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Nonparametric tolerance limits for pair trading

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ABSTRACT

Tolerance interval is an important statistical tool for determining the threshold of a certain reference. We propose to utilize nonparametric one-sided tolerance limits with three look-back window sizes for return spreads in order to find trading entry and exit signals. We illustrate how the proposed method help uncover arbitrage opportunities via the daily return spreads of 12 stock pairs in the U.S. markets and then report the performance of pair trading for two out-of-sample periods. The empirical results suggest that combining the minimum squared distance method and nonparametric one-sided tolerance limits generates positive excess returns, relative to the underlying stocks.

1. Introduction

"Pair trading" is Wall Street's quantitative method of speculation that was pioneered by Nunzio Tartaglia's quant group at Morgan Stanley in the 1980s and continues to be an important statistical arbitrage technique used by hedge funds. The strategy behind pair trading is to find similar assets with dissimilar valuations, by analyzing stocks that are relatively correlated or similar (same industry or subsector), but are valued differently by the market. Investors can buy the cheap asset and sell (go short) the rich asset, with the target being a convergence in the value of the spread (the difference between the prices of the two traded assets). One attractive feature of pair trading is the ability to profit whether the market is going up, down, or sideways, because the strategy does not depend on market direction, but on the relationship between the two assets. A major task in developing a pair trading strategy is to identify potential stock pairs.

There are many ways to find stocks that move together. Gatev et al. (2006), Vidyamurthy (2004), and Elliott et al. (2005) are examples of studies on pair trading in the literature. Krauss (2016) broadly reviews the literature on pair trading frameworks, categorizing them into five groups. Some pair traders search for securities with a high degree of correlation so that they can attempt to profit when prices behave outside this statistical norm. Correlation analysis helps identify interesting potential pairs, but one must be aware of spurious correlation. The simplest one is the minimum squared distance method (MSD), which involves calculating the sum of squared deviations between two normalized stock prices and choosing the one with the minimum value. Chen et al. (2014) use MSD to choose pairs.

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In this paper we employ two methods to find matching stocks. One is the MSD method, as noted above. Highly-correlated pairs often come from the same sector, because they face similar systematic risks. The second way is to look at the fundamentals of firms to select two stocks that have almost the same risk factor exposure. Some common pairs are suggested for various trading markets, in which the selected similar companies usually operate in the same sector. As noted, the spread is the difference between the prices of two traded assets. In order to obtain appropriate investment decisions, traders can compare observations of the spread with predictions from a calibrated model.

Some works on pair trading rely on constructing mean-reverting spreads that maintain a certain degree of predictability. Elliott et al. (2005) propose a mean reverting Gaussian Markov chain model for the spread. Chen et al. (2014) set up a three-regime threshold autoregressive model with GARCH effects (TAR-GARCH model) for a return spread, in which the upper and lower thresholds of TAR-GARCH represent trading entry and exit signals for the next trading day. Chen et al. (2016) employ smooth transition heteroskedastic models with a second-order logistic function to generate trading entry and exit signals and suggest two pair trading strategies: the first uses the upper and lower threshold values in the proposed model as trading entry and exit signals, while the second strategy instead takes one-step-ahead quantile forecasts obtained from the same model. In practice, one may not be aware of the assumption of a population probability distribution function. Thus, this study suggests implementing tolerance intervals to identify potential arbitrage trading opportunities. Several works contribute findings on the issue of distribution-free option bounds. For example, Laurence and Wang (2008) derive closed-form distribution-free bounds and optimal hedging strategies for spread options.

A tolerance interval can be a statistical version of a probability interval, whereby the endpoints of a tolerance interval are called tolerance limits. Hahn and Meeker (1991) point out that a one-sided nonparametric tolerance limit is equivalent to a one-sided nonparametric confidence limit for a percentile of that population. A one-sided tolerance interval consists of a single upper or lower limit under which a given proportion p of the population falls with a given confidence level $1-\alpha$. It provides information on the entire population; to be specific about the information content, a tolerance interval should capture a certain proportion or more of the population, under a given confidence level. Many various practical applications arising in process capability studies and quality control, pharmaceutical bioequivalence, environmental monitoring, plant or animal inbreeding, and other areas utilize meaningful applications for tolerance intervals.

Some well-known empirical characteristics concerning financial time series are non-Gaussian and asymmetry. In this paper we propose to implement nonparametric one-sided tolerance limits, and with specified content, calculate nonparametric one-sided tolerance limits for a trading pair's spread in order to design trading entry and exit signals - that is, one-sided tolerance limits for low and high percentiles. We estimate the tolerance by the rolling window approach.

