

Hidden Sector on Magnetized D7-brane Model

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August 14, 2024

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Summer School 2024 @ Karuizawa

Around the end of the semester, I begin to consider the **hidden sector** of the magnetized D7-brane model, which was constructed by **Chin-san** in his master's thesis.

6.2 ポテンシャル解析

簡単のために磁場が 0 の場合でポテンシャルの解析をする。例としてオービフォルドの射影演算子と磁場が

$$P_{++-}, \quad P'_{+--}, \quad M^{(1)} = \left(\begin{array}{c|c} \underline{0} & \\ \hline \underline{0} \times \mathbf{1}_N & \underline{0} \end{array} \right), \quad M^{(2)} = \left(\begin{array}{c|c} H & \\ \hline \underline{0} \times \mathbf{1}_N & \underline{0} \end{array} \right), \quad M^{(3)} = \left(\begin{array}{c|c} \underline{0} & \\ \hline H \times \mathbf{1}_N & H \end{array} \right) \quad (6.5)$$

の場合について具体的に考える。ただし実線は $D7_C, D7_D$ の区別を表している。このとき、場の構成は

$$\Phi_1 = \left(\begin{array}{c|c} & \\ \hline & Q \end{array} \right), \quad \Phi_2 = \left(\begin{array}{c|c} & X \\ \hline & \end{array} \right), \quad \Phi_3 = \left(\begin{array}{c|c} & \\ \hline \tilde{Q} & \end{array} \right) \quad (6.6)$$

となる。

He already studied the **hidden sector** but he **set the all flux to zero**.

My goals are

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となる。

He already studied the **hidden sector** but he **set the all flux to zero**.

My goals are

- Introduce the flux in the hidden sector and develop the model.
- Upgrade to SUGRA and stabilize the moduli, which related to my *Sotsuron*.

Introduction & Review

There are several problems in the **Standard Model**. I list them though briefly.

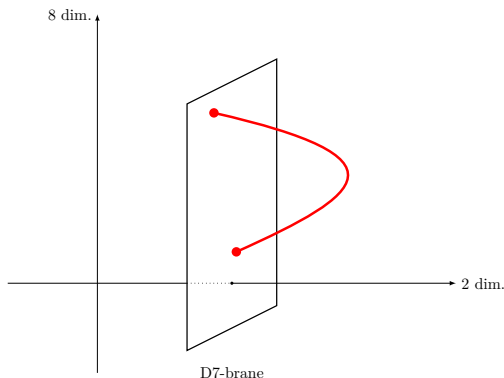
- Quantization of the gravity
- Origin of $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Hierarchy problem
- Origin of three-flavors (generations)
- Strong (QCD) CP violation, etc.

(Most of them can be found in **Kikuchi-san's master's thesis**, Hokkaido univ..)

Superstring theory is one of the candidate for solving these problems. We are interested in its effective field theory, **Super Yang-Mills theory**.

In D7-brane model,

- the endpoints of a string behave SYM particles.
- the brane is an 8-dimensional objects and the particles appear to be localized when viewed from other dimensions.



Lagrangian

$$\mathcal{L} = \frac{1}{g^2} \text{Tr} \left[-\frac{1}{4} F^{MN} F_{MN} + \frac{i}{2} \bar{\lambda} \Gamma^M D_M \lambda \right]$$

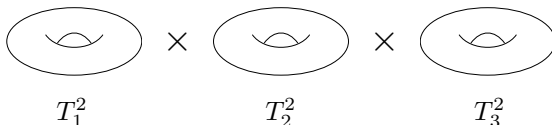
- M, N run $0, 1, \dots, 9$.
- F_{MN} is a field strength defined as $F_{MN} \equiv \partial_M A_N - \partial_N A_M - i[A_M, A_N]$.
(We assume the gauge group G is non-abelian, then the commutator does not vanish.)
- λ is a majorana-weyl spinor satisfying a **majorana condition** and a **positive chirality condition**:

$$\lambda^C = \lambda, \quad \Gamma \lambda = +\lambda.$$

To obtain an effective four-dimensional theory, we need to compactify the extra dimensions. Now \mathbb{R}^6 to $T^2 \times T^2 \times T^2$.

