Relativity - Report 4

Itsuki Miyane   ID: 5324A057-8

Last modified: June 8, 2024

(1) Let us consider the variation of the action

$$S_m = \int \mathrm{d}^4 \sqrt{-g} K(\phi, X) \tag{0.1}$$

where X denotes

$$X \equiv -\frac{1}{2} g^{\mu\nu} \partial_{[\mu]} \phi \partial_\nu \phi. \tag{0.2}$$

It obeys

$$\begin{aligned} \delta S_m &= \int \mathrm{d}^4 \left[(\delta\sqrt{-g}) K + \sqrt{-g} (\delta K) \right] \\ &= -\frac{1}{2} \int \mathrm{d}^4 x \sqrt{-g} \left\{ g_{\mu\nu} K + K_X \partial_\mu \partial_\nu \phi \right\} \delta g^{\mu\nu} \end{aligned} \tag{0.3}$$

where $K_X \equiv \partial_X K(\phi, X)$. The definition of the energy-momentum tensor so far is

$$\delta S_m = -\frac{1}{2} \int \mathrm{d}^4 x \, \delta g^{\mu\nu} \sqrt{-g} T_{\mu\nu} \tag{0.4}$$

and by comparing this definition with the equation (0.3), we find

$$T_{\mu\nu} = g_{\mu\nu} K + K_X \partial_\mu \partial_\nu \phi \qquad (K_X \equiv \partial_X K(\phi, X)). \tag{0.5}$$

(2) The perturbed FLRW metric

$$g_{\mu\nu} = a^2(\eta) \begin{pmatrix} -(1+2A) & \partial_i B \\ \partial_i B & (1+2\psi)\delta_{ij} + 2\partial_i \partial_j E \end{pmatrix} \tag{0.6}$$

is obtained by metric perturbation. The energy-momentum tensor becomes

$$T^\mu_{\ \nu} = g^{\mu\rho} T_{\rho\nu} = \delta^\mu_\nu K(\phi, X) + g^{\mu\rho} K_X \partial_\rho \phi \partial_\nu \phi \tag{0.7}$$

from the previous result. By perturbing $\phi(\eta, \boldsymbol{x})$ as $\phi(\eta) + \delta\phi(\eta, \boldsymbol{x})$[*1], we find

[*1] I follow the instructions and omit the bar on the background field.

(3) The perturbed Einstein equation is given by

$$\delta R^{\mu}{}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu}\delta R = 8\pi G \delta T^{\mu}{}_{\nu} \tag{0.8}$$

in Lecture 14. Let us follow the instruction and put $(\mu, \nu) = (0, 0)$ and $(0, i)$.

(4)

References

[1] A. Vilma, "*K-essence: cosmology, causality and emergent geometry*," 2007.