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SEQUENTIAL SPARSE BAYESIAN LEARNING FOR DOA

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ABSTRACT

Sparse Bayesian learning (SBL) can effectively and accurately solve the direction-of-arrival (DOA) estimation problem. In this paper, we introduce a sequential SBL method for time-varying DOA estimation. Statistical information provided from previous time steps is modeled by a zero-mean multivariate Gaussian that is characterized by variance parameters. The presented method propagates statistical information across time by means of a prediction and an update step. The prediction step computes the prior distribution of current variance parameters from previous variance parameters and the update step incorporates current observations. A performance evaluation based on simulated and experimental data demonstrates that the proposed sequential SBL method can provide the capability of tracking time-varying sources with a high resolution.

Index Terms— Direction-of-arrival (DOA) estimation, Compressive beamforming, Sparse Bayesian learning

1. INTRODUCTION

Direction-of-arrival (DOA) estimation is the task of determining the direction of signals transmitted by several sources from noisy measurements provided by an array of sensors [1, 2, 3]. Sequential sparse DOA estimation represents a time-recursive DOA estimation by propagating information from the estimated sources of the previous time step to the next time step. Sparse Bayesian learning (SBL) shows high resolution sparse DOA estimation performance [4, 5, 6, 7, 8]. We propose an SBL-based scheme that has sequential processing to estimate time-varying DOAs.

In SBL [4, 9], the source signal is modeled as a random vector with Gaussian prior distribution with zero-mean and time-varying variance with DOA-dependent hyperparameter. [5, 6, 7, 8] Sequential processing is achieved by plugging statistical information related to the time-varying source signal from the previous time step in the next time step. For sequential SBL, we utilize a conventional sequential state-space model to propagate the statistical information. The suggested sequential SBL has two steps: prediction and update. In the prediction step, the DOA-dependent variance parameter is predicted based on the state-space model from the estimated variance of the previous time step. In the update step, the predicted variance is updated based on the current measurement by using the fixed point SBL update [5, 6, 7].

For single time step data processing where a time sequence of multiple measurements is available, sequential SBL has improved DOA performance over conventional beamforming (CBF) and non-sequential SBL. Also, the suggested scheme has no limitation to a specific array geometry. Here, we examine sequential SBL under real-world scenarios, and we apply it to experimental data with a uniformly configured array as well as a non-uniformly configured array.

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2. SYSTEM FRAMEWORK

At time $t = 1, 2, \dots,$ we consider K_t narrowband far-field sources with complex signal amplitude $s_{t,k} \in \mathbb{C}$, $k = 1, \dots, K_t$, arriving at an array of M sensors from DOAs $\theta_{t,k} \in \Theta \triangleq [-90°, 90°)$, $k = 1, \dots, K_t$. The sensors form a linear array, the sources are assumed to be in the same plane as the array, and plane waves arrive at the array with DOAs with respect to the array axis. The observation $\mathbf{y}_t \in \mathbb{C}^M$ at time t is modeled as

$$y_{t,m} = \sum_{k=1}^{K_t} a_m(\theta_{t,k}) s_{t,k} + v_{t,m}, \;\; m = 1, \dots, M,$$

where $\mathbf{y}_t = [y_{t,1} \dots y_{t,M}]^\mathsf{T}$ is the vector of recorded by the M sensor array; $\mathbf{s}_t = [s_{t,1} \dots s_{t,K_t}]^\mathsf{T} \in \mathbb{C}^{K_t}$ is the vector of K_t source amplitudes; $a_m(\theta_{t,k})$ is the steering vector that contains DOA of the kth source on the array and has phase delay information of the kth source to the mth sensor in its phase; $\boldsymbol{\theta}_t = \{\theta_{t,1}, \dots, \theta_{t,K_t}\}$ is a set with K_t elements whose elements are kth source DOAs; and $\mathbf{v}_t = [v_{t,1} \dots v_{t,M}]^\mathsf{T} \in \mathbb{C}^M$ is the vector of measurement noise at the M sensors, and $\mathbf{v}_t$ have statistically-independent, circular-symmetric, and zero-mean Gaussian entries with variance σ_t^2 and are assumed statistically independent across time t. The steering vector $\mathbf{a}(\theta_{t,k}) \in \mathbb{C}^M$ is given by

$$\mathbf{a}(\theta_{t,k}) = \left[1 \; e^{-j\frac{2\pi}{\lambda} d_2 \sin\theta_{t,k}} \dots e^{-j\frac{2\pi}{\lambda} d_M \sin\theta_{t,k}}\right]^\mathsf{T}$$

where λ is the signal wavelength and d_m is the distance from sensor 1 to sensor m.