Suppose that the return spread is defined as return A minus return B. When the return spread is above an $h\%$ one-sided tolerance limit as the trading entry signal, we short one share of stock A and long one share of stock B, and vice versa, where $h\%$ is designed by the users. We unwind the position when the previous return spread crosses over the same tolerance limit again, which is denoted as the exit signal. Young (2010) provides a useful R package for obtaining tolerance intervals with "content" involving discrete and continuous cases as well as regression tolerance intervals. This paper determines reference thresholds by using the *R* package "tolerance" for estimating nonparametric one-sided tolerance limits of spread values (Young, 2010).

Compared to existing research, our paper offers several innovations. First, we develop nonparametric one-sided tolerance limits to generate trading entry and exit signals. To the best of our knowledge, this is the first work on tolerance limits in the domain of finance application that implements tolerance intervals for determining trading thresholds to detect potential arbitrage trading opportunities. Second, the trading strategy can be designed based on investors' attitude to risk - that is, the percentage of information content can help decide the frequency of trading.

We use the MSD method to select ten stock pairs and choose an additional three pairs of two companies in the same industrial sector. We consider pair trading profits for two out-of-sample periods in 2014 within a six-month time frame. This study's key findings are as follows. The suggested contents (70% and 80%) yield average annualized returns of at least 17.9% without a transaction cost and at least 16.9% with a transaction cost.

2. Nonparametric tolerance intervals

The key to the empirical approach is employing nonparametric one-sided tolerance limits. The nonparametric methods used herein are based on the result of Wilks (1941), who states that if a sample is from a continuous distribution, then the distribution of the proportion of the population between two order statistics is independent of the population sampled and is a function of only the particular order statistics chosen. Let $X_1, \ldots, X_n$ be a sample from a continuous distribution F, and let $X_{(1)}, \ldots, X_{(n)}$ be the order statistics for the sample. The standard approach to construct nonparametric tolerance intervals is to choose the appropriate order statistics, provided a minimum sample size requirement is met. In order to construct a nonparametric $(p, 1-\alpha)$ tolerance interval, we need to find the positive integers r and s, $r<s$, so that:

$$P_{X_{(r)},\ X_{(s)}}\left\{P_X(X_{(r)} \le X \le X_{(s)} | X_{(r)}, X_{(s)}) \ge p\right\} = 1-\alpha.$$

One may similarly define a one-sided tolerance limit based on a single-order statistic. A nonparametric $(p,\ 1-\alpha)$ lower tolerance limit requires finding a positive integer k such that:

$$P_{X_{(k)}}\left\{P_X(X \ge X_{(k)} | X_{(k)}) \ge p\right\} = 1-\alpha. \tag{1}$$

Table 1
List of 12 pairs: pairs 1–9 are from the MSD criteria and pairs 10–12 are from the same industrial sectors.

Pair	Code	Name	2014 Ave r_t*	Code	Name	2014 Ave r_t*
1	DIS	Walt Disney	1.7617	MMM	3M	1.4472
2	DIS	Walt Disney	1.7617	JNJ	Johnson & Johnson	1.2688
3	DIS	Walt Disney	1.7617	TRV	Travelers	1.4217
4	PG	Procter & Gamble	1.1388	PEP	PepsiCo	1.2638
5	TRV	Travelers	1.4217	HD	The Home Depot	2.0966
6	DIS	Walt Disney	1.7617	PEP	PepsiCo	1.2638
7	UTX	United Technologies Corporation	0.2524	MMM	3M	1.4472
8	PEP	PepsiCo	1.2638	JNJ	Johnson & Johnson	1.2688
9	DIS	Walt Disney	1.7617	HD	The Home Depot	2.0966
10	HD	Home Depot	2.0966	LOW	Lowe's Companies	2.7374
11	KO	Coca-Cola	0.4069	PEP	PepsiCo	1.2638
12	MSFT	Microsoft	1.9328	AAPL	Apple	2.7060

* : monthly average return in 2014.

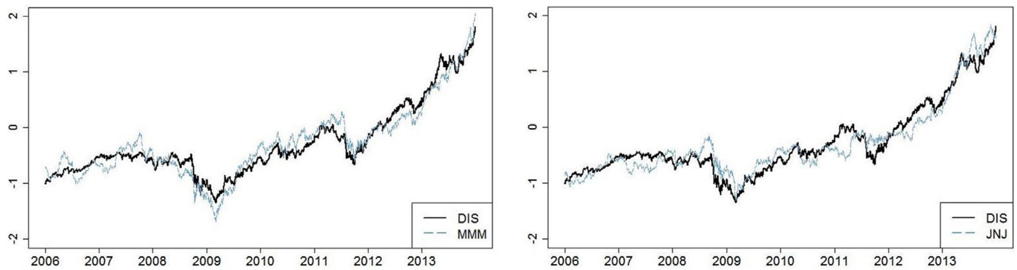

Fig. 1. Normalized prices for pair 1 (DIS/MMM) and pair 2 (DIS/JNJ) from Jan. 2006 to Dec. 2013.

