

Spring School 2024 @Izukawana

Moduli stabilization

on (for/at/in ?) supersymmetric magnetized D9-brane
model

Abe Lab. M1

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Sunday, April 7th, 2024

*I will speak in Japanese though this slide is written in English.

Topics

- Reviewing the senior thesis
- Reporting progress and difficulties I met

Introduction

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In general, these theories contain extra fields related to

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Such a field is called **moduli fields**.

Moduli stabilization

The metrics in 10-dimensional spacetime are dynamical fields:

$$\begin{aligned} ds^{10} &= G_{MN}(X) dX^M dX^N \\ &= g_{\mu\nu}(x, y) dx^\mu dx^\nu + g_{mn}(x, y) dy^m dy^n \end{aligned}$$

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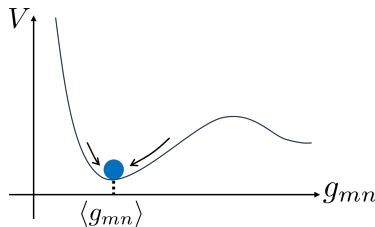
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How?

What we have to do is just

- Write down the potential for moduli (e.g. g_{mn})
- Compute the minimum and identify the value $\langle g_{mn} \rangle$



These procedures are called **moduli stabilization**.

Purpose of our study

On the other hand, it is known that **magnetized torus model**.

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We will discuss the **moduli stabilization** on **magnetized torus model**.

Magnetized torus model

Torus compactification and Magnetic flux

Torus compactification

- Compactifying 6d extra dimensions for three tori $(T^2)^3$.


 $\mathcal{A}^{(1)}$
 $\mathcal{A}^{(2)}$
 $\mathcal{A}^{(3)}$

: area of the i -th torus

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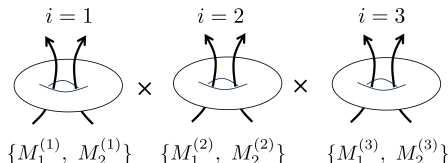
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Magnetic flux

- Assigning the magnetic flux

$$M_a^{(i)} \quad (a = 1, 2)$$

for two gauge fields on each tori.



Finding the VEVs by fluxes

By magnetic fluxes in an extra dimension, moduli $\mathcal{A}^{(i)}(x)$ obtain its potential $V^{(D)}$.



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Potential by magnetic fluxes

$$F^{MN}F_{MN} = F^{\mu\nu}F_{\mu\nu} + \underbrace{F^{mn}F_{mn}} + \dots$$



$$V^{(D)} = \pi^2 \prod_i \mathcal{A}^i \times \left\{ \underbrace{\left(\sum_i \frac{M_1^{(i)}}{\mathcal{A}^{(i)}} \right)^2} + \underbrace{\left(\sum_i \frac{M_2^{(i)}}{\mathcal{A}^{(i)}} \right)^2} \right\}$$

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The ratios of the moduli's VEVs are determined by the fluxes potential.

Summary so far

- We could stabilize the moduli
and obtain the ratio of its VEVs $\langle \mathcal{A}^{(1)} \rangle / \langle \mathcal{A}^{(2)} \rangle$ & $\langle \mathcal{A}^{(1)} \rangle / \langle \mathcal{A}^{(3)} \rangle$.

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Introducing a potential that **has a different origin** than the magnetic fluxes
to stabilize **overall factor T** .

Determination of the overall factor

F -term potential

Effective potential for moduli T

- Its effective theory remains supersymmetric.
- Supersymmetric action is determined
by **super potential W** and **Kähler potential K** .
- We will study the following potential now[3]:

$$\begin{cases} W = w_0 - Ae^{-aT} + BX \\ K = -3 \ln(T + \bar{T}) + |X|^2 \end{cases}$$

Introducing a new scalar field X and w_0, A, B, a are parameters.

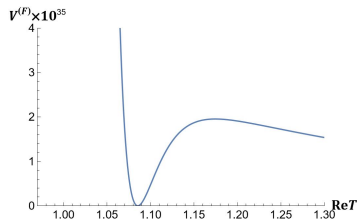
F -term potential

*We set the Planck constant as $M_{\text{Pl}} (\sim 2.4 \times 10^{18} \text{ GeV}) = 1$.

