My Study Notes 2024 / Quantum Field Theory

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Contents

1 Introduction

2 Functional Method

3 Geometry of the Spacetime

We will discuss how to treat the spinor field on the curved spacetime. Naively, we can not handle the spinor field in the same way as the spinor or vector since spinors are defined on the flat Minkowski spacetime. To obtain the covariant derivative for spinor fields, we will consider the tangent space at any spacetime point and define the spinor on it. Then introduce the new field to connect two different tangent spaces.

3.1 Vielbein

4 Perturbation Theory Anomalies

In specific circumstances, quantum corrections can destroy symmetries of the classical equations of motion.

4.1 Intuitive Example: The Axial Current in Two Dimensions

A Some notes

A.1 Normalization of Maxwell Lagrangian

The Maxwell lagrangian is the form

$$\mathcal{L} = NF^{\mu\nu}F_{\mu\nu} \tag{A.1}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is a field strength. We will determine the constant N to lagrangian contains the term

$$\mathcal{L} = \frac{1}{2}\dot{A}_1^2 + \frac{1}{2}\dot{A}_2^2 + \frac{1}{2}\dot{A}_3^2 + \cdots \ . \tag{A.2}$$

Now expanding the term $F^{\mu\nu}F_{\mu\nu}$ carefully, we obtain

$$\begin{aligned} F^{\mu\nu}F_{\mu\nu} &= (\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ &= 2((\partial^\mu A^\nu)(\partial_\mu A_\nu) - (\partial_\mu A^\nu)(\partial_\nu A_\mu)) \\ &= -2(\dot{A}_1^2 + \dot{A}_2^2 + \dot{A}_3^2) + \cdots \end{aligned} \tag{A.3}$$

and thus, the constant N satisfies the condition $-2N = 1/2$ and $N = -1/4$. That's why we get Maxwell Lagrangian as[*1]

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \ . \tag{A.4}$$

[*1] aaa