A nonparametric $(p,1-\alpha)$ upper tolerance limit also requires finding a positive integer m such that:

$$P_{X_{(m)}}\left\{P_X(X \le X_{(m)}|X_{(m)}) \ge p\right\} = 1-\alpha. \qquad (2)$$

In practical applications, the choices of p and $1-\alpha$ may be left to the subject matter expert. For the purposes of this study, the focus of interest is $(X_{(k)}, X_{(m)})$, based on the choices of $(p,1-\alpha)$, which represent tomorrow's trading entry and exit thresholds.

3. Data and the pair trading procedure

We consider the daily closing prices of 36 companies in the Dow Jones Industrial Average (DJIA), New York Stock Exchange (NYSE), and NASDAQ stock markets and obtain the data from Yahoo Finance US from January 2, 2006 to December 31, 2014. We calculate the series of returns by taking differences of the logarithms of the daily closing price $r_t^i = (\ln P_t^i - \ln P_{t-1}^i) \times 100$, where P_t^i is the closing price of asset i on day t. We employ a learning period from January 2, 2006 to December 31, 2013 to choose the trading pairs. We consider two validation periods: the first half of 2014 (denoted by H1) and the second half of 2014 (denoted by H2).

We calculate all MSD values between any two normalized price series during the learning period, and the number of possible pairs is 630 (C_2^{36}) combinations. The formula of MSD is:

$$\mathrm{MSD} = \sum_{t=1}^{n}(p_t^i - p_t^j)^2, \text{ where } p_t^i = \frac{P_t^i - E(P^i)}{\sigma^i},$$

where P_t^i is the closing price of asset i at time t, E(P^i) is the average of P^i, and σ^i is the standard deviation of the respective stock price. In this set-up, p_t^i is the normalized price of asset i at time t. Prices should be normalized, because using original prices would be a problem for the case of the minimum squared distance rule since two assets can move together, but still have a high squared distance between them. The transformation employed is the normalization of the price series based on its mean and standard deviation.

We form the pairs based on the MSD method over the learning period. Table 1 lists the selected 12 pairs, which include nine with the smallest MSD values. Note that the first nine pairs are chosen by the MSD method, while pairs 10–12 are suggested by certain webpages. The last three pairs are some common pairs that include two companies in the same industrial sector. In Fig. 1 we only present two time series plots for the normalization of the price series for Pair 1 (DIS/MMM) and

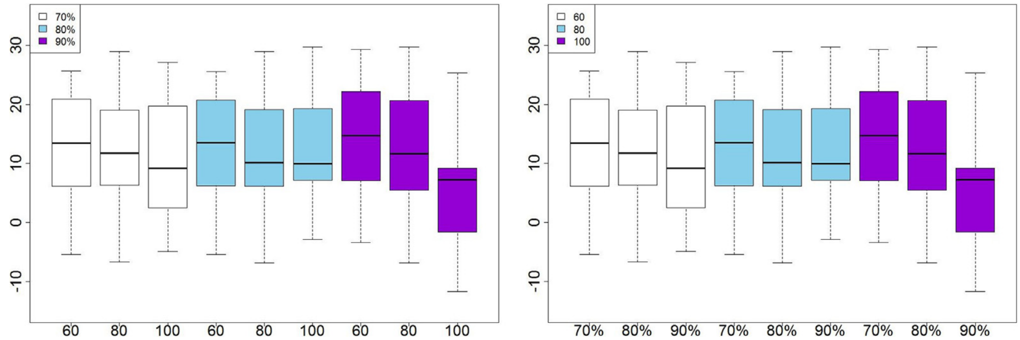

Fig. 2. Return profits for the three specification limits and sample sizes (60, 80, 100) during the first half of 2014.

Pair 2 (DIS/JNJ). These two pairs are highly correlated during the learning period. We also note that Disneyland has been selected in five pairs (see Table 1). We then trade each pair over the next six months based on three specified nonparametric one-sided tolerance limits with three look-back window sizes. We describe the procedure as follows.

Step 1: To identify stock pairs we calculate MSD between any two normalized price series during the learning period among all 36 stocks. We select nine pairs with the smallest MSD values, plus three pairs from the same industrial sector.

Step 2: We calculate the return spread between the selected pairs $X_t = r_t^i - r_t^j$. This study's in-sample period is a look-back window of size N, where N is 60, 80, and 100 trading days.

