Spring School 2024 @Izukawana

Moduli stabilization on (for/at/in ?) supersymmetric magnetized D9-brane model

Abe Lab. M1

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*I will speak in Japanese though this slide is written in English.

Topics

- Review the senior thesis
- Report progress and difficulties I met

Introduction

Motivation

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Such a field is called **moduli fields**.

Moduli stabilization

The metrics in 10-dimensional spacetime are dynamical fields:

$$\begin{aligned} \mathrm{d}s^{10} &= G_{MN}\mathrm{d}X^M\mathrm{d}X^N \\ &= g_{\mu\nu}(x,y)\mathrm{d}x^\mu\mathrm{d}x^\nu + g_{mn}(x,y)\mathrm{d}y^m\mathrm{d}y^n \end{aligned}$$

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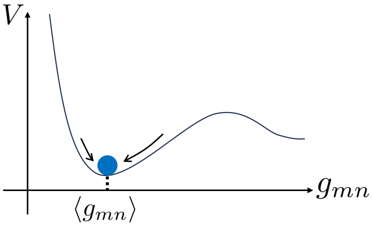
How?

What we have to do is just

- Write down the potential for moduli (e.g. g_{mn})
- Compute the minimum and identify the value $\langle g_{mn} \rangle$



These procedures are called moduli stabilization.

Purpose of my study

We will discuss the moduli stabilization on magnetized torus model.

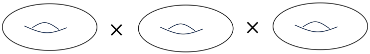
$\cdots\cdots$

Magnetized torus model

Torus compactification and Magnetic flux

Torus compactification

- Compactifying 6d extra dimensions for three tori $(T^2)^3$.



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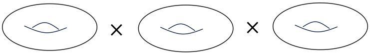

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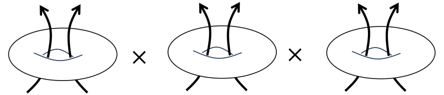
Magnetic flux

- Assigning the magnetic flux
  $M_a^{(i)}$ $(a = 1, 2)$
  for two gauge fields on each tori.



Finding the VEVs by fluxes

By magnetic fluxes in an extra dimension, moduli $\mathcal{A}^{(i)}(x)$ obtain its potential $V^{(D)}$.

$$\downarrow$$

Finding minima and determining the VEVs of $\langle \mathcal{A}^i \rangle$.

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Potential by magnetic fluxes

$$F^{MN}F_{MN} = F^{\mu\nu}F_{\mu\nu} + \underset{\sim\sim\sim\sim}{F^{mn}F_{mn}} + \cdots$$

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$$V^{(D)} = \pi^2 \prod_i \mathcal{A}^i \times \left\{ \underline{\left(\sum_i \frac{M_1^{(i)}}{\mathcal{A}^{(i)}} \right)^2} + \underline{\left(\sum_i \frac{M_2^{(i)}}{\mathcal{A}^{(i)}} \right)^2} \right\}$$

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$$\frac{\langle \mathcal{A}^{(1)} \rangle}{\langle \mathcal{A}^{(2)} \rangle} = \frac{M_1^{(3)} M_2^{(1)} - M_1^{(1)} M_2^{(3)}}{M_1^{(2)} M_2^{(3)} - M_1^{(3)} M_2^{(2)}} \, , \; \frac{\langle \mathcal{A}^{(1)} \rangle}{\langle \mathcal{A}^{(3)} \rangle} = -\frac{M_1^{(2)} M_2^{(1)} - M_1^{(1)} M_2^{(2)}}{M_1^{(2)} M_2^{(3)} - M_1^{(3)} M_2^{(2)}}$$

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The ratios of the moduli's VEVs are determined by the fluxes potential.

Summary so far

- We could stabilize the moduli
  and obtain the ratio of its VEVs $\langle \mathcal{A}^{(1)} \rangle / \langle \mathcal{A}^{(2)} \rangle$ & $\langle \mathcal{A}^{(1)} \rangle / \langle \mathcal{A}^{(3)} \rangle$.

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- But the **overall factor $\boldsymbol{T}$** is undetermined.

$$\boldsymbol{T} \propto \langle \boldsymbol{\mathcal{A}}^{(1)} \rangle, \langle \boldsymbol{\mathcal{A}}^{(2)} \rangle, \langle \boldsymbol{\mathcal{A}}^{(3)} \rangle$$

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- But the **overall factor T** is undetermined.

$$T \propto \langle \mathcal{A}^{(1)} \rangle, \langle \mathcal{A}^{(2)} \rangle, \langle \mathcal{A}^{(3)} \rangle$$

Introducing a potential that has a different origin than the magnetic fluxes to stabilize **overall factor T**.

