Spring School 2024 @Izukawana

Moduli stabilization

for

on (~~for~~/at/in ?) supersymmetric magnetized D9-brane model

Abe Lab. M1

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Sunday, April 7th, 2024

*I will speak in Japanese though this slide is written in English.

Topics

- Reviewing the senior thesis
- Reporting progress and difficulties I met

Introduction

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in its 4d effective field theory.

Such a field is called **moduli fields**.

Moduli stabilization

The metrics in 10-dimensional spacetime are dynamical fields:

$$
\begin{aligned}
\mathrm{d}s^{10} &= G_{MN}(X)\mathrm{d}X^M \mathrm{d}X^N \\
&= g_{\mu\nu}(x,y)\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}(x,y)\mathrm{d}y^m \mathrm{d}y^n
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Thus, its vacuum expectation value (VEV) should be determined **by its dynamics**.

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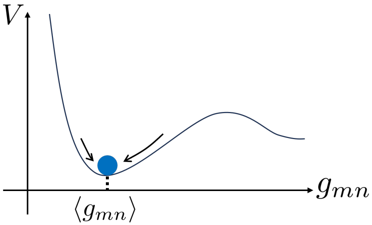
Thus, its vacuum expectation value (VEV) should be determined **by its dynamics**.

How?

What we have to do is just

- Write down the potential for moduli (e.g. g_{mn})
- Compute the minimum and identify the value $\langle g_{mn}\rangle$



These procedures are called **moduli stabilization**.

[1] H. Abe, T. Kobayashi, H. Ohki, and K. Sumita, Nucl. Phys. B (2012).
[2] H. Abe, T. Kobayashi, K. Sumita, and S. Uemura, Phys. Rev. D (2017).

Purpose of our study

On the other hand, it is known that **magnetized torus model**.

- This model realize the generation structure of the Standard Model [1, 2].

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On the other hand, it is known that **magnetized torus model**.

- This model realize the generation structure of the Standard Model [1, 2].
- But the VEVs of the moduli are not determined dynamically.

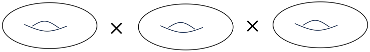
We will discuss the moduli stabilization on magnetized torus model.

Magnetized torus model

Torus compactification and Magnetic flux

Torus compactification

- Compactifying 6d extra dimensions for three tori $(T^2)^3$.



$\mathcal{A}^{(1)}$ $\mathcal{A}^{(2)}$ $\mathcal{A}^{(3)}$: area of the i-th torus

- $\mathcal{A}^{(i)}(x)$ are also moduli since it is related to the metric.

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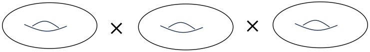

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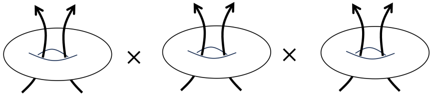
Magnetic flux

- Assigning the magnetic flux

$$M_a^{(i)} \ (a = 1, 2)$$

for two gauge fields on each tori.

$i = 1$ $i = 2$ $i = 3$



$\{M_1^{(1)},\ M_2^{(1)}\}$ $\{M_1^{(2)},\ M_2^{(2)}\}$ $\{M_1^{(3)},\ M_2^{(3)}\}$

Finding the VEVs by fluxes

By magnetic fluxes in an extra dimension, moduli $\boldsymbol{\mathcal{A}^{(i)}(x)}$ obtain its potential $\boldsymbol{V^{(D)}}$.

$$\downarrow$$

Finding minima and determining the VEVs of $\langle \boldsymbol{\mathcal{A}^{i}} \rangle$.