Step 3: We next calculate one-sided tolerance limits with $(p, 1-\alpha)$=(0.7,0.95), (0.8,0.95), and (0.9,0.95) in Eqs. (1) and (2). We take the lower and upper tolerance limits as trading entry and exit signals.

Step 4: We check whether we can borrow some treasury shares of stock A at time t. When the return spread is above the upper tolerance limit at time t, we sell one share of stock A and buy one share of stock B. Conversely, we check whether we can borrow some treasury shares of stock B. If the return spread is below the lower tolerance limit at time t, then we sell one share of stock B and buy one share of stock A at time t.

Step 5: We run a full iterative calibration procedure every time a new datapoint is observed and the window has shifted one-step ahead and set two out-of-sample periods: H1 and H2.

Step 6: We close the trading position when the given six-month period ends. We calculate the average trading return from the above positions as follows.

$$\text{Sell stock A and buy stock B} : r_1 = \left[-\ln \frac{P_{ss}^A}{P_{lb}^A} + \ln \frac{P_{ss}^B}{P_{lb}^B} \right].$$

$$\text{Sell stock B and buy stock A} : r_2 = \left[\ln \frac{P_{ss}^A}{P_{lb}^A} - \ln \frac{P_{ss}^B}{P_{lb}^B} \right],$$

where P_{ss}^i stands for the short-selling price of stock i and P_{lb}^i stands for the long-buying price of stock i. We consider two scenarios: without a transaction cost and with a transaction cost. In order to use a pair trading strategy, we buy both stocks A and B traded on their first day's price during the validation period.

4. Results and discussion

Without pair trading, Table 1 presents the monthly average returns of each company during the entire year of 2014. The ranges of these individual stock returns are (0.2, 2.73). Most companies' second-half profits exceed their first-half results, which are not listed in Table 1. Tables 2–5 report pair trading returns based on one-sided tolerance limits for the 12 pairs. We consider three settings of tolerance limits: $(p, 1-\alpha)$=(0.7,0.95), (0.8,0.95), and (0.9,0.95). We summarize the results below.

1 For MSD selection pairs (Pairs 1–9), one-sided tolerance limits with $(p, 1-\alpha)$=(0.7,0.95), (0.8,0.95) generate positive profits for both semi-annual periods, whereas we obtain some negative profits from $(p, 1-\alpha)$=(0.9,0.95).

2 After matching the industrial sector's pairs (Pairs 10–12), the most profitable is HD vs. LOW in the second half of 2014. The performance is quite extraordinary for the two. The results in Table 6 show that this pair has the highest profit of 62% under $(p, 1-\alpha)$=(0.7,0.95) for the second half of 2014, while the same pair has negative profit in the first half of 2014; none of the contents and sample size combinations gain positive profits in period H1.

3 Figs. 2–3 display the boxplots of profits for the combinations of three specification limits with three sample sizes for periods H1 and H2. The average profits under the $(p, 1-\alpha)$=(0.7,0.95) tolerance limits are the best among the three settings during period H1. It seems that the average profits under the $(p, 1-\alpha)$=(0.7,0.95) and (0.8,0.95) tolerance

Table 2
Pair trading returns based on one-sided tolerance limits for pairs 1–3.

Sample size	P-content	H1 2014			H2 2014		
		Pairs' return	0.2% cost	Round-trip trades*	Pairs' return	0.2% cost	Round-trip trades*
DIS vs. MMM							
60	70%	16.6011	13.4011	16	23.7593	21.3593	12
80	70%	16.6011	13.6011	15	23.7593	21.1593	13
100	70%	16.6011	15.4011	6	23.7593	22.3593	7
60	80%	14.7290	13.2290	7.5	24.6472	23.0472	8
80	80%	14.7290	13.1290	8	24.6472	23.0472	8
100	80%	17.9337	17.1337	4	24.6472	23.6472	5
60	90%	9.3548	9.0548	1.5	18.5847	17.9847	3
80	90%	9.3016	8.9016	2	18.5847	17.7847	4
100	90%	−11.6552	−11.7552	0.5	18.5847	18.1847	2
DIS vs. JNJ							
60	70%	27.0294	24.9294	10.5	9.9063	7.4063	12.5
80	70%	27.0294	25.1294	9.5	9.9063	7.2063	13.5
100	70%	27.0294	25.5294	7.5	9.9063	8.9063	5
60	80%	26.8779	25.3779	7.5	8.5165	7.1165	7
80	80%	26.8779	25.3779	7.5	8.5165	7.3165	6
100	80%	27.6570	26.4570	6	10.4358	9.6358	4
60	90%	22.7245	22.3245	2	4.7317	4.3317	2
80	90%	22.7245	21.9245	4	4.7317	4.3317	2
100	90%	17.6638	17.4638	1	4.7317	4.5317	1
DIS vs. TRV							
60	70%	18.0487	15.6487	12	21.5230	18.8230	13.5
80	70%	18.0487	15.4487	13	21.5230	18.4230	15.5
100	70%	18.0487	16.4487	8	21.5230	20.6230	4.5
60	80%	15.8876	14.0876	9	16.3484	15.3484	5
80	80%	15.8876	14.0876	9	18.8983	17.6983	6
100	80%	17.3782	16.4782	4.5	16.3484	15.9484	2
60	90%	17.5395	17.1395	2	−12.1236	−12.8236	3.5
80	90%	17.3782	16.6782	3.5	−17.2939	−18.1939	4.5
100	90%	7.5640	7.3640	1	−11.4820	−11.8820	2

