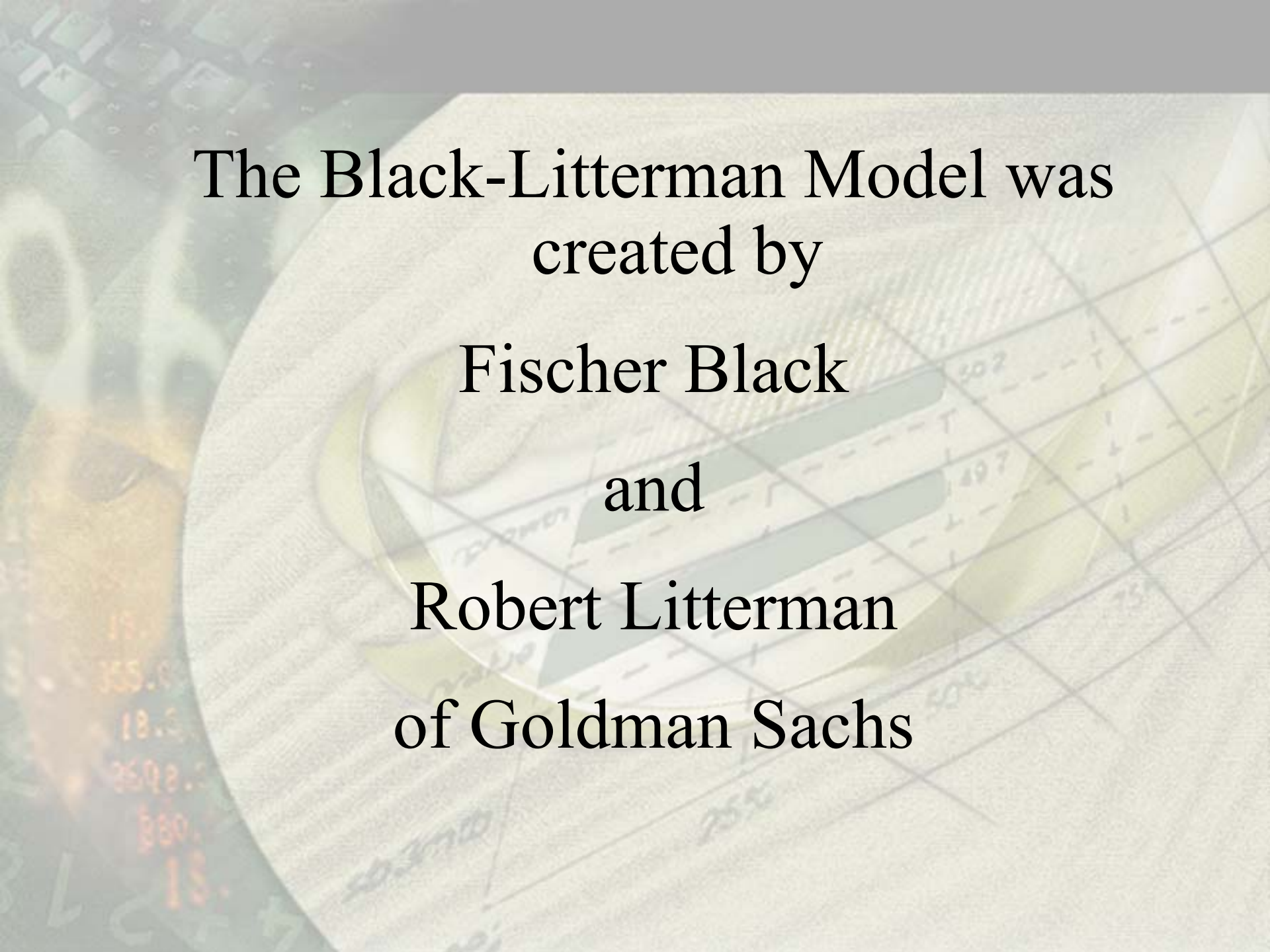# Black-Litterman Return Forecasts in

Tom Idzorek and Jill Adrogue

Zephyr Associates, Inc.

September 9, 2003

Using Black-Litterman Return Forecasts for Asset Allocation Results in Diversified Portfolios

The Black-Litterman Model was created by

Fischer Black

and

Robert Litterman

of Goldman Sachs

Allocation**ADVISOR**™

is Asset Allocation Software

Goal: To create a diversified portfolio of assets that performs well given an acceptable level of risk

How?

Asset Allocation = Mean-Variance Optimization

- Mean-Variance Optimization was developed by Nobel Laureate Harry Markowitz in 1952
    - "Portfolio Selection." *Journal of Finance* 7, no. 1 (March 1952): 77-91.
    - *Portfolio Selection: Efficient Diversification of Investments.* 1959. Reprint. 1970

- Markowitz created a technique for creating efficient diversified portfolios

Principles of Mean-Variance Optimization

- Diversification: The risk of a portfolio can be decreased by combining assets whose returns move in different directions under certain market conditions.

- Markowitz discovered that that an investor can reduce the volatility of a portfolio and increase its return at the same time.

Mean-Variance Optimization (MVO)

- Inputs
  - Returns
  - Risks
  - Correlations
- Calculations--Create Efficient Frontier
- Output--Select Portfolios from the Efficient Frontier

# Forecasts are the cornerstone of good Asset Allocation

- Three forecasts are needed for MVO:
  - Expected Returns
  - Expected Risks
  - Correlations
- AllocationADVISOR currently allows you to use historical values as forecasts or to create your own forecasts

Historical Values as Forecasts

- Expected Return = Arithmetic Mean Return

- Expected Risk = Standard Deviation

- Correlation = Pair-wise Correlation

Historical Forecasts

Analysis Inputs

Assets	Forecast		Date		Constraint	
	Return	Risk	Start	End	Min	Max
US Bonds	8.34%	3.43%	9501	0306	0%	100%
Int'l Bonds	6.49%	8.14%	9501	0306	0%	100%
US Large Growth	10.69%	20.94%	9501	0306	0%	100%
US Large Value	12.87%	15.36%	9501	0306	0%	100%
US Small Growth	7.76%	27.57%	9501	0306	0%	100%
US Small Value	12.89%	15.66%	9501	0306	0%	100%
Int'l Dev. Equity	4.01%	15.19%	9501	0306	0%	100%
Int'l Emerg. Equity	2.29%	24.75%	9501	0306	0%	100%

