Microwave Ranging via Least-Squares Estimation of Spectrally Sparse Signals in Software-Defined Radio

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***Abstract*— We present a least-squares algorithm for time delay (range) estimation of dual-tone spectrally sparse signals that minimizes bias errors. Dual-tone waveforms obtain near-optimal delay estimation performance by maximizing the mean-square bandwidth of the signal spectra, reducing the error bound. However, the choice of estimator may introduce bias, particularly for dual-tone waveforms with bandwidth (tone separation) that is small or is close to the Nyquist rate, and when the delay yields discretization errors. We address this problem by combining a matched filter with least-squares (MF-LS) optimization. We compare this with a simple matched filter and interpolation approach, and with a matched filter and sinc-function nonlinear least squares (NL-LS) fit. We demonstrate that the MF-LS algorithm has lower bias errors than interpolation and NL-LS over both bandwidth and delay. We present experimental 2.8-GHz measurements of two-tone delay estimation implemented in a software-defined radio and demonstrate that the MF-LS algorithm achieves a reduction in root-mean-square error (RMSE) of nearly an order of magnitude compared with interpolation or NL-LS.**

***Index Terms*— Microwave ranging, pulse compression, radar, spectral sparsity.**

I. Introduction

ESTIMATION of the time delay of a signal is necessary in a broad range of applications, including radar [1] and sonar target detection [2], localization in wireless networks [3], indoor tracking of people [4], and many others. Emerging applications in wireless networks and distributed wireless systems are driving an increased need for more accurate range measurements obtained through estimating the delay of a transmitted signal. New developments in coherent distributed array technologies necessitate range estimates between nodes with errors of a fraction of a wavelength using techniques that can scale to an arbitrary number of nodes [5], [6]. The most common waveforms used for such high-accuracy range estimation are linear frequency-modulated, ultra-wideband, pseudo-noise coded waveforms, and phase-modulated continuous wave, among others [7]–[9]. However, two-tone waveforms obtain the best ranging accuracy [10], [11] and are spectrally sparse, which, in general, minimizes interference. Two-tone waveforms are not generally used for non-cooperative ranging due to the highly ambiguous nature of the matched filter response. However, it is a perfect candidate for cooperative ranging, where an active retransmission can yield a strong point-like response [12].

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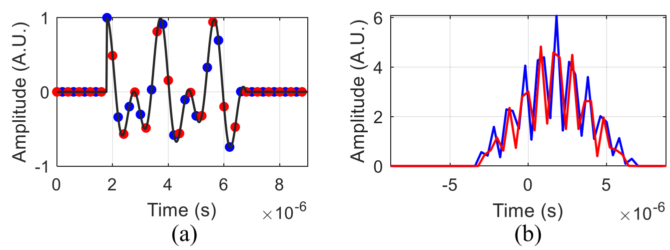

Fig. 1. (a) In-phase part of a two-tone signal with $f_1 = 53$ kHz, $f_2 = 1.03$ MHz, and a sampling frequency of $f_s = 2.5$ MHz showing discretization equivalent to two different delays (red and blue). (b) Output of the matched filter for both scenarios, showing the error due to discretization.

The traditional approach to estimating the delay of a signal is via cross correlation processing of the delayed signal with an ideal representation of the signal, or matched filtering, and estimating the time of the peak of the output signal. Matched filtering with the ideal signal realizes the maximum signal-to-noise ratio (SNR), improving the estimation accuracy. The lower bound on delay estimation accuracy is affected by multiple factors including bandwidth, SNR, and the spectral shape of the waveform [10]; however, the realized estimation accuracy is further impacted by the estimation algorithm which cannot always obtain the lower bound. Signal discretization introduces additional errors in the estimation of the matched filter peak, as illustrated in Fig. 1: depending on the delay, the discretized filter output may truncate the signal peak, leading to increased variance and bias.

We present a novel microwave ranging technique that focuses on reducing the ranging biases for high-accuracy two-tone waveforms. The new approach uses the residual sum of squares (RSS) which is obtained by comparing the calculated matched filter output with a filter bank of expected outputs. By minimizing the least-square error, the range of the target is estimated. We compare the ranging performance of the new matched filter least-squares (MF-LS) ranging technique with the performance of two commonly used approaches: interpolation and nonlinear least squares (NL-LS) sinc fitting. Through simulation and experiments in Ettus USRP X310 software-defined radios (SDRs) at 2.8 GHz, we demonstrate a reduction in range estimation bias error of nearly an order of magnitude using MF-LS compared with interpolation or NL-LS.

