Project 1: Martingale

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1) ***In Experiment 1, estimate the probability of winning \$80 within 1,000 sequential bets.***

With infinite capital, the probability of winning \$80 within 1,000 sequential bets is approximately 1.0. Every simulation, both for the 10 simulation (Figure 1) and the 1,000 simulation (Figure 2) exercises, resulted in winning \$80 well before the 1,000 bet limit. This provides strong empirical evidence that if capital were not a constraint (how nice!) the participant would almost always reach the \$80 target based on the martingale betting strategy. I attempted to corroborate this finding by calculating the probability as one minus the probability of not reaching the \$80 target.

$$\Pr(X >= 80) = 1 - \Pr(X < 80) \tag{1}$$

I then calculated the probability of not reaching \$80 as being the sum of the combinatorial probability for each losing outcome (i.e. \$0 winnings, \$1 winnings, ... \$79 winnings).

$$\Pr(X < 80) = \sum_{n=0}^{1000} \binom{1000}{n} p^n (1-p)^{1000-n} \tag{2}$$

where p is the probability of winning a bet, $1-p$ is the probability of losing a bet, n is the number of wins in a simulation, and $1,000-n$ is the number of losses in a simulation. The probability of winning a given bet with an American-style layout (2 green zeros) is the number of black spaces out of the total number of spaces.

$$\Pr(win) = \frac{black}{black + red + green} = \frac{18}{38} \tag{3}$$

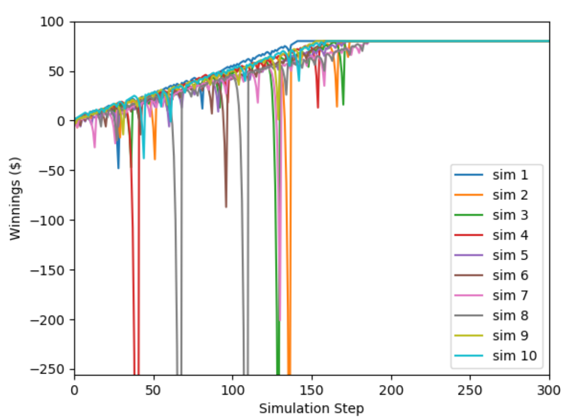

Fig 1. 10 simulations w/o bank roll

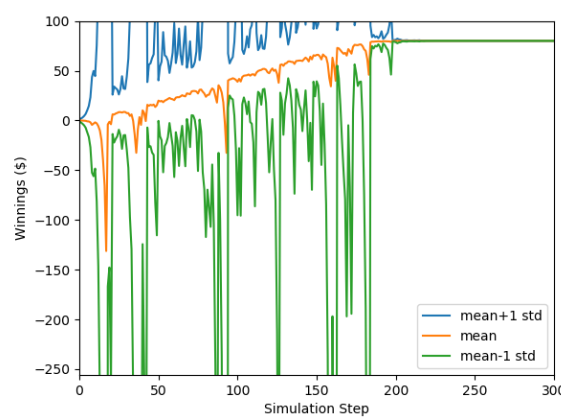

Fig 2. 1,000 simulations w/o bank roll

2) ***In Experiment 1, what is the expected value of our winnings after 1,000 sequential bets?***

The expected value after 1,000 sequential bets in Experiment 1 is \$80 since the probability of winning \$80 is 1.0 with infinite capital. I empirically confirmed this by taking the average of the last final bet for the 1,000 simulation scenario.

3) ***In Experiment 1, does the standard deviation reach a maximum value and then stabilize or converge as the number of sequential bets increases?***

The standard deviation converges to zero since the probability of reaching \$80 converges to 1.0 with infinite capital. There is a wide variance in the standard deviation as bets are placed sequentially, which reflects the random sequence of wins and losses until the \$80 threshold is reached (Figure 3). It should also be noted that $\mu + \sigma$ is relatively misleading since the bet will never increase winnings more than the initial wager size. Winnings are skewed to the downside until the \$80 threshold is reached.