Correlations

	1	2	3	4	5	6	7	8
1. US Bonds	1.0000							
2. Int'l Bonds	0.3972	1.0000						
3. US Large Growth	0.2325	-0.0320	1.0000					
4. US Large Value	0.2895	-0.0598	0.8242	1.0000				
5. US Small Growth	0.1334	-0.0785	0.8450	0.6979	1.0000			
6. US Small Value	0.2150	-0.1371	0.7244	0.8166	0.8681	1.0000		
7. Int'l Dev. Equity	0.1879	0.3856	0.6154	0.5838	0.5475	0.4964	1.0000	
8. Int'l Emerg. Equity	-0.0182	-0.1144	0.6516	0.5908	0.6736	0.6197	0.7069	1.0000

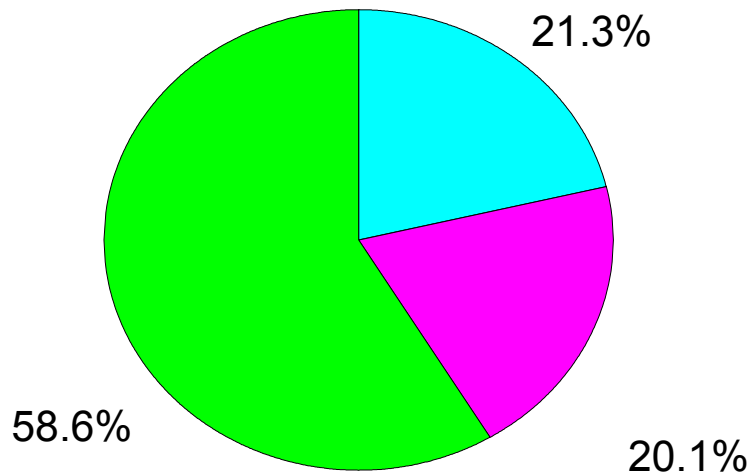# Create Efficient Frontier
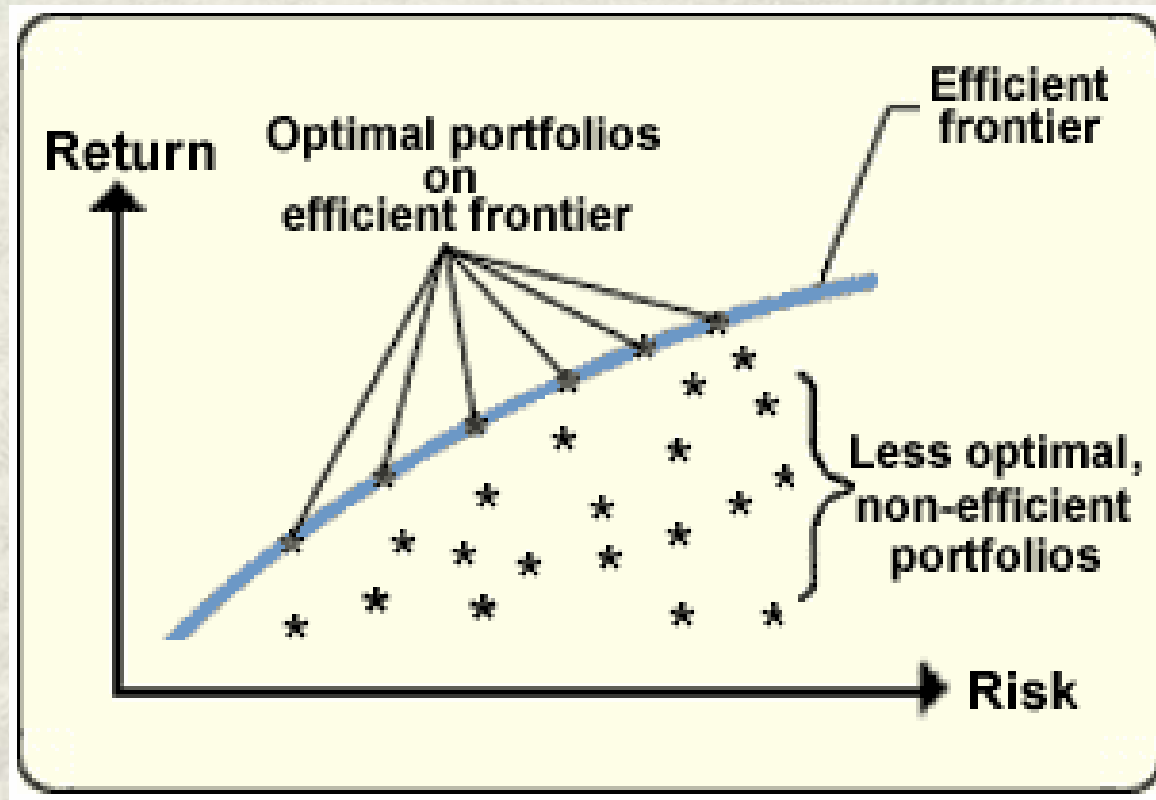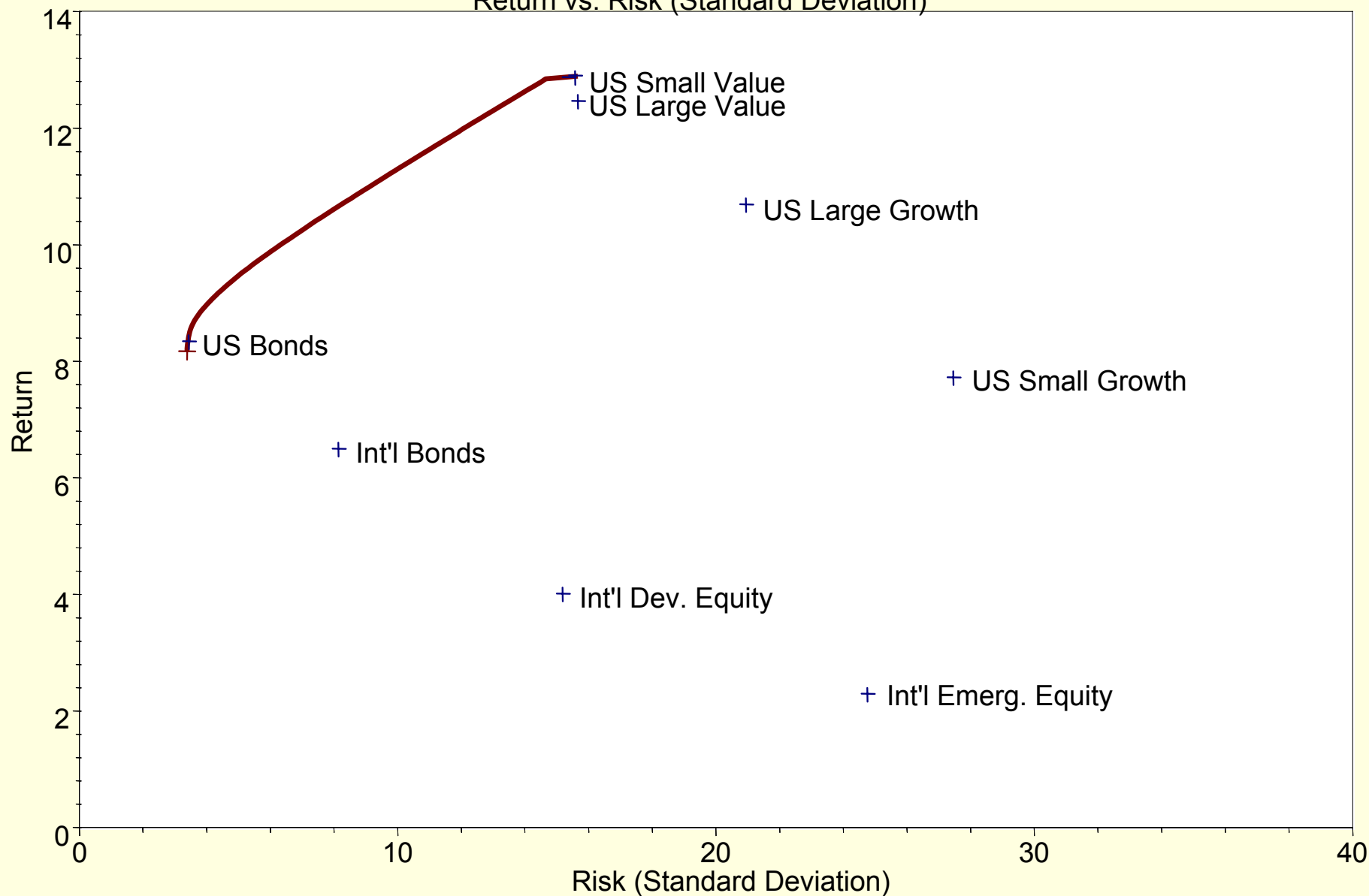
MVO with Historical Data often yields unreasonable results