* : If there is a one-way trade at the end, then we count 0.5 for the round-trip trade.

Table 3
Pair trading returns based on one-sided tolerance limits for pairs 4–6.

Sample size	P-content	H1 2014			H2 2014		
		Pairs' return	0.2% cost	Round-trip trades*	Pairs' return	0.2% cost	Round-trip trades*
PG vs. PEP							
60	70%	9.0098	5.8098	16	22.7327	20.3327	12
80	70%	9.0098	6.2098	14	22.7327	20.5327	11
100	70%	9.0098	7.2098	9	22.7327	21.1327	8
60	80%	8.1895	6.5895	8	22.0003	20.6003	7
80	80%	8.1895	6.1895	10	22.0003	20.4003	8
100	80%	5.0637	4.0637	5	22.0003	21.2003	4
60	90%	5.0637	4.6637	2	22.1672	21.5672	3
80	90%	8.5093	7.5093	5	22.0003	21.4003	3
100	90%	7.1973	6.9973	1	26.2039	25.8039	2
TRV vs. HD							
60	70%	6.1961	3.7961	12	37.6006	35.2006	12
80	70%	6.1961	3.7961	12	37.6006	35.2006	12
100	70%	6.1961	4.5961	8	37.6006	36.3006	6.5
60	80%	5.0293	3.4293	8	34.6618	33.2618	7
80	80%	3.6186	2.0186	8	34.5673	33.0673	7.5
100	80%	5.0293	4.0293	5	34.6618	33.8618	4
60	90%	-0.2379	-0.5379	1.5	37.6006	37.4006	1
80	90%	5.0293	4.2293	4	34.6618	34.4618	1
100	90%	1.8365	1.6365	1	8.1427	8.0427	0.5
DIS vs. PEP							
60	70%	21.6069	19.3069	11.5	17.1009	14.5009	13
80	70%	21.6069	19.1069	12.5	17.1009	14.5009	13
100	70%	21.6069	20.5069	5.5	17.1009	15.9009	6
60	80%	12.8748	11.5748	6.5	12.1720	10.9720	6
80	80%	12.8748	11.5748	6.5	12.1720	10.1720	10
100	80%	12.8748	12.1748	3.5	11.9263	11.3263	3
60	90%	7.5557	7.4557	0.5	11.9263	11.5263	2
80	90%	11.4133	10.9133	2.5	11.9263	11.5263	2
100	90%	7.6451	7.5451	0.5	13.2955	12.9955	1.5

* : If there is a one-way trade at the end, then we count 0.5 for the round-trip trade.

Table 4
Pair trading returns based on one-sided tolerance limits for pairs 7–9.

