MIS Homework 3

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Problem 5.2

Solution: The intern is right, we can use the data of stocks as a vector group, and each stock as a row and each trading day as a column, then the return of stock i on trading day j could be written as $a_{i,j}$, and any set of i + 1 or more i vectors is linearly dependent, which can be expressed as a linear combination.

Problem 5.3

Solution:

5.3.

$$\alpha_1 e_1 + \alpha_1 l_1 + \cdots + \alpha_n l_{n-1}$$

$$= \begin{bmatrix} \alpha_1 + \alpha_2 \\ \alpha_3 - (1+r)\alpha_2 \\ \vdots \\ \alpha_n - (1+r)\alpha_{n-1} \\ -(1+r)\alpha_n \end{bmatrix}$$

Therefore. $\alpha_n = -\frac{c_n}{1+r}$

$$\alpha_{n-1} = -\frac{c_{n-1}}{1+r} - \frac{c_n}{(1+r)^2}$$

$$\alpha_{n-2} = -\frac{c_{n-2}}{1+r} - \frac{c_{n-1}}{(1+r)^2} - \frac{c_{n-2}}{(1+r)^3}$$

$$\vdots$$

$$\alpha_1 = c_1 + \frac{c_2}{1+r} + \cdots + \frac{c_n}{(1+r)^{n-1}}$$

Problem 5.4

Solution:

5.4.

$$\|x\|^2 = x^T x$$
$$= (\beta_1 a_1 + \cdots + \beta_k a_k)^T (\beta_1 a_1 + \cdots + \beta_k a_k)$$
$$= \beta_1^2 + \cdots \beta_k^2$$
$$= \|\beta\|^2$$

Therefore $\|x\| = \|\beta\|$

Problem 5.9

Solution:

5.9.

The total flop count is

$$2nk^2 = 2 \times 1 \times 10^4 \times 10^6 = 2 \times 10^{10}$$

Therefore, the speed of computer is

$$2 \times 10^{10} \div 2 = 1 \times 10^{10} \text{ flop per sec}$$

Time to $\tilde{k} = 500$, $\tilde{n} = 1000$

$$2\tilde{n}\tilde{k}^2 \div 10^{10} = 0.05 \text{ sec}$$

Problem 6.8

Solution:

6.8 $b_1 = c_1$

$b_2 = (1+r)c_1 + c_2$

$b_3 = (1+r)b_2 + c_3$

$= (1+r)[(1+r)c_1 + c_2] + c_3$

$= (1+r)^2 c_1 + (1+r)c_2 + c_3$

$b_t = (1+r)^{t-1} c_1 + \cdots (1+r)c_{t-1} + c_t$

$$A = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ (1+r) & 1 & 0 & \cdots & 0 \\ (1+r)^2 & (1+r) & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ (1+r)^{T-1} & (1+r)^{T-2} & (1+r)^{T-3} & \cdots & 1 \end{bmatrix}$$

$$A = \begin{cases} 0 & i < j \\ (1+r)^{i-j} & i \geq j \end{cases}$$

Problem 6.17

Solution:

6.17.

① Assume S has linearly independent columns, $Sx = (Ax, x)$, therefore $x=0$

② Assume S does not has linearly independent rows. S is an $(m+n)\times n$ matrix, and $n<m+n$, therefore by independence-dimension inequality rows are dependent.