Spring School 2024 @Izukawana

# Moduli stabilization on (for/at/in ?) supersymmetric magnetized D9-brane model

Abe Lab. M1

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*I will speak in Japanese though this slide is written in English.

# Topics

- Review the senior thesis
- Report progress and difficulties I met

# Introduction

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Such a field is called **moduli fields**.

# Moduli stabilization

The metrics in 10-dimensional spacetime are dynamical fields:

$$\begin{aligned} \mathrm{d}s^{10} &= G_{MN}\mathrm{d}X^M \mathrm{d}X^N \\ &= g_{\mu\nu}(x,y)\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}(x,y)\mathrm{d}y^m \mathrm{d}y^n \end{aligned}$$

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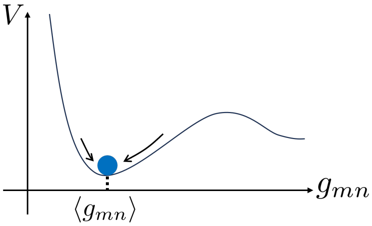
How?

What we have to do is just

- Write down the potential for moduli (e.g. $g_{mn}$)
- Compute the minimum and identify the value $\langle g_{mn} \rangle$




These procedures are called moduli stabilization.

# Purpose of my study

We will discuss the moduli stabilization on magnetized torus model.

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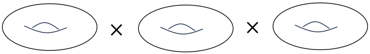
# Magnetized torus model

# Torus compactification and Magnetic flux

Torus compactification

- Compactifying 6d extra dimensions for three tori $(T^2)^3$.



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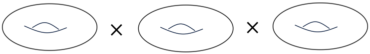

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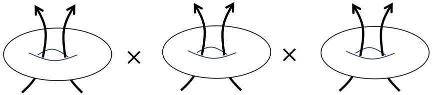
Magnetic flux

- Assigning the magnetic flux $M_a^{(i)}$ $(a = 1, 2)$ for two gauge fields on each tori.

$i = 1$ $\quad i = 2$ $\quad i = 3$



$\{M_1^{(1)},\ M_2^{(1)}\}$ $\quad\{M_1^{(2)},\ M_2^{(2)}\}$ $\quad\{M_1^{(3)},\ M_2^{(3)}\}$

# Finding the VEVs by fluxes

By magnetic fluxes in an extra dimension, moduli $\mathcal{A}^{(i)}(x)$ obtain its potential $V^{(D)}$.

$$\downarrow$$

Finding minima and determining the VEVs of $\langle \mathcal{A}^i \rangle$.

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Potential by magnetic fluxes

$$F^{MN}F_{MN} = F^{\mu\nu}F_{\mu\nu} + F^{mn}F_{mn} + \cdots$$

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$$V^{(D)} = \pi^2 \prod_i \mathcal{A}^i \times \left\{ \underline{\left( \sum_i \frac{M_1^{(i)}}{\mathcal{A}^{(i)}} \right)^2} + \underline{\left( \sum_i \frac{M_2^{(i)}}{\mathcal{A}^{(i)}} \right)^2} \right\}$$

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$$\longrightarrow \quad \frac{M_a^{(1)}}{\langle \mathcal{A}^{(1)} \rangle} + \frac{M_a^{(2)}}{\langle \mathcal{A}^{(2)} \rangle} + \frac{M_a^{(3)}}{\langle \mathcal{A}^{(3)} \rangle} = 0 \quad \text{for } a = 1, 2$$

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Relation of the VEVs $\langle \boldsymbol{\mathcal{A}^{(1)}} \rangle$ , $\langle \boldsymbol{\mathcal{A}^{(2)}} \rangle$ , $\langle \boldsymbol{\mathcal{A}^{(3)}} \rangle$

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$$\frac{\langle \mathcal{A}^{(1)} \rangle}{\langle \mathcal{A}^{(2)} \rangle} = \frac{M_1^{(3)} M_2^{(1)} - M_1^{(1)} M_2^{(3)}}{M_1^{(2)} M_2^{(3)} - M_1^{(3)} M_2^{(2)}}, \quad \frac{\langle \mathcal{A}^{(1)} \rangle}{\langle \mathcal{A}^{(3)} \rangle} = -\frac{M_1^{(2)} M_2^{(1)} - M_1^{(1)} M_2^{(2)}}{M_1^{(2)} M_2^{(3)} - M_1^{(3)} M_2^{(2)}}$$

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The ratios of the moduli's VEVs are determined by the fluxes potential.

# Summary so far

- We could stabilize the moduli
  and obtain the ratio of its VEVs $\langle \mathcal{A}^{(1)} \rangle / \langle \mathcal{A}^{(2)} \rangle$ & $\langle \mathcal{A}^{(1)} \rangle / \langle \mathcal{A}^{(3)} \rangle$.

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$$T \propto \langle \mathcal{A}^{(1)} \rangle, \langle \mathcal{A}^{(2)} \rangle, \langle \mathcal{A}^{(3)} \rangle$$

Introducing a potential that has a different origin than the magnetic fluxes to stabilize **overall factor $T$**.

# Determination of the overall factor

# $F$-term potential

Effective potential for the moduli $T$

- Its effective theory remains supersymmetric.
- Supersymmetric action is determined

  by **super potential $W$** and **Kähler potential $K$**.
- We will study the following potential now[1]:

$$\begin{cases} W = w_0 - Ae^{-aT} + BX \\ K = -\ln\left(T + \bar{T}\right) + |X|^2 \end{cases}$$

$X$ は新たに導入したスカラー場，$w_0, A, B, a$ は実パラメター

# $F$-term potential

# Summary

# Reference


[1] H. Abe, T. Higaki, T. Kobayashi, and Y. Omura, *Moduli stabilization, $\boldsymbol{F}$-term uplifting and soft supersymmetry breaking terms,* Physical Review D **75** (2007) 025019, arxiv:hep-th/0611024.

[2] J. Wess and J. Bagger, *Supersymmetry and Supergravity*. Princeton University Press, Princeton, N.J, 1992.