- Portfolios are often unintuitive
- Portfolios are often highly concentrated
- Portfolio allocations change drastically with small changes in the forecasts

Most of the Portfolios Created With Historical Returns Use Three Asset Classes or Less

Sample Portfolio 1
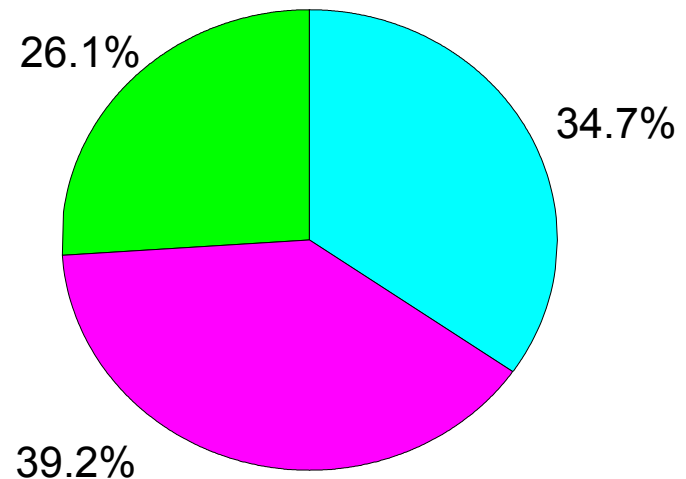
21.3%

58.6%

20.1%

Sample Portfolio 2

26.1%

34.7%

39.2%

Legend:
- US Bonds
- Int'l Bonds
- US Large Growth
- US Large Value
- US Small Growth
- US Small Value
- Int'l Dev. Equity
- Int'l Emerg. Equity

Why does this happen?

- If two asset classes are similar, but one has a slightly higher forecasted return, MVO allocates everything to the asset with the higher forecasted return and nothing to the other asset.

Why is it important?

- Unintuitive portfolios make it very tempting to "tweak" the portfolio using constraints.

- If you are going to use lots of constraints, you might as well skip using MVO in your asset allocation process.

So should we forget about MVO?

NO!

Long-term data can help

- Using long-term data, like that created by Global Financial Data, generally leads to more diversified portfolios than 5 or 10 years of data.

The Black-Litterman Model creates better return forecasts to use with MVO

- Black, Fischer, and Robert Litterman. "Global Portfolio Optimization." *Financial Analysts Journal*, September/October 1992, pp. 28-43.

- He, Guangliang, and Robert Litterman. "The Intuition Behind Black-Litterman Model Portfolios*." Investment Management Research*, Goldman, Sachs & Company, December 1999.

- Litterman, Robert, and the Quantitative Resources Group, Goldman Sachs Asset Management. *Modern Investment Management: An Equilibrium Approach*. New Jersey: John Wiley & Sons, 2003.

Black-Litterman Return Forecasts are coming to

Allocation**ADVISOR**™ ZEPHYR

How does it work?

- Start with the Market Returns
- Apply your own unique views of how certain markets are going to behave
- The end result is a set of return forecasts that give rise to diversified portfolios when used with MVO

Market Portfolio

- The Market Returns are the returns that are implied by the Market Portfolio.

- The Market Portfolio is the capitalization-weighted portfolio of the assets.