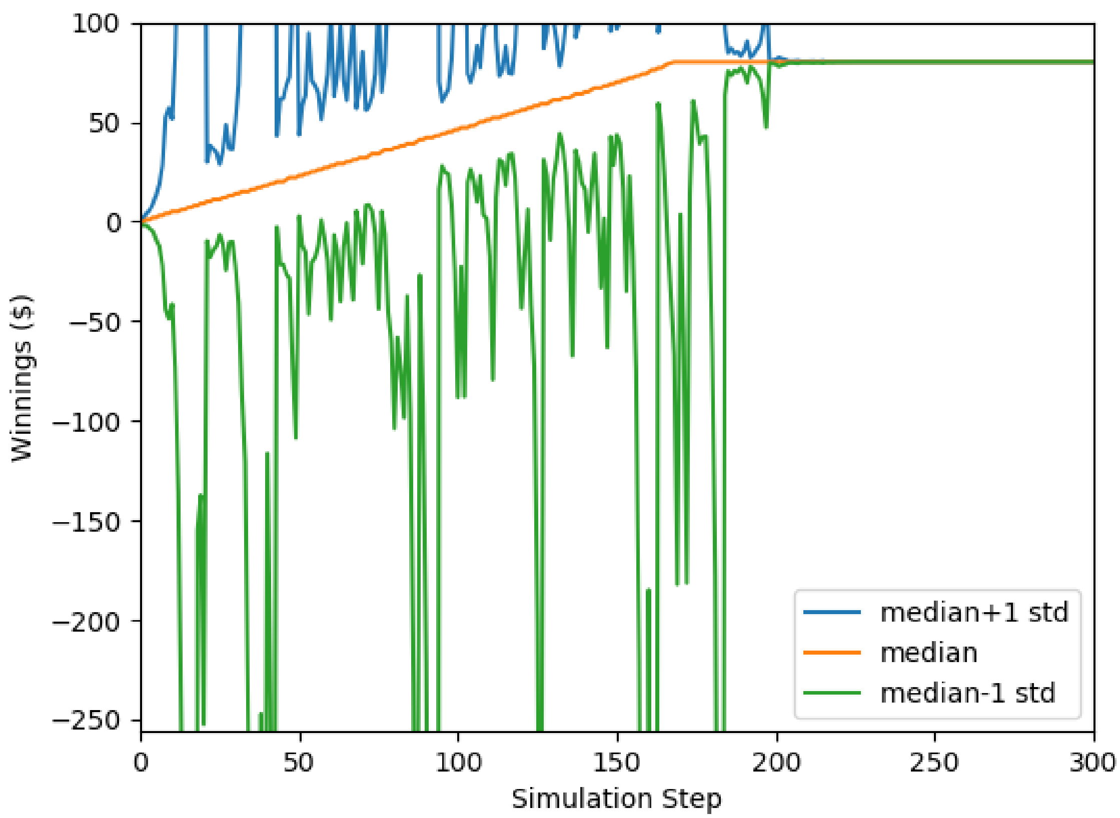

Fig 3. 1,000 simulations w/o bank roll

4) ***In Experiment 2, estimate the probabiilty of winning \$80 within 1,000 sequential bets.***

Incorporating a \$256 bank roll constraint into the betting strategy should materially reduce the probability of winning \$80 by both limiting the number of potential bets as well as the bet size at various points. The number of potential bets is limited since hitting the \$256 bank roll prevents subsequent wagers. The constraint also prevents the bet size from exceeding the bank roll thereby limiting the bettor's ability to recoup prior losses on a subsequent bet and forcing the bettor to dig out of the hole with the odds already inherently stacked against him or her. This intuition was supported by empirically measuring the probability by dividing the number of simulations with winnings greater than or equal to $80 by the total number of simulations, which resulted in a probability of 0.62.

