investment in social infrastructure (February).
- Ducoulombier, F., Lixia, L., and S. Stoyanov. What asset-liability management strategy for sovereign wealth funds? (February).
- Amenc, N., Cocquemas, F., and S. Sender. Shedding light on non-financial risks – a European survey (January).
- Amenc, N., F. Cocquemas, R. Deguest, P. Foulquier, Martellini, L., and S. Sender. Ground Rules for the EDHEC-Risk Solvency II Benchmarks. (January).
- Amenc, N., F. Cocquemas, R. Deguest, P. Foulquier, Martellini, L., and S. Sender. Introducing the EDHEC-Risk Solvency Benchmarks – Maximising the Benefits of Equity Investments for Insurance Companies facing Solvency II Constraints - Synthesis -. (January).
- Amenc, N., F. Cocquemas, R. Deguest, P. Foulquier, Martellini, L., and S. Sender. Introducing the EDHEC-Risk Solvency Benchmarks – Maximising the Benefits of Equity Investments for Insurance Companies facing Solvency II Constraints (January).
- Schoeffler.P. Les estimateurs de marché optimaux de la performance de l'immobilier de bureaux en France (January).

2012 Position Papers

- Till, H. Who sank the boat? (June).
- Uppal, R. Financial Regulation (April).
- Amenc, N., F. Ducoulombier, F. Goltz, and L. Tang. What are the risks of European ETFs? (January).

2011Publications

- Amenc, N., F. Goltz, Martellini, L., and D. Sahoo. A long horizon perspective on the cross-sectional risk-return relationship in equity markets (December 2011).
- Amenc, N., F. Goltz, and L. Tang. EDHEC-Risk European index survey 2011 (October).

- Deguest,R., Martellini, L., and V. Milhau. Life-cycle investing in private wealth management (October).
- Amenc, N., F. Goltz, Martellini, L., and L. Tang. Improved beta? A comparison of index-weighting schemes (September).
- Le Sourd, V. Performance of socially responsible investment funds against an Efficient SRI Index: The Impact of Benchmark Choice when Evaluating Active Managers (September).
- Charbit, E., Giraud J. R., F. Goltz, and L. Tang Capturing the market, value, or momentum premium with downside Risk Control: Dynamic Allocation strategies with exchange-traded funds (July).
- Scherer, B. An integrated approach to sovereign wealth risk management (June).
- Campani, C. H., and F. Goltz. A review of corporate bond indices: Construction principles, return heterogeneity, and fluctuations in risk exposures (June).
- Martellini, L., and V. Milhau. Capital structure choices, pension fund allocation decisions, and the rational pricing of liability streams (June).
- Amenc, N., F. Goltz, and S. Stoyanov. A post-crisis perspective on diversification for risk management (May).
- Amenc, N., F. Goltz, Martellini, L., and L. Tang. Improved beta? A comparison of index-weighting schemes (April).
- Amenc, N., F. Goltz, Martellini, L., and D. Sahoo. Is there a risk/return tradeoff across stocks? An answer from a long-horizon perspective (April).
- Sender, S. The elephant in the room: Accounting and sponsor risks in corporate pension plans (March).
- Martellini, L., and V. Milhau. Optimal design of corporate market debt programmes in the presence of interest-rate and inflation risks (February).

2011 Position Papers

- Amenc, N., and S. Sender. Response to ESMA consultation paper to implementing measures for the AIFMD (September).
- Uppal, R. A Short note on the Tobin Tax: The costs and benefits of a tax on financial transactions (July).
- Till, H. A review of the G20 meeting on agriculture: Addressing price volatility in the food markets (July).

2010 Publications

- Amenc, N., and S. Sender. The European fund management industry needs a better grasp of non-financial risks (December).
- Amenc, N., S, Focardi, F. Goltz, D. Schröder, and L. Tang. EDHEC-Risk European private wealth management survey (November).
- Amenc, N., F. Goltz, and L. Tang. Adoption of green investing by institutional investors: A European survey (November).
- Martellini, L., and V. Milhau. An integrated approach to asset-liability management: Capital structure choices, pension fund allocation decisions and the rational pricing of liability streams (November).