A Simple Three Asset Example: The Market Portfolio

	Market Cap	Weight
US Equity	$ 11,498	38.2%
US Bonds	$ 8,280	27.5%
Int'l Equity	$ 10,350	34.4%

Market Returns

- The Market Returns are derived from known information using Reverse Optimization:
  - Risk Premium
  - Covariance Matrix
  - Market Capitalization of the assets

Market Returns are derived using Reverse Optimization

$$\Pi = \lambda \Sigma w_{mkt}$$

Π (Pi) is the excess market returns over the risk free rate

λ (Lambda) is the risk aversion coefficient

Σ (Sigma) is the covariance matrix of returns

w_{mkt} is the market capitalization weight of the assets

To Reverse Engineer the returns for this portfolio we need:

- The Risk Aversion Coefficient λ

- The Covariance Matrix for the three assets Σ

- The market weights for the three assets w_{mkt}

The Risk Aversion Coefficient is the rate at which more return is required for more risk

$$\lambda = \frac{E(r) - r_f}{\sigma^2} = \frac{\text{Risk Premium}}{\text{Variance}}$$

The Risk Aversion Coefficient (λ)

$$\lambda = \frac{\text{Risk Premium}}{\text{Variance}}$$

- Assume a Risk Premium of 4%
- The Historical (Long-term) Standard Deviation for the Market Portfolio is 10.84%. The Variance is the Standard Deviatio.n Squared, 1.117%

The Risk Aversion Coefficient

$$\lambda = \frac{\text{Risk Premium}}{\text{Variance}}$$

$$\lambda = \frac{4.00\%}{1.117\%}$$

$$\lambda = 3.404$$

Covariance Matrix

The covariance of each pair of assets is calculated using the historical (long-term) correlations and standard deviations.

Covariance (A,B) =

Correlation (A,B) * Std Dev (A) * Std Dev (B)

Covariance Matrix (Σ)

Covariance	US Equity	US Bonds	Int'l Equity
US Equity	0.036	0.002	0.010
US Bonds	0.002	0.003	0.001
Int'l Equity	0.010	0.001	0.025

Cov(*US Equity, US Equity*) = 0.036

Cov(*US Equity, US Bonds*) = 0.002

Cov(*US Equity, Int'l Equity*) = 0.010

Market Portfolio Weights (w_{mkt})

Market Portfolio Weights are based on the market capitalization for each of the three assets

	Market Cap	Weight
US Equity	$ 11,498	38.2%
US Bonds	$ 8,280	27.5%
Int'l Equity	$ 10,350	34.4%

$$W_{\text{US Equity}} = .382$$

$$W_{\text{US Bonds}} = .275$$

$$W_{\text{Int'l Equity}} = .344$$

Implied Excess Market Returns

$$\Pi = \lambda \Sigma w_{mkt}$$

$$\Pi_{US\ Equity} = \lambda * [\text{Cov(US Equity, US Equity)} * w_{US\ Equity}$$

$$+ \text{Cov(US Equity, US Bonds)} * w_{US\ Bonds}$$

$$+ \text{Cov(US Equity, Int'l Equity)} * w_{US\ Int'l\ Equity}]$$

$$\Pi_{US\ Equity} = 3.404 * [.036 * .382 + .002 * .275 + 0.010 * .344]$$

$$\Pi_{US\ Equity} = 6.05\%$$

Implied Total Market Return

- The Total Return is the Excess Return plus the Risk-Free Rate

- Forecast Return for US Equity = 6.05 + 4.5 = 10.55%

Market Return Forecasts

- The same calculations are made for US Bonds and Int'l Equity:
  - Forecast Return for US Equity = 10.6 %
  - Forecast Return for US Bonds = 5.2 %
  - Forecast Return for Int'l Equity = 8.9 %

- These returns can be used in AllocationADVISOR, or you can combine them with your own unique views

Market Return Forecasts for Three Assets

Analysis Inputs

	Forecast		Date		Constraint	
	Return	Risk	Start	End	Min	Max
Assets						
US Equity	10.6%	19.0%	2601	0307	0%	100%
US Bond	5.2%	5.5%	2601	0307	0%	100%
Intl Equity	8.9%	15.7%	2601	0307	0%	100%

Correlations

	1	2	3
1. US Equity	1.00		
2. US Bond	0.19	1.00	
3. Intl Equity	0.34	0.17	1.00

A more complete set of assets

	Market Cap	Weight
Lehman US Universal	$ 8,280	19.34%
Lehman Global Ex US	$11,184	26.13%
Russell 1000 Growth	$ 5,174	12.09%
Russell 1000 Value	$ 5,174	12.09%
Russell 2000 Growth	$ 575	1.34%
Russell 2000 Value	$ 575	1.34%
Citigroup World x US	$10,350	24.18%
Citigroup EM World B	$ 1,496	3.49%

Market Returns

Analysis Inputs

Assets	Forecast Return	Risk	Date Start	End	Constraint Min	Max
US Bonds	5.07%	3.43%	9501	0306	0%	100%
Int'l Bonds	5.90%	8.14%	9501	0306	0%	100%
US Large Growth	12.20%	20.94%	9501	0306	0%	100%
US Large Value	9.89%	15.36%	9501	0306	0%	100%
US Small Growth	13.34%	27.57%	9501	0306	0%	100%
US Small Value	9.21%	15.66%	9501	0306	0%	100%
Int'l Dev. Equity	10.66%	15.19%	9501	0306	0%	100%
Int'l Emerg. Equity	12.27%	24.75%	9501	0306	0%	100%

Correlations

	1	2	3	4	5	6	7	8
1. US Bonds	1.0000							
2. Int'l Bonds	0.4081	1.0000						
3. US Large Growth	0.2168	-0.0335	1.0000					
4. US Large Value	0.2754	-0.0602	0.8241	1.0000				
5. US Small Growth	0.1070	-0.0837	0.8445	0.6971	1.0000			
6. US Small Value	0.1869	-0.1413	0.7242	0.8160	0.8684	1.0000		
7. Int'l Dev. Equity	0.1879	0.3856	0.6154	0.5838	0.5475	0.4964	1.0000	
8. Int'l Emerg. Equity	-0.0182	-0.1144	0.6516	0.5908	0.6736	0.6197	0.7069	1.0000

Add your own unique views

- Investors generally have opinions, or views, about how certain markets will behave in the future.

# Each view includes a measure of certainty

Example of an Absolute View

- Opinion: "I think that International Equity is going to do well."

- View: International Developed Equity will have a return of 11%

- Confidence of View: 55%

Example of a Relative View

- Opinion: "I believe that Growth is going to outperform Value."

- View: US Large Growth and US Small Growth will outperform US Large Value and US Small Value by 3%.