The DOA estimation is formulated into the linear system by introducing an angular search-grid of $N \gg K_t$ potential source DOAs $\bar{\boldsymbol{\theta}} = [\bar{\theta}_1 \dots \bar{\theta}_N]^\mathsf{T} \in \Theta^N$ and corresponding amplitude vector $\mathbf{x}_t \in \mathbb{C}^N$. The dictionary consists of the steering vectors as the angular search-grid, i.e., $\mathbf{A} = \left[\mathbf{a}(\bar{\theta}_1) \dots \mathbf{a}(\bar{\theta}_N)\right] \in \mathbb{C}^{M \times N}$, and the resulting DOA estimation is obtained as

$$\mathbf{y}_t = \mathbf{A}\mathbf{x}_t + \mathbf{v}_t. \tag{1}$$

In sparse signal recovery, $\mathbf{x}_t$ is assumed sparse with K_t non-zero entries ($K_t \ll N$) and the goal is to recover the K_t-sparse vector $\mathbf{x}_t$ and the corresponding K_t columns of $\mathbf{A}$ that best express the measurement $\mathbf{y}_t$.

In the SBL framework [4, 5], the source amplitudes $\mathbf{x}_t$ is assumed multi-variate Gaussian with zero-mean and variance with DOA-dependent hyperparameter $\gamma_{t,n}$, $n = 1, \dots, N$,

$$p(x_{t,n}; \gamma_{t,n}) = \begin{cases} \delta(x_{t,n}), & \text{for } \gamma_{t,n} = 0, \\ \frac{1}{\pi \gamma_{t,n}} e^{-|x_{t,n}|^2/\gamma_{t,n}}, & \text{for } \gamma_{t,n} > 0, \end{cases}$$

$$p(\mathbf{x}_t; \boldsymbol{\gamma}_t) = \mathcal{CN}(\mathbf{x}_t; \mathbf{0}, \boldsymbol{\Gamma}_t),$$

where the unknown covariance matrix $\boldsymbol{\Gamma}_t \in \mathbb{R}^{N \times N}$ is assumed to be diagonal, i.e., $\boldsymbol{\Gamma}_t = \mathrm{diag}(\boldsymbol{\gamma}_t)$ and the entries of $\boldsymbol{\gamma}_t$ are assumed statistically independent across time t.

The time-varying source amplitudes are modeled by a linear state-transition model, i.e.,

$$\mathbf{x}_t = \mathbf{F}\mathbf{x}_{t-1} + \mathbf{w}_t. \tag{2}$$

Here, $\mathbf{F} \in \mathbb{C}^{N \times N}$ is a known matrix relating the state vector at time t to that at time $t-1$ and $\mathbf{w}_t \sim \mathcal{CN}\left(\mathbf{w}_t; \mathbf{0}, \sigma_{w_t}^2 \mathbf{I}_N\right)$ is Gaussian process noise assumed statistically independent across time t. For dynamic sources, $\mathbf{x}_t$ is likely to have non-zero entries in the vicinity of the non-zero entries of $\mathbf{x}_{t-1}$. Thus, a banded symmetric Toeplitz matrix is a suitable choice for matrix $\mathbf{F}$ (see [10] for details).

For the likelihood, assuming the noise $\mathbf{v}_t$ in (1) has statistically-independent, circular-symmetric, and zero-mean Gaussian entries with variance σ_t^2, the data likelihood for the source $\mathbf{x}_t$ given the measurement $\mathbf{y}_t$ is modeled, i.e.,

$$p\big(\mathbf{y}_t | \mathbf{x}_t; \sigma_t^2\big) = \mathcal{CN}\left(\mathbf{y}_t; \mathbf{A}\mathbf{x}_t, \sigma_t^2 \mathbf{I}_M\right). \tag{3}$$

3. SPARSE BAYESIAN LEARNING WITH SEQUENTIAL PROCESSING

The proposed sequential SBL approach predicts variances $\hat{\boldsymbol{\gamma}}_{t|t-1}$ based on the state-transition model (2) from the variances $\hat{\boldsymbol{\gamma}}_{t-1}$ estimated at time $t-1$. Then the variance $\hat{\boldsymbol{\gamma}}_t$ at time t is updated by incorporating the current measurement $\mathbf{y}_t$ with fixed point SBL update.

