Spring School 2024 @Izukawana

Moduli stabilization

for

on (~~for~~/at/in ?) supersymmetric magnetized D9-brane model

Abe Lab. M1

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Sunday, April 7th, 2024

*I will speak in Japanese though this slide is written in English.

Topics

- Reviewing the senior thesis
- Reporting progress and difficulties I met

Introduction

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Such a field is called **moduli fields**.

Moduli stabilization

The metrics in 10-dimensional spacetime are dynamical fields:

$$\begin{aligned} \mathrm{d}s^{10} &= G_{MN}(X)\mathrm{d}X^M \mathrm{d}X^N \\ &= g_{\mu\nu}(x,y)\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}(x,y)\mathrm{d}y^m \mathrm{d}y^n \end{aligned}$$

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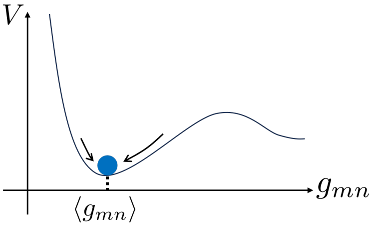
How?

What we have to do is just

- Write down the potential for moduli (e.g. g_{mn})
- Compute the minimum and identify the value $\langle g_{mn}\rangle$



These procedures are called **moduli stabilization**.

Purpose of our study

On the other hand, it is known that **magnetized torus model**.

- This model realize the generation structure of the Standard Model [1, 2].

[1] H. Abe, T. Kobayashi, H. Ohki, and K. Sumita, Nucl. Phys. B (2012).
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On the other hand, it is known that **magnetized torus model**.

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We will discuss the moduli stabilization on magnetized torus model.

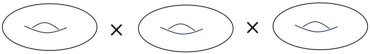
Magnetized torus model

[1] H. Abe, T. Kobayashi, H. Ohki, and K. Sumita, Nucl. Phys. B (2012).

Torus compactification and Magnetic flux

Torus compactification

- Compactifying 6d extra dimensions for three tori $(T^2)^3$.



$\mathcal{A}^{(1)}$ $\quad\mathcal{A}^{(2)}$ $\quad\mathcal{A}^{(3)}$: area of the i-th torus

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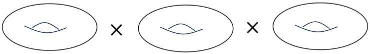

$\boldsymbol{\mathcal{A}^{(1)}}$ $\boldsymbol{\mathcal{A}^{(2)}}$ $\boldsymbol{\mathcal{A}^{(3)}}$: area of the $\boldsymbol{i}$-th torus

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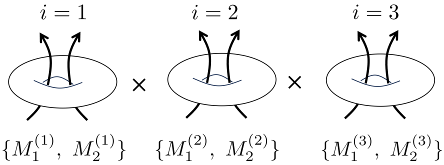
Magnetic flux

- Assigning the magnetic flux

  $\boldsymbol{M_a^{(i)}}$ $\boldsymbol{(a = 1, 2)}$

  for two gauge fields on each tori.



Finding the VEVs by fluxes

By magnetic fluxes in an extra dimension, moduli $\boldsymbol{\mathcal{A}^{(i)}(x)}$ obtain its potential $\boldsymbol{V^{(D)}}$.

$$\downarrow$$

Finding minima and determining the VEVs of $\langle \boldsymbol{\mathcal{A}^{i}} \rangle$.

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Potential by magnetic fluxes $\quad F^{MN}F_{MN} = F^{\mu\nu}F_{\mu\nu} + F^{mn}F_{mn} + \cdots$

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$$V^{(D)} = \pi^2 \prod_i \mathcal{A}^i \times \left\{ \underline{\left(\sum_i \frac{M_1^{(i)}}{\mathcal{A}^{(i)}} \right)^2} + \underline{\left(\sum_i \frac{M_2^{(i)}}{\mathcal{A}^{(i)}} \right)^2} \right\}$$

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Relation of the VEVs $\langle \mathcal{A}^{(1)} \rangle$, $\langle \mathcal{A}^{(2)} \rangle$, $\langle \mathcal{A}^{(3)} \rangle$

