Relativity - Report 3

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(1) If we put $\theta = \pi/2$, the quantity Σ becomes r^2 and the line element is obtained as

$$\mathrm{d}s^2 = -c^2 \left(1 - \frac{2\mu}{r}\right) \mathrm{d}t^2 - \frac{4\mu ac}{r} \mathrm{d}t\mathrm{d}\varphi + \frac{r^2}{\Delta} \mathrm{d}r^2 + \left(r^2 + a^2 + \frac{2\mu a^2}{r}\right) \mathrm{d}\varphi^2 \tag{0.1}$$

and the metric also is as

$$g_{\mu\nu} = \begin{pmatrix} -c^2(1 - 2\mu/r) & 0 & -2\mu ac/r \\ 0 & r^2/\Delta & 0 \\ -2\mu ac/r & 0 & r^2 + a^2 + 2\mu a^2/r \end{pmatrix} \tag{0.2}$$

where Δ still remain $r^2 - 2\mu r + a^2$. Therefore the conserved quantities p_t and p_φ are obtained as

$$\begin{aligned} p_t &= -g_{tt}\dot{t} - g_{t\varphi}\dot{\varphi} \\ &= c^2(1 - 2\mu/r)\dot{t} + (2\mu ac/r)\dot{\varphi} \end{aligned} \tag{0.3}$$

$$\begin{aligned} p_\varphi &= g_{\varphi t}\dot{t} + g_{\varphi\varphi}\dot{\varphi} \\ &= (2\mu ac/r)\dot{t} + (r^2 + a^2 + 2\mu a^2/r)\dot{\varphi} \end{aligned} \tag{0.4}$$

and we will solve these equations to $\dot{t}$ and $\dot{\varphi}$.

References

[1] Chapter 22 Geodesic motion in Kerr spacetime. (Last accessed: May 23, 2024)

[2] Kerr Geometry and Rotating Black Hole. (Last accessed: May 23, 2024)