

## PS8: Content Addressable Memory

For the purpose of this assignment, we were given parts of a Hopfield neural network designed to correct erroneous memories (cues) of images. We were provided code that would read in correct bitmap images to a memory bank using the sum of outer products of themselves. We were to write code that could fix a provided erroneous bitmap image (cue) using the provided memory bank and some randomness. The process to fix the erroneous memory is the main part of the Hopfield network that were desired to implement. The reason that the Hopfield network can be applied to correct erroneous memory is that it acts as a content addressable memory. Given properties of content addressable memory, one can use outer products of the sum of correct memories to retrieve the correct representation of a memory (or image) given some time and randomness along with a bipolar activation function.
















### Number of Recoverable Images:

I have found that I can encode and recover up to 5 memories perfectly from flawed cues.

The memories that I was able to recover perfectly given the completed Hopfield network were the following:

1. clubspade.bmp
2. handheart.bmp
3. happyworld.bmp
4. printtrash.bmp
5. winhelp.bmp

When testing, I found that leaving out “computersum.bmp” and “notespell.bmp” were necessary for the rest of the images to be recovered. This phenomenon has to do with the premise of orthogonality between memories in the Hopfield network. A Hopfield network requires that memories be quite orthogonal to each other or they will wipe each other out. When the memories wipe each other out, this means that those memories will only be partially recoverable at best. With this in mind, here are the results of the content addressable memory algorithm on each of the erroneous cues of these images, provided that all but the two mentioned images were in the memory bank:

Image Name	Erroneous Image	Corrected Image	Actual Image
clubspade.bmp			
handheart.bmp			
happyworld.bmp			
printtrash.bmp			
winhelp.bmp			

One can see that the corrected images are exactly the same as the actual images. The reasoning behind this is that all of these images are above a certain threshold of orthogonality which is

optimal for the Hopfield network's maximum benefit. I have found that ~35% (~0.65) is the optimal ratio for this when comparing angles between two images with a cosine function. A zero output from the cosine function means the two images are fully orthogonal. We desire the orthogonality that divides the images into separate areas of the matrix memory stored in the Hopfield network. We do not wish to wipe out memories that overlap with condensing values. The images of "computersum.bmp" and "notespell.bmp" were closely related to each other. These images had a low orthogonality with the other images in the provided set.

### Intermediate Steps to Perfect Convergence:

First we have my initial results of reading all memories into the memory bank of the Hopfield network.

Image Name	Erroneous Image	Corrected Image	Actual Image
clubspade.bmp			
computersum.bmp			
handheart.bmp			
happyworld.bmp			
notespell.bmp			
printtrash.bmp			
winhelp.bmp			

These results can be viewed as partially converged. Two of the corrections have converged to the correct image and some have not. The ratio of correct to incorrect images is unacceptable using all possible memories in our memory bank. We can see that some memories are wiping others out due to orthogonality. The two images that were corrected must have been most orthogonal from the others in the memory bank. We must continue to remove one memory from the bank. We will choose the computer sum memory because it is less orthogonal to the others when compared to the rest.

Image Name	Erroneous Image	Corrected Image	Actual Image
clubspade.bmp			
computersum.bmp			
handheart.bmp			
happyworld.bmp			
printtrash.bmp			
winhelp.bmp			

We are getting closer to the correct results but we have not attained them yet. We can see that all of the images except "happyworld.bmp" have nearly converged. If we remove the

“notespell.bmp” image from our network’s memory, we will attain the perfect results presented above. We can see that elimination of the two least orthogonal memories from memory bank has allowed the Hopfield network to converge to the correct solutions.

Properties of memories that can be recalled and memories that interfere with each other:

In conclusion, we have found that Hopfield networks successfully work as content addressable memories so long as the memories have a certain threshold of orthogonal encodings. More memories can be stored provided greater orthogonality. We see that the Hopfield network operates correctly in our situation when the encoded memories are quite orthogonal (~35%). We conclude that the Hopfield network is sensitive to the orthogonality of memories, giving it an “all or nothing” ability to fix erroneous memories with an encoded and correct memory matrix.