II. Delay Estimation of Dual-Tone Waveforms

Estimating the range of a cooperative target is equivalent to the estimate of the delay of a signal transmitted to and reflected back from the target, which is obtained by the peak of the response of the matched filter output

$$y[n] = r[n] \circledast s^*[-n] = \mathrm{IFT}\big\{R[k]S^*[k]\big\} \tag{1}$$

where $r[n]$ is the received baseband signal, $s[n]$ is the transmitted baseband signal, $R[k]$ and $S[k]$ are the discrete Fourier transforms of $r[n]$ and $s[n]$, respectively, $\circledast$ represents convolution, $\langle^*\rangle$ represents complex conjugate, and $\mathrm{IFT}\{\cdot\}$ is the inverse Fourier transform. Accurate peak estimation from a sampled received signal can be challenging due to discretization error. Conventionally, a refinement step is used to improve the estimation accuracy. Typical refinement approaches include interpolation, NL-LS, quadratic least squares, and spectral phase slope, among other methods [13]–[16]. Of these, interpolation and NL-LS often provide the best performance and the lowest bias for conventional ranging waveforms.

A waveform of two discrete frequency tones yields the lowest delay estimation variance for a given bandwidth [10]. In this work, the transmitted and noiseless received signals are given by

$$s[n] = (u[n] - u[n - n_T + 1])\left(e^{j2\pi f_1 \frac{n}{f_s}} + e^{j2\pi f_2 \frac{n}{f_s}}\right) \tag{2}$$

$$r[n] = (u[n - \lceil \tau f_s \rceil] - u[n - \lfloor \tau f_s \rfloor - n_T + 1]) \times \left(e^{j2\pi f_1 \left(\frac{n}{f_s} - \tau\right)} + e^{j2\pi f_2 \left(\frac{n}{f_s} - \tau\right)}\right) \tag{3}$$

where $u[\cdot]$ is the unit step function, n_T is the sample representing the end of the pulse with a pulsewidth $T = n_T/f_s$, f_1 and f_2 are the two tones of the transmitted and received signals, f_s is the sampling frequency, τ is the time delay, $\lceil . \rceil$ maps its argument to the least integer greater than or equal to its value, and $\lfloor . \rfloor$ maps its argument to the greatest integer less than or equal to its value. While matched filtering maximizes the SNR, it results in delay ambiguities for two-tone waveforms. These ambiguities are resolved in this work using the main peak disambiguation approach in [17], where a pulse consisting of one cycle of a sinusoidal wave is transmitted to disambiguate the output of the matched filter.

A. Estimation Using Interpolation

Initially, the received signal is discretized with the time interval $T_s = 1/f_s$. This limits the range estimation ability to a coarse grid of T_s, e.g., for a 10-MS/s sampling rate, $T_s =$ 100 ns, yielding a grid of 15-m increments. Refinement of the matched filter output can be accomplished by interpolating L samples preceding and following the initial (coarse) estimate of the discretized delay from the matched filter output peak n_{pk}, which is given by $\tau_c = n_{\mathrm{pk}} T_s$. Spline interpolation with K points between each two samples is appropriate since the underlying functions are sinusoidal. The output of a spline interpolation is a set of third-order polynomials with $2LK + 1$ samples. The refined delay estimate is obtained from the peak of the interpolated waveform at sample k_0. The refined value $\tau_r = k_0 T_s / K$ is added to the initial estimate τ_c to obtain the range of the target from $r = c(\tau_c + \tau_r)/2$. When the main lobe of the matched filter output is detected, L can be set to 1 to minimize the computation time. Nevertheless, this is not feasible if a rough estimate of the peak of the matched filter output is not available.

Algorithm 1 MF-LS

Input: $n_T,\ T,\ f_1,\ f_2,\ f_s,\ s[n],\ y[n],\ H$.
Output: τ_e.
Obtain the initial peak estimate n_{pk};
for $h = 1$ *to* H **do**
$dt_h = n_{pk} T_s - 0.5/(f_2 - f_1) + (h/H)/(f_2 - f_1)$;
$t_h[n] = -dt_h : 1/f_s : 2(n_T - 1)/f_s - dt_h$;
$\tilde{r}[n] = (u[t_h] - u[t_h - T])\left(e^{j2\pi f_1 t_h} + e^{j2\pi f_2 t_h}\right)$;
$\tilde{y}[n] = \tilde{r}[n] \circledast s^*[-n]$;
$RSS[h] = \sum_{n=0}^{N} (y[n]/\max(y) - \tilde{y}[n]/\max(\tilde{y}))^2$;
end
Using the interpolation algorithm, estimate h_{min}, the refined value that minimizes RSS;
Local time delay:
$dt_{loc} = -0.5/(f_2 - f_1) + (h_{min}/H)/(f_2 - f_1)$;
Estimate the time of arrival using: $\tau_e = n_{pk} T_s + dt_{loc}$;
Return τ_e

