Exercise-1 for Numerical Methods

Deadline: 2022-9-28

注意事项：

1. **每次作业用 Microsoft WORD 写，然后转成 PDF 上交；**
2. **作业上交文件命名：学号-中文姓名-Wi，i 为作业次数。如 M2022xxxxx-周某某-W1；**
3. **做完后在 QQ 群的作业栏上传 PDF 文件。**

1. Use a computer to accumulate the following sums. The intent is to have the computer do repeated subtractions. Do not use the multiplication shortcut.

   a) $10000-\sum_{k=1}^{100000}0.1$, $1000-\sum_{k=1}^{10000}0.1$, $10-\sum_{k=1}^{100}0.1$.

   b) $10000-\sum_{k=1}^{80000}0.125$, $0.3-\sum_{k=1}^{3}0.1$, $0.2-\sum_{k=1}^{2}0.1$.

2. P23:9(a)

For the following seven-digit binary approximations, find the error in the approximation $R - 0.d_1d_2d_3d_4d_5d_6d_{7\,\text{two}}$.

(a) $1/10 \approx 0.0001100_{\text{two}}$

3. P23:13(c)

Use Table 1.3 to determine what happens when a computer with a 4-bit mantissa performs the following calculations.

(c) $\left(\frac{3}{17}+\frac{1}{9}\right)+\frac{1}{7}$

4. Find the error E_x and relative error R_x. Also determine the number of significant digits in the approximation:

   a) x=2.71828182, x'=2.717276.
   b) y=83500000, y'=83499884.
   c) z=0.000068, z'=0.00006666666.

5. P50: 5

Let $g(x) = x\cos(x)$. Solve $x = g(x)$ and find all the fixed points of g (there are infinitely many). Can fixed-point iteration be used to find the solution(s) to the equation $x = g(x)$? Why?

6. P37: 5

Sometimes the loss of significance error can be avoided by rearranging terms in the function using a known identity from trigonometry or algebra. Find an equivalent formula for the following functions that avoids a loss of significance.

(a) $\ln(x+1) - \ln(x)$ for large x

(b) $\sqrt{x^2+1} - x$ for large x

(c) $\cos^2(x) - \sin^2(x)$ for $x \approx \pi/4$

(d) $\sqrt{\dfrac{1+\cos(x)}{2}}$ for $x \approx \pi$

7. P39: 2

Follow Example 1.25 and generate the first ten numerical approximations for each of the following three difference equations. In each case a small initial error is introduced. If there were no initial error, then each of the difference equations would generate the sequence $\{1/2^n\}_{n=1}^{\infty}$. Produce output analogous to Tables 1.4 and 1.5 and Figures 1.8, 1.9, and 1.10.

(a) $r_0 = 0.994$ and $r_n = \frac{1}{2}r_{n-1}$, for $n = 1, 2, \ldots$

(b) $p_0 = 1$, $p_1 = 0.497$, and $p_n = \frac{3}{2}p_{n-1} - p_{n-2}$, for $n = 2, 3, \ldots$

(c) $q_0 = 1$, $q_1 = 0.497$, and $q_n = \frac{5}{2}q_{n-1} - q_{n-2}$, for $n = 2, 4, \ldots$

8. P98: 5 (Aitken's formula (4))

Let $p_n = 1/(2^n - 1)$. Show that $q_n = 1/\left(4^{n+1} - 1\right)$ for all n.

9. P99: 8

The sequence $\{p_n\}$ generated by fixed-point iteration, starting with $p_0 = 3.14$, and using the function $g(x) = \ln(x) + 2$ converges linearly to $p \approx 3.1419322$. Use Aitken's formula (4) to find q_1, q_2, and q_3, and hence speed up the convergence.

10. Use the Newton-Raphson method and the false position method to solve one of the following problems, and compare the time costs of the two methods:

(a) Find the point on the parabola $y = x^2$ that is closest to the point $(3, 1)$ accurate to 10 decimal places.

(b) Find the point on the graph of $y = \sin(x - \sin(x))$ that is closest to the point $(2.1, 0.5)$ accurate to 10 decimal places.

(c) Find the value of x at which the minimum vertical distance between the graphs of $f(x) = x^2 + 2$ and $g(x) = (x/5) - \sin(x)$ occurs accurate to 10 decimal places.

11. For equation $f(x) = (x+4)^2(x+2)(x-2)(x-4)^3 = 0$,

a) Run the Newton-Raphson program with different initial points in [-6, 6].

b) Show what solution r_i the program converges to for each of the initial points with 2D plot functions in MATLAB.