Scalar potential

$$V^{(F)} = e^K (K^{I\bar{J}} (D_I W)(D_{\bar{J}} \bar{W}) - 3|W|^2)$$

$$\begin{cases} D_I W \equiv \partial_I W + K_I W \\ K^{I\bar{J}}: \text{inverse matrix of } K_{I\bar{J}} \end{cases} \quad (I = X, T)$$



Parameters

$$w_0 \sim 2.17 \times 10^{-18}, \quad a = 4\pi^2, \quad A = 1, \quad B = e^{-4\pi^2}$$

$$\text{and } \langle X \rangle = \sqrt{3} - 1$$

$$\rightarrow \langle T \rangle \sim 1.085$$

Report progresses

In my senior thesis, we

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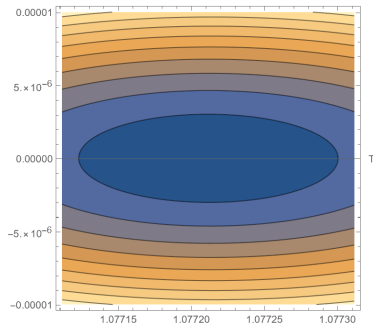
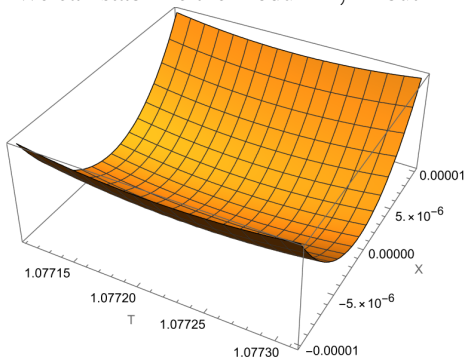
During this vacation, I mainly studied the F -term potential more general form:

$$\begin{cases} W = w_0 - Ae^{-aT} + \underbrace{Be^{-bT}} \\ K = -3 \ln(T + \bar{T}) + |X|^2 - \underbrace{\frac{1}{\Lambda^2} |X|^4} \end{cases}$$

Wavy factors are the difference from the previous one.

Report progresses

We can stabilize the moduli T , X but



Parameters

$$A = B = 1, a = b = 4\pi^2, w_0 = 10^{-17}, \Lambda = 10^{-4}$$
$$\longrightarrow \langle T \rangle \sim 1.07, \langle X \rangle \sim 10^{-8}, V_{\min} \sim -10^{-35}$$

Difficulties

- We should tune the parameters A, B, a, b, w_0, Λ
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I want to find **more efficient ways to search the set of parameters.**

$$\text{e.g. } \frac{\partial V}{\partial T} = \frac{\partial V}{\partial X} = 0 \quad \& \quad \frac{\partial V}{\partial a} = \frac{\partial V}{\partial b} = \frac{\partial V}{\partial A} = \frac{\partial V}{\partial B} = \frac{\partial V}{\partial w_0} = 0$$

(I tried it but it seems not to converge the Newtonian method...)

Summary & Future Works

Summary

- Discussing moduli stabilization in magnetized torus model
- Determining the ratio of the moduli by potential $V^{(D)}$ with fluxes
- Stabilizing the overall moduli T by potential $V^{(F)}$ that has a different origin

Future Works

- [new!] Finding more efficient ways to search the set of parameters.
- Discussing soft SUSY breaking and mass of supersymmetric particles, etc.

Reference

- [1] H. Abe, T. Kobayashi, H. Ohki, and K. Sumita, *Superfield description of 10D SYM theory with magnetized extra dimensions*, **Nucl. Phys. B** **863** (2012) 1–18, [arxiv:1204.5327 \[hep-ph, physics:hep-th\]](#).
- [2] H. Abe, T. Kobayashi, K. Sumita, and S. Uemura, *Kähler moduli stabilization in semi-realistic magnetized orbifold models*, **Phys. Rev. D** **96** (2017) 026019, [arxiv:1703.03402 \[hep-ph, physics:hep-th\]](#).
- [3] H. Abe, T. Higaki, T. Kobayashi, and Y. Omura, *Moduli stabilization, \mathbf{F} -term uplifting and soft supersymmetry breaking terms*, **Phys. Rev. D** **75** (2007) 025019, [arxiv:hep-th/0611024](#).
- [4] H. Abe, T. Higaki, and T. Kobayashi, *More about \mathbf{F} -term uplifting*, **Phys. Rev. D** **76** (2007) 105003, [arxiv:0707.2671 \[hep-ph, physics:hep-th\]](#).
- [5] J. Wess and J. Bagger, *Supersymmetry and Supergravity*. Princeton University Press, Princeton, N.J, 1992.