

Correctness of selection sort:

Pseudo code.

```
n = A.length  
for i = 0 to n - 2
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 minIndex = i

 for j = i + 1 to n - 1

 if A[j] < A[minIndex]

 minIndex = j

 if minIndex ≠ i

 swap A[i] with A[minIndex]

Loop invariant for inner loop (4-6)

Initialization: Before the first iteration of the inner loop (when $j = i+1$), minIndex is initialized to i . This correctly identifies the smallest element in the subarray $A[i \dots j]$ because a single element is the smallest.

Maintenance: Each iteration of the inner loop, if $A[j]$ is smaller than $A[\text{minIndex}]$, the algorithm updates minIndex to j ensuring that minIndex always holds the index of the smallest element in $A[i \dots j]$.

Termination: When the inner loop terminates (when $j = n$) 'minIndex' points to the index of the smallest element in $A[i \dots n-1]$.

Outerloop (2-8):

Initialization: Before the first iteration of the outer loop (when $i=0$) the subarray $A[0 \dots i-1]$ is empty. An empty array is sorted.

Maintenance: Assume that the loop invariant holds at the start of iteration i of the outer loop. The inner loop finds the smallest element in the subarray $A[i \dots n-1]$ and swaps it with $A[i]$. As a result, the subarray $A[0 \dots i]$ now contains the smallest $i+1$ elements in sorted order, and each element in $A[0 \dots i]$ is less than or equal to each element in $A[i+1 \dots n-1]$. Thus loop invariant is maintained.

Termination: When the outer loop terminates after $i=n-1$ the loop invariant guarantees that the subarray $A[0 \dots n-2]$ is sorted and that every element $A[0 \dots n-2]$ is less than or equal

to the remaining element $A[n-1]$. Since $A[n-1]$ is already the largest element in the array, the entire array is sorted.

Since loop invariant holds true for both inner & outer loops, and the algorithm terminates, we can conclude that selection sort correctly sorts the input array.

Time complexity

The time complexity is $O(n^2)$ on the worst case. The outer loop runs $n-1$ times and the inner loop performs a linear search over the unsorted part of the array for each iteration of the outer loop.