- Confidence of View: 80%

Combine the Market Returns with your Views

Market Returns   Views

Black-Litterman Forecast Returns
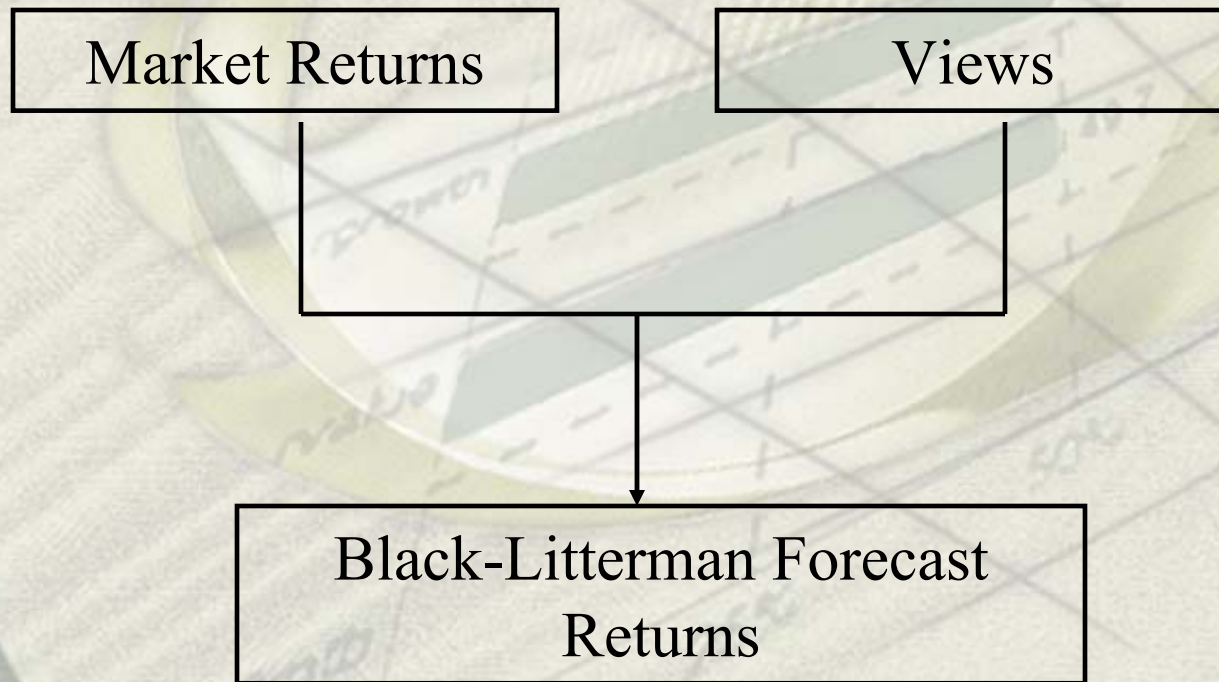
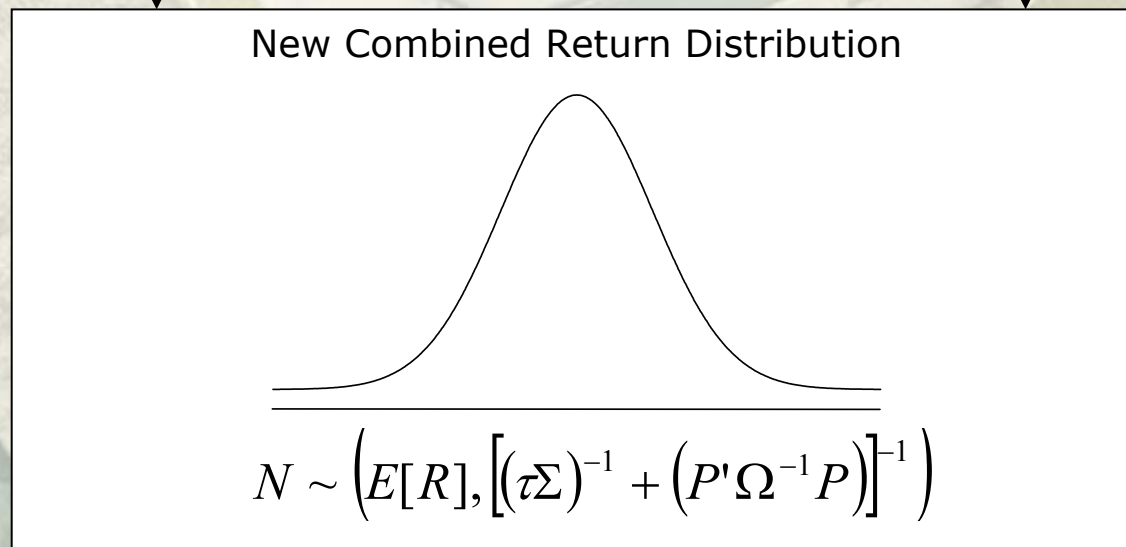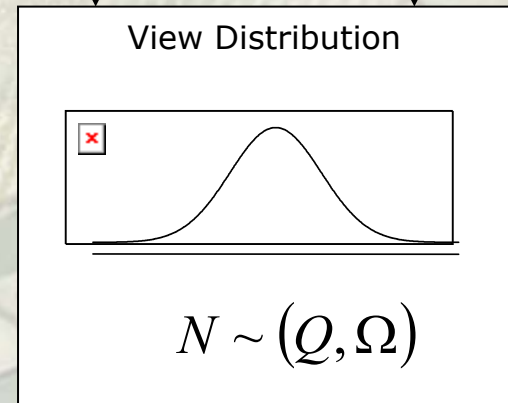| Risk Aversion Coefficient	Covariance Matrix	Market Capitalization Weights	Views	Uncertainty of Views
$\lambda = \left(E(r) - r_f\right)/\sigma^2$	(Σ)	(w_{mkt})	(Q)	(Ω)