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Potential by magnetic fluxes

$$F^{MN}F_{MN} = F^{\mu\nu}F_{\mu\nu} + F^{mn}F_{mn} + \cdots$$

$\downarrow$

$$V^{(D)} = \pi^2 \prod_i \mathcal{A}^i \times \left\{ \underline{\left(\sum_i \frac{M_1^{(i)}}{\mathcal{A}^{(i)}} \right)^2} + \underline{\left(\sum_i \frac{M_2^{(i)}}{\mathcal{A}^{(i)}} \right)^2} \right\}$$

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$$\longrightarrow \quad \boxed{\frac{M_a^{(1)}}{\langle \mathcal{A}^{(1)} \rangle} + \frac{M_a^{(2)}}{\langle \mathcal{A}^{(2)} \rangle} + \frac{M_a^{(3)}}{\langle \mathcal{A}^{(3)} \rangle} = 0 \quad \text{for } a = 1, 2}$$

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Relation of the VEVs $\langle \mathcal{A}^{(1)} \rangle$, $\langle \mathcal{A}^{(2)} \rangle$, $\langle \mathcal{A}^{(3)} \rangle$

$$\frac{M_a^{(1)}}{\langle \mathcal{A}^{(1)} \rangle} + \frac{M_a^{(2)}}{\langle \mathcal{A}^{(2)} \rangle} + \frac{M_a^{(3)}}{\langle \mathcal{A}^{(3)} \rangle} = 0 \quad \text{for } a = 1, 2$$

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$$\longrightarrow \quad M_a^{(1)}+M_a^{(2)}\frac{\langle\mathcal{A}^{(1)}\rangle}{\langle\mathcal{A}^{(2)}\rangle}+M_a^{(3)}\frac{\langle\mathcal{A}^{(1)}\rangle}{\langle\mathcal{A}^{(3)}\rangle}=0$$

$$\downarrow$$

$$\frac{\langle\mathcal{A}^{(1)}\rangle}{\langle\mathcal{A}^{(2)}\rangle}=\frac{M_1^{(3)}M_2^{(1)}-M_1^{(1)}M_2^{(3)}}{M_1^{(2)}M_2^{(3)}-M_1^{(3)}M_2^{(2)}}\,,\ \frac{\langle\mathcal{A}^{(1)}\rangle}{\langle\mathcal{A}^{(3)}\rangle}=-\frac{M_1^{(2)}M_2^{(1)}-M_1^{(1)}M_2^{(2)}}{M_1^{(2)}M_2^{(3)}-M_1^{(3)}M_2^{(2)}}$$

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Relation of the VEVs $\langle \mathcal{A}^{(1)} \rangle$, $\langle \mathcal{A}^{(2)} \rangle$, $\langle \mathcal{A}^{(3)} \rangle$

$$\frac{M_a^{(1)}}{\langle \mathcal{A}^{(1)} \rangle} + \frac{M_a^{(2)}}{\langle \mathcal{A}^{(2)} \rangle} + \frac{M_a^{(3)}}{\langle \mathcal{A}^{(3)} \rangle} = 0 \quad \text{for } a = 1, 2$$

$$\longrightarrow \quad M_a^{(1)} + M_a^{(2)} \frac{\langle \mathcal{A}^{(1)} \rangle}{\langle \mathcal{A}^{(2)} \rangle} + M_a^{(3)} \frac{\langle \mathcal{A}^{(1)} \rangle}{\langle \mathcal{A}^{(3)} \rangle} = 0$$

$$\downarrow$$

$$\frac{\langle \mathcal{A}^{(1)} \rangle}{\langle \mathcal{A}^{(2)} \rangle} = \frac{M_1^{(3)} M_2^{(1)} - M_1^{(1)} M_2^{(3)}}{M_1^{(2)} M_2^{(3)} - M_1^{(3)} M_2^{(2)}} \, , \; \frac{\langle \mathcal{A}^{(1)} \rangle}{\langle \mathcal{A}^{(3)} \rangle} = - \frac{M_1^{(2)} M_2^{(1)} - M_1^{(1)} M_2^{(2)}}{M_1^{(2)} M_2^{(3)} - M_1^{(3)} M_2^{(2)}}$$

The ratios of the moduli's VEVs are determined by the fluxes potential.

Summary so far

- We could stabilize the moduli
  and obtain the ratio of its VEVs $\langle \mathcal{A}^{(1)} \rangle / \langle \mathcal{A}^{(2)} \rangle$ & $\langle \mathcal{A}^{(1)} \rangle / \langle \mathcal{A}^{(3)} \rangle$.