Sample size	P-content	H1 2014			H2 2014		
		Pairs' return	0.2% cost	Round-trip trades*	Pairs' return	0.2% cost	Round-trip trades*
UTX vs. MMM							
60	70%	8.5186	6.4186	10.5	14.2439	11.1439	15.5
80	70%	8.5186	6.2186	11.5	14.2439	11.1439	15.5
100	70%	8.5186	6.9186	8	14.2439	11.8439	12
60	80%	7.9830	6.3830	8	14.9474	12.9474	10
80	80%	10.1862	8.5862	8	17.3467	14.8467	12.5
100	80%	9.4861	8.8861	3	14.9474	13.3474	8
60	90%	9.5416	9.2416	1.5	−3.1498	−3.6498	2.5
80	90%	9.4861	8.8861	3	5.6956	4.7956	4.5
100	90%	−5.0011	−5.2011	1	−3.1498	−3.6498	2.5
PEP vs. JNJ							
60	70%	25.2302	22.5302	13.5	7.3668	4.7668	13
80	70%	25.2302	22.3302	14.5	7.3668	4.7668	13
100	70%	25.2302	23.9302	6.5	7.3668	6.0668	6.5
60	80%	25.2302	23.9302	6.5	9.2260	8.1260	5.5
80	80%	25.2302	24.1302	5.5	9.2260	7.9260	6.5
100	80%	24.9391	24.1391	4	10.2849	9.2849	5
60	90%	25.5851	25.2851	1.5	10.2849	9.8849	2
80	90%	25.8578	25.2578	3	10.2849	9.4849	4
100	90%	25.5851	25.2851	1.5	10.0072	9.7072	1.5
DIS vs. HD							
60	70%	11.5535	9.4535	10.5	35.7181	33.0181	13.5
80	70%	11.5535	9.4535	10.5	35.7181	33.0181	13.5
100	70%	11.5535	10.5535	5	35.7181	34.5181	6
60	80%	7.1939	6.1939	5	35.7181	34.2181	7.5
80	80%	7.1939	5.9939	6	35.7181	34.0181	8.5
100	80%	7.1939	6.7939	2	30.5435	29.5435	5
60	90%	0.5373	0.2373	1.5	30.7891	30.5891	1
80	90%	7.1939	6.7939	2	30.2187	29.6187	3
100	90%	3.7578	3.6578	0.5	16.1995	16.0995	0.5

* : If there is a one-way trade at the end, then we count 0.5 for the round-trip trade.

Table 5
Pair trading returns based on one-sided tolerance limits for pairs 10–12.

Sample size	P-content	H1 2014			H2 2014		
		Pairs' return	0.2% cost	Round-trip trades*	Pairs' return	0.2% cost	Round-trip trades*
HD vs. LOW							
60	70%	−2.3617	−5.4617	15.5	62.5537	60.5537	10
80	70%	−2.3617	−5.4617	15.5	62.5537	60.4537	10.5
100	70%	−2.3617	−3.3617	5	62.5537	61.8537	3.5
60	80%	−5.5127	−6.7127	6	55.1548	53.8548	6.5
80	80%	−5.5127	−6.9127	7	62.2177	60.8177	7
100	80%	−6.1635	−6.8635	3.5	51.6109	51.1109	2.5
60	90%	−4.5305	−4.9305	2	54.9045	54.7045	1
80	90%	−2.3617	−2.8617	2.5	51.6109	51.3109	1.5
100	90%	−4.5305	−4.9305	2	20.7014	20.6014	0.5
KO vs. PEP							
60	70%	15.5184	13.4184	10.5	8.5200	5.8200	13.5
80	70%	15.5184	13.4184	10.5	8.5200	5.6200	14.5
100	70%	15.5184	14.0184	7.5	8.5200	7.0200	7.5
60	80%	13.6526	11.9526	8.5	9.4463	7.6463	9
80	80%	9.7532	8.0532	8.5	8.8048	7.1048	8.5
100	80%	11.9140	11.1140	4	1.9835	0.8835	5.5
60	90%	11.9140	11.3140	3	6.0281	5.5281	2.5
80	90%	11.9140	11.1140	4	6.0281	5.5281	2.5
100	90%	7.9823	7.4823	2.5	4.2847	3.9847	1.5
MSFT vs. AAPL							
60	70%	28.1827	25.6827	12.5	29.1296	26.3296	14
80	70%	28.1341	25.5341	13	29.1296	26.1296	15
100	70%	30.3414	29.3414	5	29.1296	27.5296	8
60	80%	30.3414	28.9414	7	35.3799	34.0799	6.5
80	80%	30.3414	28.9414	7	32.4618	30.8618	8
100	80%	30.3414	29.7414	3	35.3799	34.2799	5.5
60	90%	27.4318	27.1318	1.5	33.6955	33.2955	2
80	90%	30.3414	29.7414	3	33.6955	33.2955	2
100	90%	10.9680	10.8680	0.5	33.6955	33.4955	1

* : If there is a one-way trade at the end, then we count 0.5 for the round-trip trade.

Table 6
Summary of pair trading profits for the combination of three sample sizes and three specification limits without a transaction cost.