Implied Equilibrium Return Vector

$$\Pi = \lambda \Sigma w_{mkt}$$

Prior Equilibrium Distribution

$$N \sim (\Pi, \tau\Sigma)$$

View Distribution

$$N \sim (Q, \Omega)$$

New Combined Return Distribution

$$N \sim \left(E[R], \left[(\tau\Sigma)^{-1} + \left(P'\Omega^{-1}P\right)\right]^{-1}\right)$$
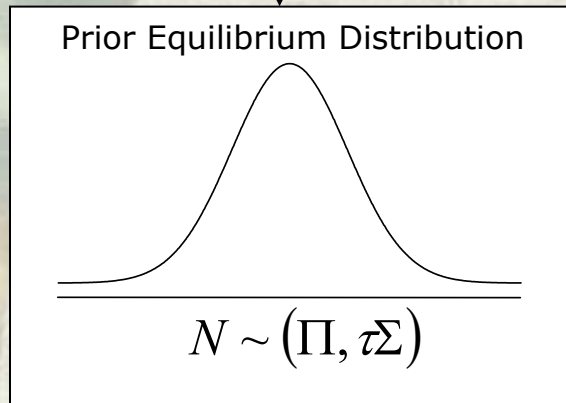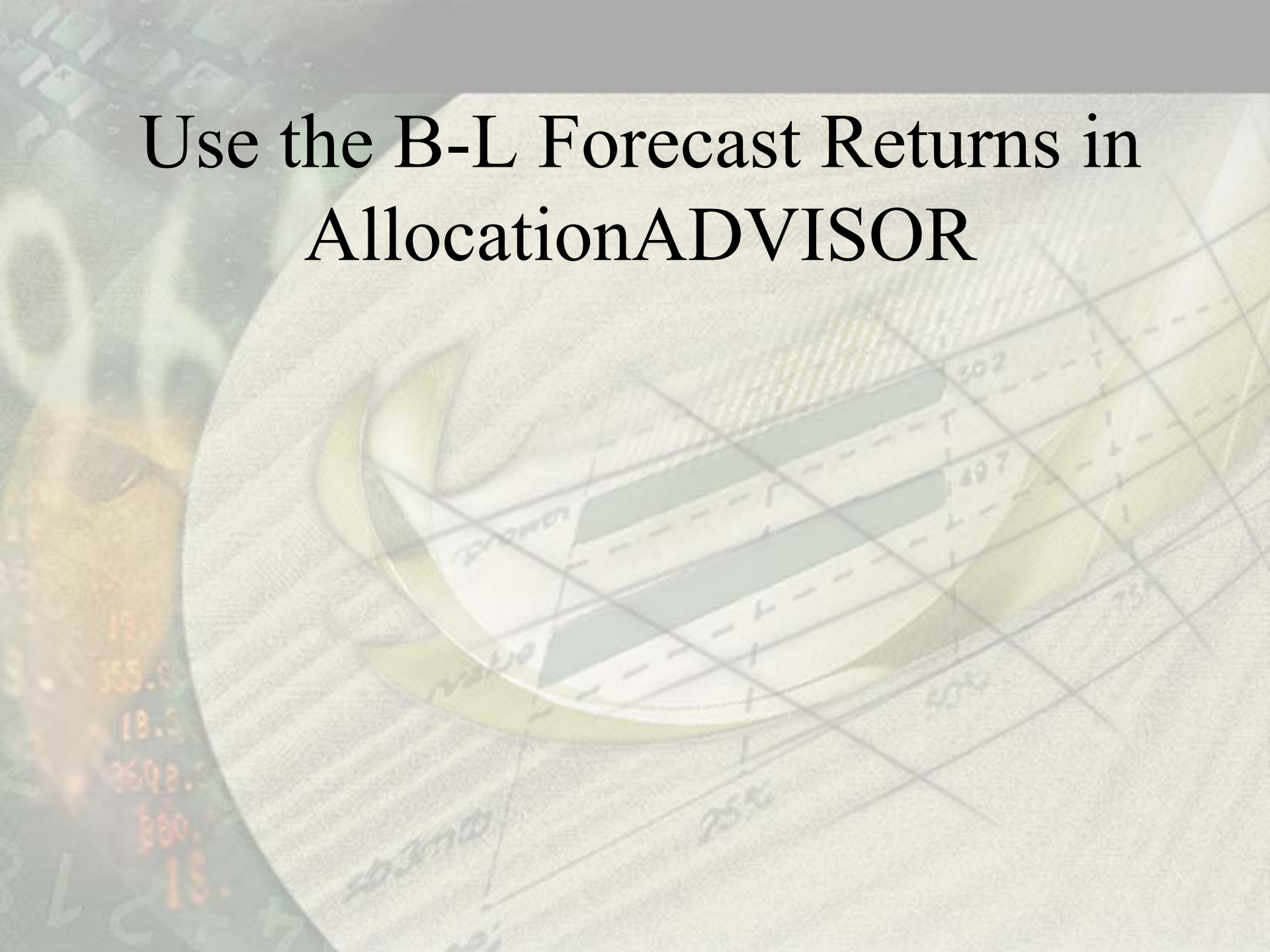
The Views move the Return Forecasts away from the Market Returns
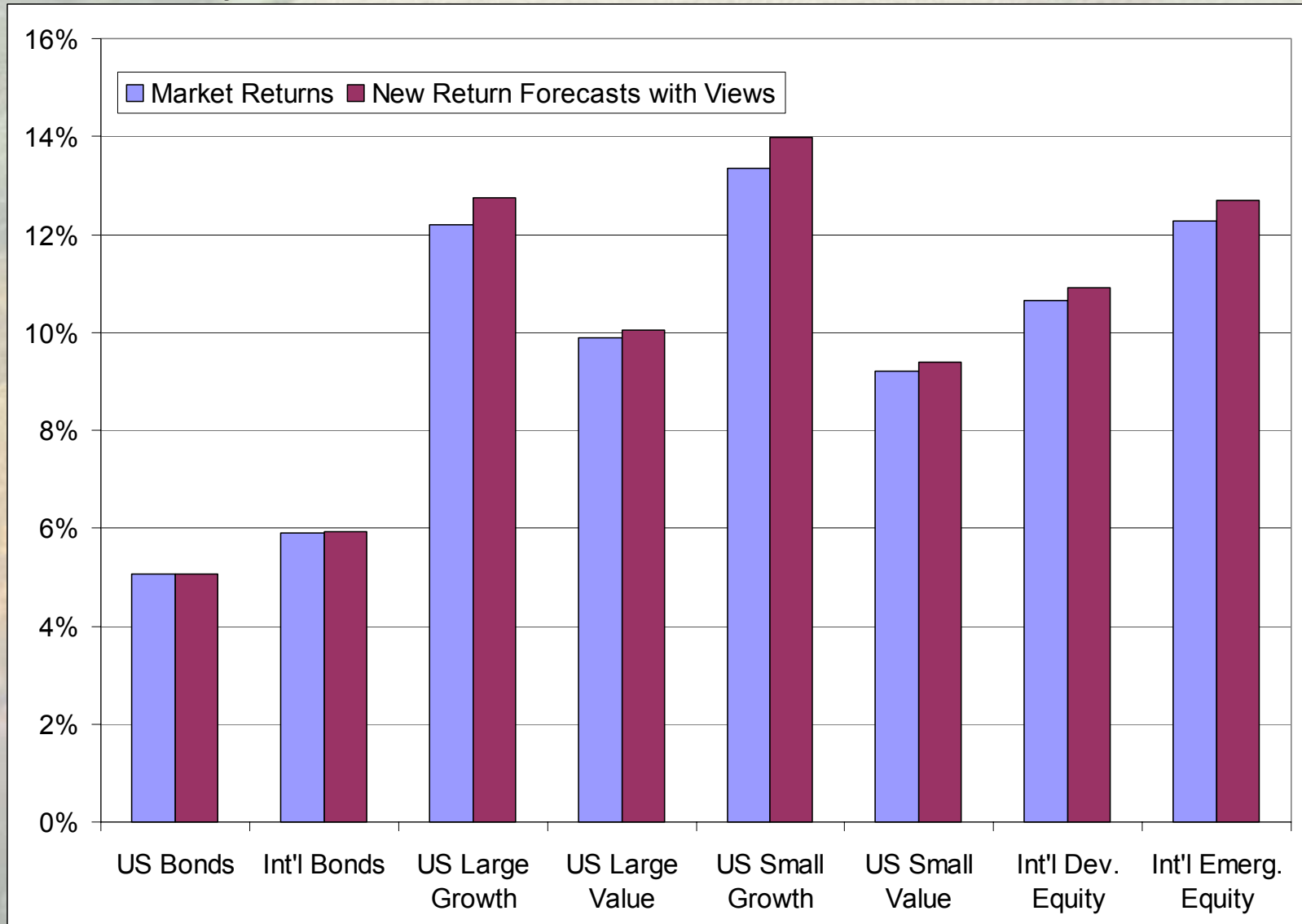
And the resulting Asset Allocations will change, reflecting your views