Let us assume at time $t-1$, we have estimated the variances $\hat{\boldsymbol{\gamma}}_{t-1}$ at time $t-1$ from all previous measurement vectors $\mathbf{y}_{1:t-1}$, which denotes the ordered sequence $(\mathbf{y}_1 \dots \mathbf{y}_{t-1})$. The corresponding approximation of the marginal distribution $p(\mathbf{x}_{t-1}|\mathbf{y}_{1:t-1})$ is given by

$$\tilde{p}(\mathbf{x}_{t-1}|\mathbf{y}_{1:t-1}) = \mathcal{CN}(\mathbf{x}_{t-1}; \mathbf{0}, \hat{\boldsymbol{\Gamma}}_{t-1}),$$

where $\hat{\boldsymbol{\Gamma}}_{t-1} = \mathrm{diag}(\hat{\boldsymbol{\gamma}}_{t-1})$. In the prediction step, we predict $\hat{\boldsymbol{\Gamma}}_{t|t-1}$ with the predicted posterior

$$\tilde{p}(\mathbf{x}_t|\mathbf{y}_{1:t-1}) = \mathcal{CN}(\mathbf{x}_t; \mathbf{0}, \hat{\boldsymbol{\Gamma}}_{t|t-1}) \tag{4}$$

based on the state-transition model (2). Since both $\mathbf{x}_{t-1}$ and $\mathbf{w}_t$ are complex Gaussians, $\hat{\boldsymbol{\Gamma}}_{t|t-1}$ is given by

$$\hat{\boldsymbol{\Gamma}}_{t|t-1} = \mathbf{F}\hat{\boldsymbol{\Gamma}}_{t-1}\mathbf{F}^\mathsf{T} + \sigma_{w_t}^2 \mathbf{I}_N. \tag{5}$$

In the update step, the predicted variances $\hat{\boldsymbol{\gamma}}_{t|t-1} = \left[\hat{\Gamma}_{t|t-1,11} \cdots \hat{\Gamma}_{t|t-1,NN}\right]^\mathsf{T}$ are updated by incorporating $\mathbf{y}_t$ with fixed point SBL update, and results in the variance $\hat{\boldsymbol{\gamma}}_t$ at time t. In the SBL framework, the likelihood (3) and the prior (4) with (5) results in the fixed point update rules, with the iterative updates [5, 6, 7]

$$\gamma_{t,n}^{\text{new}} = \gamma_{t,n}^{\text{old}} \frac{\|\mathbf{y}_t^\mathsf{H} \boldsymbol{\Sigma}_{\mathbf{y}_t}^{-1} \mathbf{a}_n\|_2^2}{\mathbf{a}_n^\mathsf{H} \boldsymbol{\Sigma}_{\mathbf{y}_t}^{-1} \mathbf{a}_n},$$

where $\boldsymbol{\Sigma}_{\mathbf{y}_t}$ is the data covariance, given as

$$\boldsymbol{\Sigma}_{\mathbf{y}_t} = \mathbf{A}\boldsymbol{\Gamma}_t^{\text{old}}\mathbf{A}^\mathsf{H} + \sigma_t^2 \mathbf{I}_M,$$

and

$$\{\sigma_t^2\}^{\text{new}} = \frac{\mathrm{tr}\left[\left(\mathbf{I}_M - \mathbf{A}_\mathcal{M}\mathbf{A}_\mathcal{M}^+\right)\mathbf{y}_t\mathbf{y}_t^\mathsf{H}\right]}{M - K_t}.$$

Here, $\mathcal{M}$ is a set of K_t active DOAs; $\mathbf{A}_\mathcal{M}$ is the corresponding active steering matrix; and $\mathbf{A}_\mathcal{M}^+$ denotes the Moore-Penrose pseudo-inverse. From the resulting variance $\hat{\boldsymbol{\gamma}}_t$, the approximation of the marginal distribution $p(\mathbf{x}_t|\mathbf{y}_{1:t})$ is given by

$$\tilde{p}(\mathbf{x}_t|\mathbf{y}_{1:t}) = \mathcal{CN}(\mathbf{x}_t; \mathbf{0}, \hat{\boldsymbol{\Gamma}}_t)$$

with $\hat{\boldsymbol{\Gamma}}_t = \mathrm{diag}(\hat{\boldsymbol{\gamma}}_t)$, which is used in the prediction step at time $t+1$.