Sample Size	Time	Content	Mean	Median	Std	Minimum	Maximum
60	H1	70%	15.4278	16.0597	9.2710	−2.3617	28.1827
		80%	13.5397	13.2637	10.1777	−5.5127	30.3414
		90%	11.0400	9.4482	10.4127	−4.5305	27.4318
	H2	70%	24.1796	22.1278	15.6700	7.3668	62.5537
		80%	23.1849	19.1743	14.4530	8.5165	55.1548
		90%	17.9533	15.2555	18.9359	−12.1236	54.9045
80	H1	70%	15.4238	16.0597	9.2648	−2.3617	28.1341
		80%	13.2808	11.5305	10.2662	−5.5127	30.3414
		90%	13.0656	10.4497	9.3558	−2.3617	30.3414
	H2	70%	24.1796	22.1278	15.6673	7.3668	62.5537
		80%	23.8814	20.4493	15.6220	8.5165	62.2177
		90%	17.6787	15.2555	18.1682	−17.2939	51.6109
100	H1	70%	15.6077	16.0597	9.5574	−2.3617	30.3414
		80%	13.6373	12.3944	10.6076	−6.1635	30.3414
		90%	5.7511	7.3807	10.0908	−11.6552	25.5851
	H2	70%	24.1796	22.1278	15.6673	7.3668	62.5537
		80%	22.0642	19.1743	13.9834	1.9835	51.6109
		90%	11.7679	11.6514	12.5129	−11.4820	33.6955

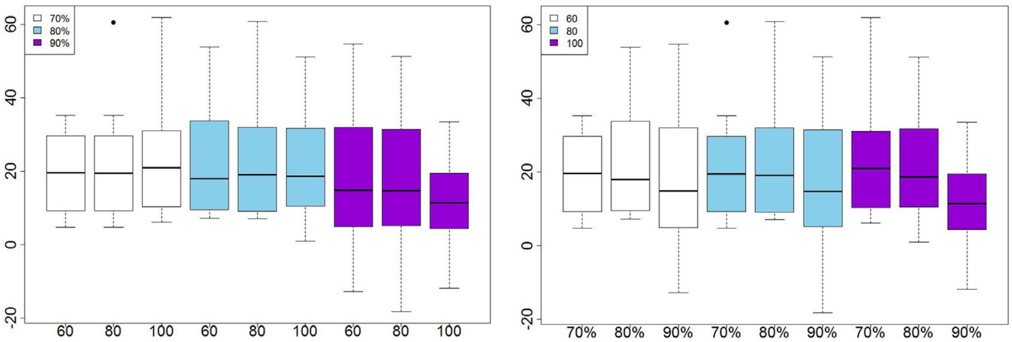

Fig. 3. Return profits for the three specification limits and sample sizes (60, 80, 100) during the second half of 2014.

limits are equally sound among the three sample sizes during period H2. In the case of (0.9,0.95) tolerance limits with size 100, the profit is the worst among all in terms of average profits.

A pair trader must naturally pay commissions to either enter or exit each trade. Depending on the particular strategy employed, this cost can add up quickly. Thus, transaction cost should be taken into consideration to determine if the strategy can turn a profit. We now consider the profit performance for the three specification limits with a 0.2% transaction cost. In other words, we assume a 0.2% cost is incurred upfront to go short, while there is no cost to go long. Tables 6 and 7 present descriptive statistics on the trade-return profits for the 12 pairs without and with a transaction cost, respectively.

Without considering any transaction cost, Table 7 shows that $(p,1-\alpha)=(0.7,0.95)$ performs the best in terms of average semi-annual profit among the three settings during H1 and H2 (see also Fig. 4). The performance of $(p,1-\alpha)=(0.8,0.95)$ is the second best. When we consider the transaction cost, the cumulative net profit mean from the first half of the year is 12.91%, 12.08%, and 10.70% for $(p,1-\alpha)=(0.7,0.95)$, (0.8,0.95), and (0.9,0.95) (with sample size 60), respectively. We observe that the results are similar for various sample sizes. For the second half of the year, it is 21.60%, 21.77%, and 17.53% for $(p,1-\alpha)=(0.7,0.95)$, (0.8,0.95), and (0.9,0.95) (with sample size 60), respectively. Performance is relatively insensitive to which look-back window sizes we use. In summary, $(p,1-\alpha)=(0.7,0.95)$ has the largest average semi-annual profit among three settings during H1 and H2.

4. Conclusion

Pair trading is the leading type of statistical arbitrage technique. We propose implementing a statistical arbitrage strategy based on nonparametric one-sided tolerance limits for return spreads, in order to generate trading entry and exit signals. We focus on $(p,1-\alpha)=(0.7,0.95)$, (0.8,0.95), and (0.9,0.95). From our empirical results, we favor the former two contents since they allow investors to obtain greater amounts of profit for most of the pair trading cases. It seems that the sample sizes are not the major factor in gaining profit. Of course, one may design any content different from our choices.

Table 7
Summary of pair trading profits for the combination of three sample sizes and three specification limits with a 0.2% transaction cost.