View 1 - Bullish view on Int'l Dev. Equity increases allocation

View 3 - Bullish view on Growth tilts portfolio toward Growth

Categories: US Bonds, Int'l Bonds, US Large Growth, US Large Value, US Small Growth, US Small Value, Int'l Dev. Equity, Int'l Emerg. Equity

Legend: ☐ Market Weights ■ New Allocations (with Views)
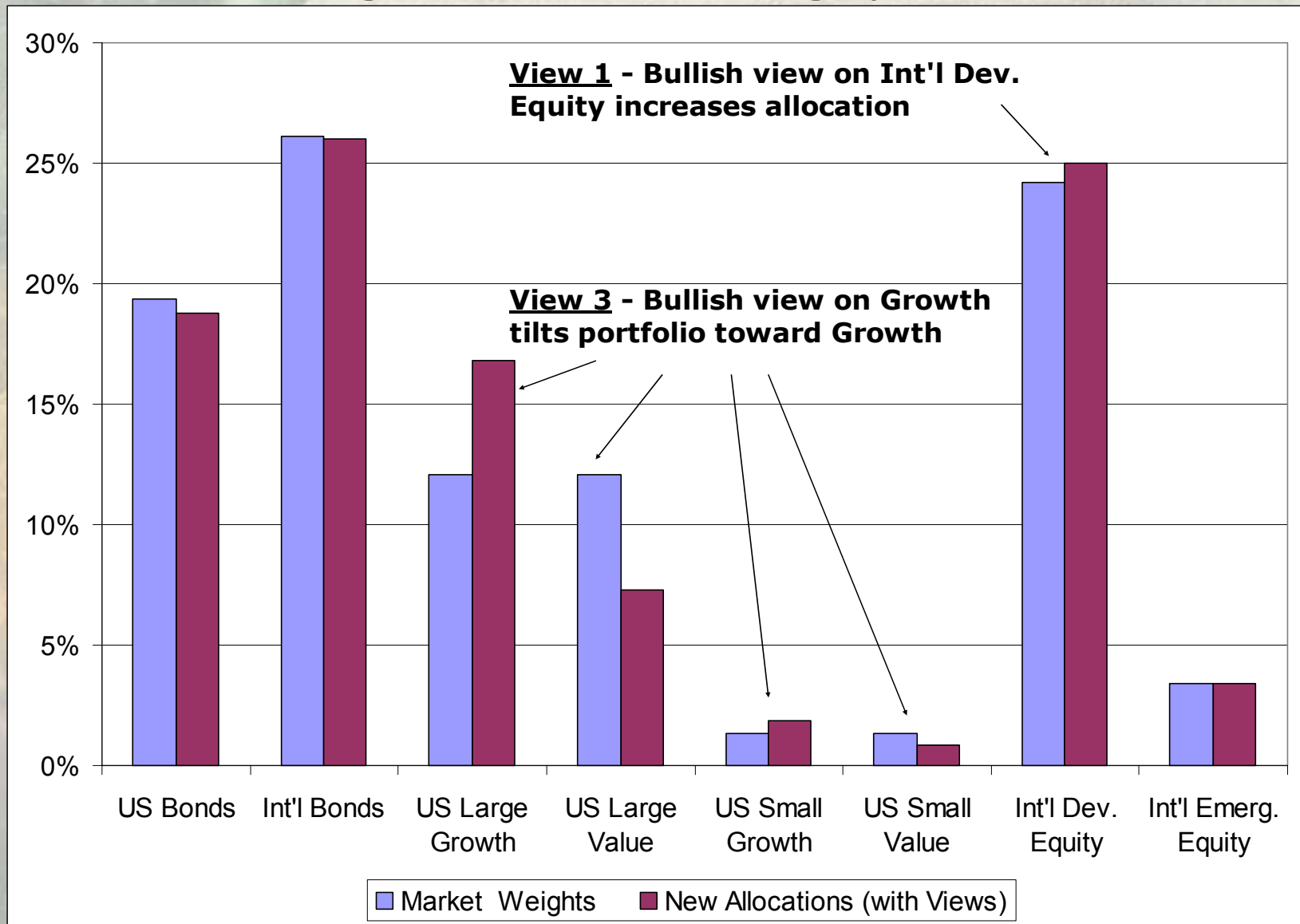
Use the B-L Forecast Returns in AllocationADVISOR