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$$\frac{\langle \mathcal{A}^{(1)} \rangle}{\langle \mathcal{A}^{(2)} \rangle} = \frac{M_1^{(3)} M_2^{(1)} - M_1^{(1)} M_2^{(3)}}{M_1^{(2)} M_2^{(3)} - M_1^{(3)} M_2^{(2)}}\,, \quad \frac{\langle \mathcal{A}^{(1)} \rangle}{\langle \mathcal{A}^{(3)} \rangle} = -\frac{M_1^{(2)} M_2^{(1)} - M_1^{(1)} M_2^{(2)}}{M_1^{(2)} M_2^{(3)} - M_1^{(3)} M_2^{(2)}}$$

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The ratios of the moduli's VEVs are determined by the fluxes potential.

Summary so far

- We could stabilize the moduli
  and obtain the ratio of its VEVs $\langle \mathcal{A}^{(1)} \rangle / \langle \mathcal{A}^{(2)} \rangle$ & $\langle \mathcal{A}^{(1)} \rangle / \langle \mathcal{A}^{(3)} \rangle$.

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- But the **overall factor T** is undetermined.

$$T \propto \langle \mathcal{A}^{(1)} \rangle, \langle \mathcal{A}^{(2)} \rangle, \langle \mathcal{A}^{(3)} \rangle$$

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$$T \propto \langle \mathcal{A}^{(1)} \rangle, \langle \mathcal{A}^{(2)} \rangle, \langle \mathcal{A}^{(3)} \rangle$$

Introducing a potential that has a different origin than the magnetic fluxes
to stabilize **overall factor T**.

Determination of the overall factor

[3] H. Abe, T. Higaki, T. Kobayashi, and Y. Omura, Phys. Rev. D (2007).
[4] H. Abe, T. Higaki, and T. Kobayashi, Phys. Rev. D (2007).

F-term potential

Effective potential for moduli T

- Its effective theory remains supersymmetric.
- Supersymmetric action is determined

  by **super potential W** and **Kähler potential K**.
- We will study the following potential now[3]:

$$\begin{cases} W = w_0 - Ae^{-aT} + BX \\ K = -3\ln\left(T + \bar{T}\right) + |X|^2 \end{cases}$$

Introducing a new scalar field X and w_0, A, B, a are parameters.

[3] H. Abe, T. Higaki, T. Kobayashi, and Y. Omura, Phys. Rev. D (2007).
[4] H. Abe, T. Higaki, and T. Kobayashi, Phys. Rev. D (2007).

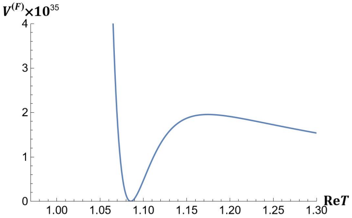
F-term potential

*We set the Planck constant as $M_{\rm Pl}$ $(\sim 2.4 \times 10^{18}$ GeV$) = 1$.

Scalar potential

$$V^{(F)} = e^K(K^{I\bar{J}}(D_I W)(D_{\bar{J}}\bar{W}) - 3|W|^2)$$

$$\begin{cases} D_I W \equiv \partial_I W + K_I W \\ K^{I\bar{J}}\text{: inverse matrix of } K_{I\bar{J}} \end{cases} \quad (I = X, T)$$



Parameters

$$w_0 \sim 2.17 \times 10^{-18}\,,\ a = 4\pi^2\,,\ A = 1\,,\ B = e^{-4\pi^2}$$
$$\text{and } \langle X \rangle = \sqrt{3} - 1$$

$$\to\ \langle T \rangle \sim 1.085$$

Report progresses

In my senior thesis, we

- fix the value of the fluxes $\boldsymbol{M_a^{(i)}}$
- and compute the area of the tori $\langle \boldsymbol{\mathcal{A}^{(i)}} \rangle$ $\boldsymbol{(i = 1, 2, 3)}$.

Report progresses

I

In my senior thesis, ~~we~~

- fix the value of the fluxes $M_a^{(i)}$
- and compute the area of the tori $\langle \mathcal{A}^{(i)} \rangle$ $(i = 1, 2, 3)$.