4. SIMULATION RESULTS

Simulation data involve a uniform linear array (ULA) with $M = 15$ sensors with half-wavelength sensor spacing, and 50 time steps are observed. The angular search-grid is discretized using a grid size $N = 361$ with 0.5° spacing, i.e., the potential DOAs $\bar{\boldsymbol{\theta}} = [-90 - 89.5 \dots 90]^\mathsf{T} \in \Theta^N$. We use the signal-to-noise ratio (SNR) definition in Ref. [11] and simulate SNR $= 20$ dB case. We simulate six stationary sources at DOAs $[-70, -55, -3, 2, 50, 65]^\circ$ in Figs. 2(a) and 2(b) and sources 3 and 4 moving in Figs. 2(c) and 2(d). The source

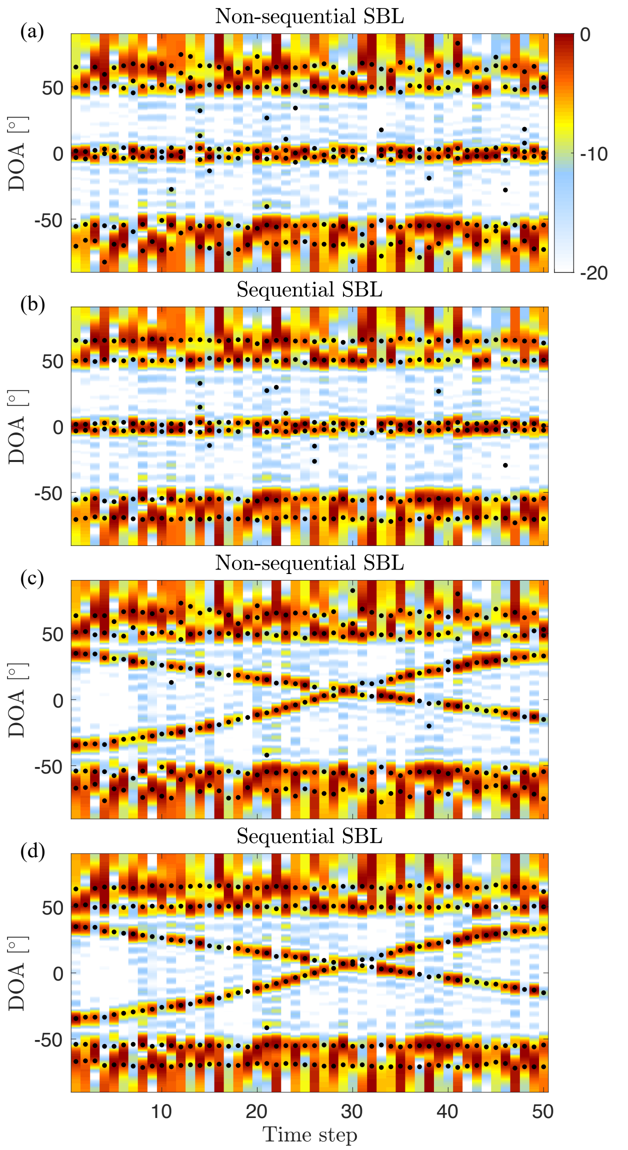

Fig. 1. DOA estimation versus time steps with stationary sources for (a) non-sequential SBL and (b) sequential SBL and with moving sources for (c) non-sequential SBL and (d) sequential SBL. DOA estimates are shown as black dots. The CBF solution is shown in the background and the solution at each time step is normalized by the maximum value of the estimated source strength.

magnitude varies $[0, 20]$ dB for each source. For sequential SBL, we used a banded symmetric Toeplitz $\mathbf{F}$ in (2) based on a rectangular 2-lags blurring window $[.2\ .2\ .2\ .2\ .2]^{\mathsf{T}}$. This matrix $\mathbf{F}$ takes source motion and measurement noise into account and is discussed in [10].

We compare sequential SBL with non-sequential SBL [6] together with CBF in the background, Fig. 1. The proposed sequential SBL can localize accurately stationary as well as moving sources, and shows high resolution near endfire of the array.

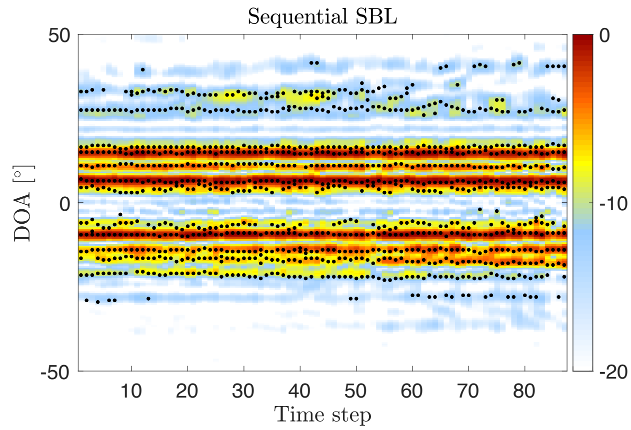

Fig. 2. DOA estimation versus time steps using SWellEx-96 data for sequential SBL. (a deep source) The CBF solution is shown in the background and the SBL solution is shown as black dots.