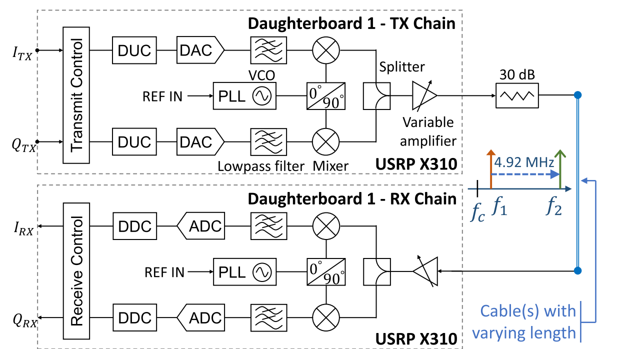

Fig. 2. Experimental SDR setup. Extra cables were added to modify the received signal time delay. DUC: digital upconverter. DDC: digital down-converter. DAC: digital-to-analog converter. ADC: analog-to-digital converter. VCO: voltage-controlled oscillator. PLL: phase-locked loop.

B. Estimation Using Sinc Nonlinear Least Squares (NL-LS)

NL-LS is a common peak estimator used for hyperbolic and Gaussian functions [18]–[20]. In [13], NL-LS was used for linear frequency-modulated and pseudo-noise coded ranging waveforms. This method fits a sinc equation over the peak of the matched filter output and it only requires three points (the peak and two points around it). This algorithm works by minimizing the residual error using

$$L = \sum_{i=0}^{2} (y_i - f(x_i, \boldsymbol{\lambda}))^2 \tag{4}$$

where $f(\boldsymbol{x}; \boldsymbol{\lambda}) = \lambda_0 \mathrm{sinc}((\boldsymbol{x} - \lambda_1)\lambda_2)$, $f(x_i, \boldsymbol{\lambda})$ is the predicted matched filter output main lobe for the samples x_i from $\boldsymbol{x} = [-1, 0, 1]^T$ around the peak, $\boldsymbol{y} : y_i = |d[n_{\mathrm{pk}-1+i}]|\ i \in \{0, 2\}$, y_i are the samples from $y[n]$ around the estimated peak, $d[n_{\mathrm{pk}-1+i}]$ represents the amplitude of the matched filter output around the peak n_{pk}, and $\boldsymbol{\lambda} = [\lambda_0, \lambda_1, \lambda_2]^T$ is the vector of the coefficients. Gauss–Newton optimization is used to solve (4). More details are shown in [13].

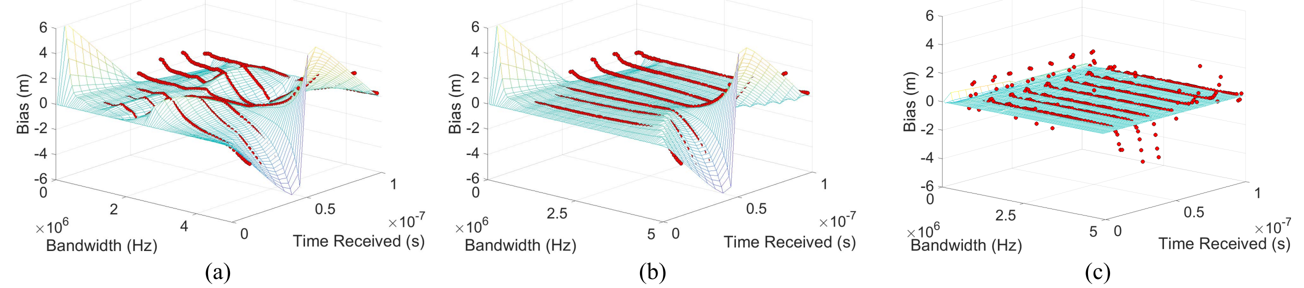

Fig. 3. Simulated and experimental results showing a comparison between the actual distances traveled by the signals and their estimates. The surfaces show the simulated results, and the red dots show the experimental results of (a) interpolation, (b) NL-LS, and (c) MF-LS. The proposed MF-LS algorithm significantly reduces the bias error across all delays and bandwidths, with errors only for very low bandwidths or bandwidths close to the Nyquist rate.