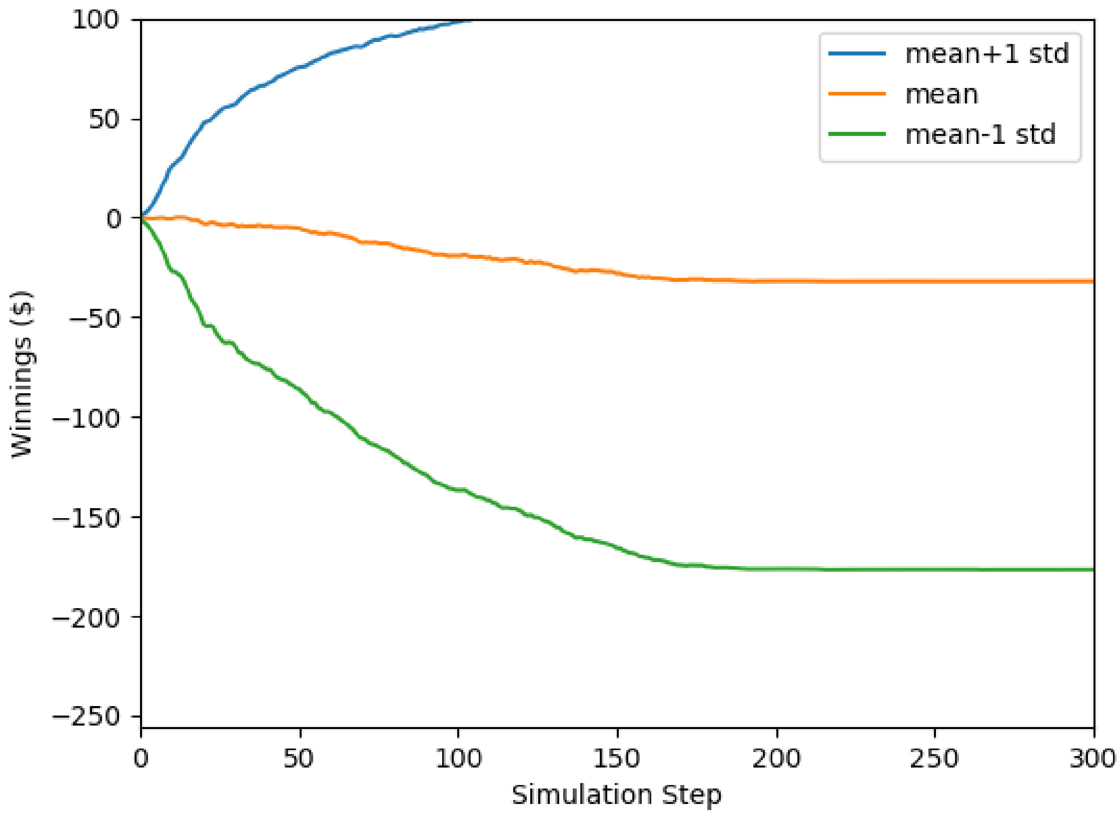

Fig 4. 1,000 simulations w/bank roll

5) ***In Experiment 2, what is the estimated expected value of oru winnings after 1,000 sequential bets?***

   The expected value drops to -$32, which more accurately reflects the probability of winning a bet. This intuitively makes sense since winnings are bounded to the downside at -$256 and there is no longer a probability of 1.0 that the $80 will be reached.

6) ***In Experiment 2, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases?***

   The standard deviation reaches a maximum value and then stabilizes as a result of the bank roll constraint. The bank roll limits the ability to wager an infinite amount to recoup prior losses, which is why the standard deviation in Experiment 2 is less volatile than in Experiment 1. Since the probability of winning a given bet remains stable through the series of bets, the standard deviation increases during the initial few bets and then stabilizes once bumping up against the bank roll constraint. It's interesting that while the expected value is steadily negative through the betting sequence, the median marches steadily upwards until leveling off, presumably reflecting the binary outcomes of reaching the $80 target or getting stopped out at $256 (Figure 5). As compared with Experiment 1, it also takes more bets for the median to stabilize presumably reflecting the instances where the bank roll limits the ability to double the wager followed by subsequent $1 bets upon a win.

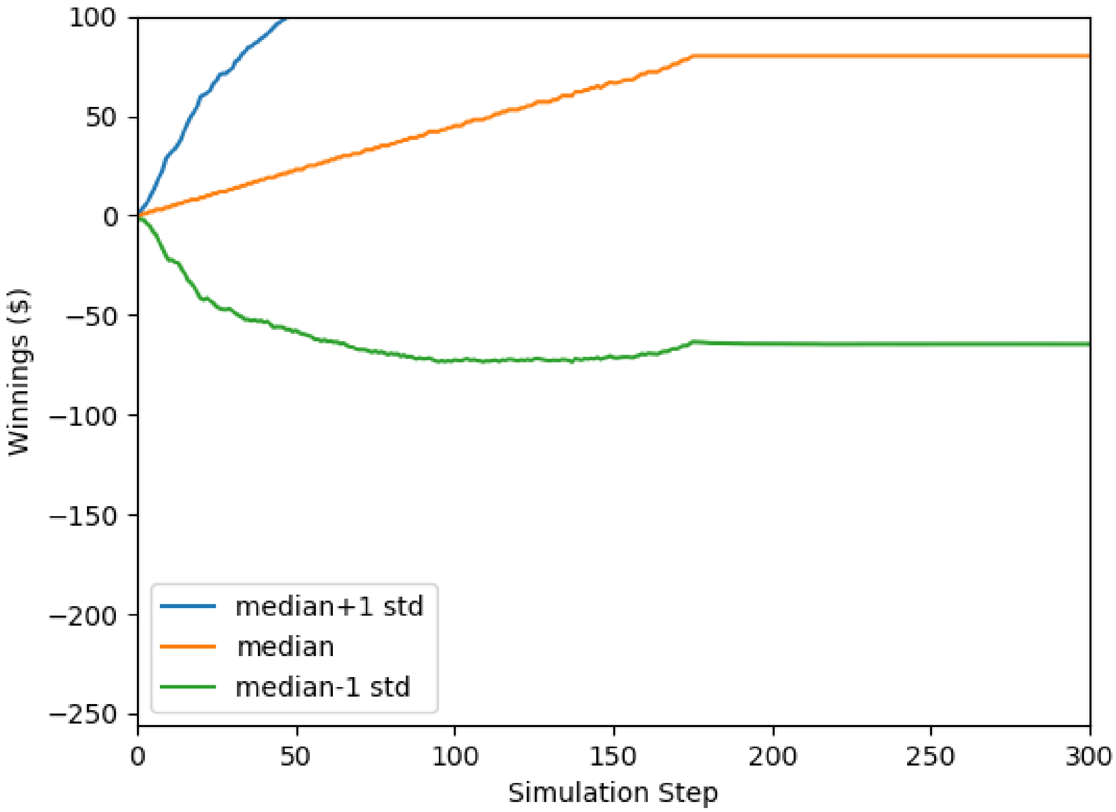

Fig 5. 1,000 simulations w/bank roll