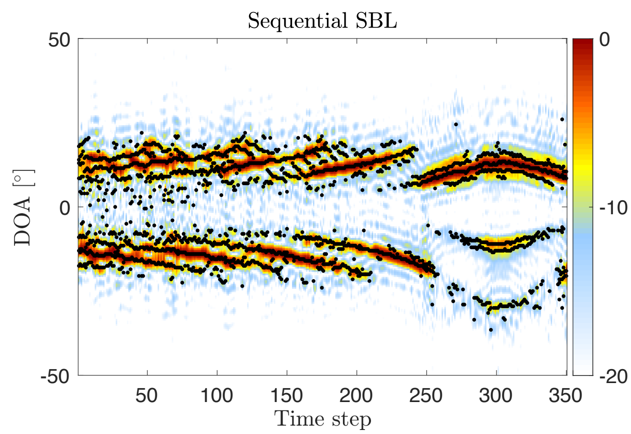

Fig. 3. Same as in Fig. 2, but for a shallow source.

5. EXPERIMENTAL RESULTS

We present the DOA estimation results of the suggested sequential SBL applied to the experimental data, the shallow water evaluation cell experiment 1996 (SWellEx-96) [12, 13]. A ULA with $M = 64$ sensors with a sensor spacing of 1.875 m at a water depth 94.125–212.25 m recorded two sources, a shallow and a deep. For the detailed description and discussions of the experiment, see Ref. [11]. We focus on the signal component of the deep source at a frequency of 235 Hz, Fig. 2. The dataset has a duration of 1.5 min (0.5 min before and 1 min after the closest point of approach (CPA)) sampled at 1500 Hz, and 87 time step measurements are obtained by the Fourier transform with 2^{12} samples (2.7 s duration) and 63 % overlap.

DOA estimation results with CBF and sequential SBL show the presence of multipath arrivals, which have almost stationary DOAs across time. CBF has low resolution and artifacts due to sidelobes and noise. Sequential SBL shows

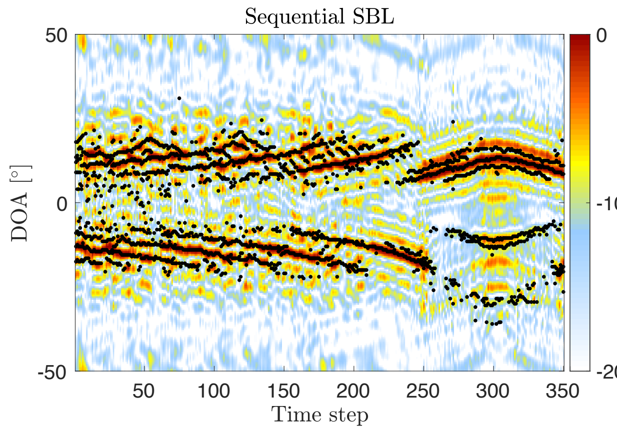

Fig. 4. Same as in Fig. 3, but with non-uniformly-spaced 32-element data. The array aperture is the same as that of the full 64-sensor array.

improved resolution by enforcing sparsity and can estimate stationary DOAs along with the time steps. We also focus on the signal component of the shallow source at a frequency of 232 Hz, Figs. 3 and 4. The recording from 23:19–00:22 on 11 May 1996 is split into 350 time step measurements, which are obtained by the Fourier transform with 2^{13} samples (5.4 s duration) without overlap.

DOA estimation results with CBF and sequential SBL show the presence of multipath arrivals, which show dynamic DOA structure. Compared to CBF, which suffers from low resolution, sequential SBL results in improved source tracking capability. The suggested sequential SBL does not require a specific array geometry, e.g., ULA. We process 32-element non-uniformly configured array data (elements 1, 3, 8, 9, 13, 14, 15, 17, 18, 20, 23, 26, 30, 31, 33, 34, 35, 38, 40, 41, 42, 44, 45, 47, 49, 51, 52, 55, 56, 57, 63, 64). Sequential SBL still can estimate time-varying DOAs with high resolution.

6. CONCLUSIONS

We presented a sequential sparse Bayesian learning method for the estimation of time-varying directions-of-arrival (DOA). The performance of the new method was evaluated based on simulated as well as experimental data. Our results showed that the proposed sequential SBL can provide the capability of tracking time-varying sources with a high resolution. A compelling feature of the presented method is that it is not limited to a particular array geometry. Possible directions for future research include a combination of the considered method with Bayesian multiobject tracking [14, 15].

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