Determination of the overall factor

F-term potential

Effective potential for moduli T

- Its effective theory remains supersymmetric.
- Supersymmetric action is determined

  by **super potential W** and **Kähler potential K**.
- We will study the following potential now[1]:

$$\begin{cases} W = w_0 - Ae^{-aT} + BX \\ K = -3\ln\left(T + \bar{T}\right) + |X|^2 \end{cases}$$

Introducing a new scalar field X and w_0, A, B, a are parameters.

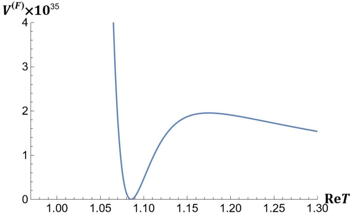
$\boldsymbol{F}$-term potential

*We set the Planck constant as $M_{\text{Pl}}(\sim \mathbf{2.4} \times \mathbf{10^{18}}\ \text{GeV}) = \mathbf{1}$.

Scalar potential

$$\boldsymbol{V^{(F)} = e^{K}(K^{I\bar{J}}(D_I W)(D_{\bar{J}}\bar{W}) - 3|W|^2)}$$

$$\begin{cases} \boldsymbol{D_I W \equiv \partial_I W + K_I W} \\ \boldsymbol{K^{I\bar{J}}}\text{: inverse matrix of } \boldsymbol{K_{I\bar{J}}} \end{cases} \qquad \boldsymbol{(I = X, T)}$$



Parameters

$$\boldsymbol{w_0 \sim 2.17 \times 10^{-18}\,,\ a = 4\pi^2\,,\ A = 1\,,\ B = e^{-4\pi^2}}$$
$$\textbf{and } \boldsymbol{\langle X \rangle = \sqrt{3} - 1}$$

$$\boldsymbol{\rightarrow\ \langle T \rangle \sim 1.085}$$

Report progresses

In my senior thesis, we

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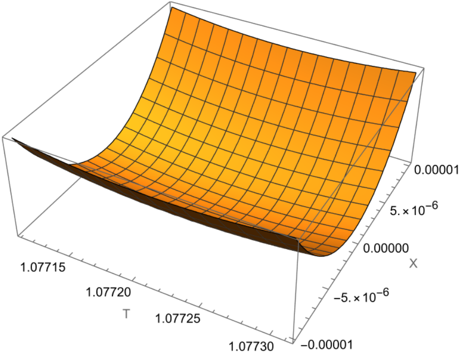
During this vacation, I mainly studied the F-term potential more general form:

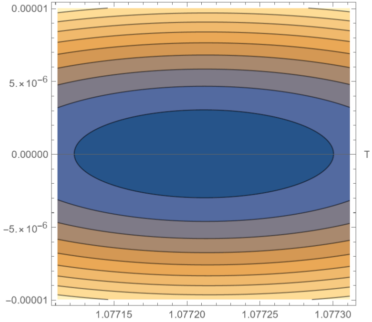
$$\begin{cases} W = w_0 - Ae^{-aT} + B\underset{\sim}{e^{-bT}}X \\ K = -3\ln(T + \bar{T}) + |X|^2 \underset{\sim}{- \frac{1}{\Lambda^2}|X|^4} \end{cases}$$

Wavy factors are the difference from the previous one.

Report progresses

We can stabilize the moduli T, X but $\cdots\cdots$





Parameters

$$A = B = 1, a = b = 4\pi^2, w_0 = 10^{-17}, \Lambda = 10^{-4}$$

$$\longrightarrow \quad \langle T\rangle \sim 1.07, \ \langle X\rangle \sim 10^{-8}, V_{\rm min} \sim 10^{-35}$$

Difficulites & Future Works

- We should tune the parameters $\boldsymbol{A, B, a, b, w_0, \Lambda}$ to **uplift the potential minima $\boldsymbol{V_{\mathrm{min}}}$ to the order $\boldsymbol{+10^{-120}}$**.
- But it is difficult (at least for me) since

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- But it is difficult (at least for me) since I tried and/or will try following the method

Summary

Reference

[1] H. Abe, T. Higaki, T. Kobayashi, and Y. Omura, *Moduli stabilization, $\boldsymbol{F}$-term uplifting and soft supersymmetry breaking terms,* Physical Review D **75** (2007) 025019, arxiv:hep-th/0611024.

[2] J. Wess and J. Bagger, *Supersymmetry and Supergravity*. Princeton University Press, Princeton, N.J, 1992.