$$ds^{10} = g_{\mu\nu} dx^\mu dx^\nu + g_{mn} dx^m dx^n$$

$$g_{mn} = \begin{pmatrix} g^{(1)} & 0 & 0 \\ 0 & g^{(2)} & 0 \\ 0 & 0 & g^{(3)} \end{pmatrix}, \quad g^{(i)} = (2\pi R_i)^2 \begin{pmatrix} 1 & \text{Re } \tau_i \\ \text{Re } \tau_i & |\tau_i|^2 \end{pmatrix}$$



An area of the i -th torus is $\mathcal{A}^{(i)} = 4\pi^2 R_i^2 (\text{Im } \tau_i)$.

Procudure

[1] H. Abe, T. Kobayashi, K. Sumita, and S. Uemura, *Nucl. Phys. B* **863** (2012) 1–18, arXiv:1204.5327 [hep-th].
 [3] N. Arkani-Hamed, T. Gregoire, and J. Wacker, *JHEP* **03** (2002) 055, arXiv:hep-th/0101233.

$$\mathcal{L} = \frac{1}{g^2} \text{Tr} \left[-\frac{1}{4} F^{MN} F_{MN} + \frac{i}{2} \bar{\lambda} \Gamma^M D_M \lambda \right]$$

$$\Downarrow \text{4-dim. } \mathcal{N} = 1 \text{ superspace [3]}$$

$$\mathcal{L} = \int d^4\theta \mathcal{K} + \left\{ \int d^2\theta \left(\frac{1}{4g^2} \mathcal{W}^\alpha \mathcal{W}_\alpha + \mathcal{W} \right) + \text{h.c.} \right\}, \quad S = \int d^{10}X \sqrt{-G} \mathcal{L}$$

$$\Downarrow \text{Compactify } \mathbb{R}^6 \text{ to } T^2 \times T^2 \times T^2$$

$$\mathcal{L} = \sum_{\text{KK modes}} \int d^4\theta \mathcal{K} + \left\{ \int d^2\theta \left(\frac{1}{4g^2} \mathcal{W}^\alpha \mathcal{W}_\alpha + \mathcal{W} \right) + \text{h.c.} \right\}, \quad S = \int d^4x \mathcal{L}$$

where

$$\begin{cases} \mathcal{K}(V, \phi_i) = \frac{2}{g^2} h^{\bar{i}j} \text{Tr} \left[\bar{\phi}_{\bar{i}} \phi_j + \sqrt{2} \left\{ \left(\bar{\partial}_{\bar{i}} \phi_j + \frac{1}{\sqrt{2}} [\langle \bar{\phi}_{\bar{i}} \rangle, \phi_j] + \text{h.c.} \right) \right\} + \dots \right. \\ \left. \mathcal{W}(\phi_i) = \frac{1}{g^2} \varepsilon^{ijk} \text{Tr} \left[\sqrt{2} \left(\partial_i \phi_j - \frac{1}{\sqrt{2}} [\langle \phi_i \rangle, \phi_j] \right) \phi_k - \frac{2}{3} \phi_i \phi_j \phi_k \right] \right\} \end{cases}$$

$$\mathcal{L} = \sum_{\text{KK modes}} \int d^4\theta \mathcal{K} + \left\{ \int d^2\theta \left(\frac{1}{4g^2} \mathcal{W}^\alpha \mathcal{W}_\alpha + \mathcal{W} \right) + \text{h.c.} \right\}$$

Some features

- This theory contains **infinite number of fields**.
 → Find the massless particles. (**Zero mode equations**)
- We will not care about Lorentz invariance on the extra dimension.
 → Assign the VEV $\langle A_m \rangle$ along the extra dimensions.



Break the gauge symmetry $U(N) \rightarrow U(n_1) \times U(n_2) \times \dots$.