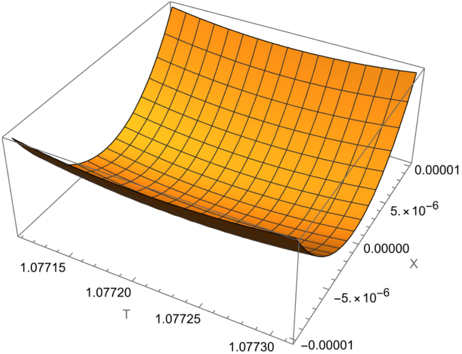
During this vacation, I mainly studied the F-term potential more general form:

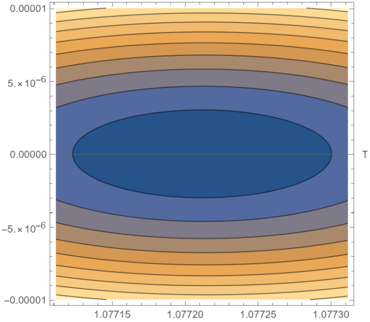
$$\begin{cases} W = w_0 - Ae^{-aT} + B\underset{\sim\sim}{e^{-bT}}X \\ K = -3\ln(T+\bar{T}) + |X|^2 \underset{\sim\sim\sim}{- \frac{1}{\Lambda^2}|X|^4} \end{cases}$$

Wavy factors are the difference from the previous one.

Report progresses

We can stabilize the moduli $\boldsymbol{T}$, $\boldsymbol{X}$ but $\cdots\cdots$





Parameters

$$\boldsymbol{A = B = 1, a = b = 4\pi^2, w_0 = 10^{-17}, \Lambda = 10^{-4}}$$

$$\longrightarrow \quad \boldsymbol{\langle T\rangle \sim 1.07,\ \langle X\rangle \sim 10^{-8},\ V_{\min} \sim -10^{-35}}$$

Difficulites

- We should tune the parameters $\boldsymbol{A, B, a, b, w_0, \Lambda}$ to **uplift the potential minima $V_{\min}$ to the order $+10^{-120}$**.

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$$\Downarrow$$

I want to find more efficient ways to search the set of parameters.

e.g. $\quad \boldsymbol{\frac{\partial V}{\partial T} = \frac{\partial V}{\partial X} = 0} \quad \& \quad \boldsymbol{\frac{\partial V}{\partial a} = \frac{\partial V}{\partial b} = \frac{\partial V}{\partial A} = \frac{\partial V}{\partial B} = \frac{\partial V}{\partial w_0} = 0}$

(I tried it but it seems not to converge the Newtonian method...)

Summary & Future Works

Summary

- Discussing moduli stabilization in magnetized torus model
- Determining the ratio of the moduli by potential $\boldsymbol{V^{(D)}}$ with fluxes
- Stabilizing the overall moduli $\boldsymbol{T}$ by potential $\boldsymbol{V^{(F)}}$ that has a different origin

Future Works

- [new!] Finding more efficient ways to search the set of parameters.
- Discussing soft SUSY breaking and mass of supersymmetric particles, etc.

Reference

[1] H. Abe, T. Kobayashi, H. Ohki, and K. Sumita, *Superfield description of 10D SYM theory with magnetized extra dimensions,* Nucl. Phys. B **863** (2012) 1–18, arxiv:1204.5327 [hep-ph, physics:hep-th].

[2] H. Abe, T. Kobayashi, K. Sumita, and S. Uemura, *Kähler moduli stabilization in semi-realistic magnetized orbifold models,* Phys. Rev. D **96** (2017) 026019, arxiv:1703.03402 [hep-ph, physics:hep-th].

[3] H. Abe, T. Higaki, T. Kobayashi, and Y. Omura, *Moduli stabilization, F-term uplifting and soft supersymmetry breaking terms,* Phys. Rev. D **75** (2007) 025019, arxiv:hep-th/0611024.

[4] H. Abe, T. Higaki, and T. Kobayashi, *More about F-term uplifting,* Phys. Rev. D **76** (2007) 105003, arxiv:0707.2671 [hep-ph, physics:hep-th].

[5] J. Wess and J. Bagger, *Supersymmetry and Supergravity*. Princeton University Press, Princeton, N.J, 1992.