

My Study Notes 2024 / QFT ver.

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1 Introduction

2 Functional Method

A Some notes

A.1 Normalization of Maxwell Lagrangian

The Maxwell lagrangian is the form

$$\mathcal{L} = N F^{\mu\nu} F_{\mu\nu} \quad (\text{A.1})$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is a field strength. We will determine the constant N to lagrangian contains the term

$$\mathcal{L} = \frac{1}{2} \dot{A}_1^2 + \frac{1}{2} \dot{A}_2^2 + \frac{1}{2} \dot{A}_3^2 + \dots \quad (\text{A.2})$$

Now expanding the term $F^{\mu\nu} F_{\mu\nu}$ carefully, we obtain

$$\begin{aligned} F^{\mu\nu} F_{\mu\nu} &= (\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ &= 2((\partial^\mu A^\nu)(\partial_\mu A_\nu) - (\partial_\mu A^\nu)(\partial_\nu A_\mu)) \\ &= -2(\dot{A}_1^2 + \dot{A}_2^2 + \dot{A}_3^2) + \dots \end{aligned} \quad (\text{A.3})$$

and thus, the constant N satisfies the condition $-2N = 1/2$ and $N = -1/4$. That's why we get Maxwell Lagrangian as

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad (\text{A.4})$$

References

- [PS95] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley Pub. Co, Reading, Mass, 1995.
- [WB92] J. Wess and J. Bagger, *Supersymmetry and supergravity*, Princeton University Press, Princeton, N.J, 1992.