We assume that “the lightest field is massless.” This assumption implies

$$\partial_i f_j^{(i)} - \frac{1}{2} \left[\langle A_i \rangle, f_j^{(i)} \right] = 0$$

where $\phi_j(x^\mu, z, \bar{z}) \equiv f_j^{(1)}(z_1, \bar{z}_1) f_j^{(2)}(z_2, \bar{z}_2) f_j^{(3)}(z_3, \bar{z}_3) \phi_j^{(0)}(x^\mu)$

for each torus, and we assign the VEV along the extra dimensions as

$$\langle A_i \rangle = \frac{\pi}{\text{Im } \tau_i} (M^{(i)} \bar{z}_i + \bar{\zeta}).$$

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for each torus, and we assign the VEV along the extra dimensions as

$$\langle A_i \rangle = \frac{\pi}{\text{Im } \tau_i} (M^{(i)} \bar{z}_i + \bar{\zeta}).$$

The “massless condition” is rewritten as

$$\left[\partial_i - \frac{\pi}{2\text{Im } \tau_i} (M^{(i)} \bar{z}_i + \bar{\zeta}^{(i)}) \right] f_j^{(i)} = 0 \quad \text{Zero mode equations}$$

Zero mode equation(s)

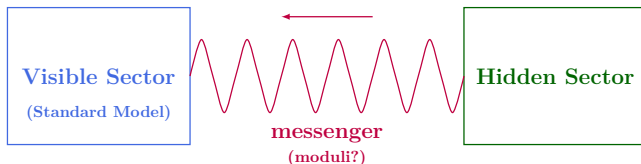
$$\left[\partial_i - \frac{\pi}{2\text{Im } \tau_i} (M^{(i)} \bar{z}_i + \bar{\zeta}^{(i)}) \right] f_j^{(i)} = 0$$

- The degeneracy of the solution $f_j^{(i)}$ depend on the backgrounds $M^{(i)}$ and ζ^i , and the orbifold conditions.
- The degeneracy corresponds to the *flavor* on the Standard Model.
- Once the solutions are obtained, we can evaluate the Yukawa matrix (Higgs & Quark coupling) and CKM matrix (mixing of flavors).
→ These facts are related to Chin-san's and Shimada-kun's works.

Hidden sector

From now on, we will consider the **hidden sector**, that is decoupled from the visible (or SM) sector.

- If supersymmetry is broken in the visible sector, some phenomenological problems will arise:
 - ▶ It is difficult to give gauginos mass.
 - ▶ Squarks and sleptons will become too light, so that they will already be observed.
- Thus, we think that the supersymmetry breaking occurs in a "hidden sector" of particles.



We assume that the gauge group is broken as

$U(N+2) \rightarrow U(1)_X \times U(N) \times U(1)_Y$ due to the flux

$$M^{(1)} = \left(\begin{array}{c|cc} M_X^{(1)} & & \\ \hline & M_N^{(1)} \times \mathbf{1}_N & \\ & & M_Y^{(1)} \end{array} \right) \begin{array}{l} X \\ N \\ Y \end{array}$$

$$M^{(2)} = \left(\begin{array}{c|cc} H & & \\ \hline & M_N^{(2)} \times \mathbf{1}_N & \\ & & M_Y^{(2)} \end{array} \right) \begin{array}{l} X \\ N \\ Y \end{array}$$

$$M^{(3)} = \left(\begin{array}{c|cc} M_X^{(3)} & & \\ \hline & H \times \mathbf{1}_N & \\ & & H \end{array} \right) \begin{array}{l} X \\ N \\ Y \end{array}$$

These flux determines the particles existing in its effective 4D theory.

There are two kinds of potentials in this theory, F -term potential and D -term potential, but the former one is already vanished by the VEV $\langle A_i \rangle$ assignment, at least perturbatively.

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The remaining one is

$$\begin{aligned} V_D = & \frac{1}{2} \left(\frac{M_X^{(1)}}{\mathcal{A}^{(1)}} + \frac{M_X^{(3)}}{\mathcal{A}^{(3)}} + (\text{matter}) \right)^2 \\ & + \frac{1}{2} \left(\frac{M_Y^{(1)}}{\mathcal{A}^{(1)}} + \frac{M_Y^{(2)}}{\mathcal{A}^{(2)}} + (\text{matter}) \right)^2 \\ & + \frac{1}{2} \left(\frac{M_N^{(1)}}{\mathcal{A}^{(1)}} + \frac{M_N^{(2)}}{\mathcal{A}^{(2)}} + \frac{M_N^{(3)}}{\mathcal{A}^{(3)}} + (\text{matter}) \right)^2 . \end{aligned}$$

Example: fluxless model

Before introducing flux, we investigate the model without the magnetic flux.