- Hitaj, A., L. Martellini, and G. Zambruno. Optimal hedge fund allocation with improved estimates for coskewness and cokurtosis parameters (October).

- Amenc, N., F. Goltz, L. Martellini, and V. Milhau. New frontiers in benchmarking and liability-driven investing (September).

- Martellini, L., and V. Milhau. From deterministic to stochastic life-cycle investing: Implications for the design of improved forms of target date funds (September).

- Martellini, L., and V. Milhau. Capital structure choices, pension fund allocation decisions and the rational pricing of liability streams (July).

- Sender, S. EDHEC survey of the asset and liability management practices of European pension funds (June).
- Goltz, F., A. Grigoriu, and L. Tang. The EDHEC European ETF survey 2010 (May).

- Martellini, L., and V. Milhau. Asset-liability management decisions for sovereign wealth funds (May).

- Amenc, N., and S. Sender. Are hedge-fund UCITS the cure-all? (March).

- Amenc, N., F. Goltz, and A. Grigoriu. Risk control through dynamic core-satellite portfolios of ETFs: Applications to absolute return funds and tactical asset allocation (January).

- Amenc, N., F. Goltz, and P. Retkowsky. Efficient indexation: An alternative to cap-weighted indices (January).

- Goltz, F., and V. Le Sourd. Does finance theory make the case for capitalisation-weighted indexing? (January).

2010 Position Papers

- Amenc, N., and V. Le Sourd. The performance of socially responsible investment and sustainable development in France: An update after the financial crisis (September).

- Amenc, N., A. Chéron, S. Gregoir, and L. Martellini. Il faut préserver le Fonds de Réserve pour les Retraites (July).

- Amenc, N., P. Schoefler, and P. Lasserre. Organisation optimale de la liquidité des fonds d'investissement (March).

- Lioui, A. Spillover effects of counter-cyclical market regulation: Evidence from the 2008 ban on short sales (March).

Notes

Founded in 1906, EDHEC Business School offers management education at undergraduate, graduate, post-graduate and executive levels. Holding the AACSB, AMBA and EQUIS accreditations and regularly ranked among Europe's leading institutions, EDHEC Business School delivers degree courses to over 6,000 students from the world over and trains 5,500 professionals yearly through executive courses and research events. The School's 'Research for Business' policy focuses on issues that correspond to genuine industry and community expectations.

Established in 2001, EDHEC-Risk Institute has become the premier academic centre for industry-relevant financial research. In partnership with large financial institutions, its team of ninety permanent professors, engineers, and support staff, and forty-eight research associates and affiliate professors, implements six research programmes and sixteen research chairs and strategic research projects focusing on asset allocation and risk management. EDHEC-Risk Institute also has highly significant executive education activities for professionals. It has an original PhD in Finance programme which has an executive track for high level professionals. Complementing the core faculty, this unique PhD in Finance programme has highly prestigious affiliate faculty from universities such as Princeton, Wharton, Oxford, Chicago and CalTech.

In 2012, EDHEC-Risk Institute signed two strategic partnership agreements with the Operations Research and Financial Engineering department of Princeton University to set up a joint research programme in the area of risk and investment management, and with Yale School of Management to set up joint certified executive training courses in North America and Europe in the area of investment management.

For more information, please contact:
Carolyn Essid on +33 493 187 824
or by e-mail to: carolyn.essid@edhec-risk.com

EDHEC-Risk Institute
393 promenade des Anglais
BP 3116 - 06202 Nice Cedex 3
France
Tel: +33 (0)4 93 18 78 24

EDHEC Risk Institute—Europe
10 Fleet Place, Ludgate
London EC4M 7RB
United Kingdom
Tel: +44 207 871 6740

EDHEC Risk Institute—Asia
1 George Street
#07-02
Singapore 049145
Tel: +65 6438 0030

EDHEC Risk Institute—North America
1230 Avenue of the Americas
Rockefeller Center - 7th Floor
New York City - NY 10020 USA
Tel: +1 212 500 6476

EDHEC Risk Institute—France
16-18 rue du 4 septembre
75002 Paris
France
Tel: +33 (0)1 53 32 76 30

www.edhec-risk.com

EDHEC-RISK
Institute