Sample Size	Time	Content	Mean	Median	Std	Minimum	Maximum
60	H1	70%	12.9111	13.4097	9.4110	−5.4617	25.6827
		80%	12.0814	11.7637	10.1464	−6.7127	28.9414
		90%	10.6983	9.1482	10.4079	−4.9305	27.1318
	H2	70%	21.6046	19.5778	15.8300	4.7668	60.5537
		80%	21.7682	17.9743	14.4820	7.1165	53.8548
		90%	17.5283	14.7555	19.0574	−12.8236	54.7045
80	H1	70%	12.8988	13.5097	9.3673	−5.4617	25.5341
		80%	11.7641	10.0805	10.3250	−6.9127	28.9414
		90%	12.4240	9.9074	9.3349	−2.8617	29.7414
	H2	70%	21.5129	19.4778	15.8471	4.7668	60.4537
		80%	22.2730	19.0493	15.6674	7.1048	60.8177
		90%	17.1121	14.6555	18.3225	−18.1939	51.3109
100	H1	70%	14.2577	14.7097	9.6030	−3.3617	29.3414
		80%	12.8456	11.6444	10.5697	−6.8635	29.7414
		90%	5.5344	7.1807	10.0751	−11.7552	25.2851
	H2	70%	22.8379	20.8778	15.8531	6.0668	61.8537
		80%	21.1725	18.5743	14.0676	0.8835	51.1109
		90%	11.4929	11.3514	12.5617	−11.8820	33.4955

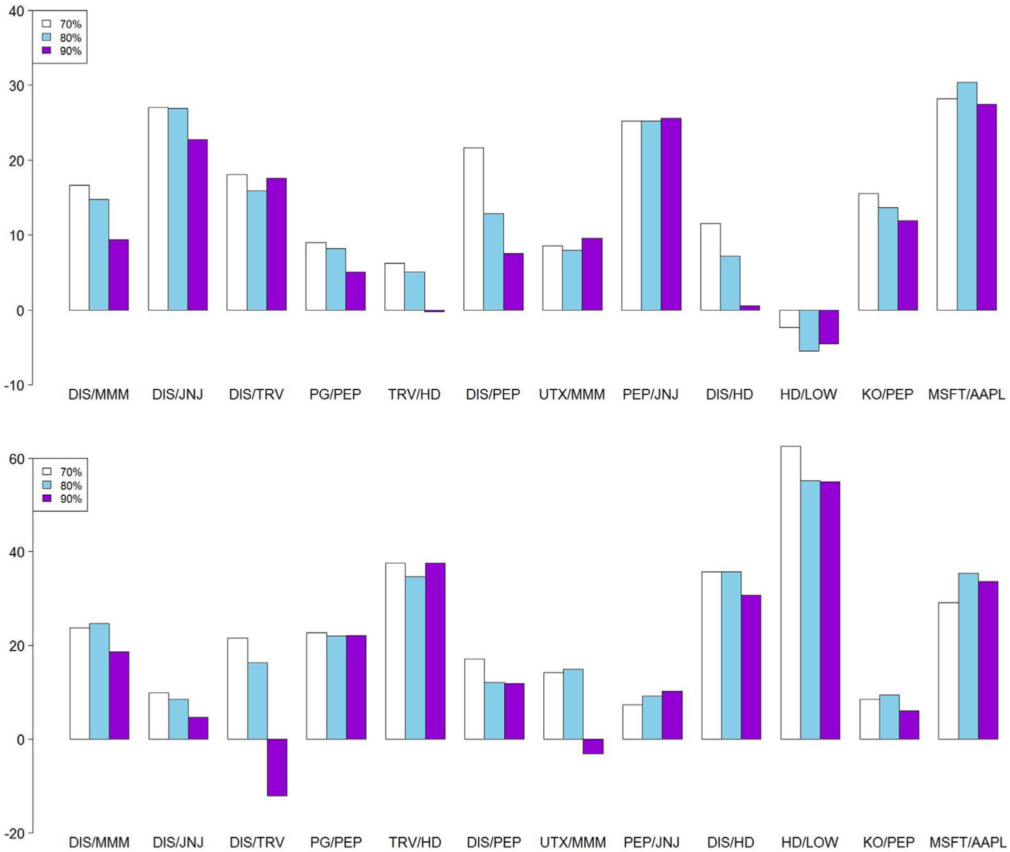

Fig. 4. Return profits for the three specification limits. Upper panel: without a transaction cost in H1. Lower panel: without a transaction cost in H2.

The empirical results suggest that combining the minimum squared distance method and nonparametric one-sided tolerance limits generates positive excess returns, relative to the underlying stocks. The nonparametric tolerance method runs free of the distributional assumption and is not very complicated to set up, which may benefit investors who do not understand all the theory behind it. The advantage of the proposed trading strategy is that it is independent of market movements, making it market neutral and thus not so risky. This is a good theoretical and practical experience that can turn out to be a profitable investment strategy.

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