The model has only three-particles, Q, \tilde{Q}, X :

$$\Phi^{(1)} = \left(\begin{array}{c|c} & \\ \hline & \\ \hline & Q \\ \hline \end{array} \right) \begin{array}{l} X \\ N \\ Y \end{array}$$

$$\Phi^{(2)} = \left(\begin{array}{c|c} & X \\ \hline & \\ \hline & \\ \hline \end{array} \right) \begin{array}{l} X \\ N \\ Y \end{array}$$

$$\Phi^{(3)} = \left(\begin{array}{c|c} & \\ \hline \tilde{Q} & \\ \hline & \\ \hline \end{array} \right) \begin{array}{l} X \\ N \\ Y \end{array}$$

Both fermions Q and \tilde{Q} couple with the gauge group $SU(N)$ and they can be confined in the low energy theory, like QCD.

To prove the above phenomena, we evaluate **realization group (RG) equation** for gauge coupling g . It is known that the **β -function**

The **β -function** for $SU(N)$ Yang-Mills theory, with n_f Weyl fermions and n_s complex scalar fields:

$$\beta(g) = -\frac{g^3}{(4\pi)^2} \left(\frac{11}{3}C(G) - \frac{2}{3}n_f C(R_f) - \frac{1}{3}n_s C(R_s) \right)$$

In this case,

$$C(SU(N)) = N, \quad n_f C(R_f) = 2N \quad \text{then} \quad \beta(g) = -\frac{7N}{48\pi^2}g$$

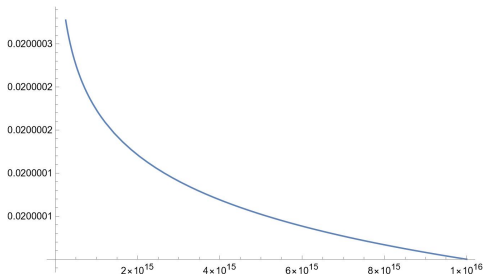
The RG equation is

$$\frac{d}{d \ln \mu / \mu_0} g = -\frac{7N}{48\pi^2} g, \quad g(\mu_0) = g_0$$

where the 2nd relation is the “initial condition” explained later.

The solution is

$$g(\mu) = \frac{g_0}{[1 + (7Ng_0^3/48\pi^2) \ln \mu/\mu_0]^{1/2}}$$



Rough plot of the solution.

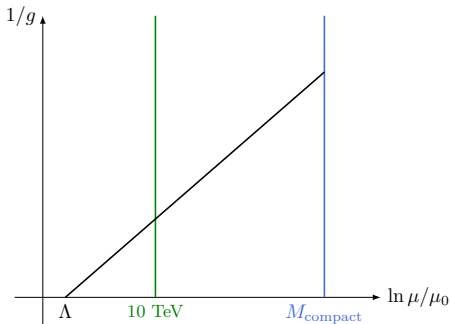
(Vertical line: $g(\mu)$, Horizontal line: μ)

$$g(\mu) = \frac{g_0}{[1 + (7Ng_0^3/48\pi^2) \ln \mu/\mu_0]^{1/2}}$$

From the solution, we can evaluate the confinement scale Λ :

$$\Lambda = \mu_0 \exp \left[-\frac{1}{7Ng_0^3/48\pi^2} \right].$$

If $N = 2$, $g_0 = 1/50$, $\mu_0 = 10^{16}$ GeV, then $\Lambda \sim 10^{-10^5} \sim 0$.



