AMC 8 Practice Test 1

1. What is $20 \times 19 \div 20 \times 20$?

2. Jack teaches a health class at Richard University where class sizes are always at least 90 students and at most 100. One day Jack decides that he wants to arrange the students in their desks in a rectangular grid with no gaps. Unfortunately for Jack he discovers that doing so could only result in one straight line of desks. How many students does Jack have in his class?

3. Rick owns a gas station called DXE. Due to the lack of profit from consumers, Rick decided to raise the price of gas. Rick raises the price of 0.02 dollars at 6 AM every day. If he starts raising prices on August 1st and the price is 2.97 dollars per gallon, how much does one-gallon cost at the end of the day of August 12th?

4. 6 people stand in a line. Richard and Jack hate each other so much that there must be at least 2 people between them, but there are no other restrictions. How many possible arrangements are there?

5. Jane wanted to tile a regular hexagon side length 10 into equilateral triangles side length 1. She will use toothpicks. In example, for a hexagon with side length 1, she uses 6 toothpicks, and for a hexagon of side length 2, she will use 30 toothpicks. Find the number of toothpicks she will use for the hexagon side length 10. Note that the hexagonal border is not made of toothpicks.

6. At Richard Meyers Elementary School, the fastest boys are the most popular in school. Each boy times themselves to see how fast they are, however they time themselves on different distances in the track. The three fastest runners in school are shown in the chart below with their respective time and distance run.

   Fastest Runners in School

	Pilot	**Time in Flight**	**Distance**
First Place	Arjun	10 seconds	100 meters
Second Place	Rick	18 seconds	150 meters
Second Place	Samad	59 seconds	400 meters

   Let the average speeds be x, y and z m/s for Arjun, Rick and Samad respectively to the nearest whole number. What is the average of x, y and z?

7. The area of a square A is 2020% larger than square B. A also has a side length $d\%$ larger than B. What is d?

8. Find the units digit of
$$3^0 + 3^1 + 3^2 + \cdots + 3^{2019} + 3^{2020}.$$

9. Jayden has a triangle $\triangle ABC$ such that $AB = BC = 40$. What is the sum of the minimum and maximum possible integer value of AC?

10. How many 6-digit numbers are there where all the digits are different and in increasing order? For example, the numbers 145689 and 256789 would be counted. (The first digit cannot be zero.)

11. If the mean of the set of numbers $5, 7, x, 8-x, y$ is equal to x, then what is the value of $\frac{5x-y}{5}$?

12. How many two-digit integers are 36 more than the sum of their digits?

13. Find the sum of the coefficients when $(2a+3b+6c)^3$ is expanded.

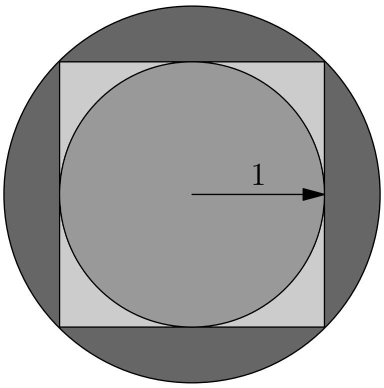
(A) 1331 **(B)** 1332 **(C)** 1333 **(D)** 1336 **(E)** 1337

14. Find the number of real numbers DXD that satisfy the equation

$$\begin{array}{rcccc} & & D & X & D \\ \times & & & & 3 \\ \hline & A & B & A & B \end{array}$$

Where A, B, D and X are distinct numbers

15. What is the probability that a randomly selected point on the region shown below is of the lightest gray.

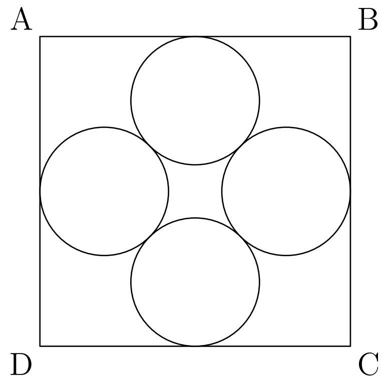


16. There are 121 marbles in a basket. Some of them are red, and some are blue, of which there are more blue marbles than red marbles. If Tsun Liu picks two marbles from the basket, the probability that they are the same color is the same as the probability that they are different colors. How many red marbles are in the basket?

17. Find the value of

$$x = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ddots}}}}.$$

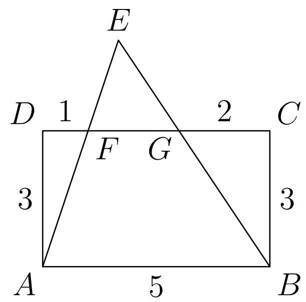
18. Four circles are inscribed in square $ABCD$ as shown in the figure below. What is the area of $ABCD$ if the radius of each circle is 2?



19. What are the number of positive integers (a, b, c) that satisfy the equation

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1?$$

20. In rectangle $ABCD$, $AB = 5$ and $BC = 3$. Points F and G are on $\overline{CD}$ so that $DF = 1$ and $GC = 2$. Lines AF and BG intersect at E. Find the area of triangle EFG.



21. For some real number a, there are 3 points in the coordinate plane A, B, C, such that $A = (12a + 3, 2)$, $B = \left(\frac{a}{3}, -2\right)$, and $C = (6, 6)$. Suppose these points form a right triangle in the plane, with right angle at C. Given that there are 2 possible values of a, find the smallest integer greater than their sum.

22. A real number α is randomly chosen in the range $-200 \leq \alpha \leq 200$. Find the probability that the equation $x^2 + (2 + \alpha)x + \alpha^2 x$ has two real roots.

23. Let a and b be positive integers such that

$$\gcd(a^2, b^2)\text{lcm}(a, b) + 5ab = 6\,\text{lcm}(a, b)$$

Assume that $\gcd(a, b)$ is positive. What is the remainder when $\gcd(a^4, b^4)$ is divided by 10?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 5 **(E)** 16

24. Find the number of ways to put 73 indistinguishable bananas into 2 boxes considering that there must be a minimum of 4 bananas in each box.

25. Deshawn Williams is extremely bored. He calculates 40^{13} and then divides this by 85 to get a remainder. He then proceeds to do the same thing but this time for 40^{12}. What is the ratio of the first remainder he found to the the second remainder?

(A)$\frac{1}{3}$ **(B)**$\frac{1}{7}$ **(C)**3 **(D)**7 **(E)**$\frac{2}{3}$