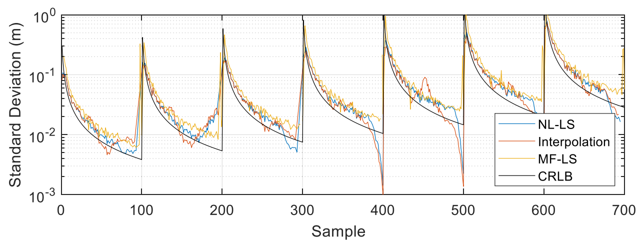

Fig. 4. Standard deviation for each ranging method along with the Cramer–Rao lower bound (CRLB). The SNR for the first 100 samples was 32 dB and then it decreased by around 2.9 dB for every added cable (every 100 samples), reaching an SNR of 14.6 dB. For some samples, the standard deviation of both the interpolation and NL-LS is smaller than the CRLB, and this is due to the bias in the estimates.

C. Estimation Using Matched Filter Least Squares (MF-LS)

In the proposed MF-LS algorithm, the output of the matched filter is compared with a set of $H = 50$ potential responses with shifted discretization points. The steps for the MF-LS time delay estimator are shown in Algorithm 1. The RSS of the normalized matched filter outputs, which is defined by

$$\mathrm{RSS}[h] = \sum_{n=0}^{N} (y[n]/\max(y) - \tilde{y}[n]/\max(\tilde{y}))^2 \tag{5}$$

is computed for all $H \geq 3$ (required for interpolation) potential responses, where $\tilde{y}[n] = \tilde{r}[n] \circledast s^*[-n]$ represents the predicted value of $y[n]$, and $\tilde{r}[n]$ represents the predicted discretized version of the received signal for every predicted time shift dt_h (as shown in Fig. 1(a)). The values of dt_h are selected such that a linear vector of candidate time delays around the coarse estimate of the matched filter output n_{pk} is considered. Using (5) for all H potential responses, a function is obtained and used to determine the time delay using the method of least squares. Afterward, the estimated time delay of the pulse τ_e is obtained using $\tau_e = n_{\mathrm{pk}} T_s + dt_{\mathrm{loc}}$. The estimates will generally improve with large H, as more potential outcomes are evaluated.

III. Experimental Implementation

The three ranging algorithms were tested through simulation and experiment. The experimental setup is shown in Fig. 2. The measurement was conducted using an Ettus USRP X310 SDR with a UBX-160 daughterboard. The time delay between the transmit and receive channels was controlled by modifying the length of the cable(s) connected between them. The transmitted and received signals were generated in terms of their in-phase and quadrature components, i.e., $s[n] = I_{\mathrm{TX}}[n] + jQ_{\mathrm{TX}}[n]$ and $r[n] = I_{\mathrm{RX}}[n] + jQ_{\mathrm{RX}}[n]$. The simulation and experimental results are shown in Fig. 3, in which we used $f_1 = 20$ kHz, $f_s = 10$ MHz, and f_2 was varied from 60 kHz to 4.94 MHz. The pulsewidth was set to 99.8 μs and the carrier frequency was 2.8 GHz. The results were obtained by averaging 100 estimates. The root-mean-square error (RMSE) over all bandwidths and delays for the interpolation, NL-LS, and MF-LS are as follows: 0.688, 0.482, and 0.407 m, respectively. These values are close since the bias for both low and very high bandwidths are common for all these methods. When only bandwidths between 2 and 4.5 MHz are considered (avoiding low frequencies and those close to Nyquist), the RMSE values are 0.78, 0.421, and 0.056 m, respectively, demonstrating that the MF-LS algorithm reduces errors by nearly an order of magnitude. A degradation in performance at low bandwidths is observed since less than one period of the beat frequency was captured. Thus, a larger T can be selected for low bandwidths to minimize the bias. The standard deviation and Cramer–Rao lower bound (CRLB) for each waveform [21] and algorithm are shown in Fig. 4. A significant reduction in the bias was achieved using the proposed MF-LS algorithm, which was the objective of this work, with only a slight increase in standard deviation compared with the interpolation and NL-LS algorithms.

IV. Conclusion

A novel MF-LS algorithm for two-tone delay estimation based on matched filtering and least-squares optimization was introduced. This method was developed to reduce the biases seen when using common estimation approaches. MF-LS showed better accuracies than interpolation or NL-LS, especially at high bandwidths. The measured results showed good agreement with simulations. For tone separations below the Nyquist rate and above 2 MHz, the error of the MF-LS algorithm had an improvement of nearly an order of magnitude.

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