Some notes

- I choose the parameter $N = 2, g_0 = 1/50, \mu_0 = 10^{16}$ GeV by referring the Nakano-san's thesis.
- This result contradict with our assumption that the fermions are confined and construct the meson $M = \text{tr } \tilde{Q}Q$. (I guessed that we assume it for F -term SUSY breaking, but I do not know why actually.)
- To obtain a desirable result, we should tune the initial gauge coupling g_0 , since other parameters change Λ very little. If Λ is around 10 TeV, the coupling should be lower than $g \sim 1.57$ at the compactified scale $\mu = 10^{16}$ GeV, with $SU(2)$.

I would like to start a discussion on introducing magnetic flux.
The flux $M_A^{(i)}$ should satisfy a *D-flat condition*, but ·····.

Discussion (?)

There are at least two ways to compute the *D-flat condition*:

1. Find potential vanishing condition naively:

$$\sum_{i=1,3} \frac{M_X^{(i)}}{\mathcal{A}^{(i)}} = \sum_{i=1,2} \frac{M_Y^{(i)}}{\mathcal{A}^{(i)}} = \sum_{i=1,2,3} \frac{M_N^{(i)}}{\mathcal{A}^{(i)}} = 0$$

2. Substitute the area of tori $\mathcal{A}^{(i)} \rightarrow T^i$ and then evaluate $\partial_i V_D = 0$.

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2. Substitute the area of tori $\mathcal{A}^{(i)} \rightarrow T^i$ and then evaluate $\partial_i V_D = 0$.

I guess the 2nd way is plausible, but

it conclude that we cannot assign the flux in this setup!

(The word “plausible” means that the first orders of the fluctuation δT^i vanish if we assume the 2nd way, rather than vanishing the zeroth order in the 1st way. Non vanishing first order terms occurs the shifts of the VEV $\langle T^i \rangle$.)

In $D7$ -brane model, the relation between Kähler moduli and the area of the torus is

$$T_i = \mathcal{A}^{(j)} \mathcal{A}^{(k)} \longrightarrow \mathcal{A}^{(i)} = \sqrt{\frac{T_j T_k}{T_i}}$$

$$\Downarrow$$

$$V_D = \frac{1}{2} (V_X + (\text{matter}))^2 + \frac{1}{2} (V_Y + (\text{matter}))^2$$

$$V_X \equiv M_X^{(1)} \sqrt{\frac{T_1}{T_2 T_3}} + M_X^{(3)} \sqrt{\frac{T_3}{T_1 T_2}}$$

$$V_Y \equiv M_Y^{(1)} \sqrt{\frac{T_1}{T_2 T_3}} + M_Y^{(2)} \sqrt{\frac{T_2}{T_1 T_3}}$$

A D -flat condition should be

$$\partial_{T_i} V_D = 0 \quad \text{for } i = 1, 2, 3$$



$$\left\{ \begin{array}{l} M_X^{(1)} T_1 - M_X^{(3)} T_3 = 0, \\ M_X^{(1)} T_1 + M_X^{(3)} T_3 = 0, \\ M_Y^{(1)} T_1 - M_Y^{(2)} T_3 = 0, \\ M_Y^{(1)} T_1 - M_Y^{(2)} T_2 = 0, \\ M_Y^{(1)} T_1 + M_Y^{(2)} T_2 = 0. \end{array} \right.$$

→ all flux are zero:

$$M_X^{(1)} = M_X^{(3)} = M_Y^{(1)} = M_Y^{(2)} = 0$$

1. Find potential vanishing condition naively:

$$\sum_{i=1,3} \frac{M_X^{(i)}}{\mathcal{A}^{(i)}} = \sum_{i=1,2} \frac{M_Y^{(i)}}{\mathcal{A}^{(i)}} = \sum_{i=1,2,3} \frac{M_N^{(i)}}{\mathcal{A}^{(i)}} = 0$$



2. Substitute the area of tori $\mathcal{A}^{(i)} \rightarrow T^i$ and then evaluate $\partial_i V_D = 0$.

Thus, to introduce the flux, we may need to take the 1st way (?)

$$\clubsuit \Rightarrow \begin{cases} \frac{\mathcal{A}^{(3)}}{\mathcal{A}^{(1)}} = -\frac{M_X^{(3)}}{M_X^{(1)}} \\ \frac{\mathcal{A}^{(2)}}{\mathcal{A}^{(1)}} = -\frac{M_Y^{(2)}}{M_Y^{(1)}} \\ M_N^{(1)} M_X^{(3)} M_Y^{(2)} - M_N^{(2)} M_X^{(3)} M_Y^{(1)} - M_N^{(3)} M_X^{(1)} M_Y^{(2)} = 0 \end{cases}$$

The ratio of the torus is determined from the visible sector.