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- But the **overall factor $\boldsymbol{T}$** is undetermined.

$$\boldsymbol{T} \propto \langle \boldsymbol{\mathcal{A}}^{(1)} \rangle, \langle \boldsymbol{\mathcal{A}}^{(2)} \rangle, \langle \boldsymbol{\mathcal{A}}^{(3)} \rangle$$

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- But the **overall factor T** is undetermined.

$$T \propto \langle \mathcal{A}^{(1)} \rangle, \langle \mathcal{A}^{(2)} \rangle, \langle \mathcal{A}^{(3)} \rangle$$

Introducing a potential that has a different origin than the magnetic fluxes
to stabilize **overall factor T**.

Determination of the overall factor

[3] H. Abe, T. Higaki, T. Kobayashi, and Y. Omura, Phys. Rev. D (2007).
[4] H. Abe, T. Higaki, and T. Kobayashi, Phys. Rev. D (2007).

F-term potential

Effective potential for moduli T

- Its effective theory remains supersymmetric.
- Supersymmetric action is determined

  by **super potential W** and **Kähler potential K**.
- We will study the following potential now[3]:

$$\begin{cases} W = w_0 - Ae^{-aT} + BX \\ K = -3\ln\left(T + \bar{T}\right) + |X|^2 \end{cases}$$

Introducing a new scalar field X and w_0, A, B, a are parameters.

[3] H. Abe, T. Higaki, T. Kobayashi, and Y. Omura, Phys. Rev. D (2007).
[4] H. Abe, T. Higaki, and T. Kobayashi, Phys. Rev. D (2007).

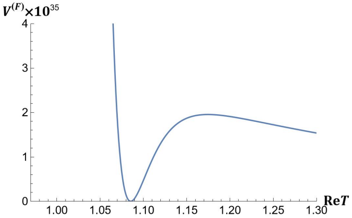
F-term potential

*We set the Planck constant as $M_{\rm Pl}$ ($\sim 2.4\times10^{18}$ GeV) $= 1$.

Scalar potential

$$V^{(F)} = e^{K}\left(K^{I\bar{J}}(D_I W)(D_{\bar{J}}\bar{W}) - 3|W|^2\right)$$

$$\begin{cases} D_I W \equiv \partial_I W + K_I W \\ K^{I\bar{J}}:\ \text{inverse matrix of } K_{I\bar{J}} \end{cases} \quad (I = X, T)$$



Parameters

$$w_0 \sim 2.17\times10^{-18}\,,\ a = 4\pi^2\,,\ A = 1\,,\ B = e^{-4\pi^2}$$
$$\text{and } \langle X\rangle = \sqrt{3}-1$$

$$\to\ \langle T\rangle \sim 1.085$$

Report progresses

In my senior thesis, we

- fix the value of the fluxes $\boldsymbol{M_a^{(i)}}$
- and compute the area of the tori $\langle \boldsymbol{\mathcal{A}^{(i)}} \rangle$ $\boldsymbol{(i = 1, 2, 3)}$.

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Report progresses

In my senior thesis, ~~we~~ I

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- and compute the area of the tori $\langle \boldsymbol{\mathcal{A}^{(i)}} \rangle$ $\boldsymbol{(i = 1, 2, 3)}$.

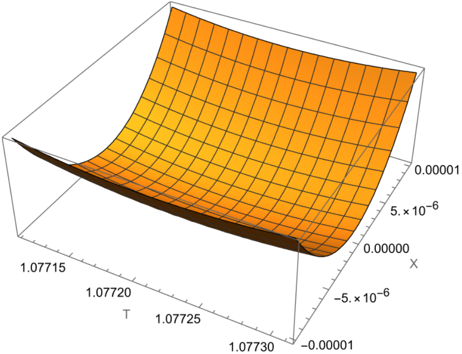
During this vacation, I mainly studied the $\boldsymbol{F}$-term potential more general form:

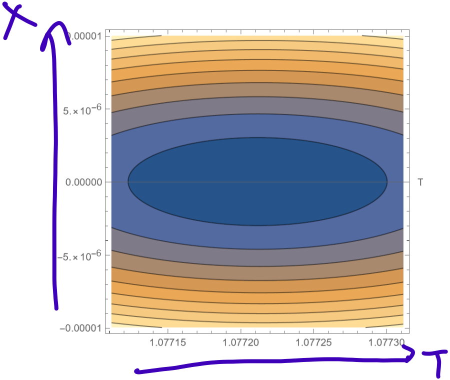
$$\begin{cases} \boldsymbol{W = w_0 - Ae^{-aT} + Be^{-bT}X} \\ \boldsymbol{K = -3\ln\left(T + \bar{T}\right) + |X|^2 - \dfrac{1}{\Lambda^2}|X|^4} \end{cases}$$

Wavy factors are the difference from the previous one.

Report progresses

We can stabilize the moduli $\boldsymbol{T}$, $\boldsymbol{X}$ but $\cdots\cdots$





Parameters

$$\boldsymbol{A = B = 1, a = b = 4\pi^2, w_0 = 10^{-17}, \Lambda = 10^{-4}}$$

$$\longrightarrow \quad \langle \boldsymbol{T} \rangle \sim \boldsymbol{1.07}, \ \langle \boldsymbol{X} \rangle \sim \boldsymbol{10^{-8}}, \ \boldsymbol{V_{\min}} \sim \boldsymbol{-10^{-35}} \times M_{Pl}^4$$

$\times M_{Pl}^2$ $\quad\quad$ $\times M_{Pl}^2$

Difficulites

- We should tune the parameters $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{w_0}, \boldsymbol{\Lambda}$
  to **uplift the potential minima $\boldsymbol{V_{\min}}$ to the order $\boldsymbol{+10^{-120}}$**.

Difficulites

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- But it is difficult (at least for me) since
  - we can not find the minimum points analytically
  - thus what we can do is only

    to choose the parameters and find the minimum for each time.

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  to **uplift the potential minima** V_{min} **to the order** $+10^{-120}$.
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to choose the parameters and find the minimum for each time.

$$\Downarrow$$

I want to find more efficient ways to search the set of parameters.

e.g. $\quad \dfrac{\partial V}{\partial T} = \dfrac{\partial V}{\partial X} = 0 \quad \& \quad \dfrac{\partial V}{\partial a} = \dfrac{\partial V}{\partial b} = \dfrac{\partial V}{\partial A} = \dfrac{\partial V}{\partial B} = \dfrac{\partial V}{\partial w_0} = 0$

(I tried it but it seems not to converge the Newtonian method...)

Summary & Future Works

Summary

- Discussing moduli stabilization in magnetized torus model
- Determining the ratio of the moduli by potential $\boldsymbol{V^{(D)}}$ with fluxes
- Stabilizing the overall moduli $\boldsymbol{T}$ by potential $\boldsymbol{V^{(F)}}$ that has a different origin

Future Works

- [new!] Finding more efficient ways to search the set of parameters.
- Discussing soft SUSY breaking and mass of supersymmetric particles, etc.

Reference

[1] H. Abe, T. Kobayashi, H. Ohki, and K. Sumita, *Superfield description of 10D SYM theory with magnetized extra dimensions,* Nucl. Phys. B **863** (2012) 1–18, arxiv:1204.5327 [hep-ph, physics:hep-th].

[2] H. Abe, T. Kobayashi, K. Sumita, and S. Uemura, *Kähler moduli stabilization in semi-realistic magnetized orbifold models,* Phys. Rev. D **96** (2017) 026019, arxiv:1703.03402 [hep-ph, physics:hep-th].

[3] H. Abe, T. Higaki, T. Kobayashi, and Y. Omura, *Moduli stabilization, F-term uplifting and soft supersymmetry breaking terms,* Phys. Rev. D **75** (2007) 025019, arxiv:hep-th/0611024.

[4] H. Abe, T. Higaki, and T. Kobayashi, *More about F-term uplifting,* Phys. Rev. D **76** (2007) 105003, arxiv:0707.2671 [hep-ph, physics:hep-th].

[5] J. Wess and J. Bagger, *Supersymmetry and Supergravity*. Princeton University Press, Princeton, N.J, 1992.