Notes on Algebra

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Last modified: June 9, 2024

Contents

Chapter 1

Representation theory

1.1 Lie group

In subsequent sections, we will discuss the representation of the groups. Before going into such a subject, let us review the properties of groups, especially the Lie group.

1.1.1 Definition of the Lie group

One of the examples of the two-dimensional Lie group is the *orthogonal group* $O(2,\mathbb{R})$. It can be written as

$$O(2,\mathbb{R}) = \left\{ \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \quad \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \middle| \, 0 \leq \theta \leq 2\pi \right\}. \tag{1.1.1}$$

The *special orthogonal group* is constituted by elements of $O(2,\mathbb{R})$ whose determinant is unit:

$$SO(2,\mathbb{R}) = \left\{ \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \middle| \, 0 \leq \theta \leq 2\pi \right\}. \tag{1.1.2}$$

This group represents the rotation about the $x - y$ plain respect to the origin, and it also isomorphic to

1. the residual group $\mathbb{R}/\mathbb{Z}$, which is obtained by dividing the additional additive group $\mathbb{R}$ by its subgroup $\mathbb{Z}$,

2. the group constituted by the complex number whose absolute value is 1 for the product.

Bibliography

[Hum72] J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Graduate Texts in Mathematics, vol. 9, Springer New York, New York, NY, 1972. http://link.springer.com/10.1007/978-1-4612-6398-2.