Break the gauge group $U(8) \rightarrow U(3)_C \times U(1)_l \times U(2)_L \times U(2)_R$

$$M^{(1)} = \left(\begin{array}{c|c} m_c^{(1)} \times \mathbf{1}_3 & \\ \hline & m_l^{(1)} \\ \hline & m_L^{(1)} \times \mathbf{1}_2 \\ & m_R^{(1)} \times \mathbf{1}_2 \end{array} \right)$$

$$M^{(2)} = \left(\begin{array}{c|c} H \times \mathbf{1}_3 & \\ \hline & H \\ \hline & m_L^{(2)} \times \mathbf{1}_2 \\ & m_R^{(2)} \times \mathbf{1}_2 \end{array} \right)$$

$$M^{(3)} = \left(\begin{array}{c|c} m_c^{(3)} \times \mathbf{1}_3 & \\ \hline & m_l^{(3)} \\ \hline & H \times \mathbf{1}_2 \\ & H \times \mathbf{1}_2 \end{array} \right)$$

with Wilson line, and orbifolds $\mathbb{Z}_2 \times \mathbb{Z}'_2$.

There are five models $(m_c^{(i)}, m_l^{(i)}, m_L^{(i)}, m_R^{(i)})$ that have successfully realized the Yukawa matrix and CKM matrix to some extent.

For example,

$$\frac{\mathcal{A}^{(2)}}{\mathcal{A}^{(1)}} = \frac{1}{10}, \quad \frac{\mathcal{A}^{(3)}}{\mathcal{A}^{(1)}} = 1.$$

Thus, we should choose flux to satisfy

$$\frac{M_X^{(3)}}{M_X^{(1)}} = -1, \quad \frac{M_Y^{(2)}}{M_Y^{(1)}} = -\frac{1}{10},$$
$$M_N^{(1)} M_X^{(3)} M_Y^{(2)} - M_N^{(2)} M_X^{(3)} M_Y^{(1)} - M_N^{(3)} M_X^{(1)} M_Y^{(2)} = 0$$



Future works!

Summary

Summary

- I study the hidden sector of the magnetized D7-brane model.
- If the flux are zero, we need *artificial* tune for the initial parameter of the RG equation, for the consistency.
- If we rewrite a D -term potential in terms of Kähler moduli T^i , it conclude that we can *not* introduce a flux.

Future Works

- Which conditions we should impose for the D -flat condition?
- Determine the flux which consistent with the visible sector.
- Identify particles which are created by the flux and study the potential.

Appendix

Questions

I list them below:

- Why particles do not appear symmetrically in the flux model?
→ Next page.
- If matters are created, its theory becomes far from *asymptotic free* in general. Thus, the flux-less is the best for the confinement?

$$\Phi^{(1)} = \left(\begin{array}{c|c} & \\ \hline & ? \\ \hline & Q \end{array} \right) \begin{array}{l} X \\ N \\ Y \end{array}$$

$$\Phi^{(2)} = \left(\begin{array}{c|c} & X \\ \hline & \\ \hline ? & \end{array} \right) \begin{array}{l} X \\ N \\ Y \end{array}$$

$$\Phi^{(3)} = \left(\begin{array}{c|c} & ? \\ \hline \tilde{Q} & \end{array} \right) \begin{array}{l} X \\ N \\ Y \end{array}$$

References

- [1] H. Abe, T. Kobayashi, H. Ohki, and K. Sumita, *Superfield description of 10D SYM theory with magnetized extra dimensions*, **Nucl. Phys. B** **863** (2012) 1–18, [arXiv:1204.5327 \[hep-th\]](#).
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