Black-Litterman Forecast Returns

Analysis Inputs

Assets	Forecast Return	Risk	Date Start	End	Constraint Min	Max
US Bonds	5.07%	3.43%	9501	0306	0%	100%
Int'l Bonds	5.92%	8.14%	9501	0306	0%	100%
US Large Growth	12.76%	20.94%	9501	0306	0%	100%
US Large Value	10.04%	15.36%	9501	0306	0%	100%
US Small Growth	13.99%	27.57%	9501	0306	0%	100%
US Small Value	9.39%	15.66%	9501	0306	0%	100%
Int'l Dev. Equity	10.92%	15.19%	9501	0306	0%	100%
Int'l Emerg. Equity	12.70%	24.75%	9501	0306	0%	100%

Correlations

	1	2	3	4	5	6	7	8
1. US Bonds	1.0000							
2. Int'l Bonds	0.4081	1.0000						
3. US Large Growth	0.2168	-0.0335	1.0000					
4. US Large Value	0.2754	-0.0602	0.8241	1.0000				
5. US Small Growth	0.1070	-0.0837	0.8445	0.6971	1.0000			
6. US Small Value	0.1869	-0.1413	0.7242	0.8160	0.8684	1.0000		
7. Int'l Dev. Equity	0.1879	0.3856	0.6154	0.5838	0.5475	0.4964	1.0000	
8. Int'l Emerg. Equity	-0.0182	-0.1144	0.6516	0.5908	0.6736	0.6197	0.7069	1.0000

Asset Allocation Analysis

Efficient Frontier
Return vs. Risk (Standard Deviation)
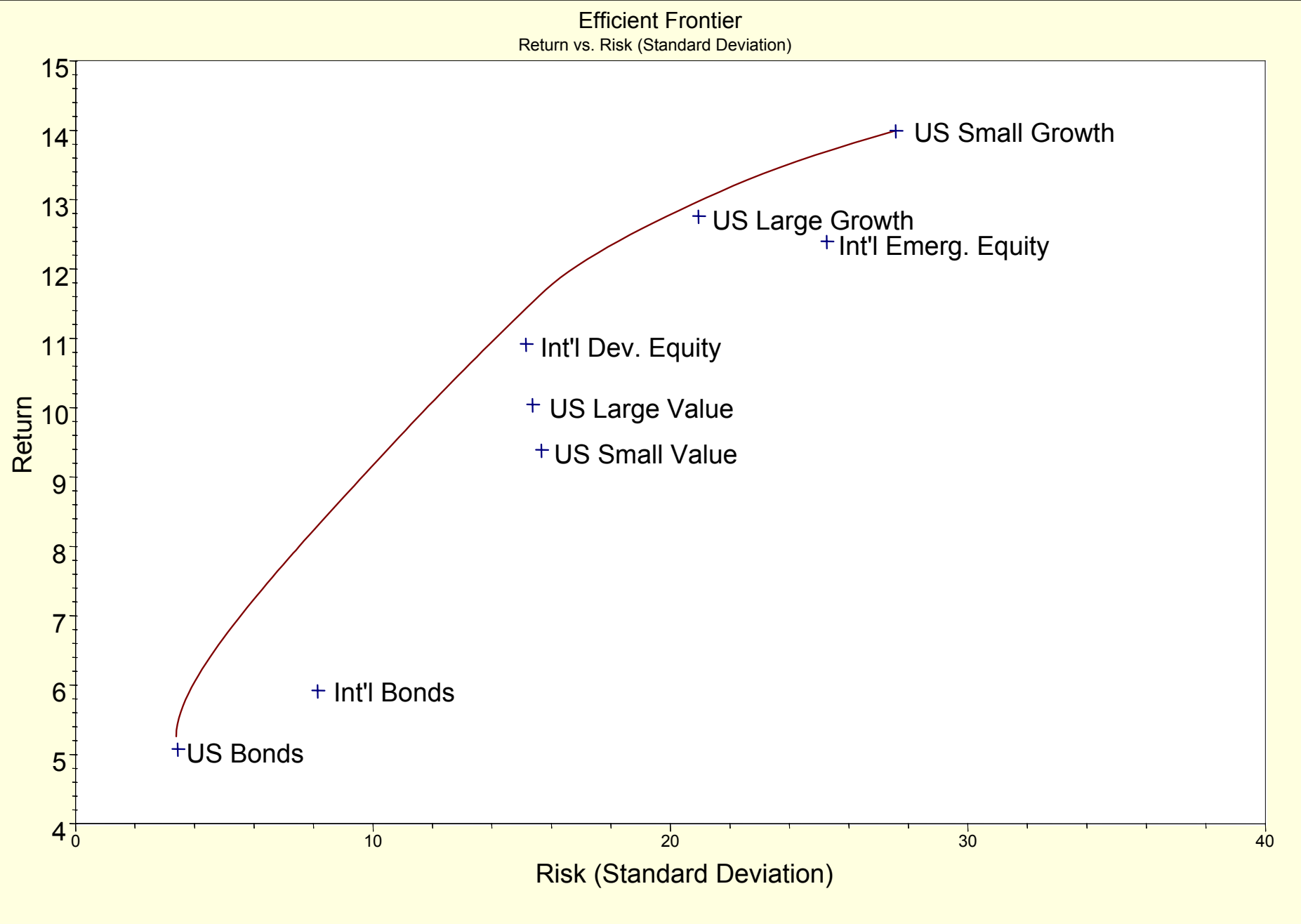
The B-L Return Forecasts yield more diversified portfolios

Sample Portfolio 1

3.5%
19.3%
24.2%
26.1%
1.3%
1.3%
12.1%
12.1%

Sample Portfolio 2
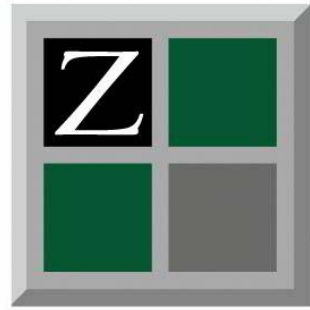
4.5% 3.0%
30.0%
30.2%
0.3%
2.4%
9.4%
20.3%

Legend:
- US Bonds
- Int'l Bonds
- US Large Growth
- US Large Value
- US Small Growth
- US Small Value
- Int'l Dev. Equity
- Int'l Emerg. Equity